

## Dynamical Systems 2020/21 Lab Test

1. Find a polynomial solution of the differential equation

$$u'' + 5u' - 7u = x^2 + 5x - 7,$$

then plot its graph on the interval  $[-10, 5]$ , and finally compute, for it and for its first order derivative, approximate values in  $\pi\sqrt{2}$ . Note that the unknown is the function denoted by  $u(x)$ .

2. a) Plot the planar curve of parametric equations  $x = \sin(t)$ ,  $y = \sin(2t)$  for  $t \in [0, 10]$ .  
b)\* Can  $\varphi(t) = (\sin(t), \sin(2t))$ ,  $t \in \mathbb{R}$ , be a solution of a linear planar system  $\dot{X} = AX$ ?
3. Introduce the matrix  $A$  corresponding to the linear system  $x' = -7x$ ,  $y' = x + 7y$ . Compute its determinant and eigenvalues. Compute  $e^{tA}$ . Specify the type and stability of the linear system.
4. We consider the nonlinear system  $x' = x - 17y + 3y^2 - 2xy$ ,  $y' = 17x + y$ . Is  $(0, 0)$  the unique equilibrium point? Is  $(0, 0)$  a hyperbolic equilibrium point?
5. We consider the map  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 0.02x(100 - x)$ . Find its fixed points. Describe your opinion on the behavior of the sequence of iterations starting with 10, 80 and, respectively, 95.