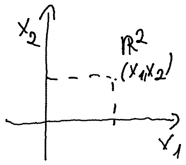
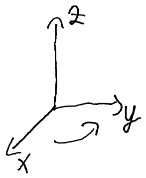
Seminar 8

Monday, February 6, 2023 11:58 PM

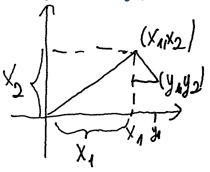




$x = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m$, $x_i \in \mathbb{R}$ $\forall i \in \{1, \dots, m\}$

$y = (y_1, y_2, \dots, y_m) \in \mathbb{R}^m$, $y_i \in \mathbb{R}$ $\forall i \in \{1, \dots, m\}$

$\langle x, y \rangle = x_1 \cdot y_1 + \dots + x_m \cdot y_m = \sum_{i=1}^{m} x_i y_i$ - scalar/linear/dot product



$\sqrt{x_1^2 + x_2^2} = \sqrt{x_1 \cdot x_1 + x_2 \cdot x_2} = \sqrt{\langle x, x \rangle}$

$x \in \mathbb{R}^m$, $\|x\| = \sqrt{x_1^2 + \dots + x_m^2} = \sqrt{\langle x, x \rangle}$ - norm of x

distance between x and y $d(x,y) = \|x - y\|$

$\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

$\alpha \langle x, y \rangle = \langle \alpha \cdot x, y \rangle = \langle x, \alpha y \rangle$

$x \perp y \Leftrightarrow \langle x, y \rangle = 0$

ex. 2) $\forall x, y \in \mathbb{R}^m$, prove the following hold:

a) $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$ (the parallelogram identity)

$\|x+y\|^2 + \|x-y\|^2 = \langle x+y, x+y \rangle + \langle x-y, x-y \rangle = \langle x, x+y \rangle + \langle y, x+y \rangle + \langle x, x-y \rangle + \langle -y, x-y \rangle =$

$= \langle x, x \rangle + \langle x, y \rangle + \langle y, x \rangle + \langle y, y \rangle + \langle x, x \rangle + \langle x, -y \rangle + \langle -y, x \rangle + \langle -y, -y \rangle =$

$= 2(\langle x, x \rangle + \langle x, y \rangle$