BABEŞ-BOLYAI UNIVERSITY
Faculty of Computer Science and Mathematics

ARTIFICIAL INTELLIGENCE

Intelligent systems

Rule-based systems – uncertainty

Content

- Intelligent systems
  - Knowledge-based systems
    - Rule-based systems in uncertain environments

Intelligent systems – knowledge-based systems(KBS)

- Computational systems – composed of 2 parts:
  - Knowledge base (KB)
    - Specific information of the domain
  - Inference engine (IE)
    - Rules for generating new information
    - Domain-independent algorithms

Intelligent systems - KBS

Knowledge base

Content

- Information (in a given representation) about environment
- Required information for problem solving
- Set of propositions that describe the environment

Classification

- Perfect information
  - Classical logic
    - *IF A is true THEN A is ┐false*
    - *IF B is false THEN B is ┐true*
- Imperfect information
  - Non-exact
  - Incomplete
  - Incommensurable

Intelligent systems - KBS

- Sources of uncertainty
  - Imperfection of rules
  - Doubt of rules
  - Using a vague (imprecise) language

- Modalities to express the uncertainty
  - Probabilities
  - Fuzzy logic
  - Bayes theorem
  - Theory of Dempster-Shafer

- Modalities to represent the uncertainty
  - By using a single value → certainty factors, confidence, truth value
    - How sure we are that the given facts are valid
  - By using more values → logic based on ranges
    - Min → lower limit of uncertainty (confidence, necessity)
    - Max → upper limit of uncertainty (plausibility, possibility)

Intelligent systems – KBS – Fuzzy systems

- Theory of possibility
- Content and design
- Classification
- Tools
- Advantages and limits

Intelligent systems – KBS – Fuzzy systems

Why fuzzy?

- Problem: translate in C++ code the following sentences:

  Georgel is tall.

  It is cold outside.

When fuzzy is important?

- Natural queries
- Knowledge representation for a KBS
- Fuzzy control – then we dead by imprecise phenomena (noisy phenomena)

Theory of possibility – a little bit of history

- Parminedes (400 B.C.)
- Aristotle
  - "Law of the Excluded Middle" – every sentence must be True or False
- Plato
  - A third region, between True and False
  - Forms the basis of fuzzy logic
- Lukasiewicz (1900)
  - Has proposed an alternative and systematic approach related to bi-valent logic of Aristotle – trivalent logic: true, false or possible
- Lotfi A. Zadeh (1965)
  - Mathematical description of fuzzy set theory and fuzzy logic: truth functions takes values in [0,1] (instead of {True, False})
    - He as proposed new operations in fuzzy logic
    - He has considered the fuzzy logic as a generalisation of the classic logic
  - He has written the first paper about fuzzy sets

Theory of possibility

Fuzzy logic

- Generalisation of Boolean logic
- Deals by the concept of partial truth
  - Classical logic – all things are expressed by binary elements
    - 0 or 1, white or black, yes or no
  - Fuzzy logic – gradual expression of a truth
    - Values between 0 and 1

Logic vs. algebra

- Logical operators are expressed by using mathematical terms (George Boole)
  - Conjunction = minimum $a \wedge b = \min(a, b)$
  - Disjunction = maximum $a \vee b = \max(a, b)$
  - Negation = difference $\neg a = 1 - a$

Content and design

Main idea

Cf. to certainty theory:

- *Popescu is tall*

Cf. to uncertainty theory

- Cf. to probability theory
  - *There is 80% chance that Popescu is young*
- Cf. fuzzy logic

Cf. teoriei informațiilor certe

- *Popescu este tânăr*

Cf. teoriei informațiilor incerte

- Cf. teoriei probabilităților:
  - *Există 80% șanse ca Popescu să fie tânăr*
- Cf. logicii fuzzy:
  - *Popescu's degree of membership to the group of young people is 0.80*

Necessity

Real phenomena involve fuzzy sets

Example

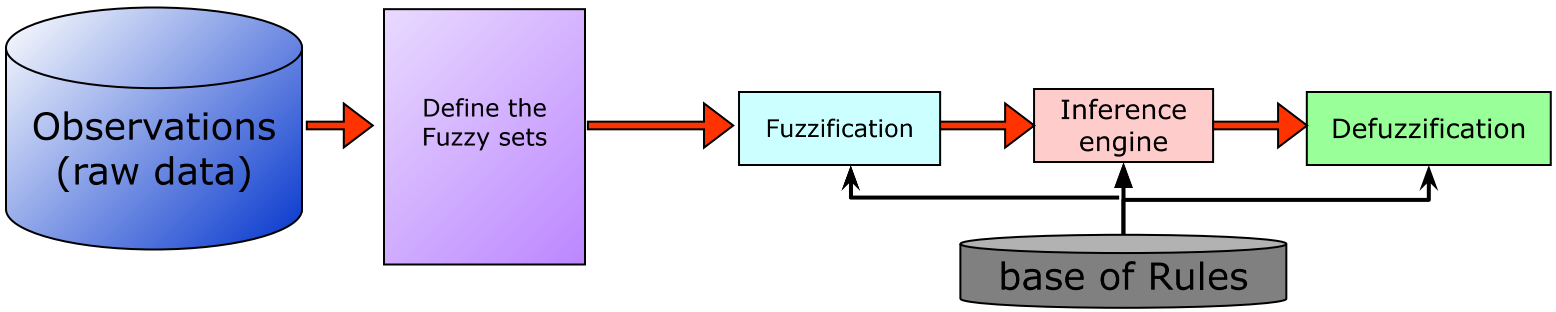
The room's temperature can be:
low,
Medium or
high

These sets of possible temperatures can overlap !

A temperature can belong to more classes (groups) depends on the person that evaluates that temperature

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzification of the result
  - Interpret the result



Elements from probability theory (fuzzy logic)

Fuzzy facts (fuzzy sets)

- Definition
- Representation
- Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
- Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
- Hedges

Fuzzy variables

- Definition
- Properties

Establish the fuzzy variables and the fuzzy sets based on membership functions

Fuzzy sets

Set definition – 2 possibilities:

- By enumeration of elements
  - Ex. *Set of students = {Ana, Maria, Ioana}*
- By specifying a property of elements
  - Ex. Set of even numbers =*{x| x = 2n, where n = 2k}*

Characteristic function *μ* for a set

- Let X a universal set and x an element of this set (x∈X)
- Classical logic
  - Let R a subset of X: R⊂X, R – regular set
  - Every element x belong to set R
  - μ_R : X {0, 1}, where $$\mu_R(x) = \begin{cases} 1, & x \in R \\ 0, & x \notin R \end{cases}$$

Fuzzy logic

- Let F a subset of X (a universe) : F⊂X, F – fuzzy set
- Every element x belongs to F by a given degree of membership $\mu_F(x)$
- μ_F : X [0, 1], $\mu_F(x)=g$, where g∈ [0,1] – membership degree of x to F
- g = 0 not-belong
- g = 1 belong

- A fuzzy set = a pair (F, μ_F), where

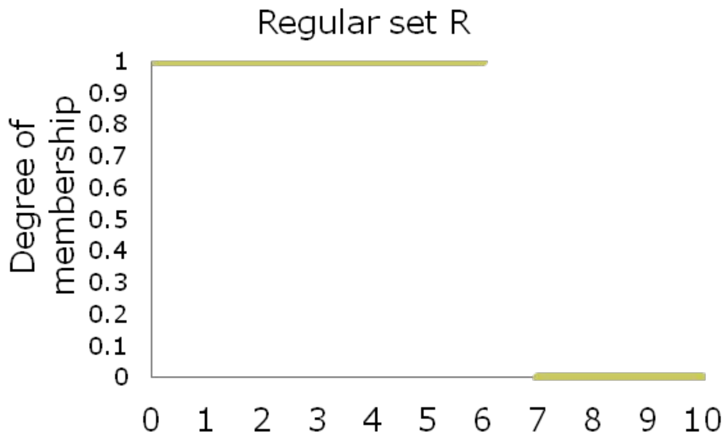
$$\mu_F(x) = \begin{cases} 1, & \text{if } x \text{ is totaly in } F \\ 0, & \text{if } x \text{ is not in } F \\ \in (0,1) & \text{if } x \text{ is part of } F (x \text{ is a fuzzy number}) \end{cases}$$

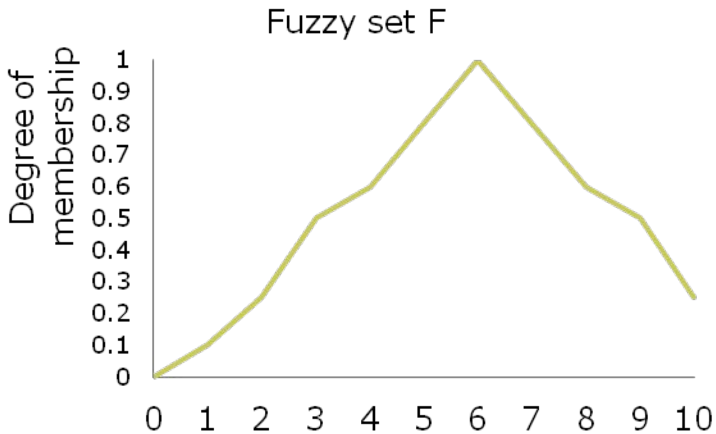
Fuzzy sets

Example 1

- X –set of natural numbers < 11
- R – set of natural numbers < 7
- F – set of natural numbers that are neighbours of 6

x	$\mu_R(x)$	$\mu_F(x)$
0	1	0
1	1	0.1
2	1	0.25
3	1	0.5
4	1	0.6
5	1	0.8
6	1	1
7	0	0.8
8	0	0.6
9	0	0.5
10	0	0.25



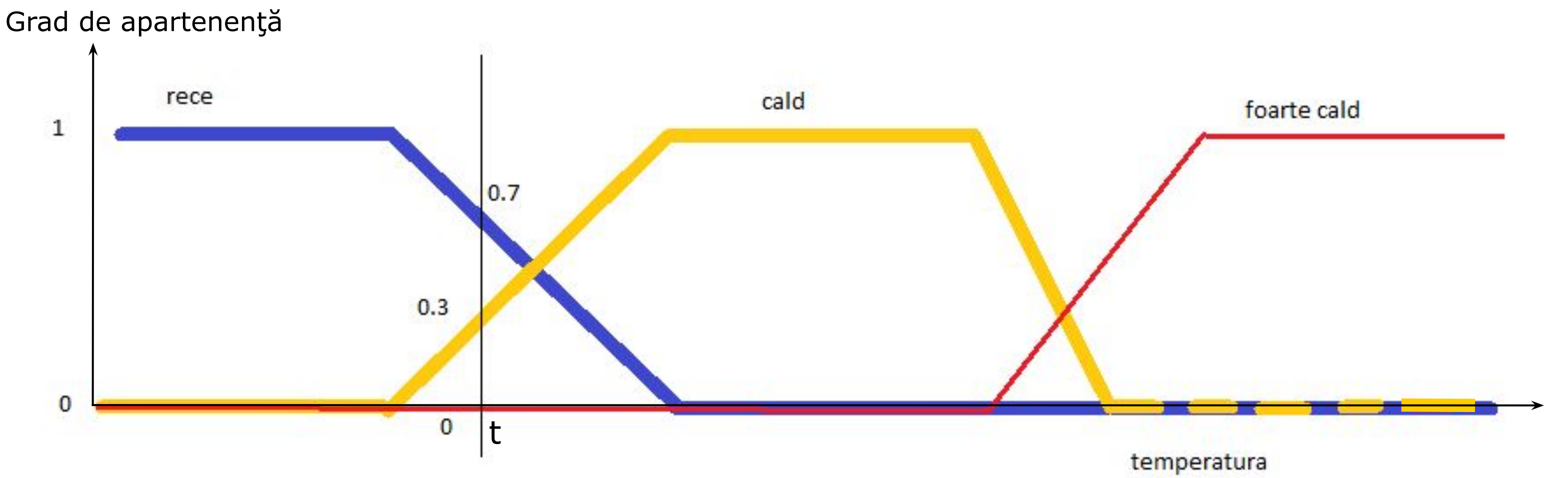


Fuzzy sets

Example 2

A temperature *t* can have 3 truth values:

- Red (0): is not hot
- Orange (0.3): warm
- Blue (0.7): cold

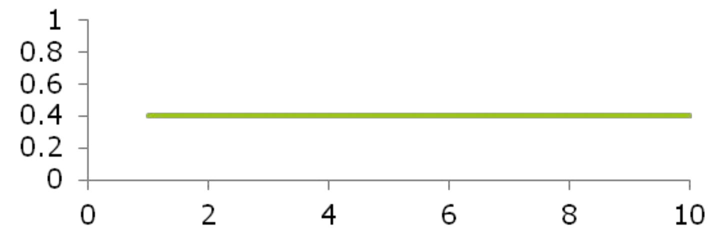


Fuzzy sets

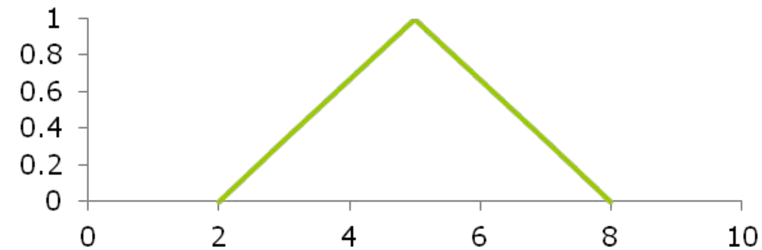
Representation:

Gradual limits representations based on membership functions

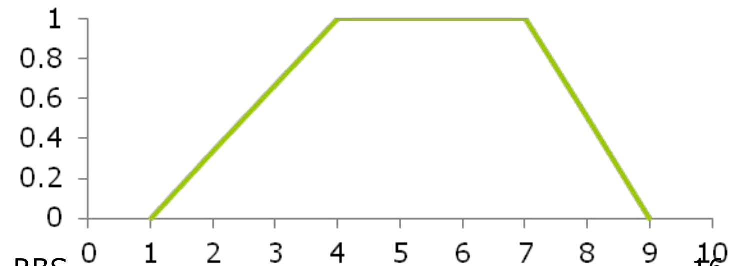
- Singular
  - $\mu(x) = s$, where s is a scalar



- Triangular
  - $$\mu(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{c-x}{c-b}\right\}\right\}$$



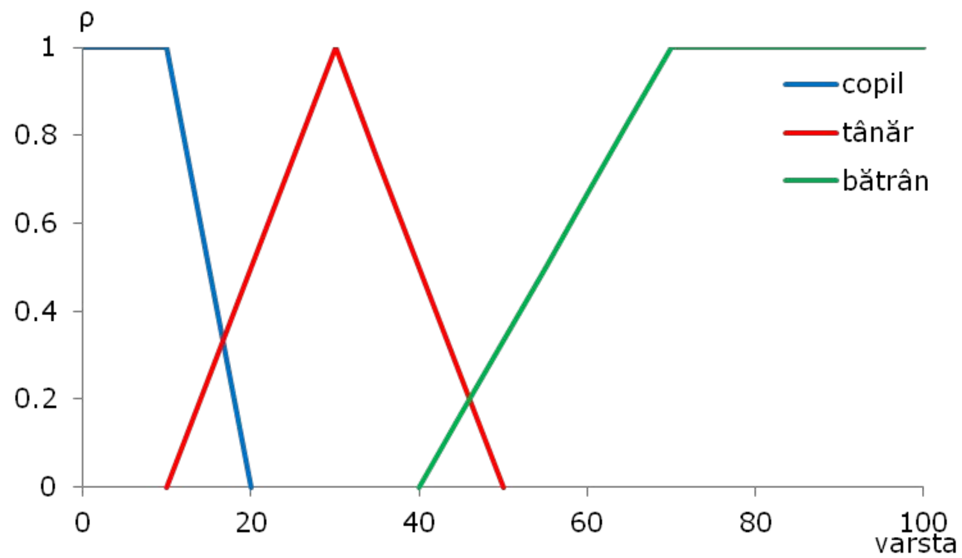
- Trapezoidal
  - $$\mu(x) = S(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right\}\right\}$$
- Z function
  - $\mu(x) = 1 - S(x)$
- Π function
  - $$\mu(x) = \Pi(x) = \begin{cases} S(x), & \text{if } x \le c \\ Z(x), & \text{if } x > c \end{cases}$$



Fuzzy sets

Example:

Age of a person



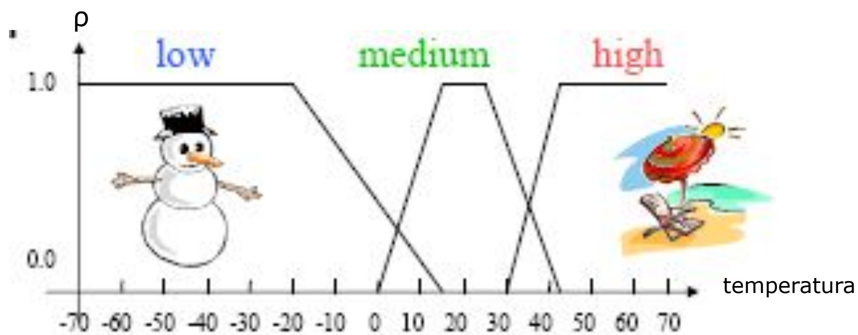
Fuzzy variable

Definitions

- A fuzzy variable is defined by *V = {x, l, u, m}*, where:
  - x – name of symbolic variable
  - L – set of possible labels for variable x
  - U – universe of the variable
  - M – semantic regions that define the meaning of labels from L (membership functions)

- Membership functions
  - Subjective assessment
    - The shape of functions is defined by experts
  - Ad-hoc assessment
    - Simple functions that can solve the problem
  - Assessment based on distributions and probabilities of information extracted from measurements
  - Adapted assessment
    - By testing
  - Automated assessment
    - Algorithms utilised for defining functions based on some training data

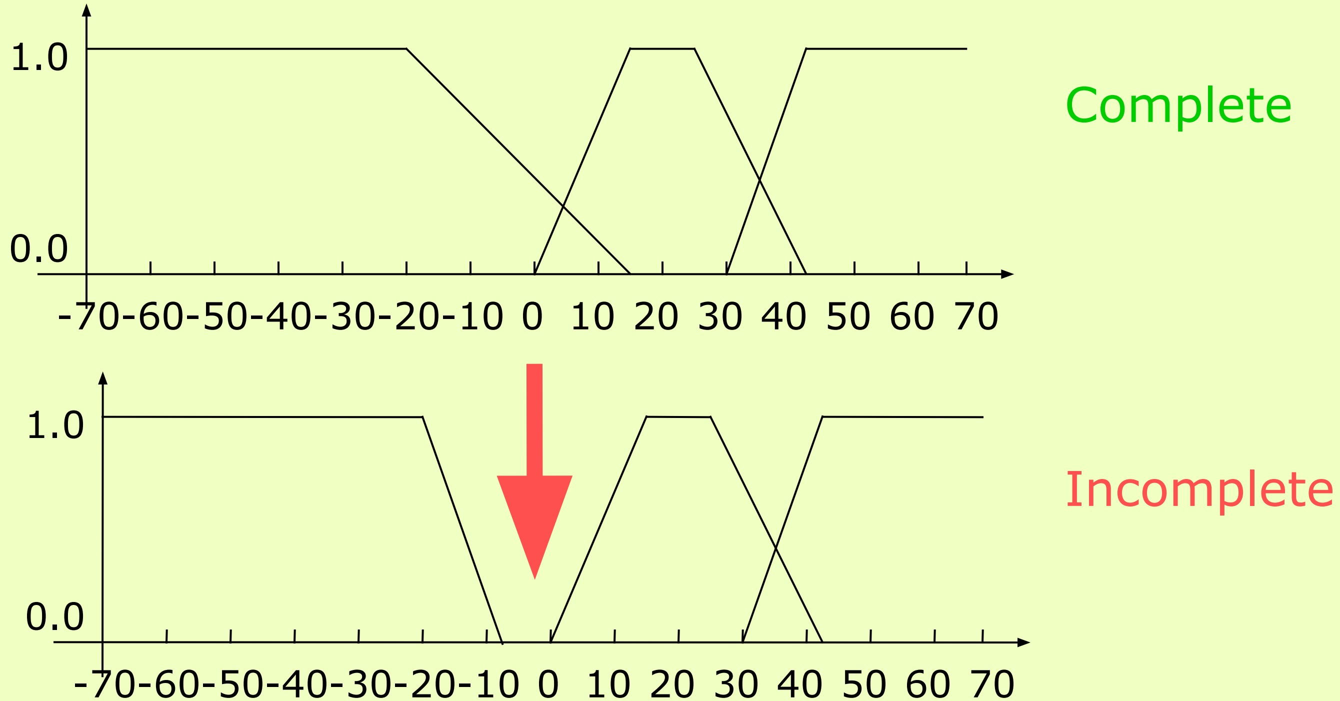
- Example
  - X = Temperature
  - L = {low, medium, high}
  - U = {x∈X | -70° ≤ x ≤+70°}
  - M =



Fuzzy variable

Properties

- Completeness
  - A fuzzy variable V is complete if for all $x \in X$ there is a fuzzy set A such as $\mu_A(x)>0$



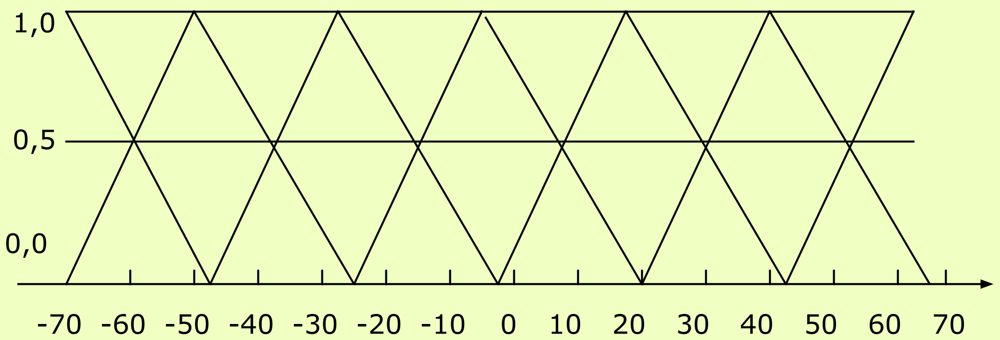
Fuzzy variable

Properties

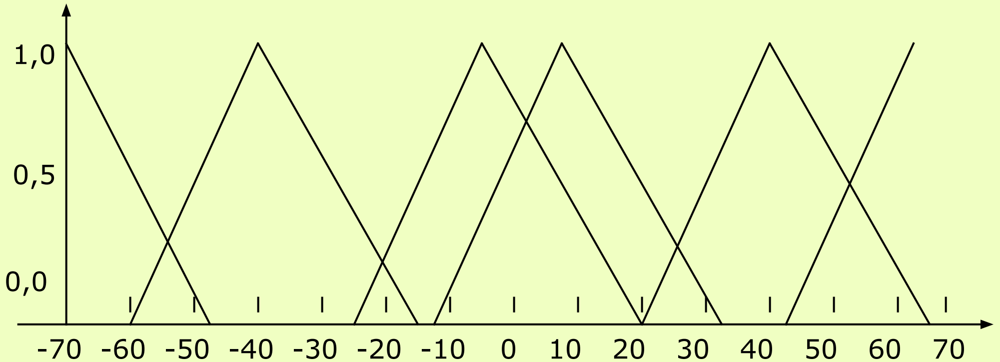
Unit partition

- A fuzzy variable V forms a unit partition if for all input values x we have
- where p is the number of sets that x belongs to
- There are no rules for defining 2 neighbour sets
  - Usually, the overlap is between 25% şi 50%

$$\sum_{i=1}^{p} \mu_{A_i}(x) = 1$$



Unit partition



Non-unit partition

Fuzzy variable

Properties

- Unit partition

A complete fuzzy variable can be transformed into a unit partition:

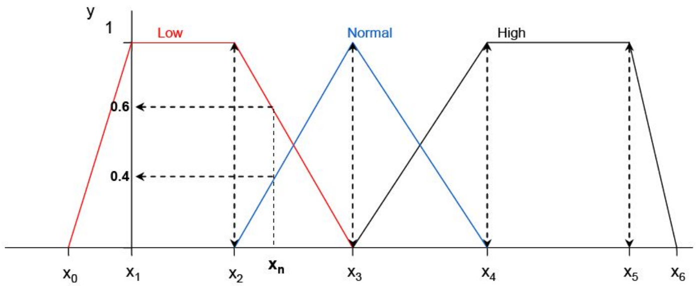
$$\mu_{\hat{A}_i}(x) = \frac{\mu_{A_i}(x)}{\sum_{j=1}^{p} \mu_{A_j}(x)} \; for \; i = 1, \ldots, p$$

Fuzzification of input data

Establish the fuzzy variables and the fuzzy sets based on membership functions

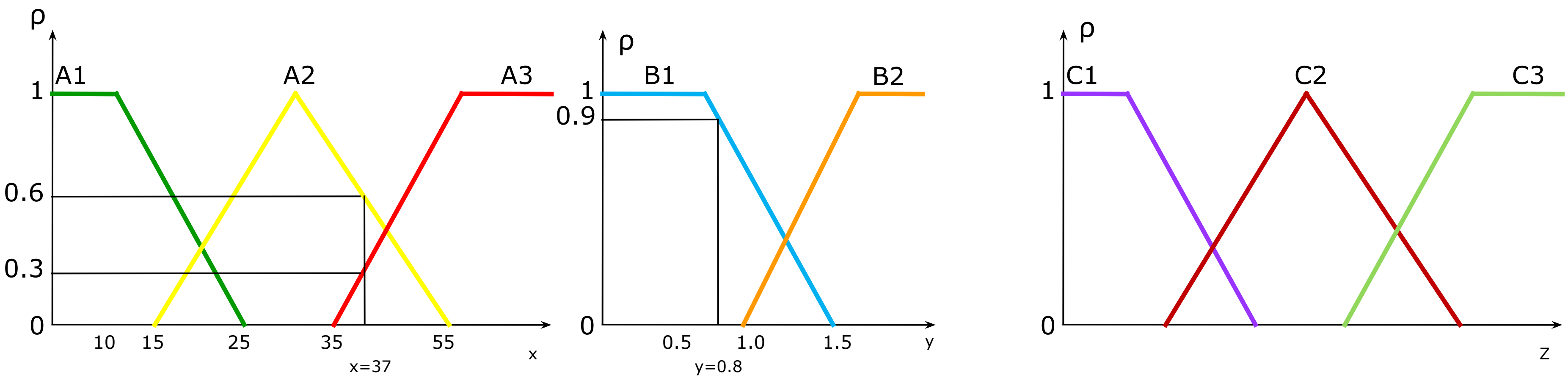
Fuzzification of input data

- Mechanism
  - Establish the raw (input and output) data of the system
  - Define membership functions for each input data
    - Each membership function has associated a quality label – linguistic variable
    - A raw variable can have associated one or more linguistic variables
    - Example
      - Raw variable: temperature T
      - Linguistic variable: low A1, medium A2, high A3
  - Transform each raw input data into a linguistic data fuzzification
    - Establish the fuzzy set of that raw input data
    - How?
      - For a given raw input determine the membership degree for each possible set
    - Example
      - T (=x_n) =5°
      - A_1 $\mu_{A1}(T) = 0.6$
      - A_2 $\mu_{A2}(T) = 0.4$



Fuzzification of input data

- Example - air conditioner device
  - Inputs :
    - x (temperature – cold, normal, hot) and
    - y (humidity – small, large)
  - Outputs:
    - z (machine power – law, medium, high)
  - Input data:
    - Temperature x = 37
      - $\mu_{A1}(x)=0$, $\mu_{A2}(x)=0.6$, $\mu_{A3}(x)=0.3$
    - Humidity y = 0.8
      - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$



Base of rules

Construct a base of rules – by an expert

Rules

- Definition
  - Linguistic constructions
    - Affirmative sentences: A
    - Conditional sentences: if A then B
  - Where A and B are (collections of) sentences that contain linguistic variables
    - A – premise of the rule
    - B – consequence of the rule
- Typology
  - Non-conditional
    - x is (in) A_i
    - Eg. *Save the energy*
  - Conditional
    - If x is (in) A_i then z is (in) C_k
    - If x is (in) A_i and y is (in) B_j, then z is (in) C_k
    - If x is (in) A_i or y is (in) B_j, then z is (in) C_k

Base of rules -example

	Rules of classical logic	Rules of fuzzy logic
R_1	*If temperature is -5, then is cold*	*If temperature is law, then is cold*
R_2	*If temperature is 15, then is warm*	*If temperature is medium, then is warm*
R_3	*If temperature is 35, then is hot*	*If temperature is high, then is hot*

Base of rules

Rules

- Database of fuzzy rules
  - R_{11}: if x is A_1 and y is B_1 then z is C_u
  - R_{12}: if x is A_1 and y is B_2 then z is C_v
  - ...
  - R_{1n}: : if x is A_1 and y is B_n then z is C_x
  - R_{21}: if x is A_2 and y is B_1 then z is C_x
  - R_{22}: if x is A_2 and y is B_2 then z is C_z
  - ...
  - R_{2n}: if x is A_2 and y is B_n then z is C_v
  - ...
  - R_{m1}: if x is A_m and y is B_1 then z is C_x
  - R_{m2}: if x is A_m and y is B_2 then z is C_v
  - ...
  - R_{mn}: if x is A_m and y is B_n then z is C_u

Base of rules

Decision matrix of the knowledge database

- Example – air conditioner device
  - Inputs :
    - x (temperature – cold, normal, hot) and
    - y (humidity – small, large)
  - Outputs:
    - z (machine power – low, constant, high)
  - Rules:
    - *If temperature is normal and humidity is small then the power is constant*

		Input data y	
		Small	Large
Input data x	Cold	Low	Constant
	Normal	Constant	High
	Hot	High	High

Rule evaluation (fuzzy inference)

Which rules are firstly evaluated?

Fuzzy inference

- Rules are evaluated in **parallel** , each rules contributing to the shape of the final result
- Resulted fuzzy sets are defuzzified **after all the rules** have been evaluated

Rule evaluation (fuzzy inference)

Evaluation of causes

- For each premise of a rule (*if s is (in) A*) establish the membership degree of raw input data to all fuzzy sets
- A rule can have more premises linked by logic operators *AND, OR* or *NOT* use fuzzy operators
  - Operator *AND* intersection (minimum) of 2 sets
    - $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$
  - Operator *OR* union (maximum) of 2 sets
    - $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$
  - Operator *NOT* negation (complement) of a set
    - $\mu_{\neg A}(x) = 1 - \mu_A(x)$
- The result of premise's evaluation
  - Degree of satisfaction
  - Other names:
    - Rule's firing strength
    - Degree of fulfillment

Rule evaluation (fuzzy inference)

Determine the results

- Establish the membership degree of variables (involved in the consequences) to different fuzzy sets

Each output region must be de-fuzzified in order to obtain crisp value

Based on the consequence's type

- **Mamdani model** – consequence of rule: "output variable belongs to a fuzzy set"
- **Sugeno model** – consequence of rule: "output variable is a crisp function that depends on inputs"
- **Tsukamoo model** – consequence of rule: "output variable belongs to a fuzzy set following a monotone membership function"

Mamdani model

Main idea:

- consequence of rule: “output variable belongs to a fuzzy set”
- Result of evaluation is applied for the membership function of the consequence

- Example
  - ***if x is in A and y is in B, then z is in C***

Classification based on how the results is applied on the membership function of the consequence:

- Clipped fuzzy sets
- Scaled fuzzy sets

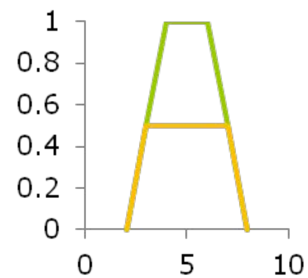
Mamdani model - Classification

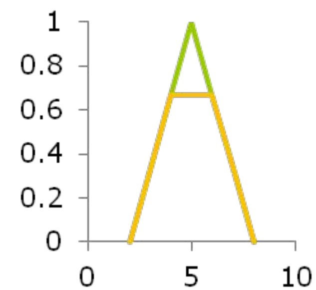
Clipped fuzzy sets

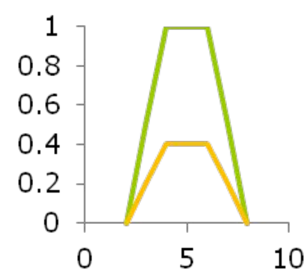
- Membership function of the consequence is cut at the level of the result’s truth value
- Advantage easy to compute
- Disadvantage some information are lost

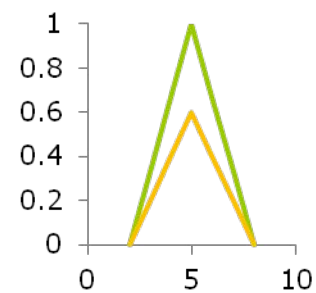
Scaled fuzzy sets

- Membership function of the consequence is adjusted by scaling (multiplication) at the level of the result’s truth value
- Advantage few information is lost
- Disadvantage complicate computing









Mamdani model

- Example – air conditioner device
  - Inputs :
    - x (temperature – cold, normal, hot) and
    - y (humidity – small, large)
  - Outputs:
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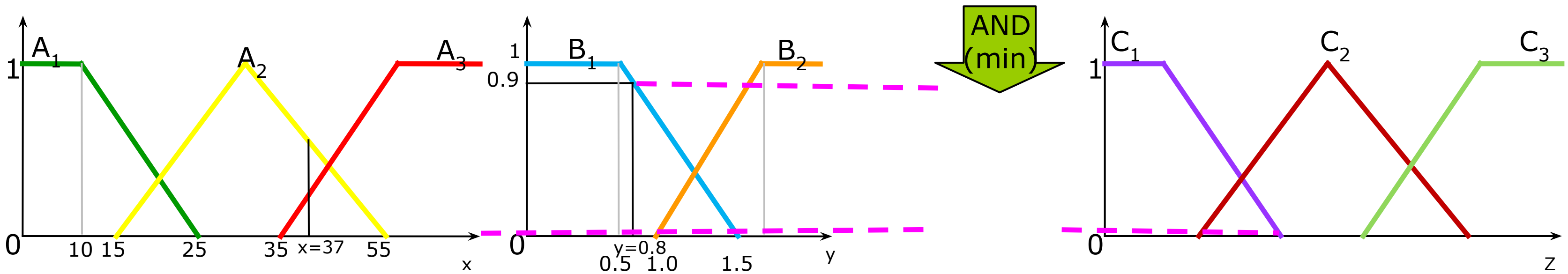
  - Input data:
    - Temperature x = 37
      - $\mu_{A1}(x)=0$, $\mu_{A2}(x)=0.6$, $\mu_{A3}(x)=0.3$
    - Humidity y = 0.8
      - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$

		Input data y	
		Small	Large
Input data x	Cold	Law	Constant
	Normal	Constant	High
	Hot	High	High

Mamdani model

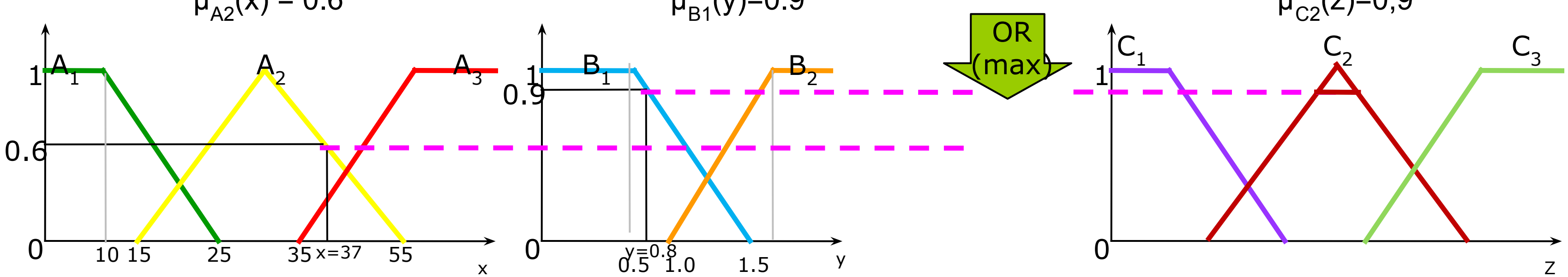
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



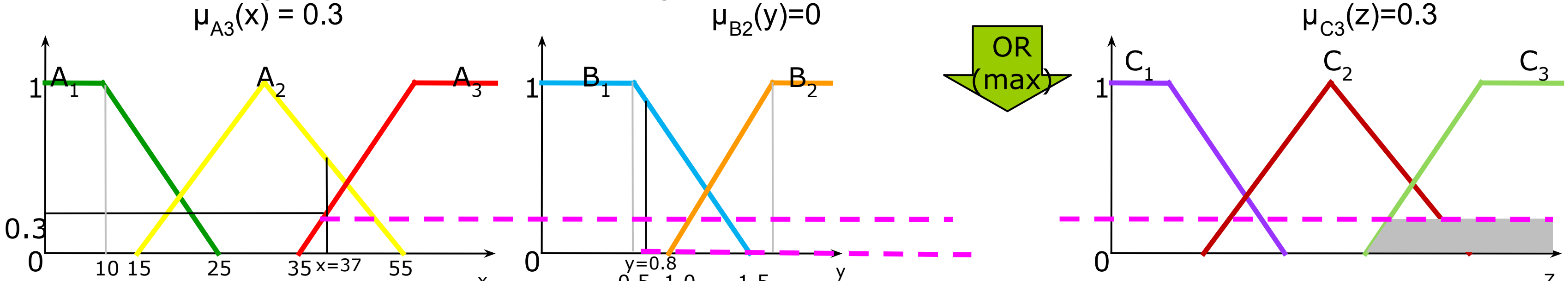
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Sugeno model

Main idea

- consequence of rule: “output variable is a crisp function that depends on inputs”

Example

If x is in A and y is in B then z is f(x,y)

Classification based on characteristics of f(x,y)

Sugeno model of degree 0

if (f(x,y) = k – constant (membership function of the consequences are singleton – a fuzzy set whose membership functions have value 1 for a single (unique) point of the universe and 0 for all other points)

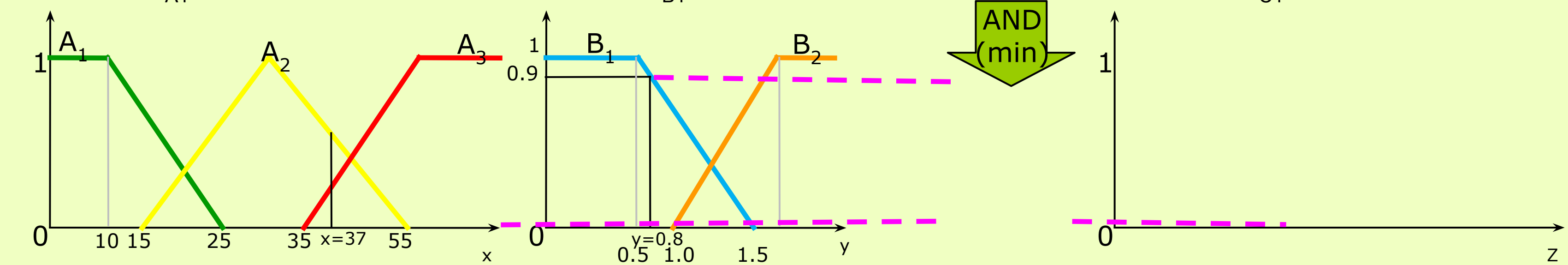
Sugeno model of degree 1

if f(x,y) = ax + by+c

Sugeno model

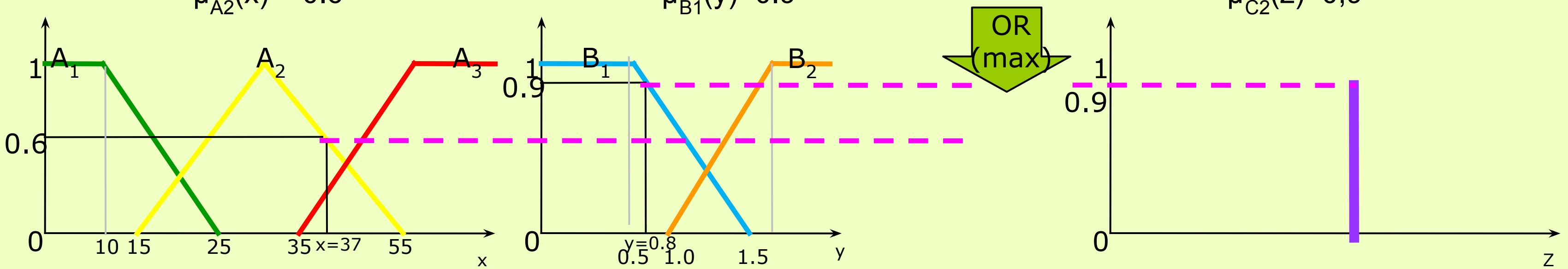
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



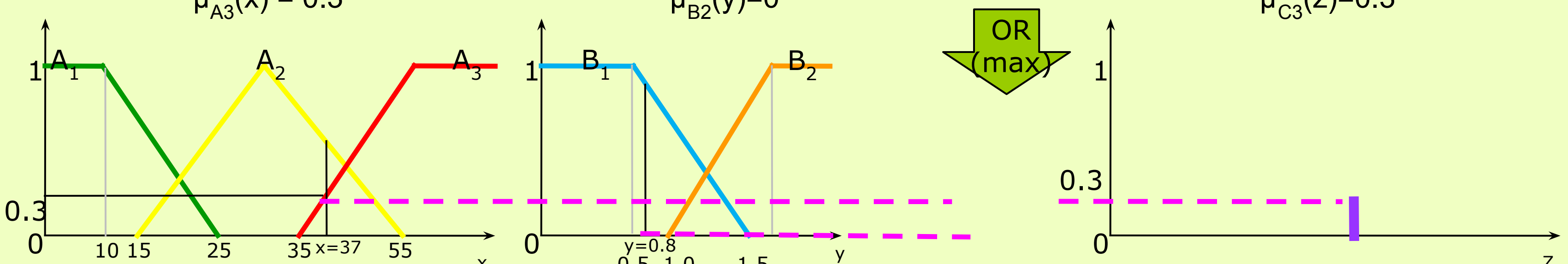
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Tsukamoto model

Main idea

- consequence of rule:

output variable belongs to a fuzzy set following a monotone membership function

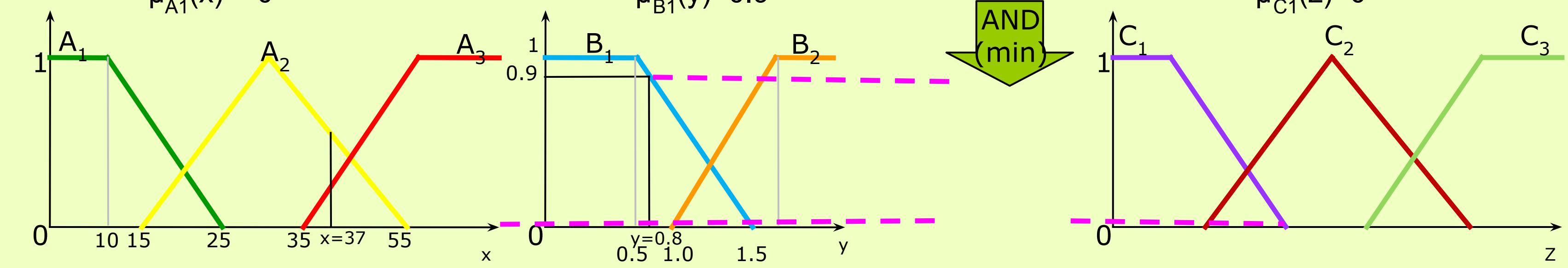
A crisp value is obtained as output *rule's firing strength*

Tsukamoto model

Content and design → rule evaluation → Evaluation of consequences → Tsukamoto model

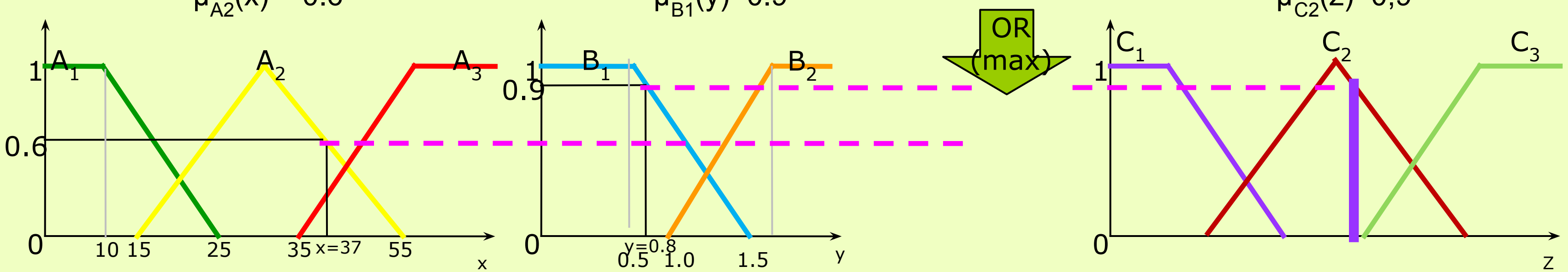
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



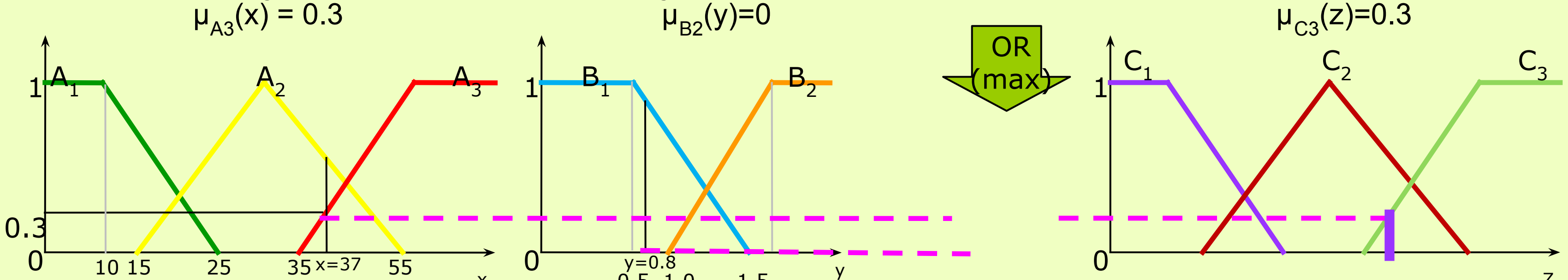
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$

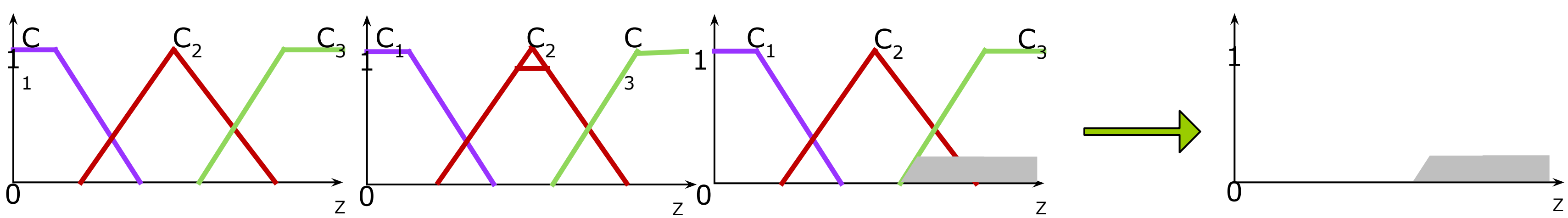


Aggregate the results

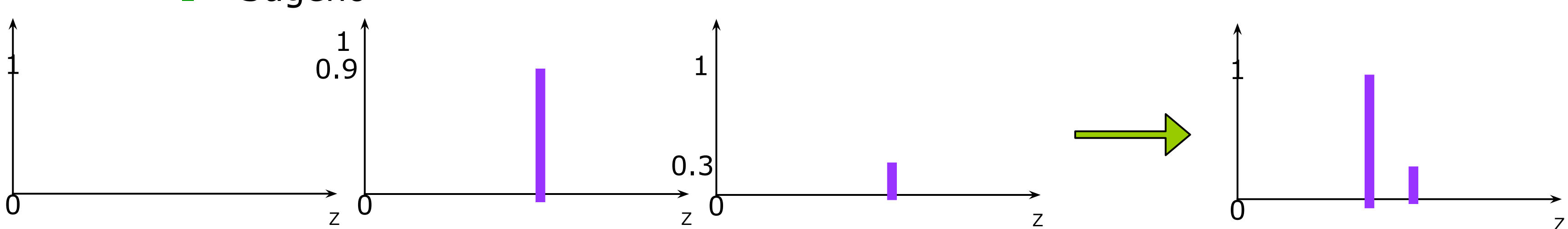
- Union of outputs for all the applied rules
- Consider the membership functions for all the consequences and combine them into a single fuzzy set (a single result)
- Aggregation process have as
  - Inputs → membership functions (clipped or scaled) of the consequences
  - Outputs → a fuzzy set of the output variable

Example

- Mamdani



- Sugeno



Defuzzification

- Transform the fuzzy result into a crisp (raw) value
- Inference obtain some fuzzy regions for each output variable
- Defuzzification transform each fuzzy region into a crisp value

Methods

- Based on the gravity center
  - COA – Centroid Area
  - BOA – *Bisector of area*
- Based on maximum of membership function
  - MOM - *Mean of maximum*
  - SOM - *Smallest of maximum*
  - LOM - *Largest of maximum*

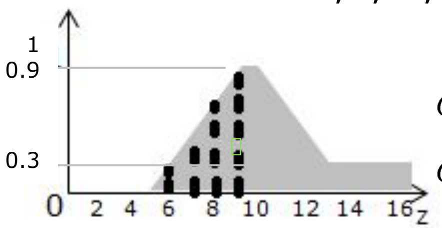
Defuzzification

COA – Centroid Area

- Identify the z point from the middle of aggregated set

$$COG = \frac{\sum_{i=0}^{n} x_i \mu_A(x_i)}{\sum_{i=0}^{n} \mu_A(x_i)} \text{ sau } COG = \frac{\int_{i=0}^{n} x_i \mu_A(x_i)}{\int_{i=0}^{n} \mu_A(x_i)}$$

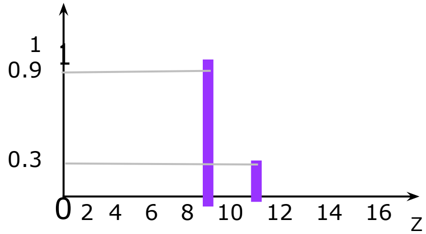
- Example
  - Mamdani model estimation of COA by using a sample of n points (x_i, i =1,2,..., n) of the resulted fuzzy set



$$COA = \frac{5*0+6*0.3+7*0.5+8*0.7+9*0.9+10*0.9+11*0.7+12*0.5+13*0.3+14*0.3+15*0.3+16*0.3}{0+0.3+0.5+0.7+0.9+0.9+0.7+0.5+0.3+0.3+0.3+0.3}$$

$$COA \cong 13.7$$

  - Sugeno or Tsukamoto model COA becomes a weighted average of m crisp values obtained by applying all m rules



$$COA = \frac{9*0.9+11*0.3}{0.9+0.3}$$

$$COA \cong 9.5$$

Defuzzification

BOA – Bisector of area

Identify the point z that determine the splitting of aggregated set in 2 parts of equal area

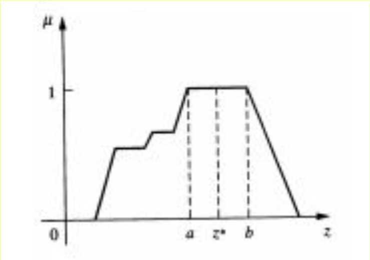
$$BOA = \int_{\alpha}^{z} \mu_A(x)dx = \int_{z}^{\beta} \mu_A(x)dx,$$

$$\text{where } \alpha = \min\{x \mid x \in A\} \text{ and } \beta = \min\{x \mid x \in A\}$$

Defuzzification

- MOM - *Mean of maximum*
  - Identify the point z that represents the mean of that points (from the aggregated set) that have a maximum membership function

$$MOM = \frac{\sum_{x_i \in \max\mu} x_i}{|max\mu|}, \text{where } max\mu = \mu^* = \{x \mid x \in A, \mu(x) = maxim\}$$

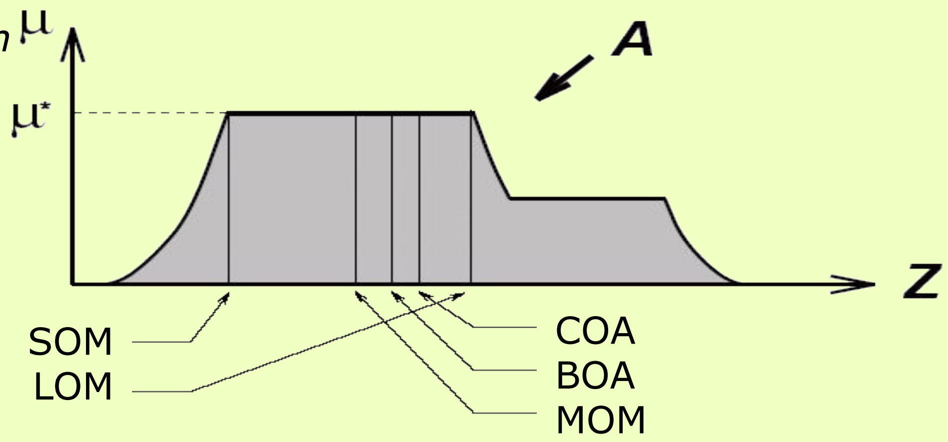


- SOM - *Smallest of maximum*
  - Identify the smallest point z (from the aggregated set) that have a maximum membership function

- LOM - *Largest of maximum*
  - Identify the largest point z (from the aggregated set) that have a maximum membership function

Defuzzification

- Main idea
  - Transform the fuzzy result into a crisp (raw) value
  - Inference obtain some fuzzy regions for each output variable
  - Defuzzification transform each fuzzy region into a crisp value
- Methods
  - Based on the gravity center
    - COA – Centroid Area
    - BOA – *Bisector of area*
  - Based on maximum of membership function
    - MOM - *Mean of maximum*
    - SOM - *Smallest of maximum*
    - LOM - *Largest of maximum*



Intelligent systems – KBS – Fuzzy systems

Advantages

- Imprecision and real-world approximations can be expressed through some rules
- Easy to understand, to test and to maintain
- Robustness can operate when rules are not so clear
- Require few rules then other KBSs
- Rules are evaluated in parallel

Disadvantages

- Require many simulations and tests
- Do not automatically learn
- It is difficult to identify the most correct rules
- There is not mathematical model

Applications

Space control

- Altitude of satellites, Setting the planes

Auto-control

- Automatic transmission, traffic control, anti-braking systems

Business

- Decision systems, personal evaluation, fond management, market predictions, etc

Industry

- Energy exchange control, water purification control
- pH control, chemical distillation, polymer production, metal composition

Electronic devices

- Camera exposure, humidity control. Air conditioner, shower setting, Freezer setting, Washing machine setting

Applications

Nourishment

- Cheese production

Military

- Underwater recognition, infrared image recognition, vessel traffic decision

Navy

- Automatic drivers, route selection

Medical

- Diagnostic systems, pressure control during anesthesia, modeling the neuropathology results of Alzheimer patients

Robotics

- Kinematics (arms)