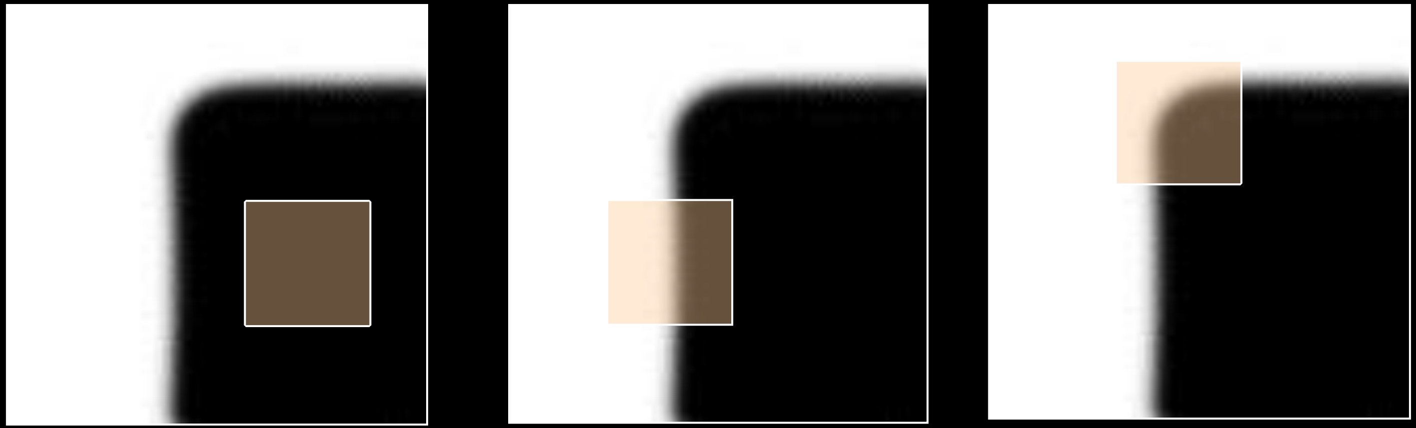
CS4495/6495
Introduction to Computer Vision

4A-L3 *Scale invariance*

Recall: Corner Detection - Basic Idea



"flat" region:
no change in
all directions

"edge":
no change
along the edge
direction

"corner":
significant change
in all directions
with small shift

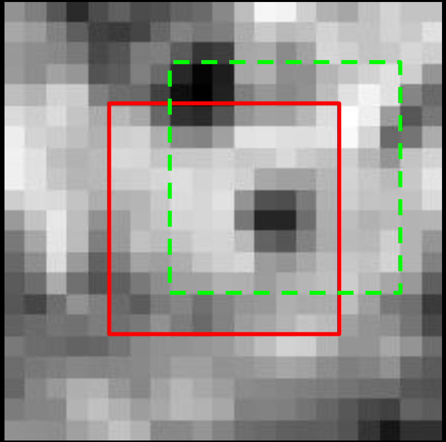
Source: A. Efros

Corner Detection: Mathematics

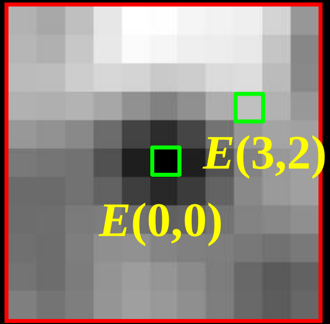
Change in appearance for the shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

The quadratic approximation simplifies to

$$\boxed{E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}}$$

where *M* is a *second moment matrix* computed from image derivatives:

$$\boxed{M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}$$

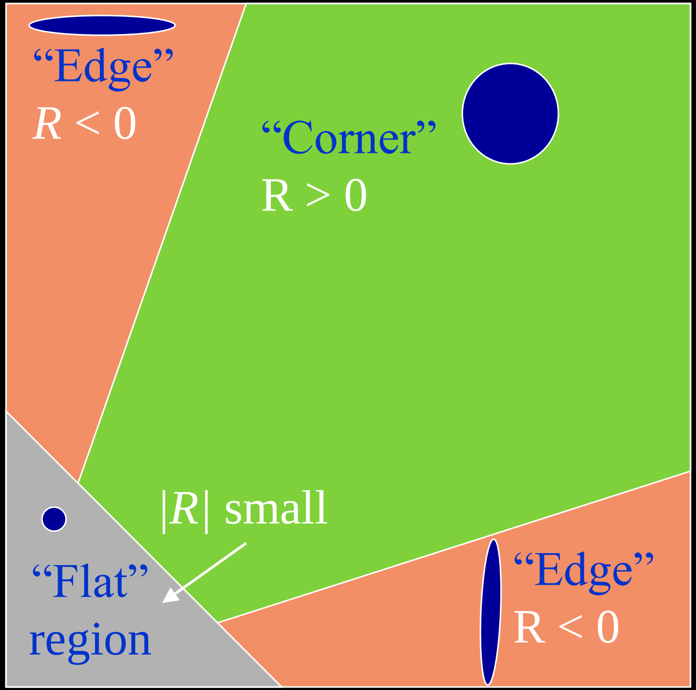
Harris corner response function

$$R = \det(M) - \alpha\ \mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

R is large for a corner

R is negative with large magnitude for an edge

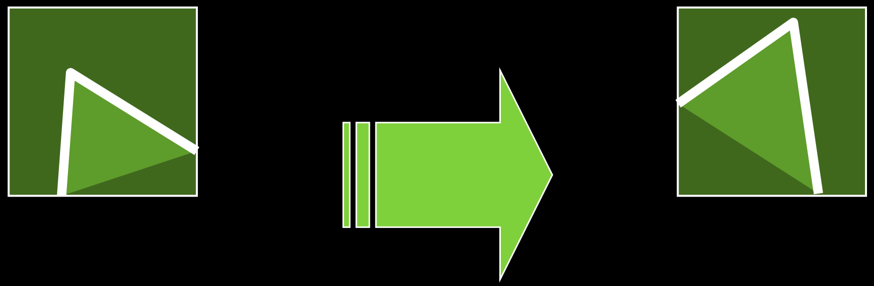
$|R|$ is small for a flat region



Harris Detector: Workflow

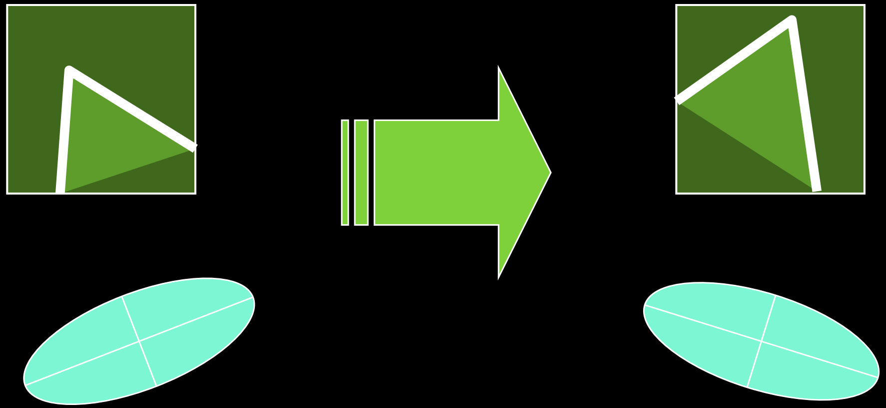
Harris Detector: Some Properties

Rotation invariance?



Harris Detector: Some Properties

Rotation invariance?

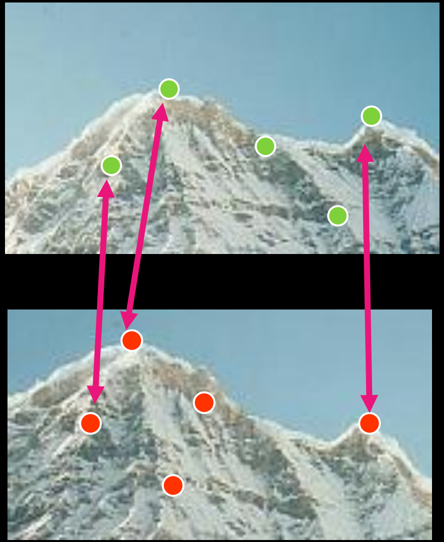


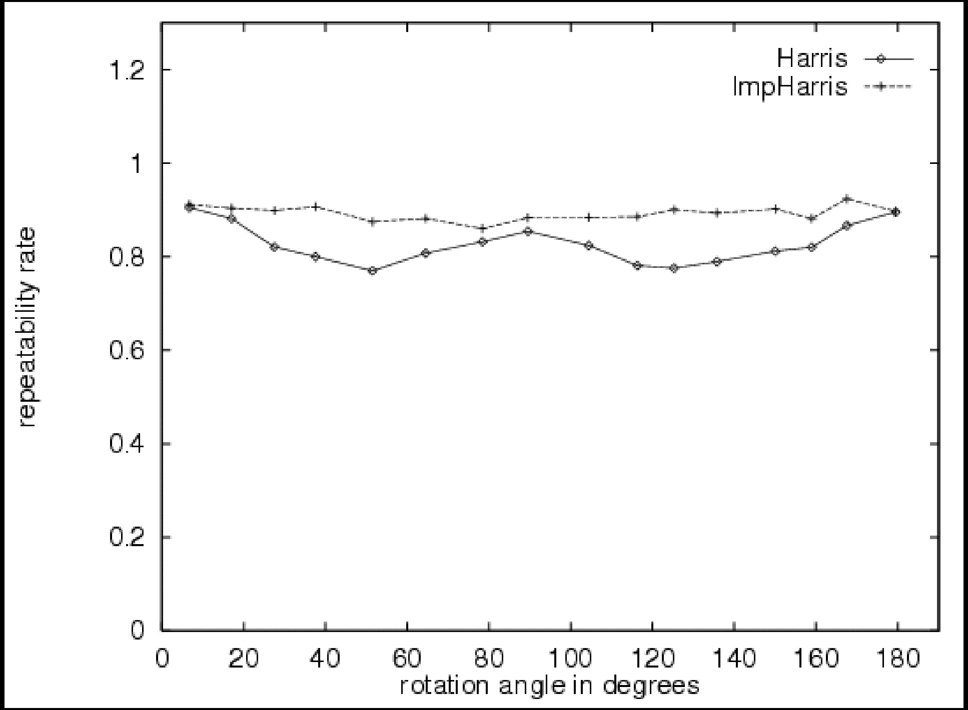
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Repeatability rate:

$$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$$





C.Schmid et.al. “Evaluation of Interest Point Detectors”. Int’l J. Computer Vision, 37(2), 2000

Evaluation plots are from this paper

International Journal of Computer Vision 37(2), 151–172, 2000

Evaluation of Interest Point Detectors

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Abstract. Many different low-level feature detectors exist and it is widely agreed that the evaluation of detectors is important. In this paper we introduce two evaluation criteria for interest points: repeatability rate and information content. Repeatability rate evaluates the geometric stability under different transformations. Information content measures the distinctiveness of features. Different interest point detectors are compared using these two criteria. We determine which detector gives the best results and show that it satisfies the criteria well.

Harris Detector: Some Properties

- Invariance to image intensity change?

Harris Detector: Some Properties

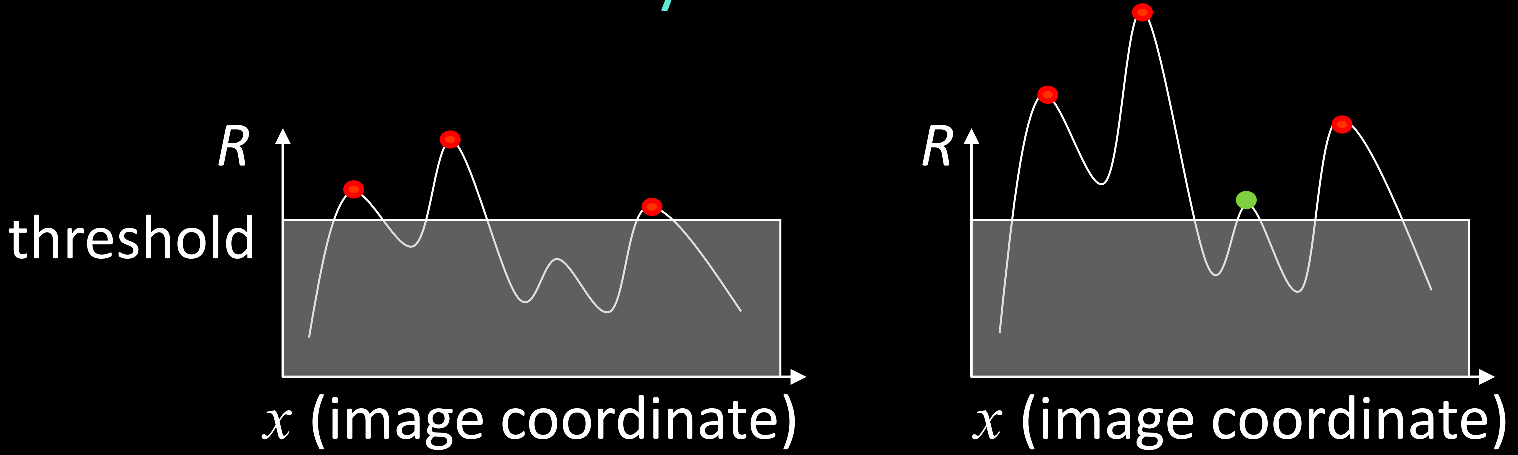
- Mostly invariant to additive and multiplicative intensity changes (threshold issue for multiplicative)

  ✓ Only derivatives are used =>
  invariance to intensity shift $I \rightarrow I + b$

Harris Detector: Some Properties

- Mostly invariant to additive and multiplicative intensity changes (threshold issue for multiplicative)

  ✓ Intensity scale: $I \rightarrow a\, I$

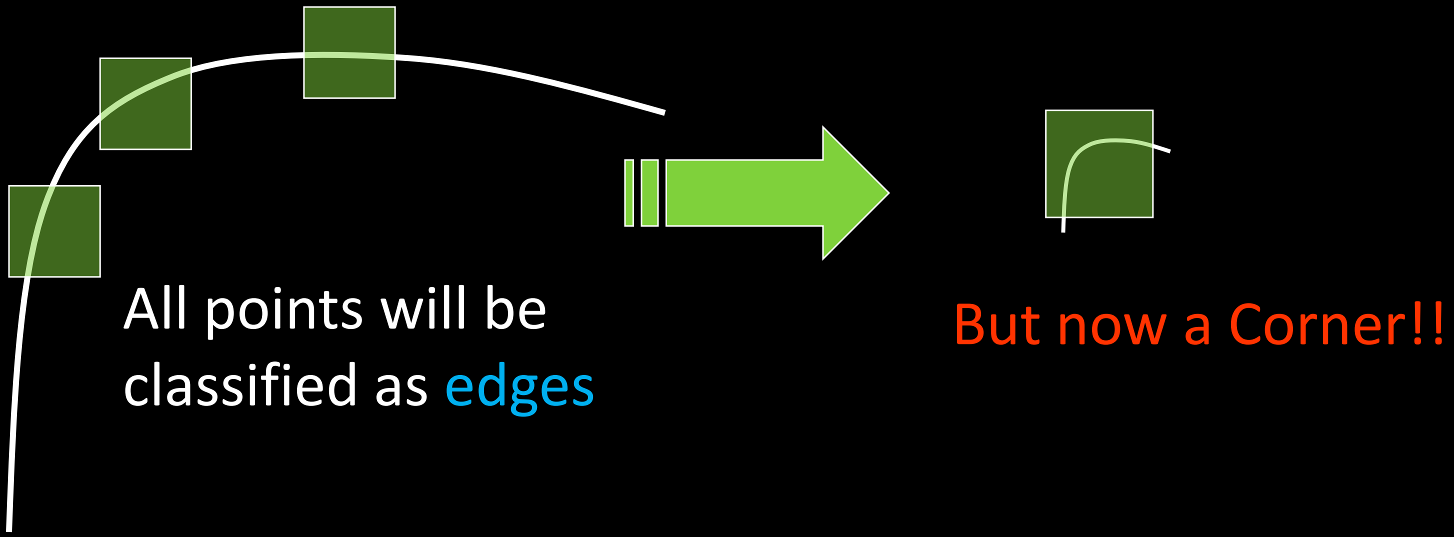


Harris Detector: Some Properties

- Invariant to image scale?

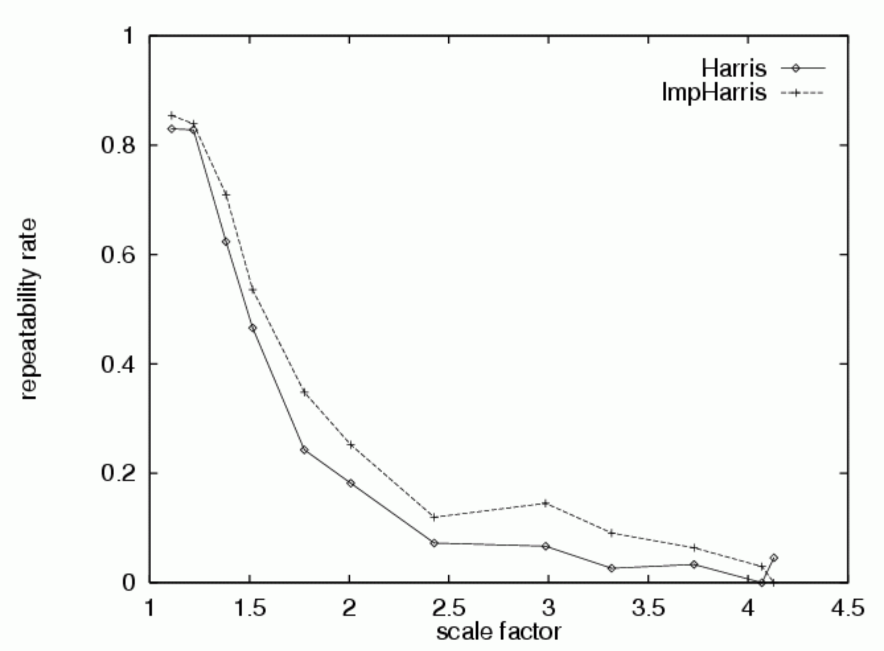
Harris Detector: Some Properties

- Not invariant to *image scale*!



Harris Detector: Some Properties

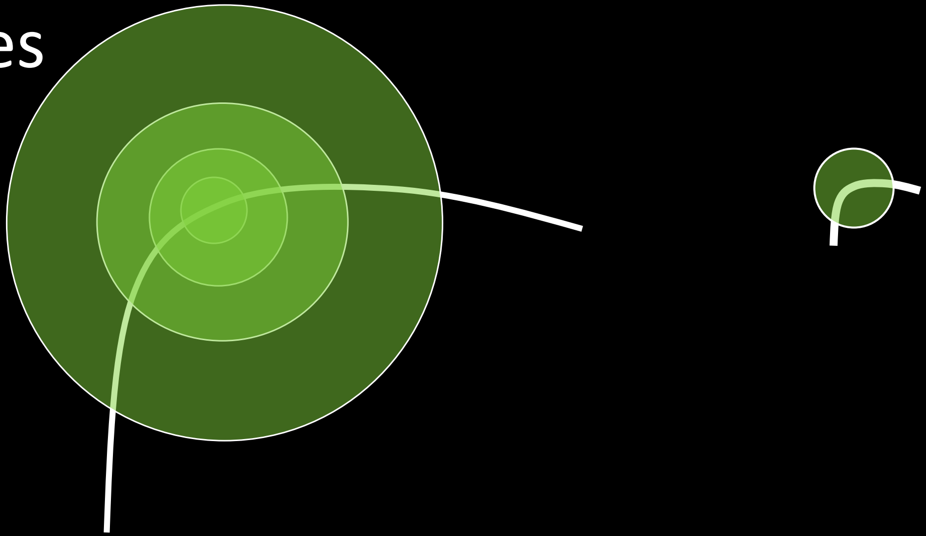
Not invariant to image scale:



IF we want scale invariance…

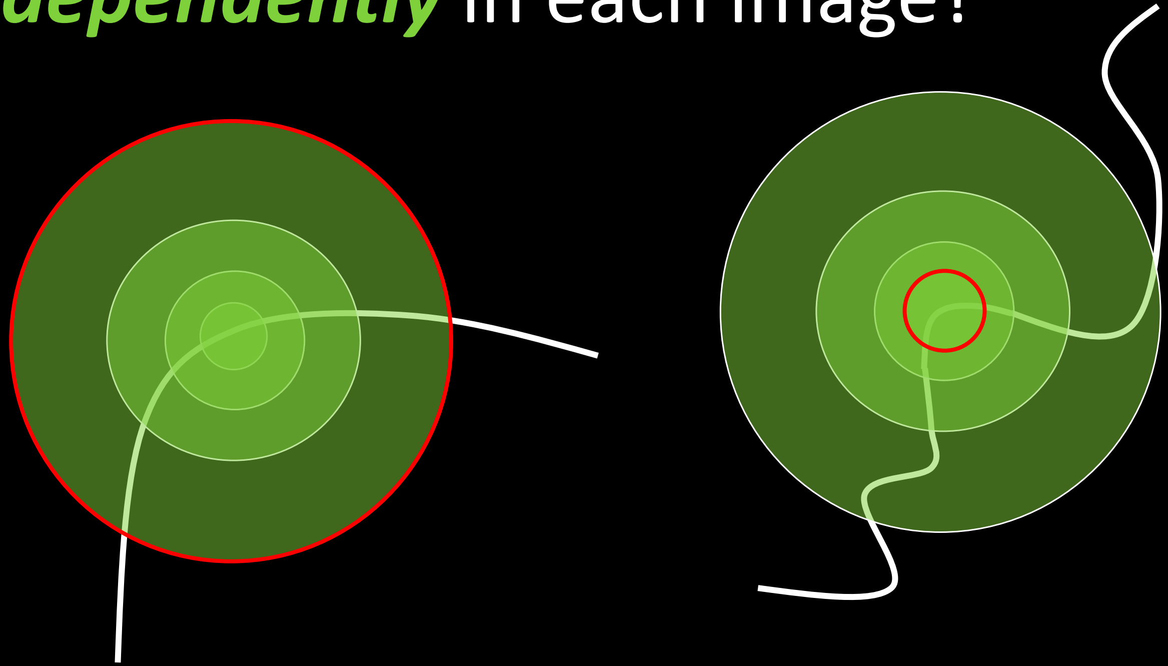
Scale Invariant Detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale Invariant Detection

- The problem: how do we choose corresponding circles ***independently*** in each image?



Scale Invariant Detection

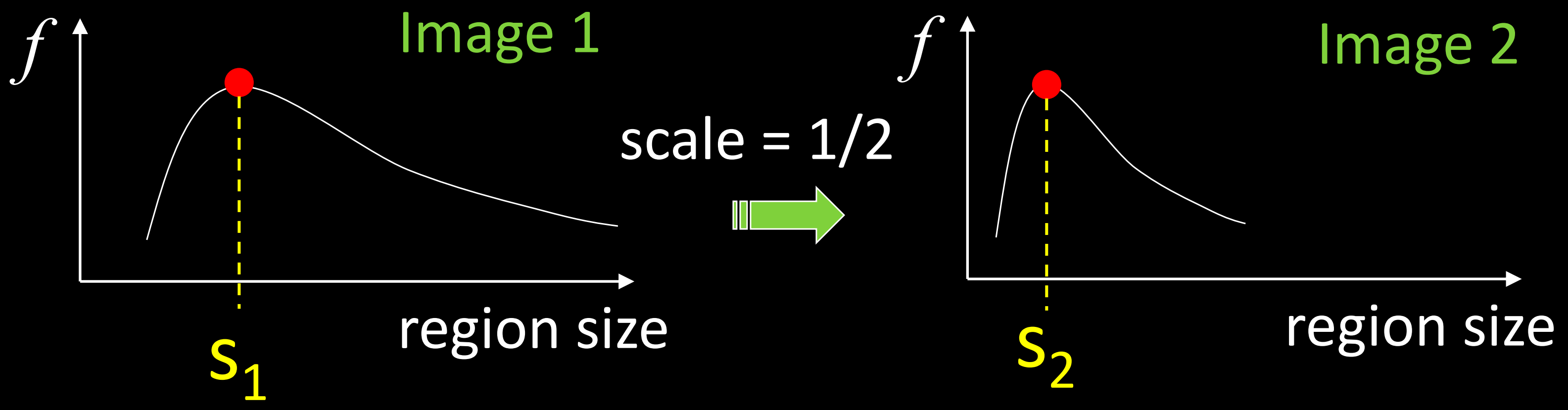
- Solution:
  - Design a function on the region (circle), which is "scale invariant" - not affected by the size but will be the same for "corresponding regions, " even if they are at different sizes/scales.

Example: Average intensity. For corresponding regions (even of different sizes) it will be the same.

Scale Invariant Detection

One method:

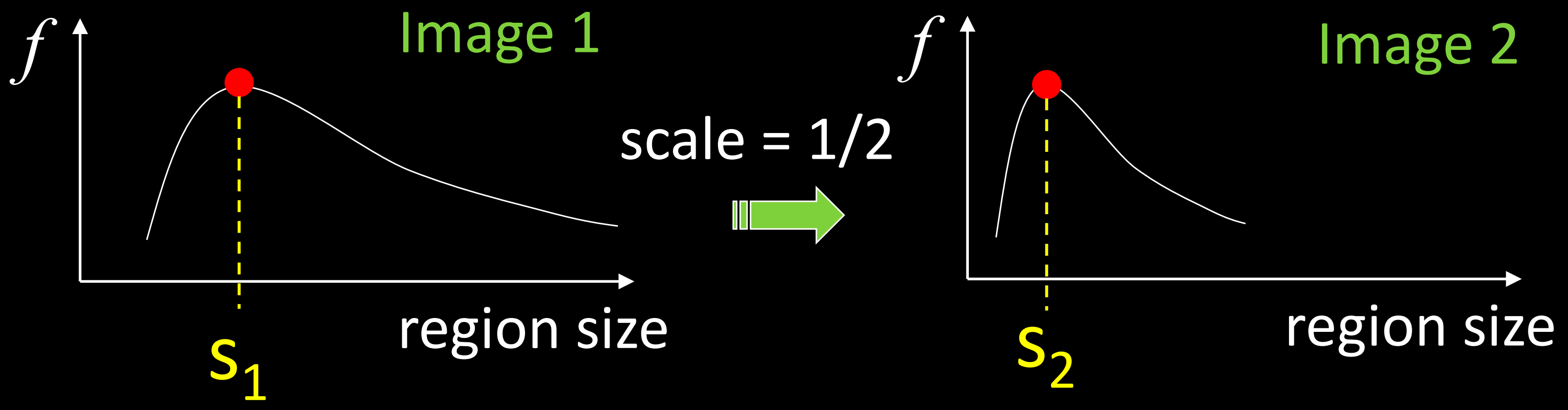
- At a point, compute the scale invariant function over different size neighborhoods (different scales).
- Choose the scale for each image at which the function is a maximum



Scale Invariant Detection

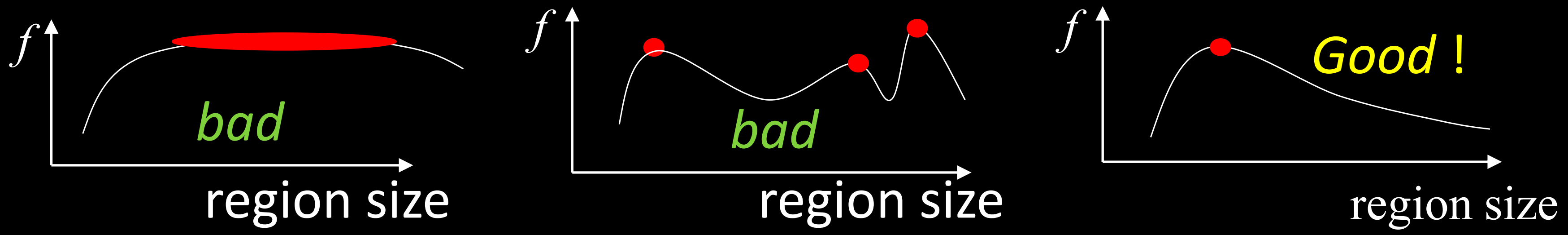
One method:

- Important: this scale invariant region size is found in each image independently



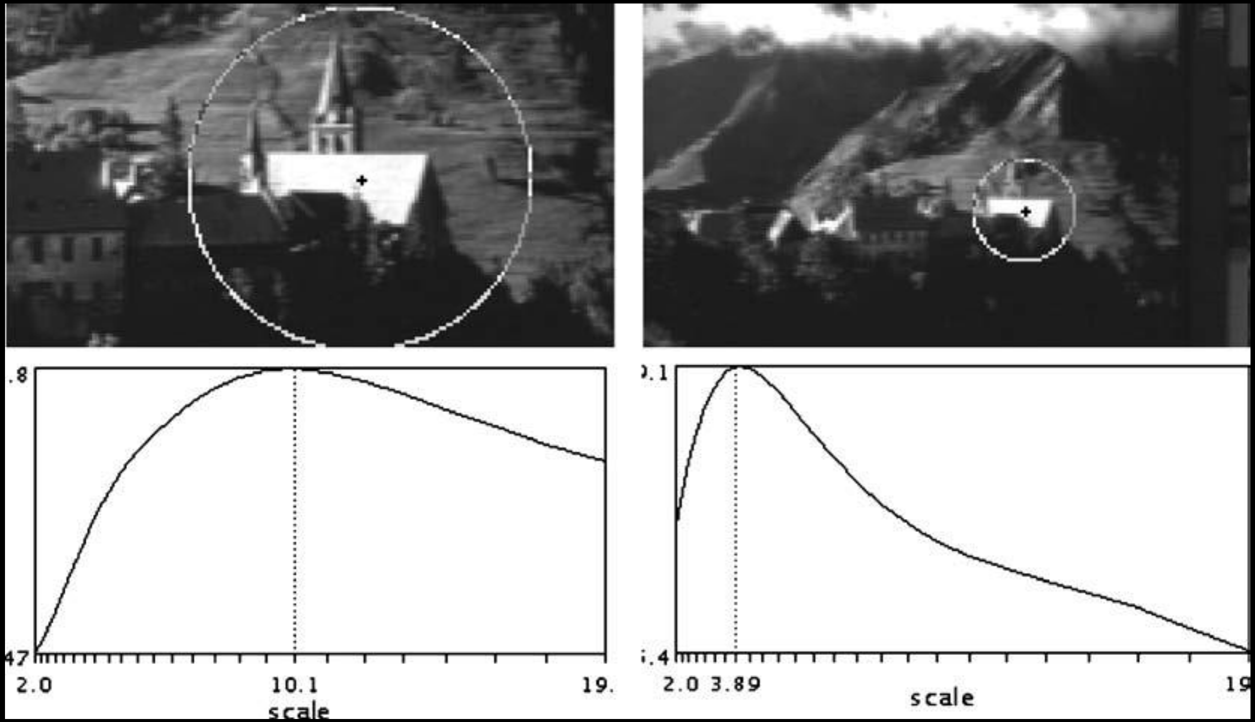
Scale Invariant Detection

- A "good" function for scale detection:
  has one stable sharp peak



For usual images: a good function would be a one which responds to contrast (sharp local intensity change)

Scale sensitive response



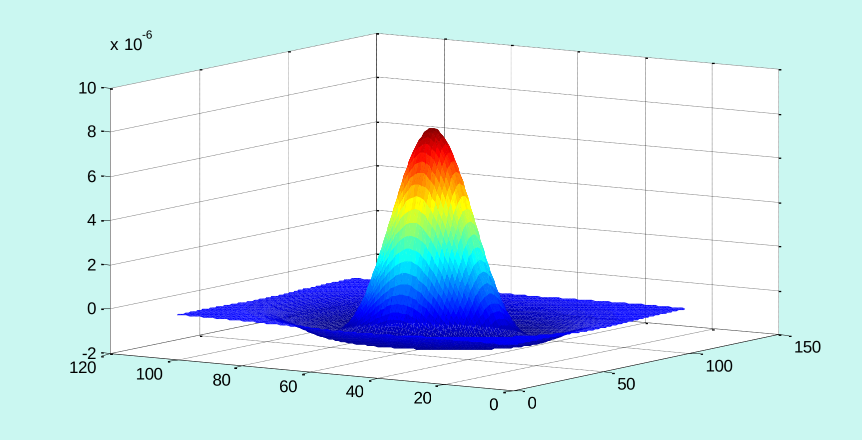
Scale Invariant Detection

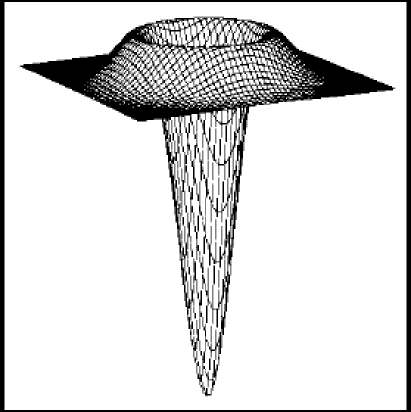
Function is just application of a kernel: $f = Kernel * Image$

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian of Gaussian - LoG)

Laplacian of Gaussian





$\nabla^2 h_\sigma(u, v)$

Scale Invariant Detection

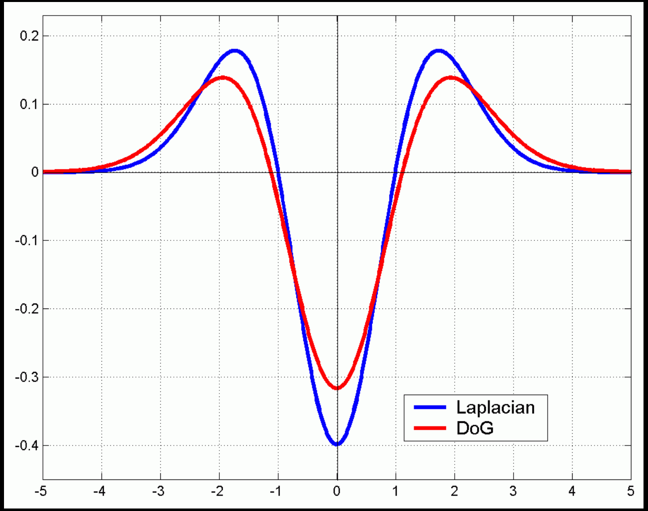
Function is just application of a kernel: $f = Kernel * Image$

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian of Gaussian - LoG)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

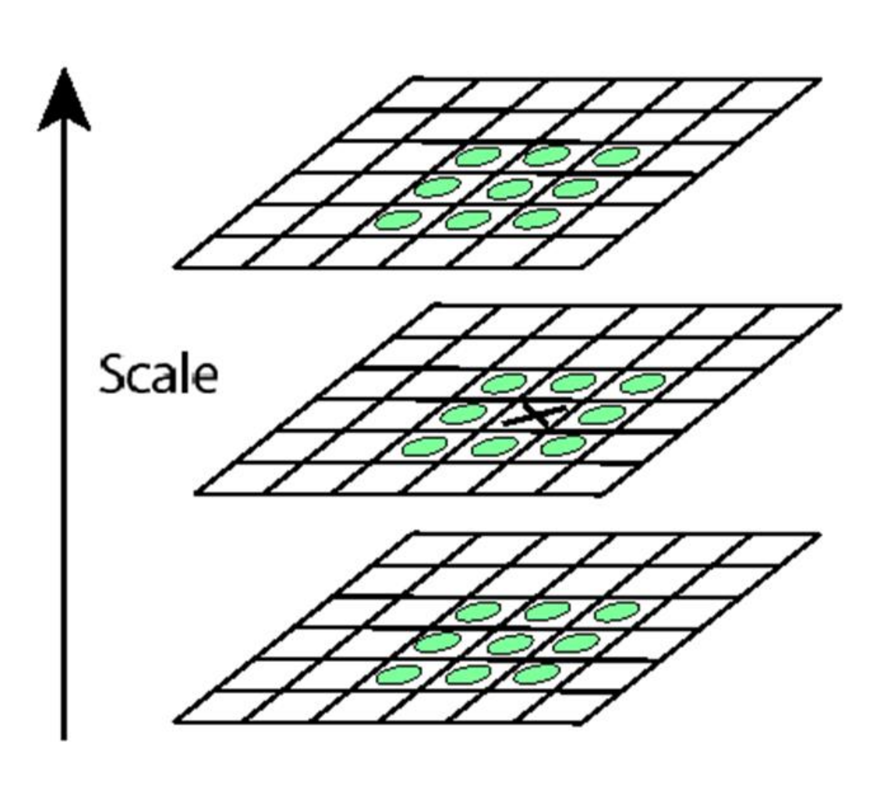
(Difference of Gaussians)



Note: both kernels are invariant to *scale* and *rotation*

Key point localization

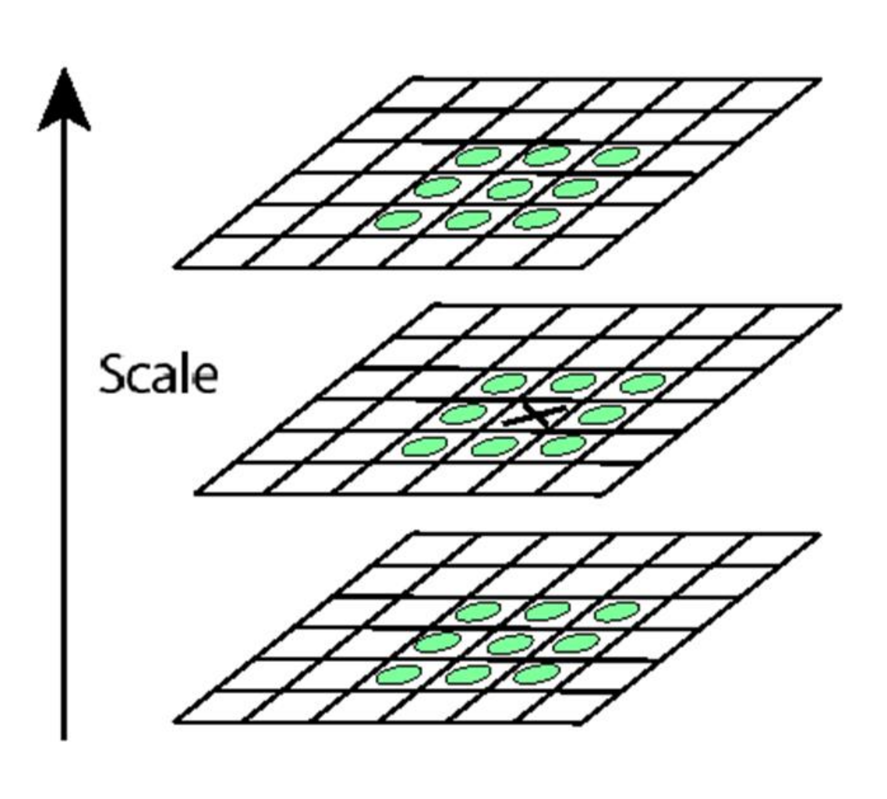
- General idea: find robust extremum (maximum or minimum) both in space and in scale.



Key point localization

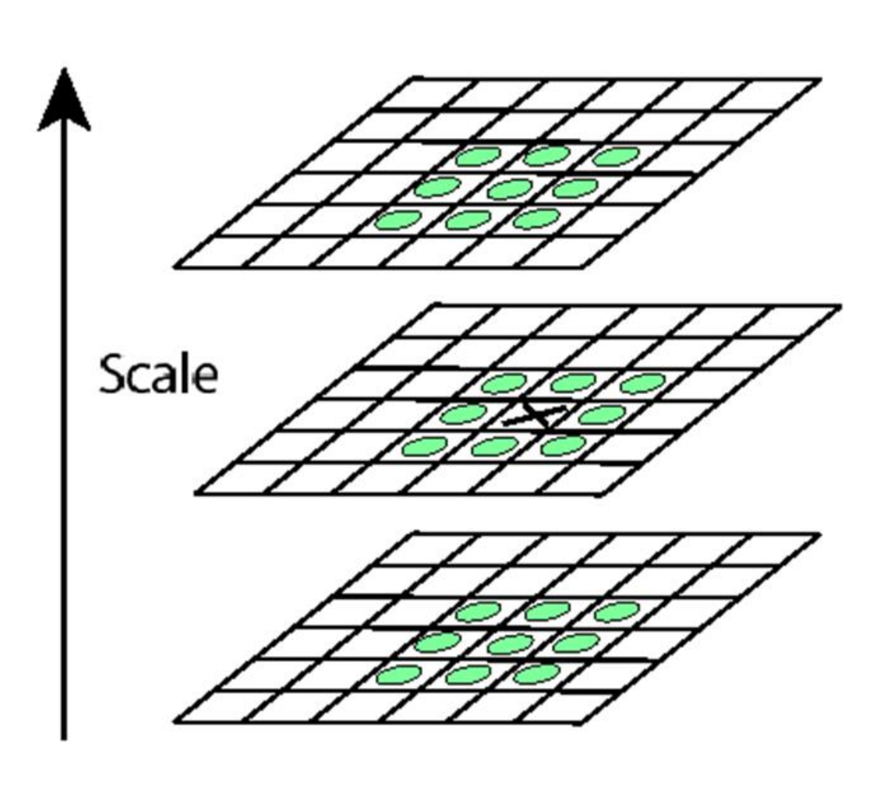
- SIFT: Scale Invariant Feature Transform

- Specific suggestion: use DoG pyramid to find maximum values (remember edge detection?) – then eliminate "edges" and pick only corners.

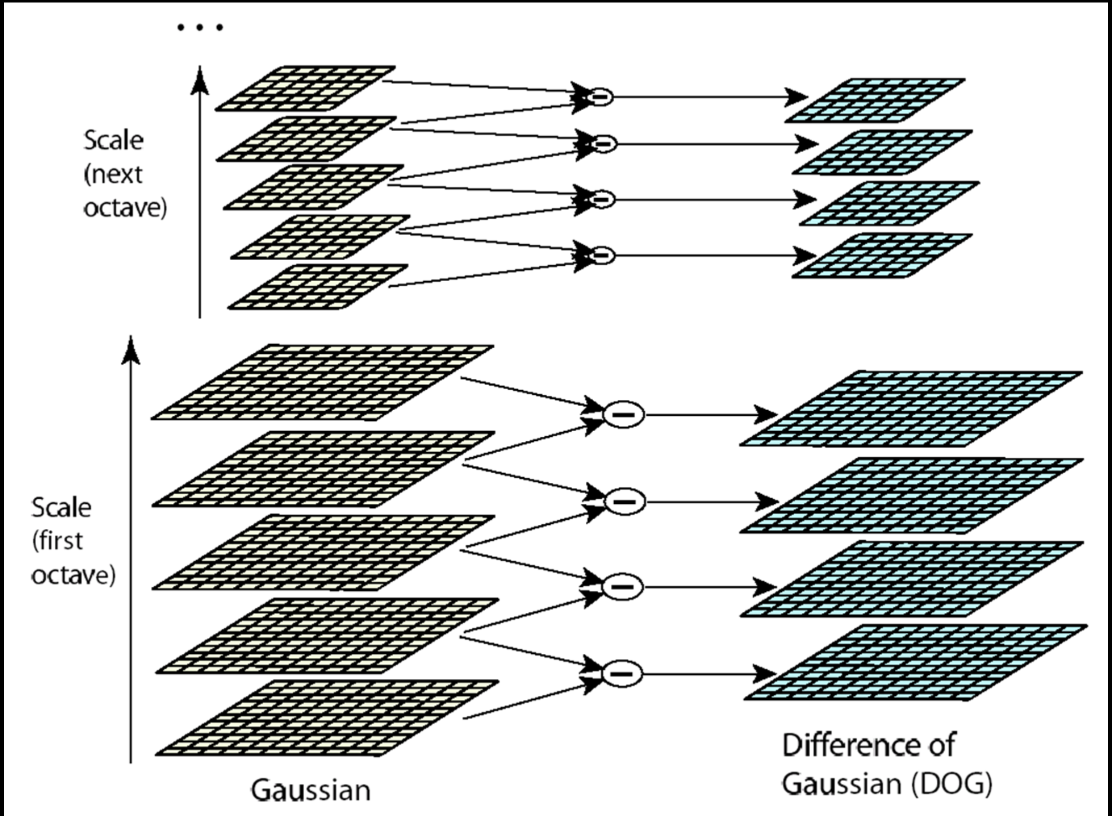


Key point localization

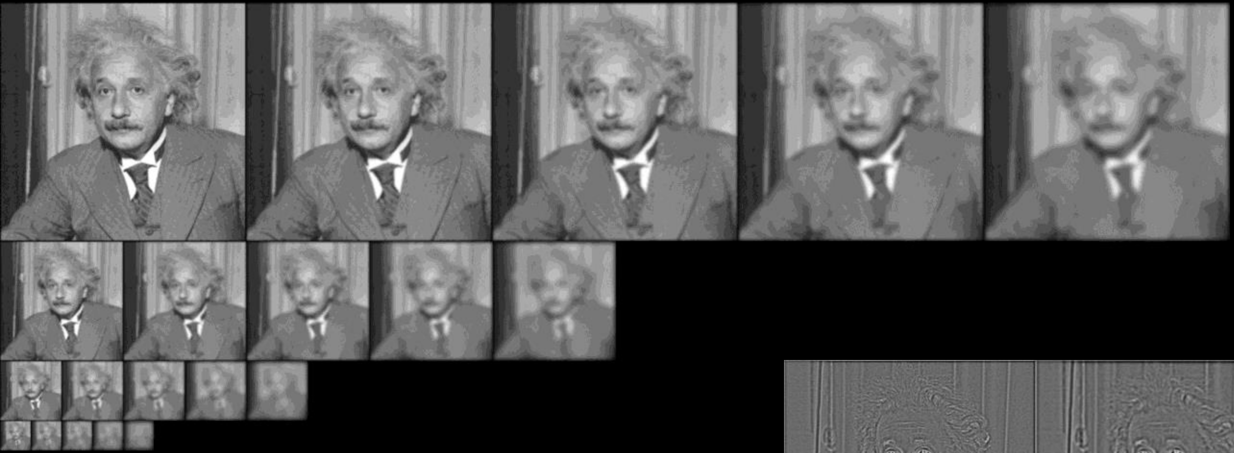
(Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below.)



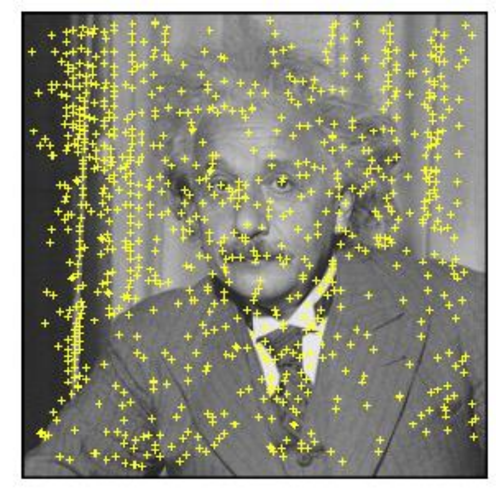
Scale space processed one octave at a time



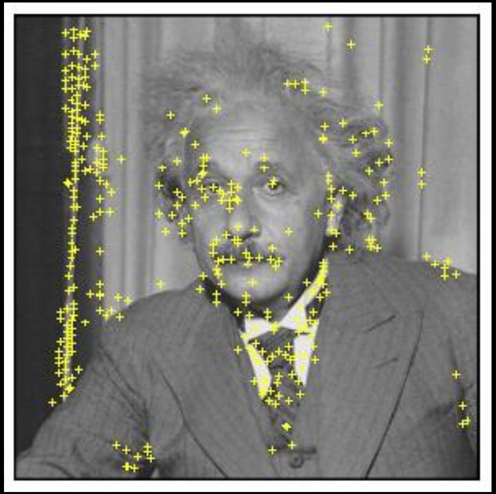
Extrema at different scales



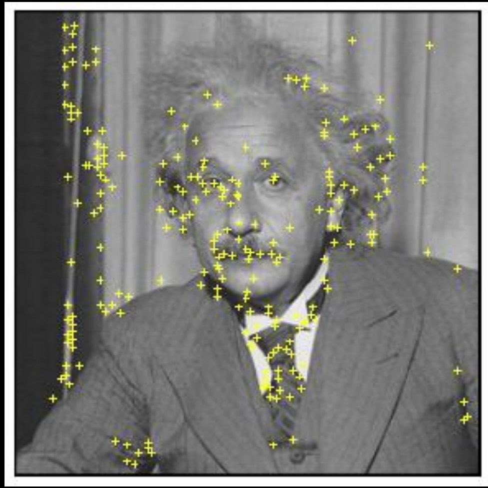
Remove low contrast, edge bound



Extrema points



Contrast > C



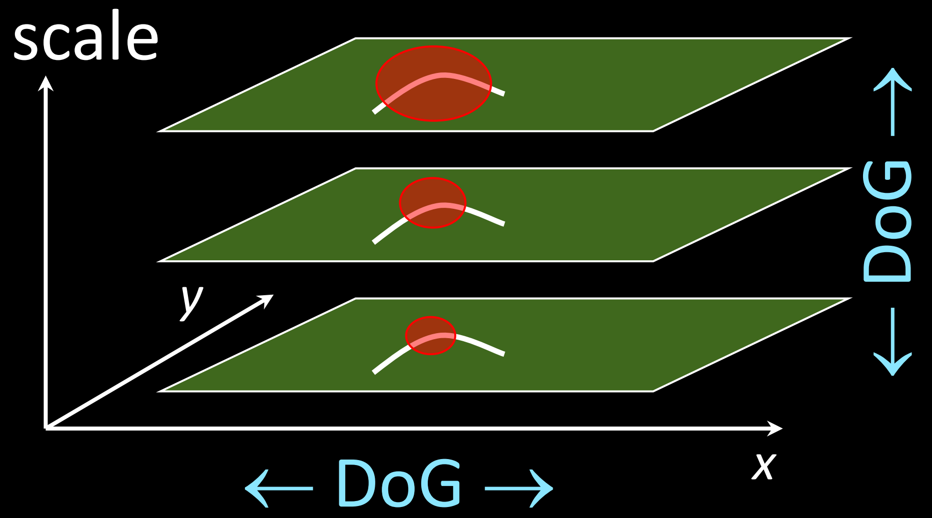
Not on edge

Scale Invariant Detectors

SIFT (Lowe)

Find local maximum of:

– Difference of Gaussians in space and scale



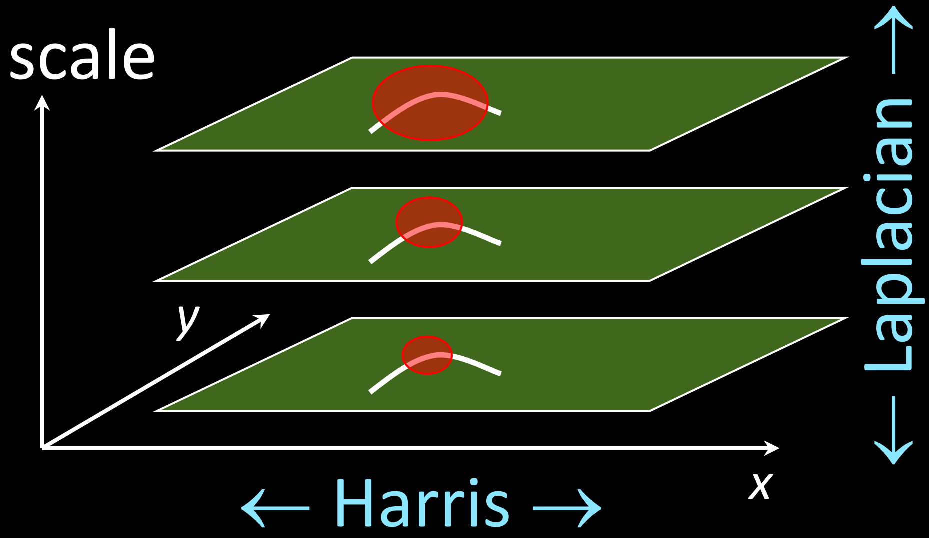
D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004

Scale Invariant Detectors

Harris-Laplacian

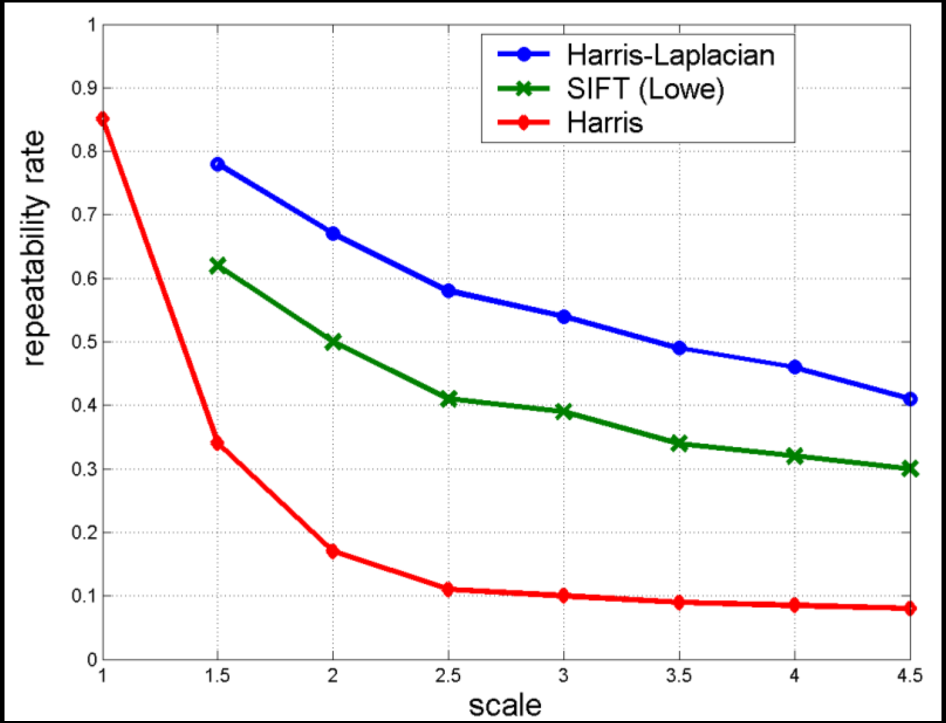
Find local maximum of:

- Harris corner detector in space (image coordinates)
- Laplacian in scale



K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

Scale Invariant Detectors



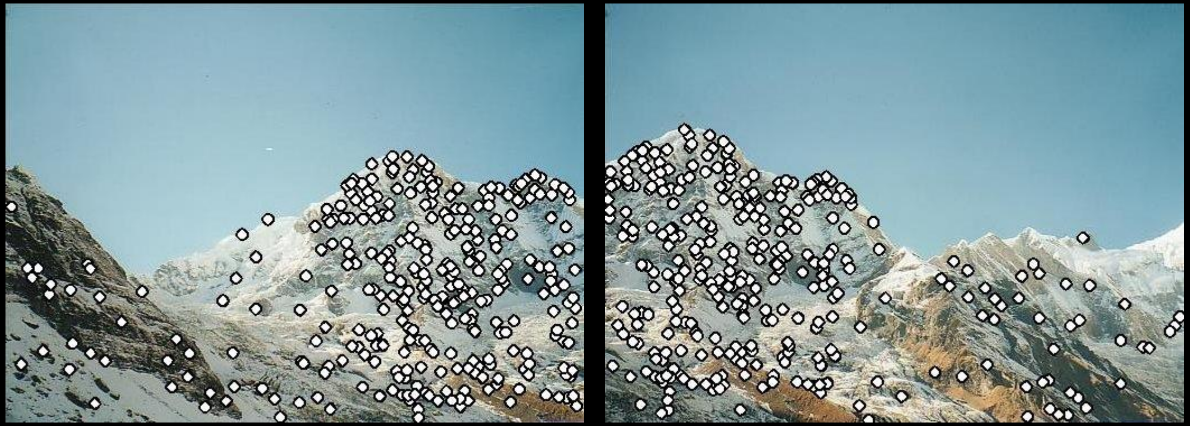
K.Mikolajczyk, C.Schmid. “Indexing Based on Scale Invariant Interest Points”. ICCV 2001

Scale Invariance: Summary

- Given: Two images of the same scene with a *large* scale difference between them
- Goal: Find the same interest points independently in each image
- Solution: Search for maxima of suitable functions in scale and in space (over the image)

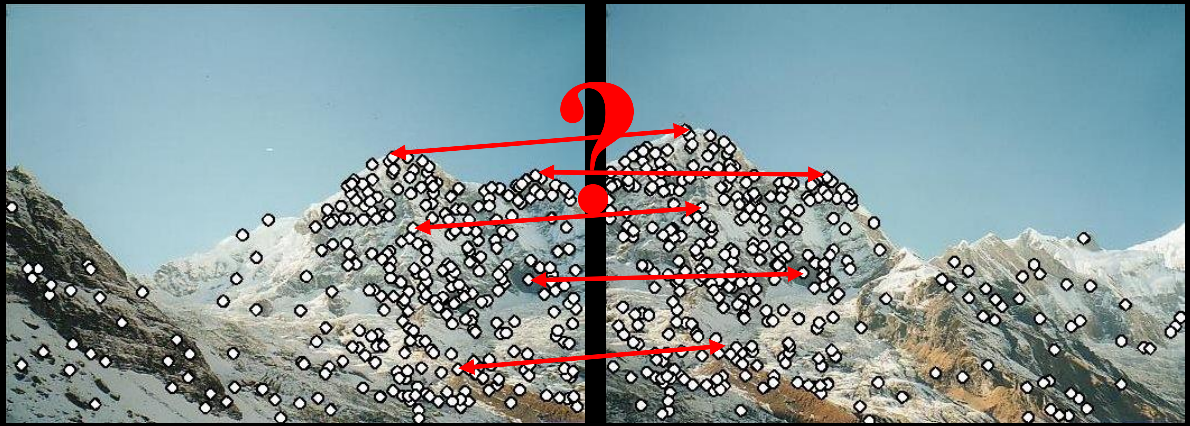
Point Descriptors

- We now know how to detect interest points



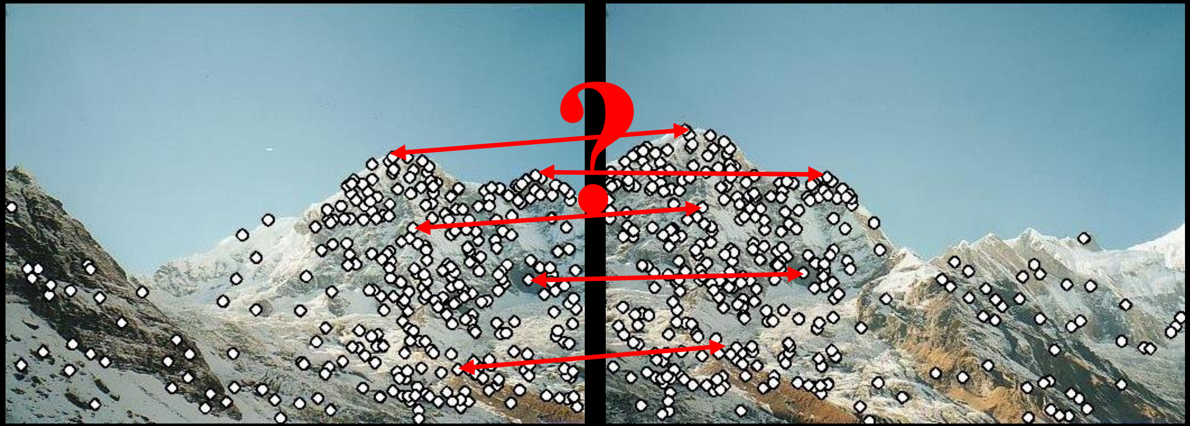
Point Descriptors

- Next question: How to match them?



Point Descriptors

- We need to describe them – a "*descriptor*"



...Next!