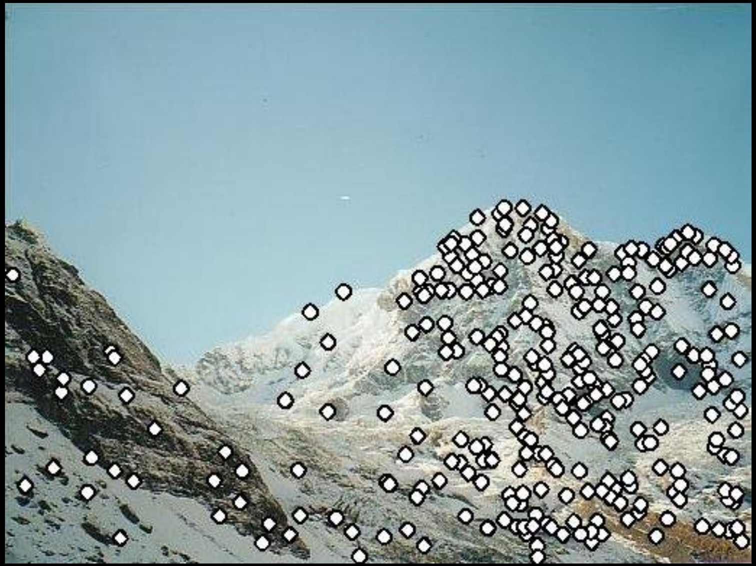
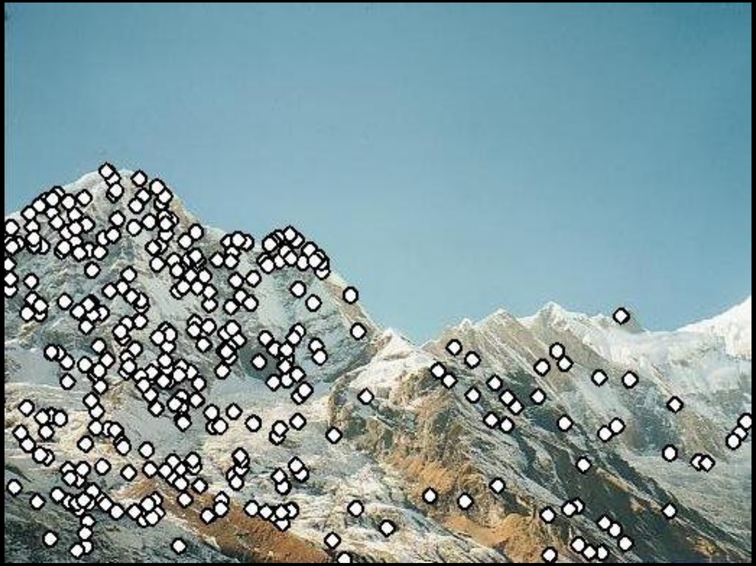
CS4495/6495
Introduction to Computer Vision

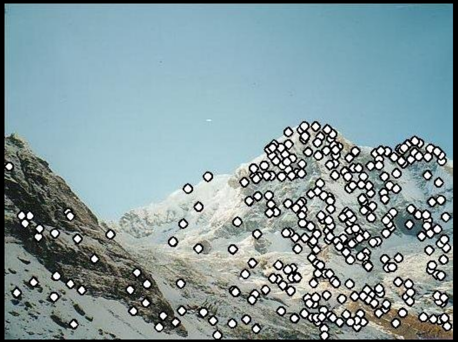
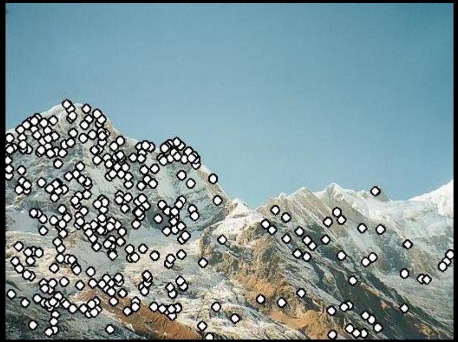
4A-L2 *Finding corners*

Feature points





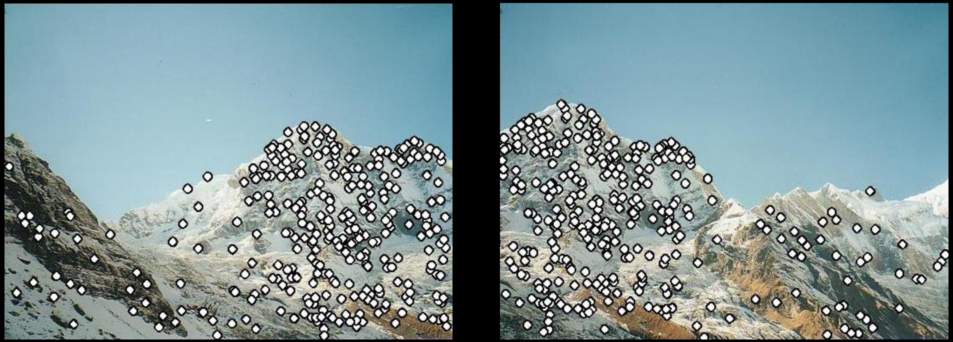
Characteristics of good features

Repeatability/Precision

- The same feature can be found in several images despite geometric and photometric transformations

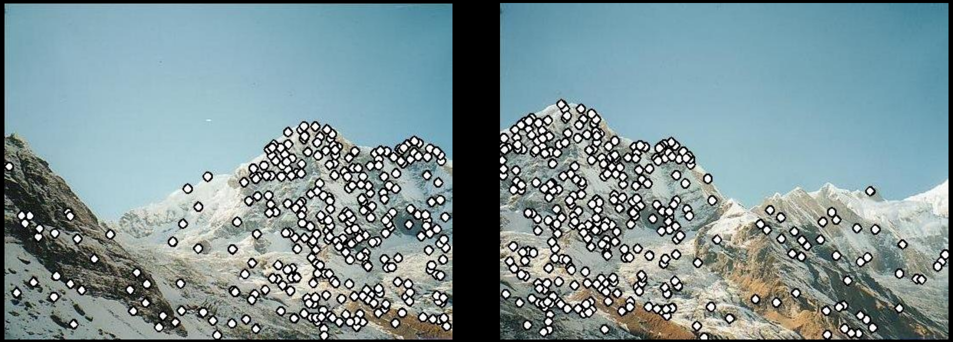
Characteristics of good features



Saliency/Matchability

- Each feature has a distinctive description

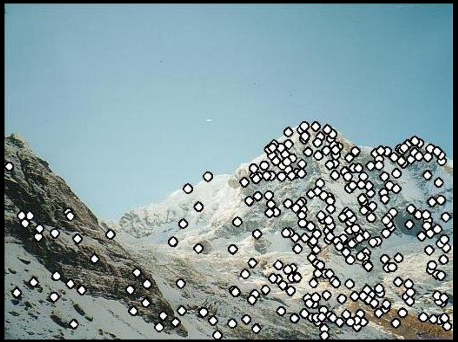
Characteristics of good features

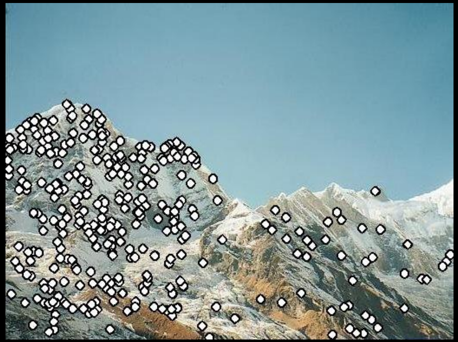


Compactness and efficiency

- Many fewer features than image pixels

Characteristics of good features

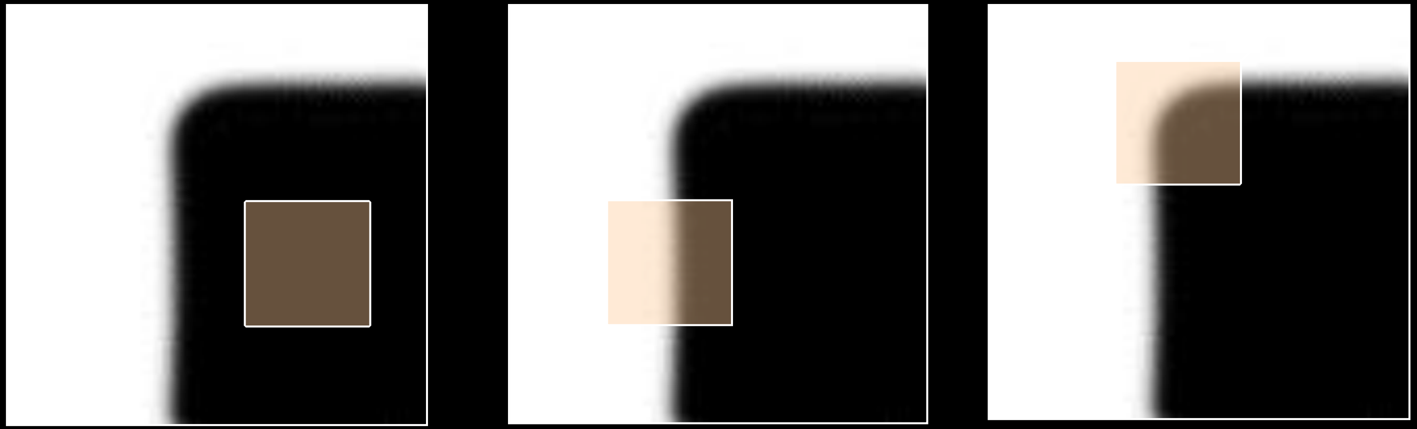




Locality

- A feature occupies a relatively small area of the image; robust to clutter and occlusion

Corner Detection: Basic Idea



"flat" region: no change in all directions

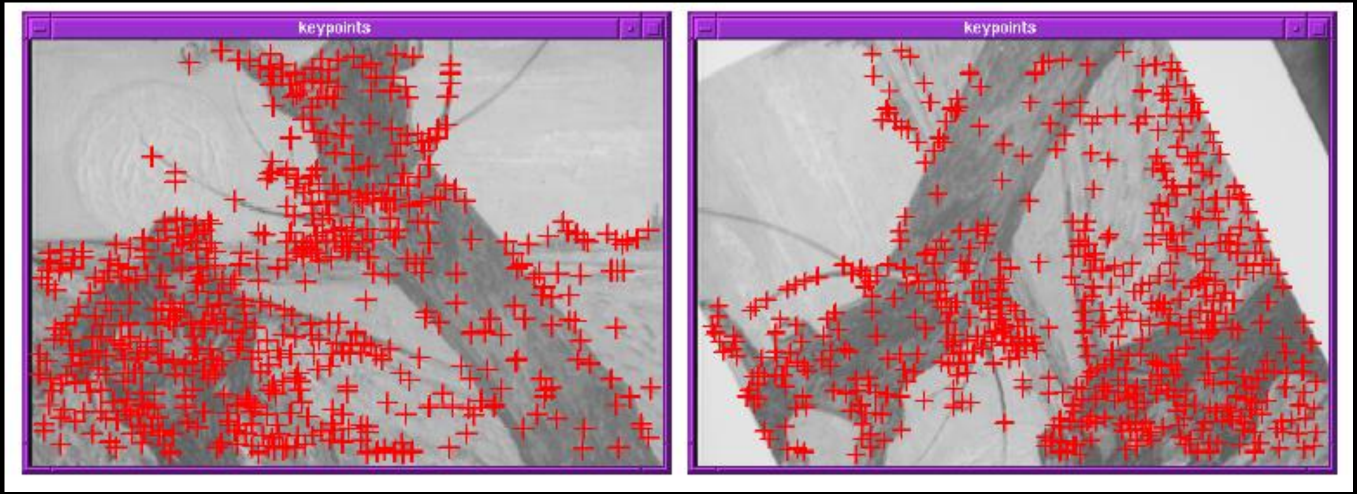
"edge": no change along the edge direction

"corner": significant change in all directions with small shift

Source: A. Efros

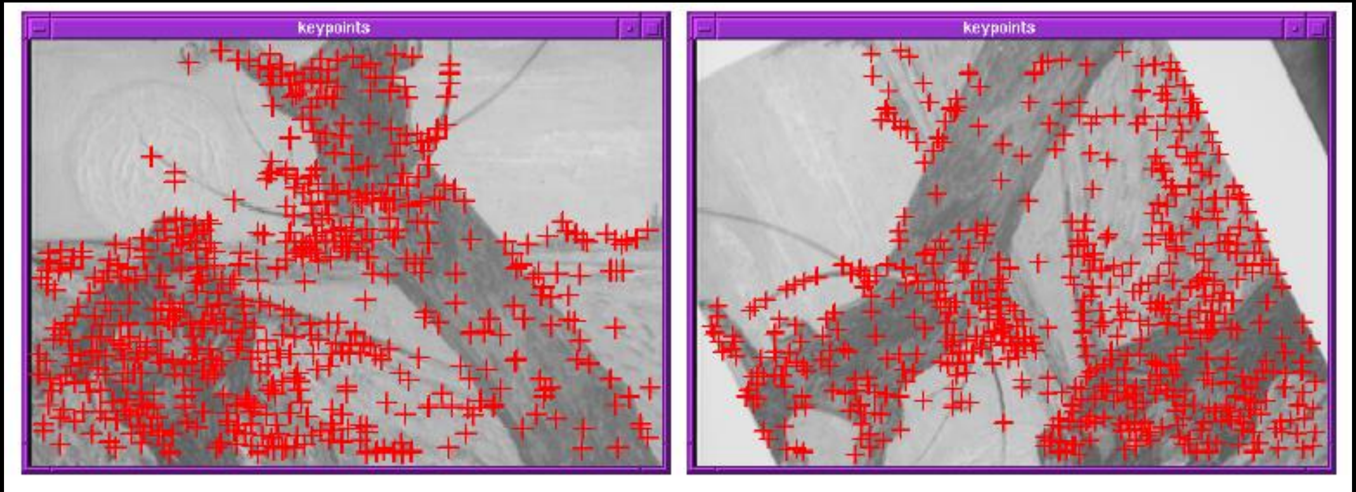
Finding Corners

- Key property: in the region around a corner, image gradient has two or more dominant directions



Finding Corners

C. Harris and M. Stephens. *"A Combined Corner and Edge Detector,"* *Proceedings of the 4th Alvey Vision Conference*: **1988**



Corner Detection: Mathematics

Change in appearance for the shift [*u*,*v*]:

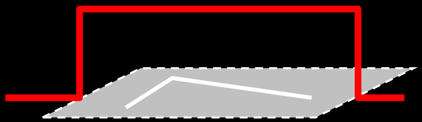
$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

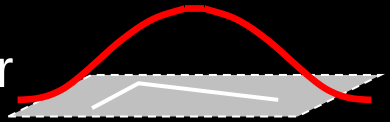
Shifted intensity

Intensity

Window function *w(x,y)* =



or



1 in window, 0 outside

Gaussian

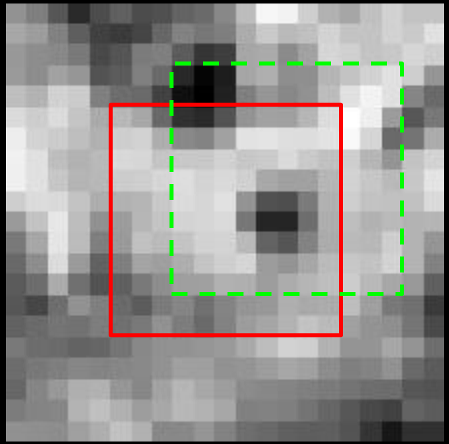
Source: R. Szeliski

Corner Detection: Mathematics

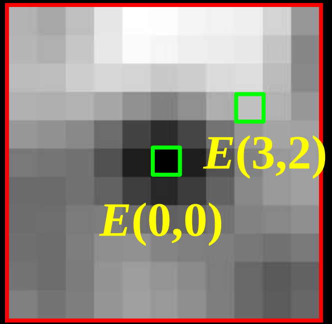
Change in appearance for the shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

Change in appearance for the shift [*u,v*]:

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2}$$

We want to find out how this function behaves for ***small*** shifts (u,v near 0,0)

Corner Detection: Mathematics

Change in appearance for the shift [u,v]:

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2}$$

Second-order Taylor expansion of $E(u,v)$ about (0,0) (local quadratic approximation for small u,v):

Corner Detection: Mathematics

Change in appearance for the shift [*u*,*v*]:

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2}$$

$$F(\delta x) \approx F(0) + \delta x \cdot \frac{dF(0)}{dx} + \frac{1}{2}\delta x^2 \cdot \frac{d^2F(0)}{dx^2}$$

Corner Detection: Mathematics

Change in appearance for the shift [*u*,*v*]:

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2}$$

$$F(\delta x) \approx F(0) + \delta x \cdot \frac{dF(0)}{dx} + \frac{1}{2}\delta x^2 \cdot \frac{d^2F(0)}{dx^2}$$

$$\boxed{E(u,v) \approx E(0,0) + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}}$$

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2}$$

Second-order Taylor expansion of *E*(*u*,*v*) about (0,0):

$$E(u,v) \approx E(0,0) + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

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Need these derivatives…

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2}$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E_u(u,v) = \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right]I_{\mathbf{x}}(x+u,y+v)$$

$$\boxed{E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2}$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_{\mathbf{x}}(x+u,y+v) I_{\mathbf{x}}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right] I_{\mathbf{xx}}(x+u,y+v)$$

$$\boxed{E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2}$$

Second-order Taylor expansion of *E*(*u*,*v*) about (0,0):

$$E_{uv}(u,v)=\sum_{x,y} 2w(x,y)I_y(x+u,y+v)I_x(x+u,y+v)$$

$$+\sum_{x,y} 2w(x,y)\left[I(x+u,y+v)-I(x,y)\right]I_{xy}(x+u,y+v)$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx E(0,0) + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u,v) = \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right]I_x(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y)I_x(x+u,y+v)I_x(x+u,y+v)$$

$$+\sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right]I_{xx}(x+u,y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y)I_y(x+u,y+v)I_x(x+u,y+v)$$

$$+\sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right]I_{xy}(x+u,y+v)$$

Evaluate E and its derivatives at **(0,0)**:

$$E(u,v) \approx \underbrace{\boxed{E(0,0)}}_{=0} + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(0,0) = \sum_{x,y} 2w(x,y)\underbrace{\boxed{\left[I(x,y) - I(x,y)\right]}}_{=0} I_x(x,y)$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_x(x,y) + \sum_{x,y} 2w(x,y) \underbrace{\boxed{\left[I(x,y) - I(x,y)\right]}}_{=0} I_{xx}(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y) I_y(x,y) I_x(x,y) + \sum_{x,y} 2w(x,y) \underbrace{\boxed{\left[I(x,y) - I(x,y)\right]}}_{=0} I_{xy}(x,y)$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx E(0,0) + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \boxed{\frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}}$$

$$E(0,0) = 0 \qquad E_{uu}(0,0) = \sum_{x,y} 2w(x,y)I_x(x,y)I_x(x,y)$$

$$E_u(0,0) = 0 \qquad E_{vv}(0,0) = \sum_{x,y} 2w(x,y)I_y(x,y)I_y(x,y)$$

$$E_v(0,0) = 0 \qquad E_{uv}(0,0) = \sum_{x,y} 2w(x,y)I_x(x,y)I_y(x,y)$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx [u \;\; v] \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2(x,y) & \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) \\ \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) & \sum_{x,y} w(x,y) I_y^2(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0 \qquad E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_x(x,y)$$

$$E_u(0,0) = 0 \qquad E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_y(x,y) I_y(x,y)$$

$$E_v(0,0) = 0 \qquad E_{uv}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_y(x,y)$$

Corner Detection: Mathematics

The quadratic approximation simplifies to

$$\boxed{E(u,v) \approx [u \quad v] \; M \begin{bmatrix} u \\ v \end{bmatrix}}$$

where *M* is a *second moment matrix* computed from image derivatives:

$$\boxed{M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}$$

The second moment matrix M:

$$\boxed{M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}$$

Can be written (without the weight):

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \left(\begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} \right) = \sum \nabla I (\nabla I)^T$$

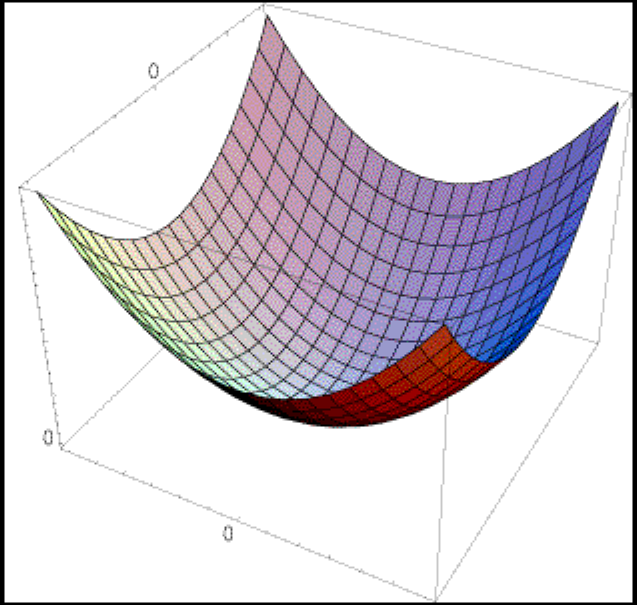
Each product is a rank 1 2x2

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form.

$$E(u,v) \approx [u \quad v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

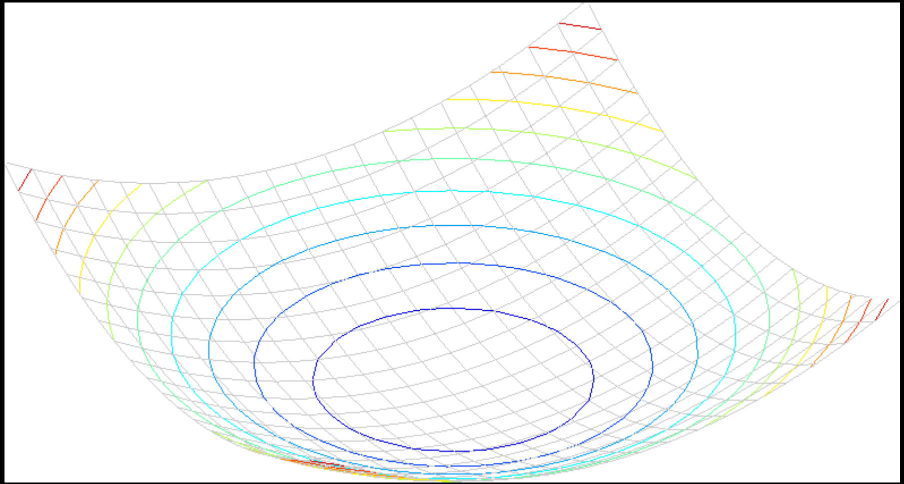


Interpreting the second moment matrix

Consider a constant "slice" of *E*(*u*, *v*):

$$\Sigma I_x^2 u^2 + 2\Sigma I_x I_y uv + \Sigma I_y^2 v^2 = k$$

This is the equation of an ellipse.



Interpreting the second moment matrix

First, consider the axis-aligned case where gradients are either horizontal or vertical

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

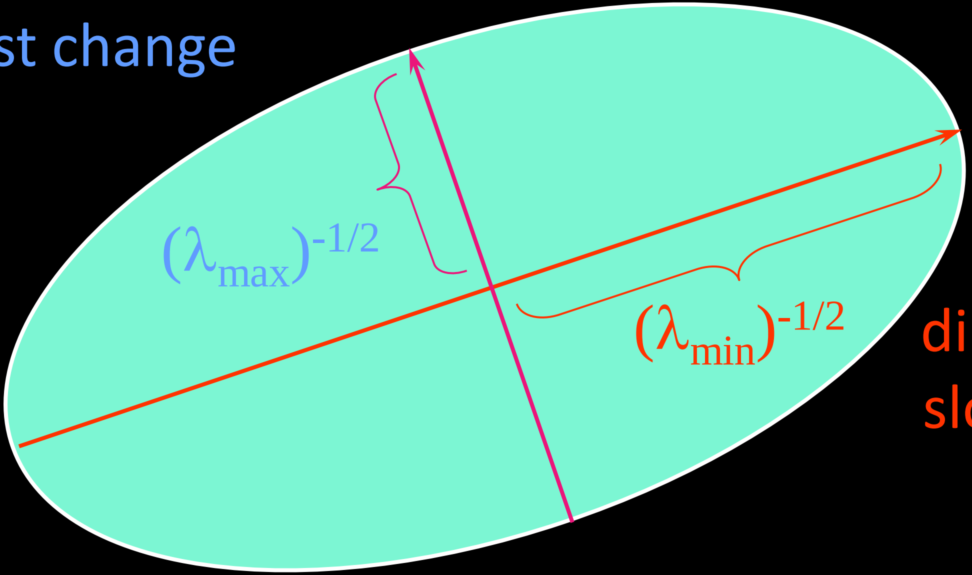
Interpreting the second moment matrix

Diagonalization of M: $$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

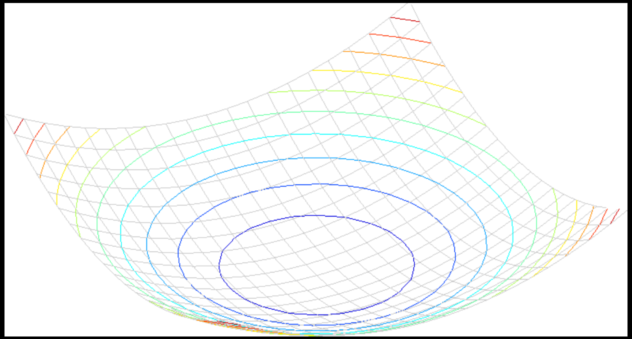
The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*

Interpreting the second moment matrix

direction of the
fastest change

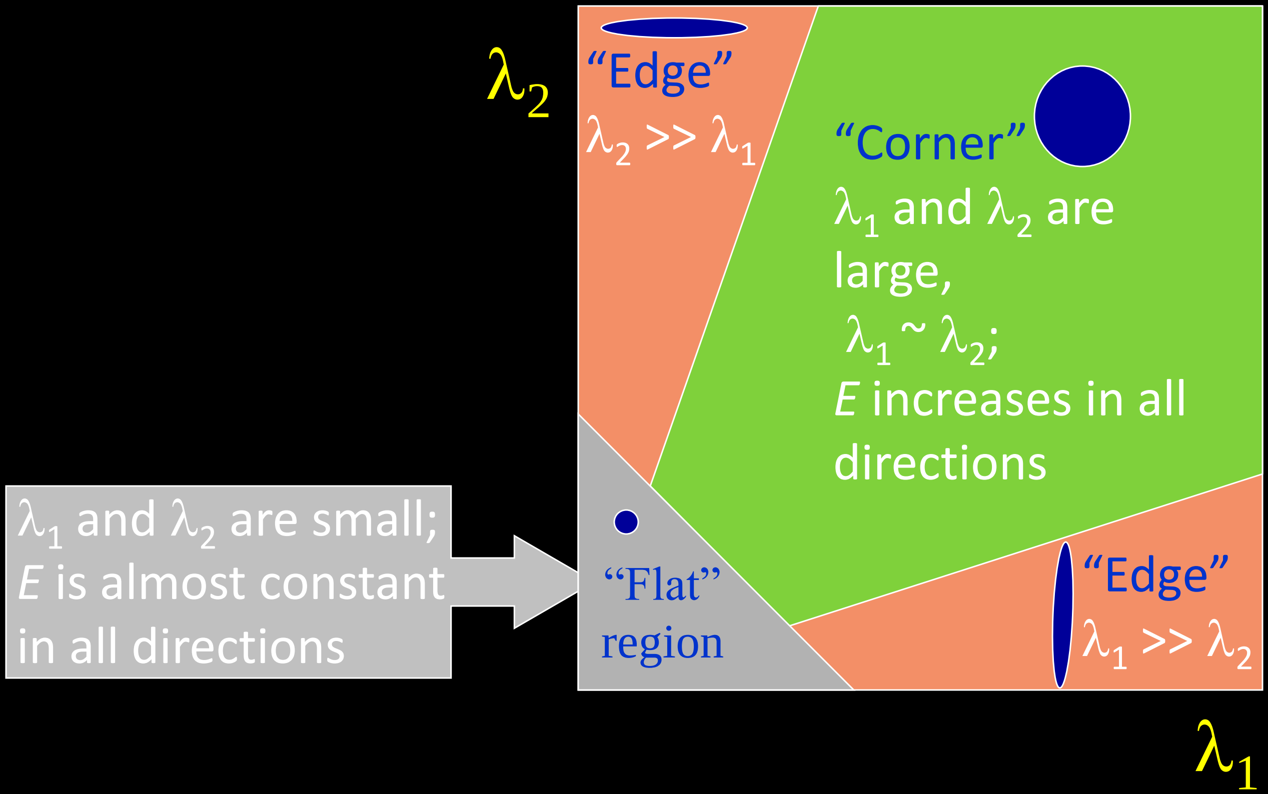


direction of the
slowest change



Interpreting the eigenvalues

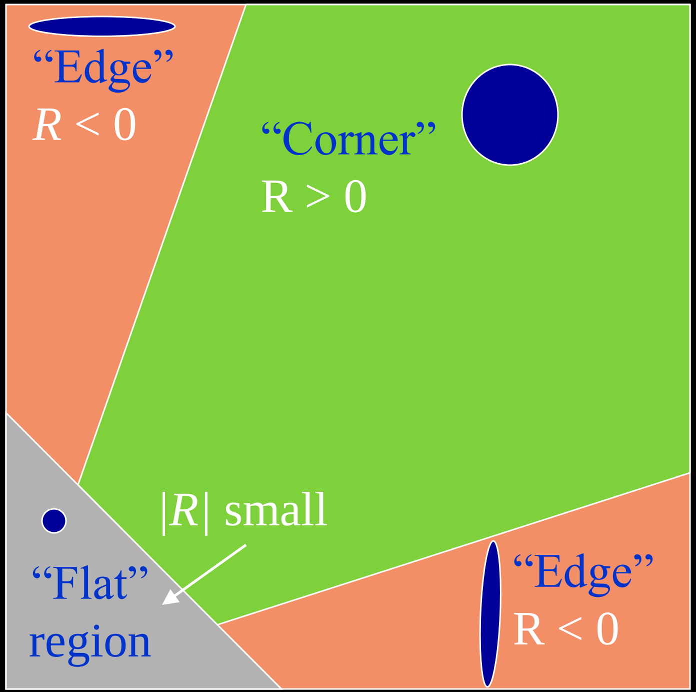
Classification of image points using eigenvalues of *M*:



Harris corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

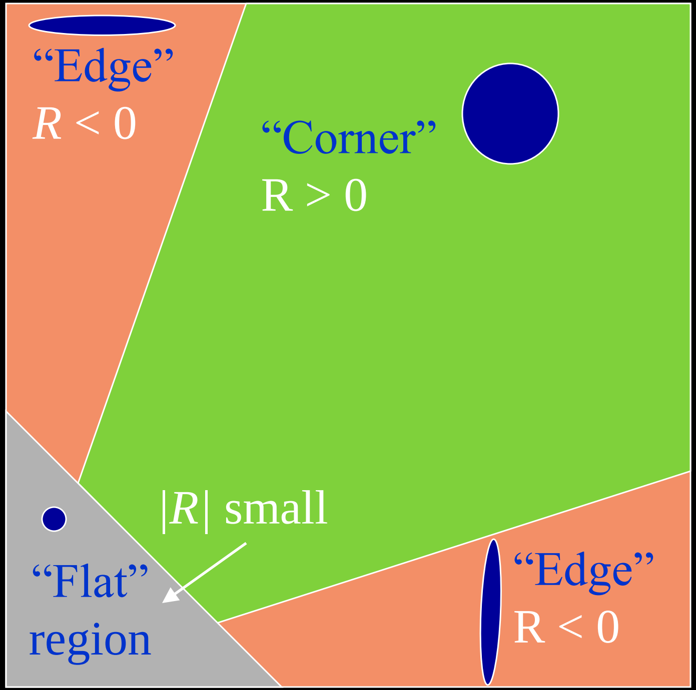
α: constant (0.04 to 0.06)



Harris corner response function

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

R depends only on eigenvalues of M, but don't compute them (no sqrt, so really fast even in the '80s).



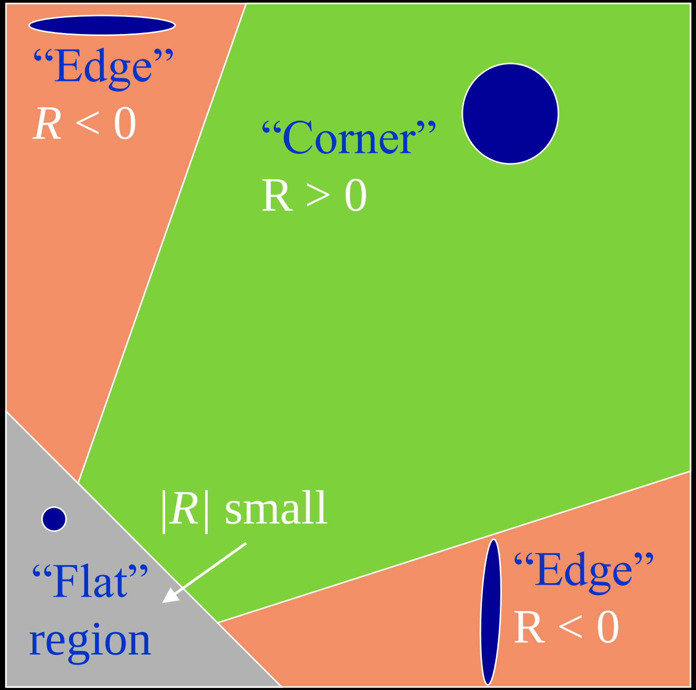
Harris corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

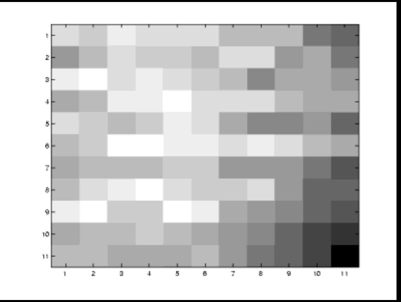
R is large for a corner

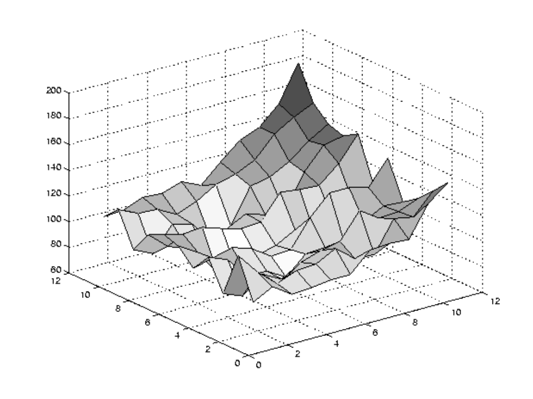
R is negative with large magnitude for an edge

$|R|$ is small for a flat region



Low texture region



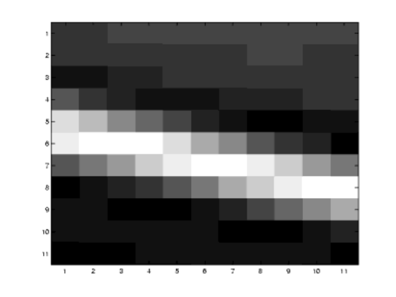


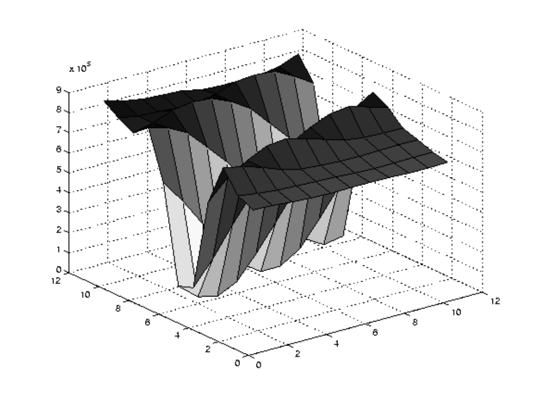
$$M = \sum \nabla I (\nabla I)^T$$

Gradients have small magnitude

=> small λ_1, small λ_2

Edge



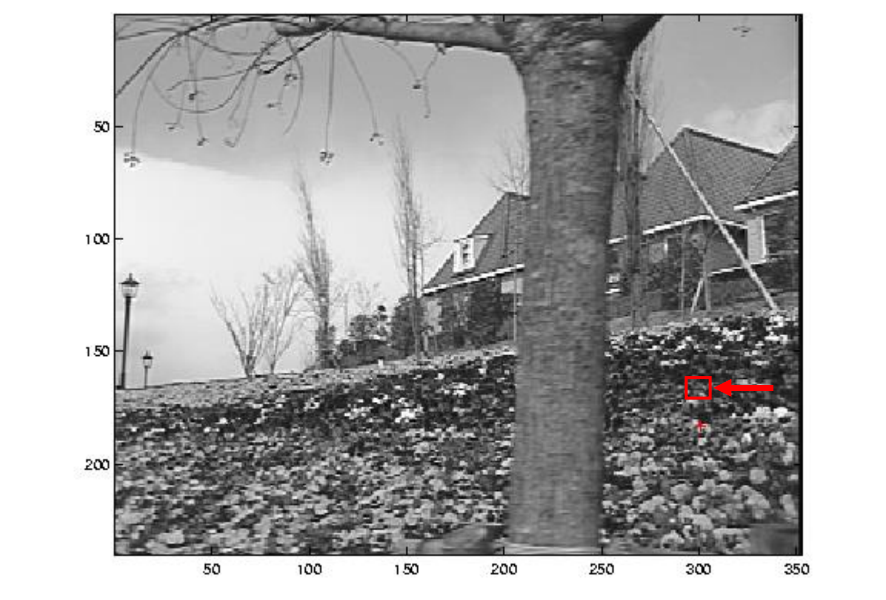


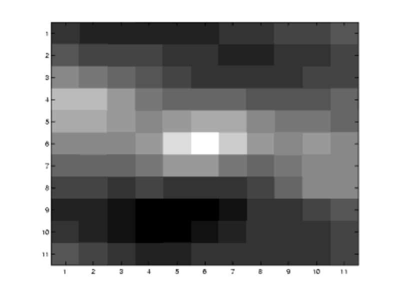
$$M = \sum \nabla I (\nabla I)^T$$

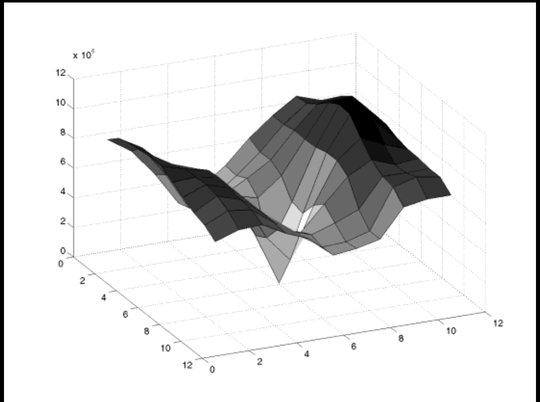
Large gradients, all the same

=> large λ_1, small λ_2

High textured region







$$M = \sum \nabla I (\nabla I)^T$$

Gradients different, large magnitudes

=> large λ_1, large λ_2

Harris Detector: Algorithm

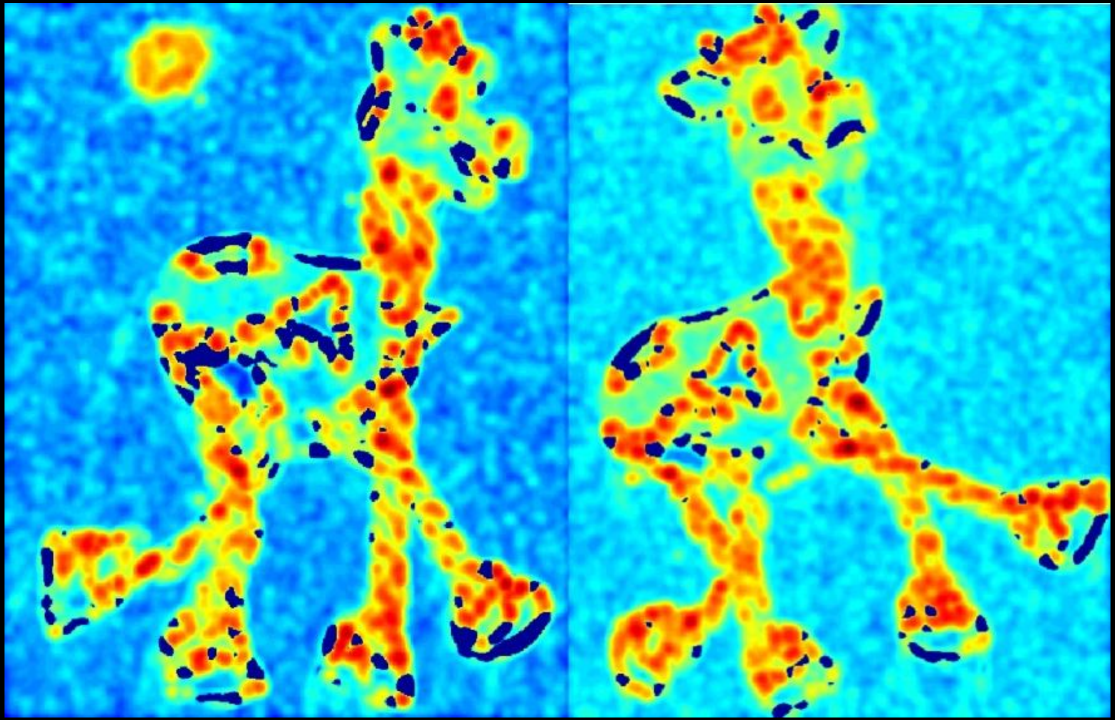
1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R
4. Threshold R
5. Find local maxima of response function (nonmaximum suppression)

C. Harris and M. Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Detector: Workflow

Harris Detector: Workflow

Compute corner response R

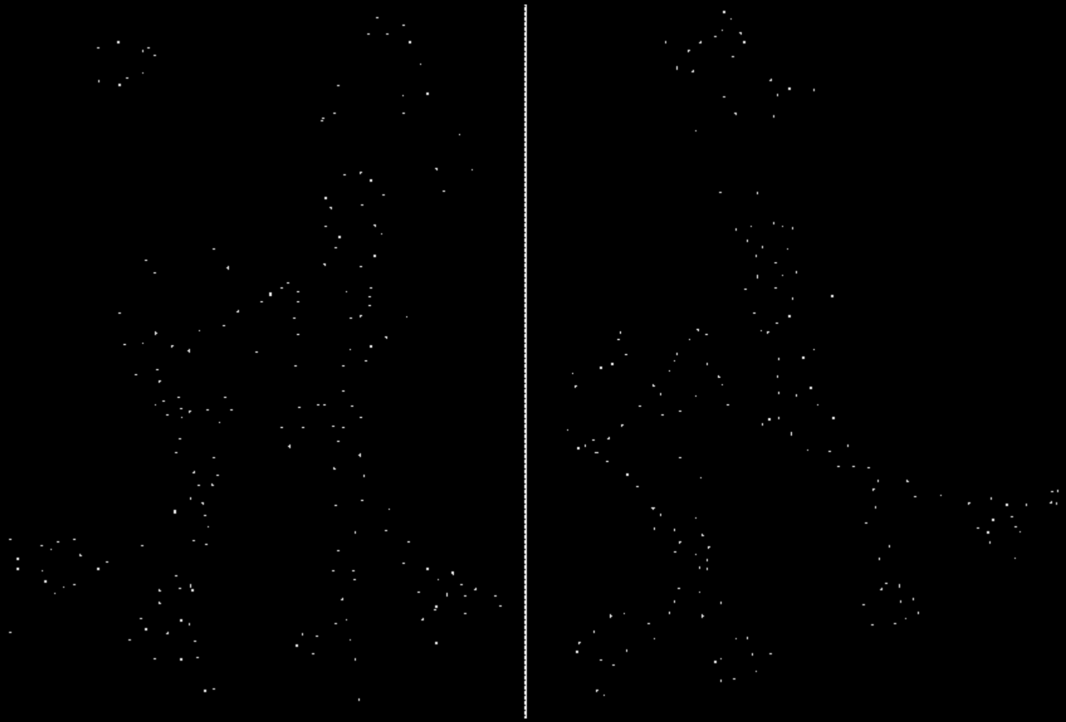


Harris Detector: Workflow

Find points with large corner response: R>threshold

Harris Detector: Workflow

Take only the points of local maxima of *R*



Harris Detector: Workflow

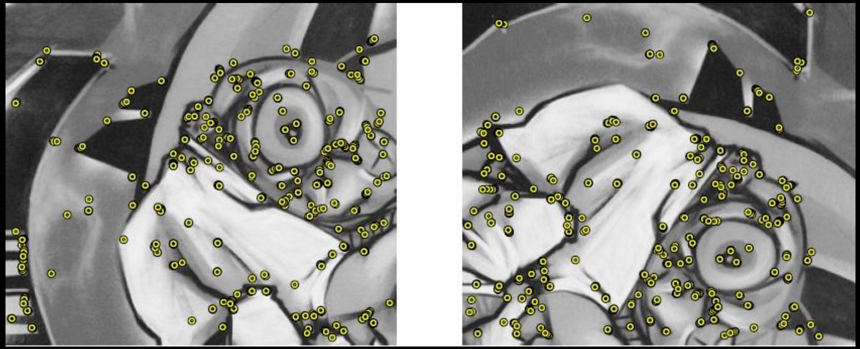
Other corners:

Shi-Tomasi '94:

"Cornerness" = min (λ_1, λ_2)   Find local maximums

cvGoodFeaturesToTrack(...)

Reportedly better for region undergoing affine deformations



Other corners:

- Brown, M., Szeliski, R., and Winder, S. (2005):

$$\frac{\det M}{\operatorname{tr} M} = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1}$$

- There are others…