Philosophy of Statistics: Homework 5

due on Gradescope by 11am on Wednesday February 10

Guidelines. Some questions ask you to justify your answers. For these questions, credit will be based on how well you justify your answers, not on whether your answers are correct. (There's often no consensus on the correct answers, even among statisticians.) However, that doesn't mean that anything goes: some answers will be hard to justify well. I give suggested word counts but these are just ballpark numbers. Don't sweat them too much. Collaboration is encouraged, but make sure to write up your answers by yourself and list your collaborators.

Problem 1 (15 points). The likelihood ratio comes up in both Neyman-Pearson testing and Likelihoodism. First, discuss what role the likelihood ratio plays in each approach. (So you should explain likelihood ratio tests, for Neyman-Pearson testing, and the Law of Likelihood, for Likelihoodism.) Second, from the perspective of the Likelihood Theorist, what do Neyman and Pearson's likelihood ratio tests amount to?

Problem 2 (25 points). The Likelihoodist says that an outcome is evidence for h over i just if the outcome has higher likelihood under h than under i. A Bayesian says that an outcome is evidence for, is neutral about, or is evidence against h according to whether the posterior is greater than, equal to, or less than the prior. Let's think about the relationship between these two theories of evidence, the Likelihoodist's and the Bayesian's.

First, imagine an experiment in which the Bayesian regards only two hypotheses, h and i, as live possibilities. (In other words, the Bayesian's priors in h and i sum to 1.) Show that in this case the Likelihoodist and the Bayesian always agree, in the sense that the Likelihoodist says the outcome is evidence for h over i if and only if the Bayesian says the outcome is evidence for h.

Second, imagine an experiment in which the Bayesian regards at least three hypotheses, h, i, and j, as live possibilities. (In other words, the Bayesian's priors in h, i, and j are all positive.) Might the Likelihoodist say the outcome is evidence for h over i but the Bayesian say the outcome is evidence against h? Either write down an example or show that none exists.

Third, might the Likelihoodist say the outcome is evidence for h over i *and* for h over j but the Bayesian say the outcome is evidence against h? Either write down an example or show that none exists.

Hint 1: remember that the Bayesian's posteriors, like their priors, always add up to 1. Hint 2: you might find the Odds Form of Bayes Theorem (class 7, slide 12) useful.

Problem 3 (20 points). Here is an argument against the Law of Likelihood:

> Assume for the sake of argument that the Law of Likelihood is true. Now imagine I'm holding two cards: either Ace-Ace, or King-King, or Ace-King. You will pick one of the cards at random.
>
> Picking an Ace is evidence for the Ace-Ace hypothesis over the Ace-King hypothesis. (Why? Because the likelihood of picking an Ace under the Ace-Ace hypothesis is 1 and under the Ace-King hypothesis is 1/2.) Therefore picking an Ace is evidence for the hypothesis that

both cards are the same over the Ace-King hypothesis. (Why? Because if e is evidence for h over i, and h entails h', then e is evidence for h' over i.)

By similar reasoning, picking a King is evidence for the King-King hypothesis over the Ace-King hypothesis, so picking a King is evidence for the hypothesis that both cards are the same over the Ace-King hypothesis.

So no matter which card you pick—Ace or King—it will be evidence for the hypothesis that both cards are the same over the Ace-King hypothesis. But that's absurd! We can't know in advance that our evidence, whatever it turns out to be, will favor one hypothesis over another. Since the Law of Likelihood led us to this absurd conclusion, the Law of Likelihood is wrong.

Evaluate this argument. Be as specific as you can. If you think a premise is false, which premise and why? If you think the conclusion is not in fact absurd, explain why not. If you think the argument is invalid, then which step is at fault? If you think the argument is sound, then can anything be salvaged from the Likelihood Theory? (300 words.)

Problem 4 (20 points, adapted from Royall (1997: 95–6).) In the Likelihood Theory, two events of interest are (i) that we get only weak evidence, and (ii) that we get misleading evidence. Let's think about these events.

Suppose we gather evidence about h and i by performing an experiment and calculating the outcome's likelihood ratio: the likelihood of the outcome under i divided by its likelihood under h. We make the following definitions:

1. M_h is the probability, assuming h, of getting an outcome with likelihood ratio at least 8.
2. M_i is the probability, assuming i, of getting an outcome with likelihood ratio at most $\frac{1}{8}$.
3. W_h is the probability, assuming h, of getting an outcome with likelihood ratio between $\frac{1}{8}$ and 8.
4. W_i is the probability, assuming i, of getting an outcome with likelihood ratio between $\frac{1}{8}$ and 8.

First, if we assume the Law of Likelihood is true and agree to count a likelihood ratio of more than 8 as "strong evidence", then what do these definitions amount to in plain English?

Second, consider the following three experiments:

	Experiment 1		Experiment 2			Experiment 3	
	0	**1**	**0**	**1**	**2**	**0**	**1**
h	.94	.06	.47	.50	.03	.47	.53
i	.02	.98	.01	.50	.49	.01	.99

Calculate M_h, M_i, W_h, W_i for each experiment, i.e. fill in the following table:

	M_h	M_i	W_h	W_i
Experiment 1				
Experiment 2				
Experiment 3				

Problem 5 (20 points). Distinguish two questions: (a) Is the outcome evidence against h? (b) Is the outcome evidence in favor i over h? The first question assumes an *absolute* concept of evidence: we ask how the evidence bears on a single hypothesis. The second question assumes a *comparative* concept of evidence: we ask how the evidence bears on one hypothesis versus another. Significance testers (one version, anyway) answer the first question. They claim that the outcome's p-value measures the strength of evidence against the null hypothesis: the lower the p-value, the stronger the evidence. So they're using an absolute concept of evidence. Likelihoodists answer the second question. They claim that the outcome is evidence in favor of i over h just if it has higher likelihood under i than under h. So they're using a comparative concept of evidence. That's one key difference between the approaches.

Likelihood Theorists complain that an absolute concept of evidence just doesn't make sense. According to them, asking "Is the outcome evidence against h?" is like asking "Does Tom prefer Chunky Monkey?" Tom may prefer Chunky Monkey to Cherry Garcia but not to Phish Food. The question "Does Tom prefer Chunky Monkey?" just doesn't make sense. Similarly, say the Likelihood Theorists, the outcome may be evidence in favor of i over h but not evidence in favor of j over h. The question "Is the outcome evidence against h?" just doesn't make sense.

Are the Likelihood Theorists correct? In your answer, you could: (i) try to give an example where whether the outcome is evidence against h doesn't depend on which alternative hypotheses we consider; or (ii) suggest why such examples are hard to find; or (iii) discuss whether we can always interpret "Is the outcome evidence against h?" as "Is e evidence in favor of $\neg h$ over h?"; or (iv) discuss how we could interpret significance tests if the Likelihood Theorist is correct; or so on. (300 words.)