Philosophy of Statistics: Homework 4

due on Gradescope by 11am on Wednesday February 3

Guidelines. Some questions ask you to justify your answers. For these questions, credit will be based on how well you justify your answers, not on whether your answers are correct. (There's often no consensus on the correct answers, even among statisticians.) However, that doesn't mean that anything goes: some answers will be hard to justify well. I give suggested word counts but these are just ballpark numbers. Don't sweat them too much. Collaboration is encouraged, but make sure to write up your answers by yourself and list your collaborators.

Problem 1 (20 points.) Here's an example of an experiment, from a Bayesian's point of view:

	1	2	3	4
$1/2 : h_1$	1/8	1/2	1/4	1/8
$1/6 : h_2$	1/4	1/8	0	5/8
$1/3 : h_3$	1/8	1/8	1/8	5/8

For each possible outcome—1, 2, 3, 4—what should the experimenter's posteriors be if they get that outcome?

Problem 2 (10 points). I, like you, tried to modify a coin to make it 75%-heads biased. But I'm not sure whether I succeeded: my prior that my coin is 75%-heads biased is .3 and my prior that it's still fair is .7. To get more evidence, I flipped it 10 times and counted the number of heads: 9. What should my posterior be that the coin is 75%-heads biased? The likelihoods to three decimal places are:

	0	1	2	3	4	5	6	7	8	9	10
50%-heads biased	.000	.010	.044	.117	.205	.246	.205	.117	.044	.010	.000
75%-heads biased	.000	.000	.000	.003	.016	.058	.146	.250	.282	.188	.056

Problem 3 (20 points). A judge is 90% reliable at telling whether defendants are guilty. That is: given that a defendant is guilty, the probability the judge says he's guilty is .9, and given that a defendant is innocent, the probability the judge says he's innocent is .9. Tom is charged with vandalism. Your prior that Tom is guilty is low: .01. You're awaiting the judge's verdict.

First, write down a table to represent the situation. Second, draw a square diagram to represent the situation. Now suppose the judge says that Tom's guilty. Third, just by looking at the square diagram, estimate what your posterior that Tom's guilty should be. Fourth, calculate the posterior exactly.

Problem 4 (15 points). You're on a gameshow. There are three doors in front of you: A, B and C. A car is behind one of the doors. Goats are behind the other two doors. The host of the gameshow knows what's behind each door. You pick a door: A, say. The host is about to reveal a goat, by opening one of the other doors. (If there are goats behind both of the other doors, the host chooses which to open at random, by flipping a fair coin.)

First, write down a table to represent the situation. Second, draw a sqaure diagram to represent the situation. Now suppose the host opens B. Third, what should your posterior be that the car is behind A?

Problem 5 (15 points). Let's consider two variations on the previous problem. You'll probably find it helpful to draw a square diagram for each case.

First variation. The set-up is the same as before, except this time you know that if there are goats behind B and C, the host always opens B. Now what should your posterior be that the car is behind A, given that the host opens B?

Second variation. There are 100 doors, A_1, A_2, ..., A_{100}, with a car behind one of them and goats behind the others. You pick a door, say A_1. The host is about to reveal some goats, by opening all but one of the other doors. (If there are goats behind all the other others, the host chooses which to leave closed at random, with all doors having equal chance of being chosen.) Suppose the host opens all the other doors except A_{27}. What should your posterior be that the car is behind A_{27}?

Problem 6 (20 points). A Bayesian says that an outcome x is evidence *for* or *against* a hypothesis h according to whether their posterior in h, $P(h \mid x)$, is *higher* or *lower* than the prior in h, $P(h)$. Let's think more about this. Consider the following scenarios:

- x is evidence for h but your posterior in h is less than .5
- x is evidence for h_1 and evidence for h_2
- x is neither evidence for h nor against h
- x is evidence for h_1 and against h_2 but your posterior in h_2 is higher than in h_1

For each scenario, draw a square diagram and indicate the actual outcome, to illustrate how it might happen.