Philosophy of Statistics: Homework 1

due on Gradescope by 11am on Wednesday January 13

Guidelines. Some questions ask you to justify your answers. For these questions, credit will be based on how well you justify your answers, not on whether your answers are correct. (There's often no consensus on the correct answers, even among statisticians.) However, that doesn't mean that anything goes: some answers will be hard to justify well. I give suggested word counts but these are just ballpark numbers. Don't sweat them too much. Collaboration is encouraged, but make sure to write up your answers by yourself and list your collaborators.

Problem 1 (10 points). Give three examples of hypotheses which *are* statistical and three examples of hypotheses which are *not* statistical. (If you're having trouble, because you think the distinction between statistical and non-statistical hypotheses is fuzzy, then you can still get full credit by explaining why the distinction is fuzzy.)

Guidelines: don't use any hypotheses discussed in class; it doesn't matter if the hypotheses are true or false; try to think of hypotheses which are of practical scientific interest (not about coin flips, or the sun rising, or what you'll eat for lunch).

Problem 2 (20 points). In class we looked at paradigm examples of statistical hypotheses, but we didn't give a general definition. Here are some attempts:

1. A statistical hypothesis is a claim about what will happen in the future.
2. A statistical hypothesis is a claim that one event is more likely than another.
3. A statistical hypothesis is a claim involving a percentage.

What do you think of these attempts? Justify your answers, giving examples or counterexamples where applicable. (200 words.)

Problem 3 (20 points.) James claims he's solved the challenge, i.e. that his taco-coin is 75%-heads biased. Suppose we try to evaluate that hypothesis by flipping the coin 100 times and counting the number of heads. What will happen? Well, we can't say for sure: we might get 0 heads, or 1 head, or 2 heads, or 3 heads, ..., or 100 heads. Let's focus on four cases:

1. We get 13 heads.
2. We get 66 heads.
3. We get 75 heads.
4. We get 84 heads.

In each case, should we believe the hypothesis is true, or should we believe it's false, or should we suspend judgment about it?

In each case, what should we do—award the prize to James? take him out of the running? do another experiment? something else?

In each case, is the result evidence for or against the hypothesis, or neither, and how does the strength of evidence compare across the four cases?

Justify your answers. (300 words total.)

Problem 4 (20 points). Suppose we're trying to evaluate a statistical hypothesis h and, to that end, have designed an experiment. (For example: the hypothesis might be that Michael's coin is equally likely to land heads as tails when flipped and the experiment might be to flip the coin 20 times and count the number of heads.) But what should we do then? Let's think about two approaches.

First approach: "Do the experiment and, assuming for the sake of argument that h is true, work out the probability p of the actual outcome. If $p < .01$, conclude that h is false."

Second approach: "Do the experiment and, assuming for the sake of argument that h is true, work out the probability p of the actual outcome. If $p < .01$, conclude that the outcome is strong evidence against h."

Discuss these approaches. Is either of them a good way of evaluating h? It will help to think through how the approaches pan out in some concrete examples. (300 words total.)

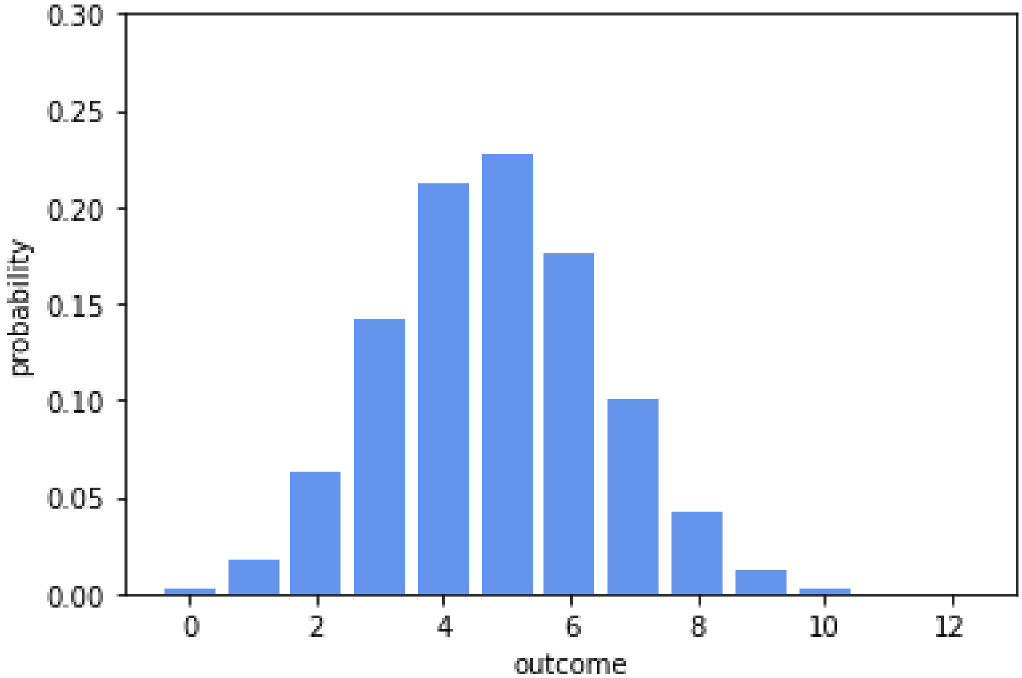
Problem 5 (15 points). Suppose we're trying to evaluate a statistical hypothesis h and, to that end, have designed an experiment. The experiment has 8 possible outcomes, labeled 1, 2, 3, ..., 8. Assuming for the sake of argument that h is true, we work out the probability of each possible outcome, and record them in a table:

outcome	1	2	3	4	5	6	7	8
probability	.001	.5	.1	.03	.1	.25	.012	.007

The following questions are designed to let you practice interpreting such tables. Assuming h is true, what is the probability of...

1. getting 5?
2. getting 2 or 6?
3. getting 1 or 4 or 5?
4. getting an even outcome?
5. not getting 3?
6. getting at least 5 but not 7?
7. getting an outcome of probability less than or equal to the probability of getting 7?

Problem 6 (15 points). Suppose we're trying to evaluate a statistical hypothesis h and, to that end, have designed an experiment. The experiment has 13 possible outcomes, labeled 0, 1, 2, ..., 12. Assuming for the sake of argument that h is true, we work out the probability of each possible outcome, and record them in a bar plot:



(Note that there are bars above 11 and 12. They're just too small to see.)

The following questions are designed to let you practice interpreting such plots. Assuming h is true...

1. which outcome has probability about 0.1?
2. how many outcomes have probability above 0.2?
3. which are the four least likely outcomes?
4. which outcomes are more likely than 7 but less likely than 4?
5. what is the sum of the heights of all the bars?