Philosophy of Statistics: Homework 1

due on Gradescope by 11am on Tuesday April 13

Guidelines. Some questions ask you to justify your answers. For these questions, credit will be based on how well you justify your answers, not on whether your answers are correct. (There's often no consensus on the correct answers, even among statisticians.) However, that doesn't mean that anything goes: some answers will be hard to justify well. I give suggested word counts but these are just ballpark numbers. Don't sweat them too much. Collaboration is encouraged, but make sure to write up your answers by yourself and list your collaborators.

Problem 1 (10 points). The following questions are about one of the readings: Alan Baker, *SEP: Simplicity*, Section 1.

(a) Baker mentions a variety of people who claimed, roughly speaking, that we should favor simpler theories. Name five of them.

(b) Baker distinguishes two kinds of simplicity: *parsimony* and *elegance*. How does he define them?

(c) Why does Baker think it's important to distinguish parsimony and elegance?

(d) What does 'OR' stand for?

(e) Baker distinguishes three questions about simplicity. Which question does he think is easier to answer for parsimony than for elegance?

Problem 2 (20 points). In class, we looked at how the average speed of a jar rolling down a slope depends on the amount of rice in the jar. You might have expected there to be a simple relationship between average speed and amount of rice: say, for the average speed to be constant, or for it to increase linearly with the amount of rice. In fact, it turned out that the relationship is somewhat complicated and surprising. (Take a look back at the plot if you've forgotten.)

Can you come up with your own experiment to determine how one quantity depends on another? Just use things you can find around the house: a kitchen or garage are typically promising settings for home experiments. Be inventive!

First, describe the set-up of your experiment and your results, including a picture or plot if appropriate. Second, state which hypothesis or hypotheses you judged most plausible, both before carrying out the experiment and after. Third, explain whether your plausibility judgments show that you favor simpler hypotheses.

Problem 3 (20 points). In "Why Favour Simplicity?", Roger White focuses on a "certain idealized model of a regularity producing mechanism", namely a box with a dial and pointer. White says:

> Movement of the dial results in movement of the pointer by some unknown mechanism inside the box. We can think of the box as computing a function from dial positions to pointer positions. Turning the dial and reading the pointer we collect data points, and from these we try to make an educated guess as to which function the machine computes.

White hopes that his simple model—the box with dial and pointer—is analogous to ordinary physical systems, so that his analysis of the one will carry over to the other. Let's put that to the test.

Choose some ordinary physical system (perhaps an example from class, or your experiment from the previous problem, or a new example) and try to spell out in detail the analogy between White's box with dial and pointer and your system. Identify any problems you face in spelling out the analogy and how White might try to get around them. (300 words)

Problem 4 (20 points). As we've seen, White imagines "a certain idealized model of a regularity producing mechanism", namely, a box with a dial and pointer. He focuses on what your credences should be about the box: which function it computes and which mechanism is inside it. This question checks your understanding of White's framework: both the concepts and the notation.

The square diagram below represents Omar's credences about the box.

<table>
<tr><td rowspan="8">F_1</td><td rowspan="4">F_1</td><td rowspan="2">F_1</td><td>F_1</td></tr>
<tr><td>F_4</td></tr>
<tr><td rowspan="2">F_2</td><td>F_6</td></tr>
<tr><td>F_7</td></tr>
<tr><td rowspan="4">F_3</td><td rowspan="2">F_3</td><td>F_8</td></tr>
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<tr><td rowspan="4">F_5</td><td rowspan="2">F_8</td><td>F_{17}</td></tr>
<tr><td>F_{18}</td></tr>
<tr><td rowspan="2">F_9</td><td>F_{19}</td></tr>
<tr><td>F_{20}</td></tr>
<tr><td>M_1</td><td>M_2</td><td>M_3</td><td>M_4</td></tr>
</table>

For each function f, write down $c(f)$ and Omar's credence that the machine computes f. Does Omar satisfy the Simple Function Favoring Principle?

Omar gathers some data points, by turning the dial and observing the pointer reading. The functions consistent with her data points are $f_2, f_3, f_{13}, f_{15}, f_{16}$. Omar updates her credences by conditionalization, as usual. For each mechanism complexity, write down Omar's new credence that the mechanism is of that complexity. Does Omar satisfy the Simple Mechanism Favoring Principle?

Problem 5 (15 points). When discussing the box with the dial and pointer, White distinguishes two kinds of complexity: the complexity of the *function* which the box computes and the complexity of the *mechanism* inside the box. White says that function complexity is hard to characterize but he'll "assume we recognize it well enough when we see it", e.g. a linear function is simpler than a quadratic. And he suggests that mechanism complexity is "roughly a matter of how many different kinds of parts it contains, all of which are intricately linked so that a change in any part would make a major difference to the workings of the mechanism." Finally, he suggests that we can define function complexity in terms of mechanism complexity, as follows: for any function f, the complexity of f is the complexity of the simplest mechanism required to compute it.

OK, so now we have two explanations of function complexity: the first explanation appeals to our intuition (you recognize it when you see it) and to examples (linear functions are simpler than quadratics); the second explanation is in terms of mechanism complexity. It would be bad if these explanations *disagreed*, i.e. f_1 counts as simpler than f_2 according to one explanation but f_2 counts as simpler than f_1 according to the other. White doesn't think there are such cases: see the middle paragraph on p.206. But can you come up with any? Discuss your attempts. (300 words)

You might find it helpful to focus on particular *kinds* of mechanisms, such as mechanisms made out of lego, or out of gears and pulleys. For example, might a gear-and-pulley mechanism computing a quadratic function be simpler than any gear-and-pulley mechanism computing a linear function?

The point of this question is to get you thinking about some subtle issues to do with simplicity: what it means for a function to be simple, what it means for a mechanism to be simple, what it means for a mechanism to compute a function. These are deep waters! But don't be daunted: I'm not looking for definitive answers. Just focus on making some clear, careful points, illustrated wherever possible with concrete examples.

Problem 6 (15 points). White aims to establish two claims: the greater the complexity of a function, the lower your credence should be that the machine computes it; and given some data points, among the mechanisms consistent with them, the greater the complexity of the mechanism, the lower your credence should be that the box contains that mechanism. He suggests that these claims follow from a few plausible assumptions:

1. you should be about equally confident that the mechanism is of complexity 1, 2, ..., n, where n is the complexity of the most complex mechanism which could fit in the box
2. the number of mechanisms of complexity k increases rapidly with k
3. given a mechanism complexity, you should be about equally confident in the various functions which mechanisms of that complexity might compute

In fact, the claims don't quite follow from these assumptions. That is, there are situations in which the assumptions are all true but one or other claim is false. For each of the two claims, write down a square diagram to represent such a situation, and explain your answer.