Bidirectional type system with $\texttt{Un}$ and Refinement types

Refinements	ϕ	$::=$	$e_1 = e_2 \mid \texttt{Un}(e) \mid \ \ldots$
Base types	$\texttt{b}$	$::=$	$\texttt{bool} \mid \texttt{int} \mid \texttt{unit}$
Ref. base types	$\texttt{R}$	$::=$	$\{x : \texttt{b} \mid \phi\}$
Types	τ	$::=$	$\texttt{R} \mid \tau_1 \to \tau_2 \mid \texttt{ref}_{x.\phi}\ \tau \mid \texttt{Un}$

$$\begin{array}{ll} \Gamma \vdash e \Rightarrow \tau & \text{inference} \\ \Gamma \vdash e \Leftarrow \tau & \text{checking} \end{array}$$

Generally applicable rules

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau} \qquad \frac{\Gamma \vdash e \Rightarrow \tau_1 \qquad \tau_1 \sqsubseteq \tau_2}{\Gamma \vdash e \Leftarrow \tau_2} \qquad \frac{\Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash e : \tau \Rightarrow \tau}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 \Rightarrow \tau_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Rightarrow \tau_2} \qquad \frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 \Leftarrow \tau_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Leftarrow \tau_2}$$

Usual rule	Un rule
$\dfrac{}{\Gamma \vdash n \Leftarrow \{x : \texttt{int} \mid x = n\}}$	$\dfrac{}{\Gamma \vdash n \Leftarrow \texttt{Un}}$
$\dfrac{\Gamma, x : \tau_1 \vdash e \Leftarrow \tau_2}{\Gamma \vdash \lambda x.e \Leftarrow \tau_1 \to \tau_2}$	$\dfrac{\Gamma, x : \texttt{Un} \vdash e \Leftarrow \texttt{Un}}{\Gamma \vdash \lambda x.e \Leftarrow \texttt{Un}}$
$\dfrac{\Gamma \vdash e_1 \Rightarrow \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 \Leftarrow \tau_1}{\Gamma \vdash e_1\ e_2 \Rightarrow \tau_2}$	$\dfrac{\Gamma \vdash e_1 \Rightarrow \texttt{Un} \qquad \Gamma \vdash e_2 \Leftarrow \texttt{Un}}{\Gamma \vdash e_1\ e_2 \Rightarrow \texttt{Un}}$
$\dfrac{\Gamma \vdash e \Leftarrow \{x : \tau \mid \phi\}}{\Gamma \vdash \mathsf{ref}\ e \Leftarrow \texttt{ref}_{x.\phi}\ \tau}$	$\dfrac{\Gamma \vdash e \Leftarrow \texttt{Un}}{\Gamma \vdash \mathsf{ref}\ e \Leftarrow \texttt{Un}}$
$\dfrac{\Gamma \vdash e_1 \Rightarrow \texttt{ref}_{x.\phi}\ \tau \qquad \Gamma \vdash e_2 \Leftarrow \{x : \tau \mid \phi\}}{\Gamma \vdash e_1 := e_2 \Rightarrow \texttt{unit}}$	$\dfrac{\Gamma \vdash e_1 \Rightarrow \texttt{Un} \qquad \Gamma \vdash e_2 \Leftarrow \texttt{Un}}{\Gamma \vdash e_1 := e_2 \Rightarrow \texttt{Un}}$
$\dfrac{\Gamma \vdash e \Rightarrow \texttt{ref}_{x.\phi}\ \tau}{\Gamma \vdash\ !e \Rightarrow \{x : \tau \mid \phi\}}$	$\dfrac{\Gamma \vdash e \Rightarrow \texttt{Un}}{\Gamma \vdash\ !e \Rightarrow \texttt{Un}}$
$\dfrac{\lvert\Gamma\rvert \Rightarrow \phi}{\Gamma \Rightarrow \mathsf{assert}(\phi) : \{\_ : \texttt{unit} \mid \phi\}}$	

The judgment $\Delta \vdash \phi$ is entailment in the refinement logic. The erasure function $|\Gamma|$ is defined as follows.

$$
\begin{array}{lcl}
|\bullet| & = & \bullet \\
|\Gamma, x : \{y : \texttt{b} \mid \phi\}| & = & |\Gamma|, \phi[x/y] \\
|\Gamma, x : \tau| & = & |\Gamma| \quad \text{when } \tau \neq \texttt{b}
\end{array}
$$

$\boxed{\text{Subtyping}}$

$$
\dfrac{}{\texttt{Un} \sqsubseteq \{x : \texttt{b} \mid \texttt{Un}(x)\}} \qquad \dfrac{}{\{x : \texttt{b} \mid \texttt{Un}(x)\} \sqsubseteq \texttt{Un}} \qquad \dfrac{|\Gamma| \vdash \forall x.\phi \Rightarrow \phi'}{\Gamma \vdash \{x : \tau \mid \phi\} \sqsubseteq \{x : \tau \mid \phi'\}}
$$

(Standard rules for subtyping are included)