How robust is a third family of compact stars against pasta phase effects?

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(Dated: November 13, 2017)

We investigate the robustness of third family solutions for hybrid compact stars with a quark matter core that correspond to the occurrence of high-mass twin stars against a softening of the phase transition by means of a construction that mimics the effects of pasta structures in the mixed phase. We consider a class of hybrid equations of state that exploits a relativistic mean-field model for the hadronic as well as for the quark matter phase. We present parametrizations that correspond to branches of high-mass twin star pairs with maximum masses between 2.05 $M_\odot$ and 1.48 $M_\odot$ having radius differences between 3.2 km and 1.5 km, respectively. When compared to a Maxwell construction with a fixed value of critical pressure P_c the effect of the mixed phase construction consists in the occurrence of a region of pressures around P_c belonging to the coexistence of hadronic and quark matter phases between the onset pressure at P_H and the end of the transition at P_Q. The maximum broadening which would still allow mass twin stars is found to be $(P_Q - P_H)_{\max} \approx P_c$ for all parametrizations within the present class of models. At least the heavier of the neutron stars of the binary merger GW170817 could have been a member of the third family of hybrid stars.

PACS numbers:

I. INTRODUCTION

Recently, large interest has been devoted to the qualitative features of the mass-radius diagram for compact stars and conclusions that could be drawn from it for the properties of the equation of state (EoS) of degenerate matter under neutron star (NS) conditions. This is nourished by the observation of high masses $\sim 2~M_\odot$ of pulsars at high accuracy [1–3], as well as constraints for the neutron star radii. A new quality will be achieved when results for simultaneous mass and radius measurements from the NICER experiment [4] will become available and when more gravitational wave signals from NS-NS mergers like GW170817 [5] will be detected that eventually will show a ringdown signal and not just point to a prompt collapse.

One of the challenging questions in this context is the possibility of a phase transition to exotic forms of matter, in particular to deconfined quark matter in the interior of massive compact stars. As was analyzed in Ref. [6], the phenomenology of hybrid star sequences in the mass-radius (M-R) diagram of compact stars may be classified into four groups in the plane of critical pressure P_c and jump in the energy density $\Delta\varepsilon$ at the deconfinement phase transition. Of particular interest is the case of stable hybrid star branches that are disconnected from the normal NS branch and thus form a "third family" of compact stars [7] that indicates a strong phase transition with a large $\Delta\varepsilon \gtrsim 0.5\varepsilon_c$ [8, 9], where ε_c is the energy density at the onset of the deconfinement phase transition. As has been shown in [10] for several realistic hybrid star EoS models, this case can occur also for pulsars with high masses where then the phenomenon of "high-mass twin" (HMT) stars is possible [11]. The HMT phenomenon is not quite as exotic as it may seem at first glance. It can also be obtained within the multi-polytrope scheme for high-density EoS [12] and meets the constraints analyzed by Hebeler et al. [13].

The question we would like to answer in this work concerns the robustness of the HMT phenomenon when the deconfinement phase transition in the compact star interior is not described by a Maxwell construction with a jump in energy density at constant pressure for which the speed of sound in matter vanishes, but when the transition proceeds rather with a finite pressure gradient. In the latter case, a mixed phase can be realized in the hybrid star structure for which the speed of sound develops a dip but does not vanish. Such a case may occur when the phase transition proceeds under global charge conservation for systems with several conserved charges [14]. A mixed phase appears provided the surface tension between quark and hadron phases does not exceed a critical value estimated as $\sigma_c \sim 60$ MeV fm^{-2}, see [15, 16]. It can lead to the formation of structures and such a mixed phase is then dubbed "pasta phase". See, e.g., Ref. [17] for more details and a recent calculation with modern hadronic and quark matter EoS.

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II. HYBRID STAR EQUATION OF STATE

A. Hadronic phase

For description of the hadronic phase we use a relativistic mean-field (RMF) model, the phenomenological KVORcut02 EoS [18]. It was constructed with hadron masses and coupling constants dependent on the scalar field. In this model all the hadron masses are decreasing functions of the mean value of the scalar field in medium with a common rate in accordance with the idea of partial restoration of the chiral symmetry and the Brown-Rho scaling conjecture. The effective coupling constants enter the expressions for the EoS only in ratios with effective masses, and the dependence of these ratios on the scalar mean field is chosen to describe experimental constraints the best way. Here we disregard the possibility of a finite hyperon abundance following the argument given in Ref. [19] that hyperons can be completely suppressed in the mixed phase because the hadronic phase is positively charged.

On the basis of the KVOR model [20] a family of models (labeled as KVORcut) was constructed in [18], implementing a version of stiffening mechanism developed in [21]. This mechanism allows to make a given EoS stiffer at suprasaturation densities without altering it at lower densities, so the saturation properties remain unchanged. KVORcut02 is a stiffest of the EoS of this family, predicting the maximum NS mass of 2.26 $M_\odot$ and the maximum radius of $R \simeq 13.9$ km.

B. Quark matter phase

For the description of the quark matter phase we choose a recently developed relativistic density functional approach [22]. It is capable of modelling effects of confinement by diverging quark masses at low densities and includes an effective screening of the string tension in the confining interaction. The density-dependent screening is described with the same functional as the excluded volume effect in a recent version [23] of the density-dependent relativistic mean-field model DD2 [24]. At the place of the excluded volume parameter of that model appears in the string-flip model the parameter α.

A repulsive vector interaction is included with a higher-order density dependence so that a relatively soft behaviour at the deconfinement transition can be reconciled with a sufficiently fast stiffening of the EoS in order to sustain the $2M_\odot$ constraint for hybrid stars with quark matter core. The former is necessary to obtain a large enough density jump at the transition which induces a gravitational instability as a condition for the appearance of a separated branch of hybrid stars in the mass-radius diagram that would form a "third family" of compact stars. The higher order density-dependent repulsion term requires a density-dependent correction factor in order to maintain causality of the EoS so that

for the speed of sound holds $c_s^2 \leq 1$. For details, see Ref. [22].

C. Phase transition construction

When the hadronic and quark matter phases are described with EoS given by relations between the pressure and chemical potential (for $T = 0$, which is relevant for the NS modeling) $P_H(\mu)$ and $P_Q(\mu)$ correspondingly, one can find the critical value of the baryochemical potential μ_c from the condition of equal pressures,

$$P_Q(\mu_c) = P_H(\mu_c) = P_c, \qquad (1)$$

for which the phases are in mechanical equilibrium with each other. The value P_c defines the Maxwell construction. Quantities characterizing the quark-, hadron-, and mixed phases are denoted by the subscripts Q, H, and M, respectively.

Assuming the surface tension to be smaller than the critical value σ_c, a mixed phase could have an influence on the compact stars structure. The adequate description of the physics of pasta phases is a complicated problem which requires to take into account sizes and shapes of structures as well as transitions between them. It has been dealt with in the literature within different methods and approximations [16, 17, 19, 25–28].
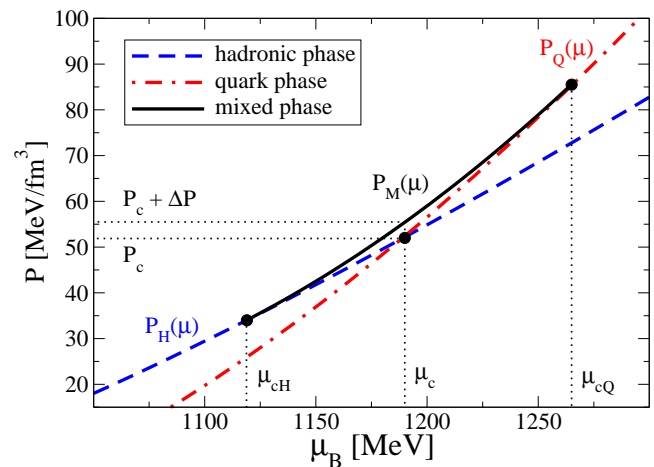

FIG. 1: The mixed phase construction (solid line) for given hadronic matter (dashed line) and quark matter (dash-dotted line) EoS models employed in this work. The Maxwell construction corresponds to switching at μ_c from the hadronic EoS for $\mu_B < \mu_c$ to the quark matter EoS for $\mu_B > \mu_c$.

Here, instead of the full solution we would like to use a simple modification of the Maxwell construction which mimics the result of pasta matter studies [29]. This means that the pressure as a function of the baryon density in the mixed phase is not constant but rather

a monotonously rising function. We are assuming that close to the phase transition point (that would be obtained using the Maxwell construction) the EoS of both phases are changing due to finite size and Coulomb effects (see Fig. 1), so that the effective mixed phase EoS $P_M(\mu)$ could be described in the parabolic form

$$P_M(\mu) = a(\mu - \mu_c)^2 + b(\mu - \mu_c) + P_c + \Delta P. \quad (2)$$

Here we have introduced the pressure shift ΔP at μ_c as a free parameter of the model which determines the mixed phase pressure at this point

$$P_M(\mu_c) = P_c + \Delta P = P_M. \quad (3)$$

According to the mixed phase construction shown in Fig. 1 we have two critical chemical potentials, μ_{cH} for transition from H-phase to M-phase and μ_{cQ} for the transition from the M-phase to Q-phase. Together with the coefficients a and b from Eq. (2) this are four unknowns which shall be determined from the four equations for the continuity of the pressure

$$P_M(\mu_{cH}) = P_H(\mu_{cH}) = P_H, \quad (4)$$
$$P_M(\mu_{cQ}) = P_Q(\mu_{cQ}) = P_Q, \quad (5)$$

and of the baryon number density $n(\mu) = dP(\mu)/d\mu$

$$n_M(\mu_{cH}) = n_H(\mu_{cH}), \quad (6)$$
$$n_M(\mu_{cQ}) = n_Q(\mu_{cQ}). \quad (7)$$

From the Eqs. (4) and (5) for the pressure a and b are found as

$$a = \frac{1}{\mu_{cQ} - \mu_{cH}} \left(\frac{P_Q - P_M}{\mu_{cQ} - \mu_c} - \frac{P_H - P_M}{\mu_{cH} - \mu_c} \right), \quad (8)$$

$$b = \frac{P_M - P_H}{\mu_c - \mu_{cH}} + \frac{P_Q - P_M}{\mu_{cQ} - \mu_c} - \frac{P_Q - P_H}{\mu_{cQ} - \mu_{cH}}, \quad (9)$$

and can be eliminated from the set of equations for the densities. By solving the remaining Eqs. (6) and (7) for the densities numerically one can find the values for the critical chemical potentials μ_{cH} and μ_{cQ}.

The procedure for "mimicking pasta phases" described in this subsection is general and can be superimposed to any hybrid EoS which was obtained using the Maxwell construction of a first-order phase transition from a low-density to a high-density phase with their given EoS. A certain limitation occurs in the case of sequential phase transitions (see, e.g., Refs. [30] and [31]) when the broadening of the phase transition region as shown in Fig. 1 would affect the adjacent transition. This case shall be discussed elsewhere [32]

III. RESULTS AND DISCUSSION

In this section we present and discuss results obtained for the case of the quark deconfinement transition when the hadronic EoS is given by the relativistic mean-field model described in Subsect. II A and the quark matter EoS is chosen to by the relativistic density-functional approach motivated by the string flip model as characterized in Subsect. II B. First we show the effect of broadening the phase transition region that the introduction of the free parameter ΔP of the mixed phase construction has on the hybrid star EoS. This is done for selected parameter values of the quark matter model. Subsequently we investigate the corresponding mass-radius relations for compact stars. We are particularly interested in the presence of a third family branch of hybrid stars with large quark matter core, disconnected from the usual hadronic branch of compact stars to which eventually a short hybrid star branch with small mixed phase cores might be connected. The possibility of such solutions may become of particular interest for the phenomenology of gravitational wave signals from NS-NS merger events. They would allow for metastable high-mass hybrid star solutions as a particular case in delayed collapse scenarios of the merger evolution.

Furthermore, a new aspect of this work is to quantify a critical broadening effect for which the third family branch of hybrid star solutions would get connected to the main neutron star branch. I.e. we will identify the maximum for the numerical value of the dimensionless broadening parameter $\Delta_P = \Delta P/P_c$ that would still allow for the appearance of a third family branch and therefore high-mass twin star solutions.

A. Hybrid star EoS with mixed phase construction

In Fig. 2 we present results of the mixed phase construction outlined in Subsect. II C where as inputs enter the fixed hadronic EoS as described in Subsect. II A and the quark matter EoS of the string-flip model characterized in Subsect. II B. For the latter, we vary the value of the excluded volume parameter α which corresponds to an effective reduction of the string tension in a dense medium. From the parameter range explored in [22] we focus on the values $\alpha = 0.18$, 0.20, 0.22, 0.24 and 0.30. We show results for pressure vs. chemical potential in Fig. 2.

With the choice $\alpha = 0.18$ (0.22), we obtain the phase transition by Maxwell construction ($\Delta_P = 0$) at $P_c = 101$ MeV/fm^3 (71 MeV/fm^3). The moderate choice of $\Delta_P \sim 5\%$ leads to a lowering of the onset pressure for the phase transition by $\sim 40\%$. The corresponding region of pressures around P_c belonging to the coexistence of hadronic and quark matter phases between the onset pressure at P_H and the end of the transition at P_Q is found to be $P_Q - P_H \approx P_c$ for all parametrisations within the present class of models.

In Fig. 3 we show the pressure vs. energy density corresponding the results shown in Fig. 2. Note that the jump in energy densities for the Maxwell construction amounts to about $70\ldots75\%$ of the critical energy density at the
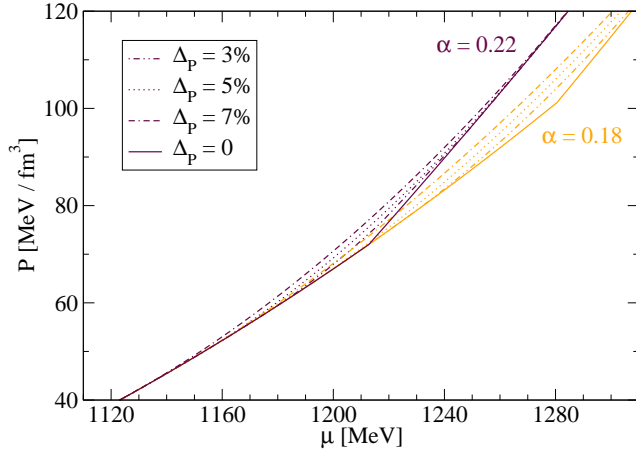
FIG. 2: Hybrid EoS according to the mixed phase construction of Fig. 1 described in the text, for the string screening parameter $\alpha = 0.18$ and 0.22. For the mixed phase parameter $\Delta_P = \Delta P / P_C$ the values 0.0 (Maxwell construction limit) 0.03, 0.05 and 0.07 are chosen.
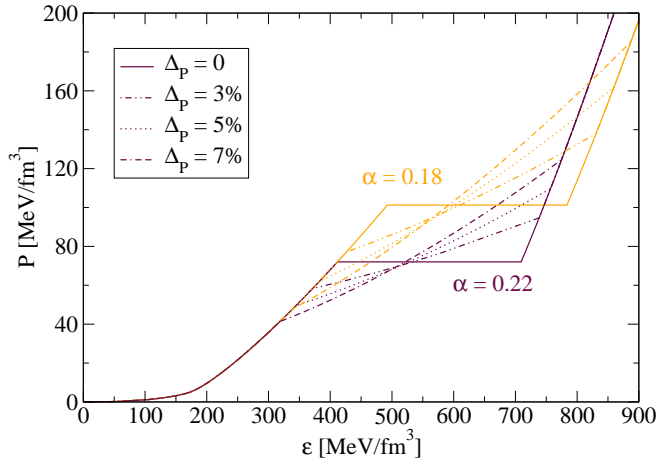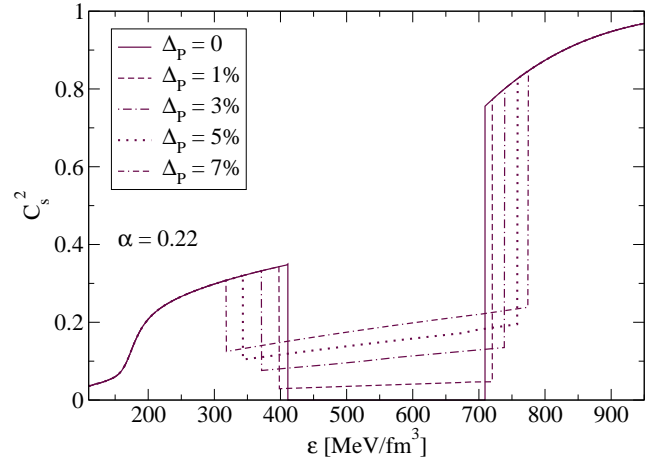

FIG. 4: Speed of sound squared as a function of energy density, derived from the set of EoS in Fig. 3 for $\alpha = 0.22$ and different mixed phase parameters $\Delta_P = 0.0 \ldots 0.07$.

FIG. 3: Same as Fig. 2 for pressure vs. energy density.

phase transition onset. The effect of the mixed phase construction is a broadening of the coexistence region of the phases and a nonzero speed of sound in it, as shown in Fig. fig:cs2.

B. Mass-radius diagram

The EoS shown in Figs. 2 - 4 has been used to solve the Tolman-Oppenheimer-Volkoff equations for structure and stability of compact star configurations. The results

for the mass-radius relationships (sequences) corresponding to the present class of EoS models are shown in Fig. 5. Increasing the α parameter from 0.18 to 0.24 leads to a lowering of the onset mass of the phase transition from 2.05 $M_\odot$ to 1.80 $M_\odot$ for the Maxwell construction case. Dotted lines correspond to unstable configurations which shall not be realized in nature and are shown only to guide the eye. The occurrence of such a gap in the mass-radius sequence defines the existence of a third family of compact stars, here they are hybrid stars with a quark matter core. The effect of the mixed phase construction is to lower the onset mass of the transition and to allow for a short sequence of hybrid stars with a small mixed phase core which is connected to the second family branch of ordinary neutron stars. In the classification scheme of Ref. [6] this would correspond to the case "B" for both, disconnected and connected hybrid star branches for the same EoS. The critical value of pressure shift which would join these two branches and thus eliminate the third family case is found to be $\Delta_P \sim 6\%$, see Fig. 5.

In Fig. 5 we show together with the M-R sequences also the 2 $M_\odot$ mass constraint of Ref. [3] and two constraints [33, 34] which have recently been extracted from GW170817 under the assumption of no direct collapse in this binary neutron star merger event.

From GW170817 a constraint for the mass ranges of the merging neutron stars has been derived for the low and high spin cases. We show the low-spin case in Fig. 6 and compare with our class of hybrid star EoS for $\alpha = 0.3$. In this case, at least one of the two neutron stars in the merger could belong to the third family branch of rather compact stars. This is an exciting possibility which actually has been overlooked in a recent analysis [35] using a "generic" multi-polytrope form of

relative to the critical pressure P_c at the critical chemical
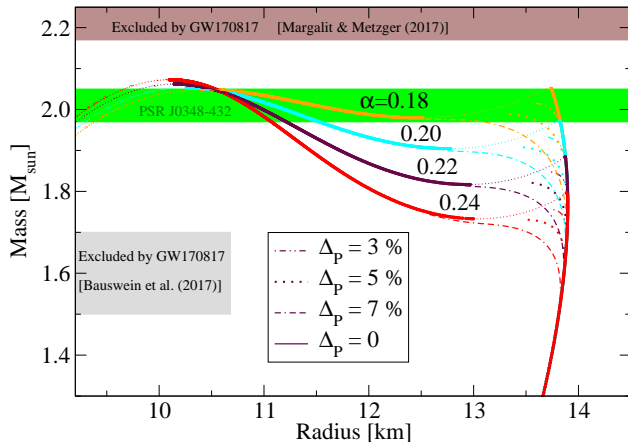

FIG. 5: Mass-radius diagram for the hybrid star sequences of this work for different values of the screening parameter $\alpha = 0.18, 0.20, 0.22, 0.24$ and varying value of the mixed phase parameter $\Delta_P = 0$ (solid lines), 0.03 (dash-double-dotted lines), 0.05 (dotted lines) and 0.07 (double-dash-dotted lines). The critical value for the transition from the third family case to a connected hybrid star branch is $\Delta_P \sim 0.06$ in all cases.
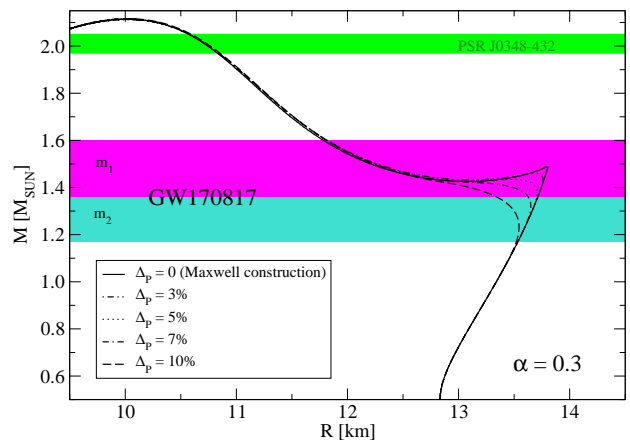

FIG. 6: Mass-radius diagram for the hybrid star sequences of this work for the screening parameter $\alpha = 0.3$ and varying value of the mixed phase parameter $\Delta_P = 0$ (solid line), 0.03 (dash-double-dotted line), 0.05 (dotted line), 0.07 (double-dash-dotted line) and 0.10 (long-dashed line). The critical value for the transition from the third family case to a connected hybrid star branch is $\Delta_P \sim 0.06$. The mass ranges (m_1 and m_2) for the two neutron stars in the merger event GW170817 [5] indicate that the heavier could be a third family member while the lighter one is a normal neutron star.

the high-density EoS. The fact that multi-polytropes can also capture the mass-twin case and at the same time fulfil the known constraints for masses and radii of compact stars has recently been demonstrated in [12].

IV. CONCLUSIONS

We have introduced a class of hybrid star EoS where both, hadronic and quark matter phases are described by a relativistic mean-field theory and would separately satisfy the $2\,M_\odot$ mass constraint for compact stars. Performing a Maxwell construction of a first-order phase transition results in a density jump at the transition which is sufficiently large to result in the occurrence of a third family branch of compact stars in the mass-radius diagram. Varying the screening parameter of the string-flip quark matter model in the range $\alpha = 0.18 \ldots 0.30$ results in a lowering of the onset density for the quark matter phase transition and thus the mass range where pairs of mass twin stars can be found, from $2.05\,M_\odot$ to $1.48\,M_\odot$, with a reduction of the radius difference from 3.2 km to 1.5 km, respectively.

We have investigated the robustness of the mass-twin (aka third family) feature against effects of finite size structures in the mixed phase, so-called "pasta phases". To this end a phase transition construction mimicking the pasta phase was employed which introduces an additional parameter, the dimensionless pressure shift Δ_P

potential μ_c of the Maxwell transition. We have found that values of Δ_P up to 5% do not exclude the existence of the third family of compact stars which has been obtaind for the Maxwell construction. This limiting value corresponds to a lowering of the onset pressure by about 40% and a broadening of the phase transition region in the pressure of the order of the critical pressure P_c for all parametrisations within the present class of models. At least the heavier of the neutron stars of the binary merger GW170817 could have been a member of the third family of hybrid stars.

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