

CI Tutorial 8

Exercise 1

Convert $r \leftrightarrow (s \leftrightarrow t)$ to CNF

$a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a)$, so

$$(r \rightarrow (s \leftrightarrow t)) \wedge ((s \leftrightarrow t) \rightarrow r)$$

And again

$$(r \rightarrow ((s \rightarrow t) \wedge (t \rightarrow s))) \wedge (((s \rightarrow t) \wedge (t \rightarrow s)) \rightarrow r)$$

$a \rightarrow b = \neg a \vee b$, so

$$(\neg r \vee ((\neg s \vee t) \wedge (\neg t \vee s))) \wedge (\neg((\neg s \vee t) \wedge (\neg t \vee s)) \vee r)$$

$(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, so

$$(((\neg s \vee t) \vee \neg r) \wedge ((\neg t \vee s) \vee \neg r)) \wedge (\neg((\neg s \vee t) \vee r) \wedge ((\neg t \vee s) \vee r))$$

Removing brackets,

$$(\neg s \vee t \vee \neg r) \wedge (\neg t \vee s \vee \neg r) \wedge (\neg(\neg s \vee t \vee r) \wedge (\neg t \vee s \vee r))$$

Exercise 2

Give an equivalent logical expression:

$$(\neg a \wedge b) \vee (a \wedge \neg b)$$

Apply the Tseytin transformation to give an equisatisfiable CNF expression:

$$y \leftrightarrow a \wedge \neg b$$

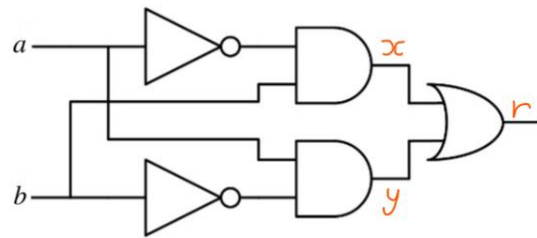
$$(y \vee \neg a \vee b) \wedge (\neg y \vee a) \wedge (\neg y \vee \neg b)$$

$$x \leftrightarrow \neg a \wedge b$$

$$(x \vee a \vee \neg b) \wedge (\neg x \vee \neg a) \wedge (\neg x \vee b)$$

$$r \leftrightarrow x \vee y$$

$$(\neg r \vee x \vee y) \wedge (r \vee \neg x) \wedge (r \vee \neg y)$$



Assume r is true:

$$(x \vee y) \wedge 1 \wedge 1$$

Merge:

$$(x \vee y) \wedge (y \vee \neg a \vee b) \wedge (\neg y \vee a) \wedge (\neg y \vee \neg b) \wedge (x \vee a \vee \neg b) \wedge (\neg x \vee \neg a) \wedge (\neg x \vee b)$$

Exercise 3

Apply the Tseytin transformation to the expression

$$(\neg a \vee c) \wedge (b \rightarrow ((a \vee c) \leftrightarrow d))$$

to give an equisatisfiable CNF expression.

$$(\neg a \vee c) \wedge (\neg b \vee ((\neg d \vee a \vee c) \wedge (d \vee \neg a) \wedge (d \vee \neg c)))$$

$(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, so

$$(\neg a \vee c) \wedge ((\neg d \vee a \vee c) \vee \neg b) \wedge (((d \vee \neg a) \wedge (d \vee \neg c)) \vee \neg b))$$

Again,

$$(\neg a \vee c) \wedge ((\neg d \vee a \vee c) \vee \neg b) \wedge ((d \vee \neg a) \vee \neg b) \wedge ((d \vee \neg c) \vee \neg b)$$

Removing brackets:

$$(\neg a \vee c) \wedge (\neg d \vee a \vee c \vee \neg b) \wedge (d \vee \neg a \vee \neg b) \wedge (d \vee \neg c \vee \neg b)$$

Exercise 4

1. 2
2. 6
3. 3

Exercise 5

1. 3
2. 5