

CL Tutorial 6

Exercise 1

1. $B \vee D$

There are two clauses in this expression, B and D.

The Karnaugh map of this expression is on the right:

The block of zeros for D is in orange, the block of zeros for B is blue.

| | | | | |
|----|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 1 | 1 | 0 |

2. $(\neg A \vee C) \wedge (A \vee \neg D)$

There are two clauses in this expression, $(\neg A \vee C)$ and $(A \vee \neg D)$.

The Karnaugh map of this expression is on the right:

The block of zeros for $(\neg A \vee C)$ is in orange, the block of zeros for $(A \vee \neg D)$ is blue.

| | | | | |
|----|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 1 | 0 | 0 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 1 | 1 |

Exercise 2

X1: $\neg D \vee \neg A$

Y1: $D \vee A$

X2: $(C \vee B) \vee A$

Y2: $(\neg C \wedge \neg B) \wedge (\neg A)$

Exercise 3

1. $r \leftrightarrow (a \wedge b)$

Karnaugh map:

CNF from map: $(\neg B \vee R \vee \neg A) \wedge (\neg B \vee \neg R \vee A) \wedge (B \vee \neg R)$

| | | | | |
|----|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

$$2. r \leftrightarrow (a \rightarrow b)$$

Karnaugh map:

CNF from map: $(R \vee A) \wedge (\neg A \vee \neg R \vee B) \wedge (\neg A \vee R \vee \neg B)$

| | | AG | | | | |
|----|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 | |
| RB | 00 | 0 | 0 | 1 | 1 | 00 |
| | 01 | 0 | 0 | 0 | 0 | 01 |
| | 11 | 1 | 1 | 1 | 1 | 11 |
| | 10 | 1 | 1 | 0 | 0 | 10 |
| | | 00 | 01 | 11 | 10 | |

Exercise 4

For each of the following pairs of clauses, draw a Karnaugh map and show the two blocks of zero states corresponding to the two clauses:

$$1. A \vee \neg B, \neg A \vee \neg D$$

$$2. A \vee \neg B, A \vee B \vee C$$

Use these Karnaugh maps to identify new clauses Δ_1 and Δ_2 , different from both premises, such that the following sequents are valid:

$$1. A \vee \neg B, \neg A \vee \neg D \models \Delta_1$$

$$\Delta_1 = \neg B \vee \neg D$$

$$2. A \vee \neg B, A \vee B \vee C \models \Delta_2$$

$$\Delta_2 = A \vee C$$

How many different solutions can you find for clauses Δ_1 and Δ_2 ?

As solutions will always form contiguous blocks of zeroes in the shapes $[1 \times 1, 1 \times 2, 2 \times 1, 2 \times 2, 4 \times 1, 1 \times 4, 2 \times 4, 4 \times 2, 4 \times 4]$, 1. has 19 valid clauses total, of which two are already excluded, so there are 17.

For 2. there are 14, of which two are already excluded, so there are 12.

| | | CD | | | | |
|----|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 | |
| AB | 00 | 1 | 1 | 1 | 1 | 00 |
| | 01 | 0 | 0 | 0 | 0 | 01 |
| | 11 | 1 | 0 | 0 | 1 | 11 |
| | 10 | 1 | 0 | 0 | 1 | 10 |
| | | 00 | 01 | 11 | 10 | |

| | | CD | | | | |
|----|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 | |
| AB | 00 | 0 | 0 | 1 | 1 | 00 |
| | 01 | 0 | 0 | 0 | 0 | 01 |
| | 11 | 1 | 1 | 1 | 1 | 11 |
| | 10 | 1 | 1 | 1 | 1 | 10 |
| | | 00 | 01 | 11 | 10 | |