# CL Tutorial 6

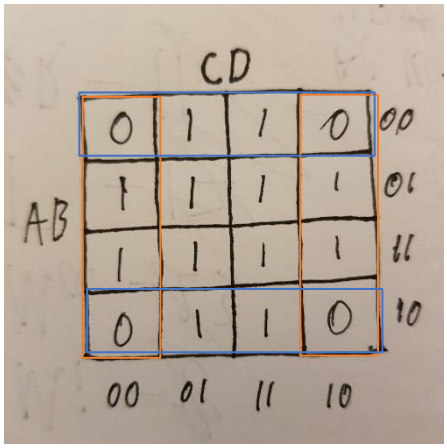
## Exercise 1

1. $B \vee D$

There are two clauses in this expression, B and D.

The Karnaugh map of this expression is on the right:

The block of zeros for D is in orange, the block of zeros for B is blue.

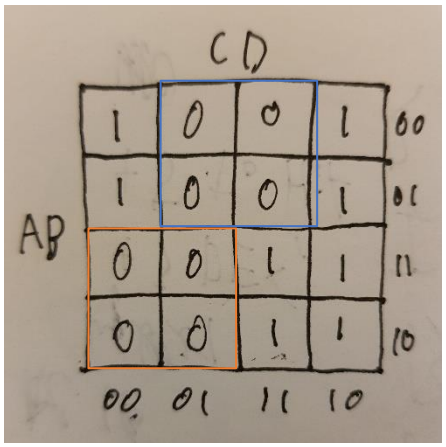



2. $(\neg A \vee C) \wedge (A \vee \neg D)$

There are two clauses in this expression, $(\neg A \vee C)$ and $(A \vee \neg D)$.

The Karnaugh map of this expression is on the right:

The block of zeros for $(\neg A \vee C)$ is in orange, the block of zeros for $(A \vee \neg D)$ is blue.




## Exercise 2

X1: $\neg D \vee \neg A$

Y1: $D \vee A$

X2: $(C \vee B) \vee A$

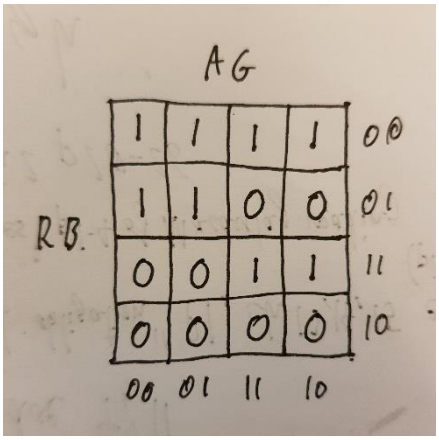
Y2: $(\neg C \wedge \neg B) \wedge (\neg A)$

## Exercise 3

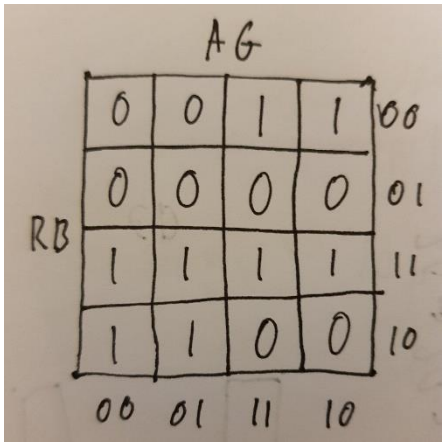
1. $r \leftrightarrow (a \wedge b)$

Karnaugh map:

CNF from map: $(\neg B \vee R \vee \neg A) \wedge (\neg B \vee \neg R \vee A) \wedge (B \vee \neg R)$



2. $r \leftrightarrow (a \rightarrow b)$

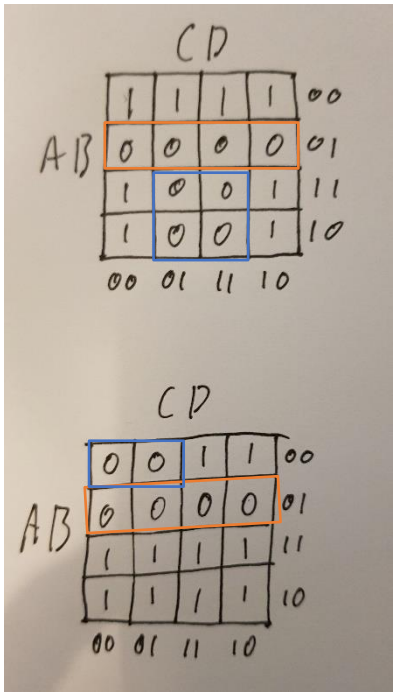



Karnaugh map:

CNF from map: $(R \vee A) \wedge (\neg A \vee \neg R \vee B) \wedge (\neg A \vee R \vee \neg B)$

## Exercise 4

For each of the following pairs of clauses, draw a Karnaugh map and show the two blocks of zero states corresponding to the two clauses:




1. $A \vee \neg B$, $\neg A \vee \neg D$

2. $A \vee \neg B$, $A \vee B \vee C$

Use these Karnaugh maps to identify new clauses Δ1 and Δ2, different from both premises, such that the following sequents are valid:

1. $A \vee \neg B, \neg A \vee \neg D \vDash \Delta 1$

Δ1 = $\neg B \vee \neg D$

2. $A \vee \neg B, A \vee B \vee C \vDash \Delta 2$

Δ2 = $A \vee C$

How many different solutions can you find for clauses Δ1 and Δ2?

As solutions will always form contiguous blocks of zeroes in the shapes [1x1, 1x2, 2x1, 2x2, 4x1, 1x4, 2x4, 4x2, 4x4], 1. has 19 valid clauses total, of which two are already excluded, so there are 17.
For 2. there are 14, of which two are already excluded, so there are 12.