CL Tutorial 4

# Exercise 1

Assuming

By contraposition,

Also,

Both of these proofs involve only equivalences, and both of them start from the same two premises. It follows that they can be combined to give the equivalence

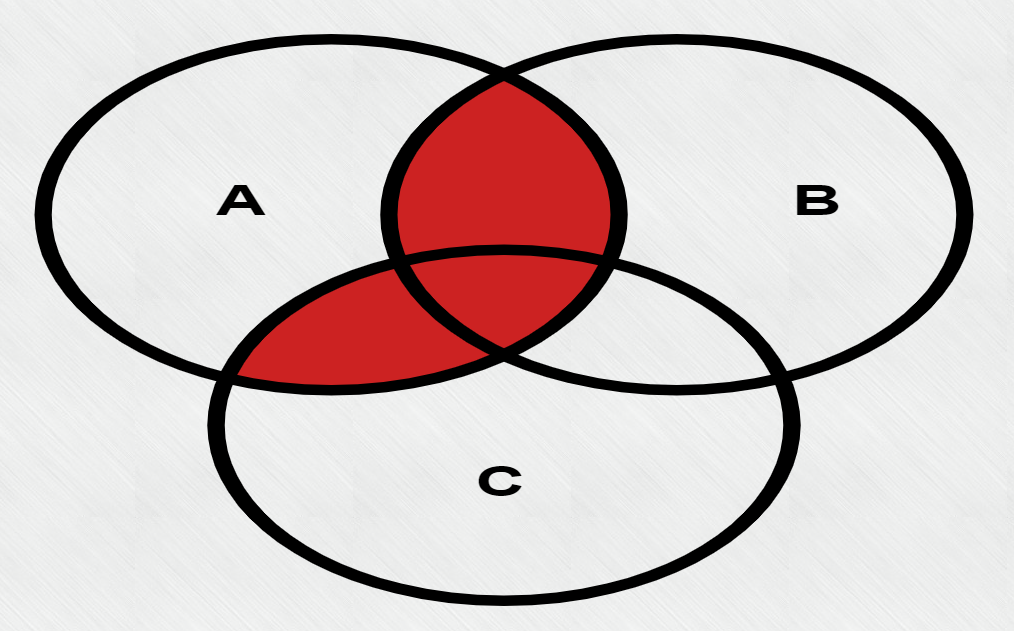
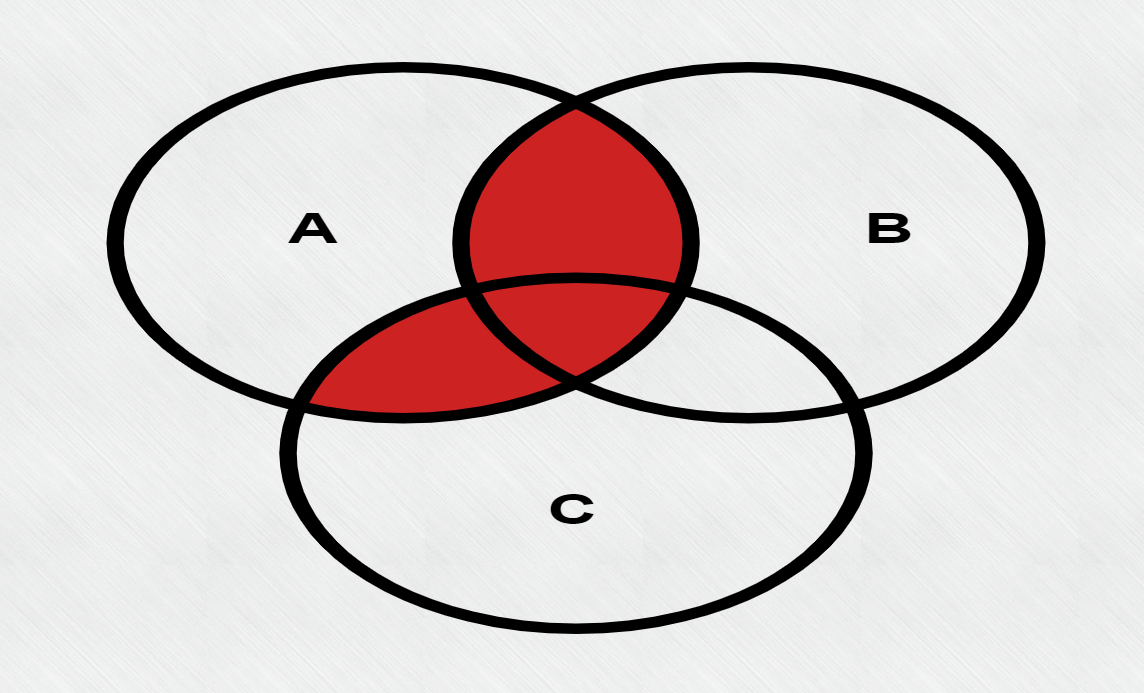
Now, we can use this equivalence in the following proofs:

Interpreting as set inclusion, this means that and, that is,

which is the second of De Morgan’s laws.

# Exercise 2

Write a proof which reduces the conclusion to premises that can’t be reduced further.

1. (the intersection of and ) conjoined with (the intersection of and ) looks like  
     
   where = A, = B, = C.
2. The intersection of and (the conjunction of and ) looks like  
     
   where = A, = B, = C.

A set of reduced premises would be .

The set is not universally valid, as a universe with no would not fill the intersection of and , so the existential assumption is required.

# Exercise 3

Write a proof which reduces the conclusion to premises that can’t be reduced further.

1. (X and not Y) or (not (X or Z) or (Y or Z))
2. If X or Z are false, truth propagates from all the way out.
3. For the other two cases (X & Z are true, Y is true or false),
   1. if Y is false, makes the statement true.
   2. if Y is true, makes the statement true.
4. The statement is always true.

The statement is a tautology as it is true by necessity.

# Exercise 4

The second cannot be reduced as it is invalid, is a superset of

Neither one is universally valid, and the counterexample for the first is a universe with no examples of and and .

The counterexample for the second is an example within but not within the intersection of and .