

## Initial conditions $SU(2)$

$$\Phi_* = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_{0*} + i\varphi_{1*} \\ \varphi_{2*} + i\varphi_{3*} \end{pmatrix} \implies \varphi_n(\mathbf{x}, t_*) \equiv \frac{|\Phi_*|}{\sqrt{2}} + \delta\varphi_{n*}(\mathbf{x}) \quad [n = 0, 1, 2, 3]$$

$$\dot{\Phi}_* = \frac{1}{\sqrt{2}} \begin{pmatrix} \dot{\varphi}_{0*} + i\dot{\varphi}_{1*} \\ \dot{\varphi}_{2*} + i\dot{\varphi}_{3*} \end{pmatrix} \implies \dot{\varphi}_n(\mathbf{x}, t_*) \equiv \frac{|\dot{\Phi}_*|}{\sqrt{2}} + \delta\dot{\varphi}_{n*}(\mathbf{x}) \quad [n = 0, 1, 2, 3]$$

$$B_i^a(\mathbf{x}, t_*) \equiv 0$$

$$\dot{B}_i^a(\mathbf{x}, t_*) \equiv \delta\dot{B}_{i*}^a(\mathbf{x}) \quad \Longleftarrow \quad \tilde{B}_i^{a'}(\mathbf{k}) = i \frac{k_i}{k^2} \frac{f_*^2}{\omega_*^2} \tilde{\mathcal{J}}_0^a(\mathbf{k})$$

$$\begin{aligned} \tilde{\mathcal{J}}_0^a(\mathbf{k} = 0) : \quad |\delta\varphi_{n1}(\mathbf{k})| &= |\delta\varphi_{n2}(\mathbf{k})| & n = 0, 1, 2, 3 \\ \theta_{n2}(\mathbf{k}) &= \theta_{02}(\mathbf{k}) + \theta_{n1}(\mathbf{k}) - \theta_{01}(\mathbf{k}) & n = 1, 2, 3 \end{aligned}$$

## Observables $SU(2)$ : fields

$SU(2)$  matter:  $\langle \tilde{\Phi}^a \rangle$ ,  $\langle (\tilde{\pi}_\Phi)^a \rangle$ ,  $\langle |\tilde{\Phi}|^2 \rangle$ ,  $\langle |\tilde{\pi}_\Phi|^2 \rangle$ .

$SU(2)$  gauge fields:  $\langle |\tilde{\mathcal{E}}^B|^2 \rangle = \sum_{a,i} \langle (\tilde{\mathcal{E}}_i^a)^2 \rangle$ ,  $\langle |\tilde{\mathcal{B}}^B|^2 \rangle = \sum_{a,i} \langle (\tilde{\mathcal{B}}_i^a)^2 \rangle$ ,

$$\tilde{\mathcal{E}}_i^a = \frac{1}{a^{1-\alpha}} (\tilde{\pi}_B)_i^a, \quad \tilde{\mathcal{B}}_i^a = \frac{\epsilon_{ijk}}{2\delta\tilde{\chi}^2 g_B Q_B^{(\tilde{\Phi})}} \text{Tr}[(iT_a)(\tilde{U}_{jk} - \tilde{U}_{kj})].$$

## Observables $SU(2)$ : energies

$$SU(2) \text{ matter: } \tilde{E}_K^\Phi = \frac{1}{a^6} \left\langle \tilde{\pi}_\Phi^\dagger \tilde{\pi}_\Phi \right\rangle, \quad \tilde{E}_G^\Phi = \frac{1}{a^2} \sum_i \left\langle (\tilde{D}_i^+ \tilde{\Phi})^* (\tilde{D}_i^+ \tilde{\Phi}) \right\rangle .$$

$$SU(2) \text{ gauge fields: } \tilde{E}_K^B = \frac{1}{2a^4} \frac{\omega_*^2}{f_*^2} \sum_{a,i} \left\langle ((\tilde{\pi}_B)_i^a)^2 \right\rangle ,$$

$$\tilde{E}_G^B = \frac{2}{a^4 \delta \tilde{\chi}^4 g_B^2 Q_B^{(\tilde{\Phi})^2}} \frac{\omega_*^2}{f_*^2} \left( 2 - \sum_{i,j < i} \left\langle \text{Tr}(\tilde{U}_{ij}) \right\rangle \right) .$$

$$\text{Potential: } \tilde{E}_V = \left\langle \tilde{V}(\tilde{\Phi}, \dots) \right\rangle .$$

## Observables $SU(2)$ : spectra

$$\tilde{\mathcal{P}}_{\tilde{\Phi}}(k(\tilde{\mathbf{n}})) = \frac{\tilde{k}^3(\tilde{\mathbf{n}})}{2\pi^2} \left( \frac{\delta\tilde{X}}{N} \right)^3 \left\langle \sum_a |\tilde{\Phi}^a(\tilde{\mathbf{n}})|^2 \right\rangle_{R(\tilde{\mathbf{n}})},$$

$$\tilde{\mathcal{P}}_{\tilde{\pi}_{\Phi}}(k(\tilde{\mathbf{n}})) = \frac{\tilde{k}^3(\tilde{\mathbf{n}})}{2\pi^2} \left( \frac{\delta\tilde{X}}{N} \right)^3 \left\langle \sum_a |(\tilde{\pi}_{\Phi})^a(\tilde{\mathbf{n}})|^2 \right\rangle_{R(\tilde{\mathbf{n}})},$$

$$\tilde{\mathcal{P}}_{\tilde{\mathcal{E}}}(k(\tilde{\mathbf{n}})) = \frac{\tilde{k}^3(\tilde{\mathbf{n}})}{2\pi^2} \left( \frac{\delta\tilde{X}}{N} \right)^3 \left\langle \sum_{i,a} |\tilde{\mathcal{E}}_i^a(\tilde{\mathbf{n}})|^2 \right\rangle_{R(\tilde{\mathbf{n}})},$$

$$\tilde{\mathcal{P}}_{\tilde{B}}(k(\tilde{\mathbf{n}})) = \frac{\tilde{k}^3(\tilde{\mathbf{n}})}{2\pi^2} \left( \frac{\delta\tilde{X}}{N} \right)^3 \left\langle \sum_{i,a} |\tilde{B}_i^a(\tilde{\mathbf{n}})|^2 \right\rangle_{R(\tilde{\mathbf{n}})},$$

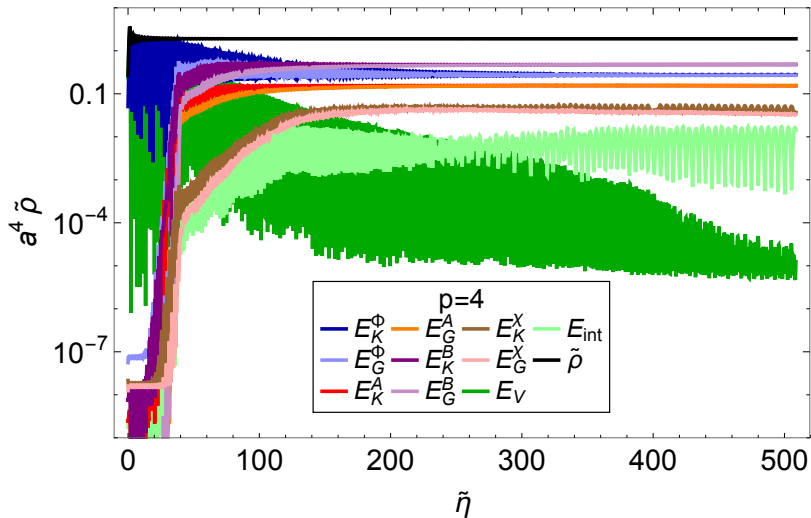
## Exercise: model to implement

Matter content:

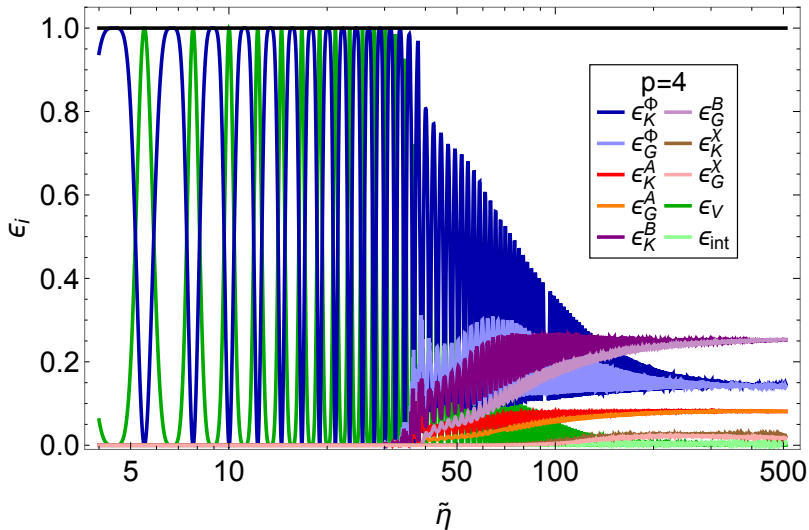
- $F_{\mu\nu}$ :  $U(1)$  gauge field
- $G_{\mu\nu}$ :  $SU(2)$  gauge field
- $\Phi$ :  $SU(2)$  doublet, inflaton. Coupled to  $F_{\mu\nu}$  and  $G_{\mu\nu}$ .
- $\chi$ : daughter scalar singlet

$$V(\Phi, \chi) = \frac{\Lambda^4}{4} \tanh^4 \left( \frac{\sqrt{2}|\Phi|}{M} \right) + g|\Phi|^2\chi^2$$

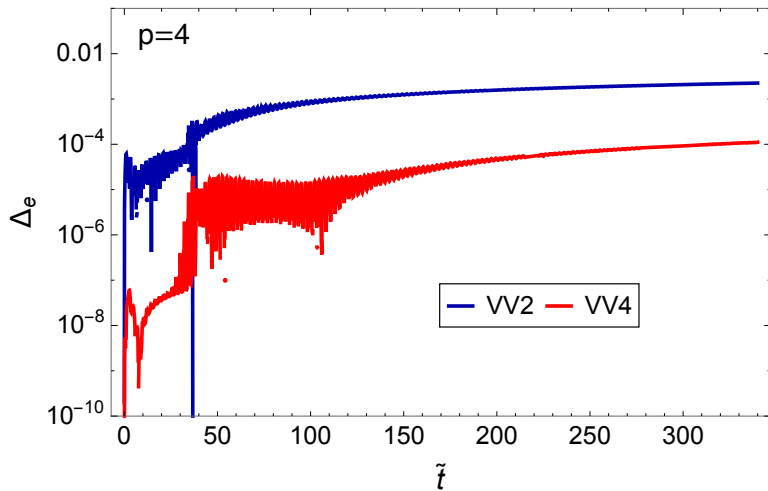
# Energies



## Energy fractions

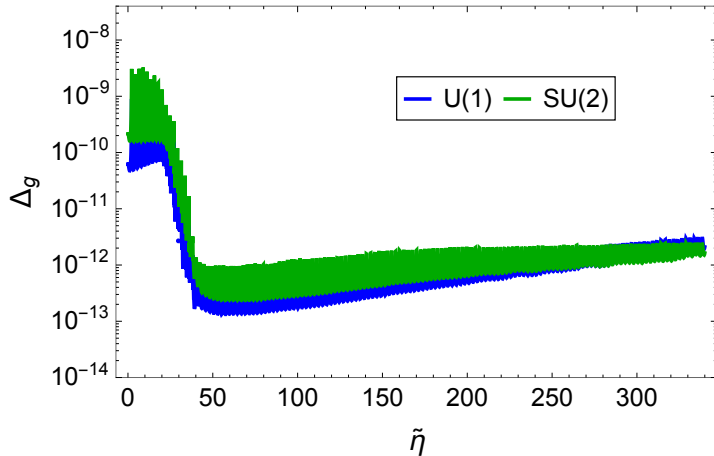


## Energy conservation

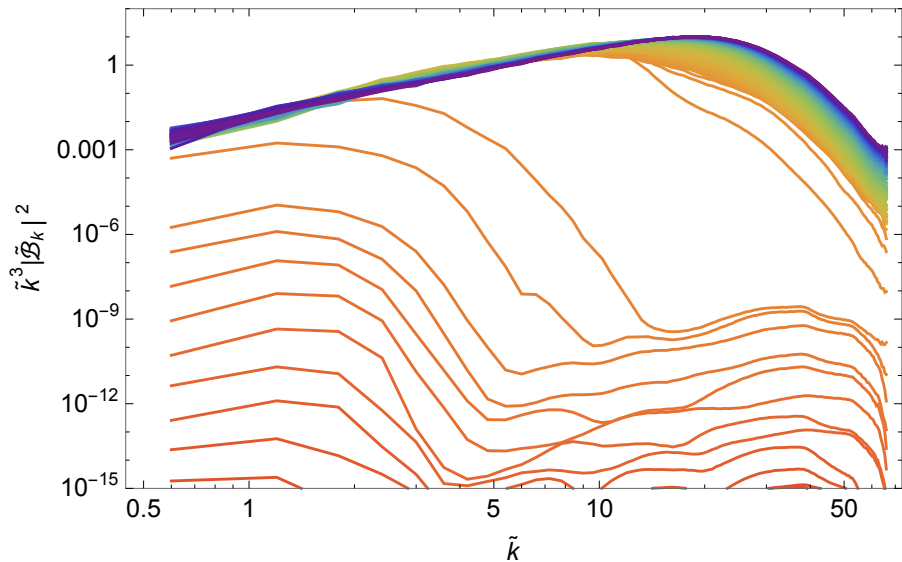




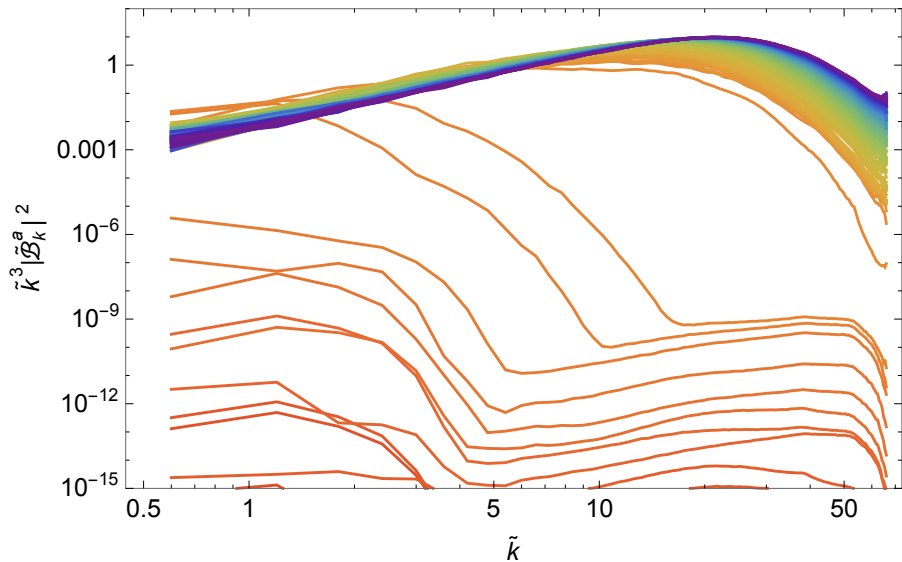
## Gauss laws



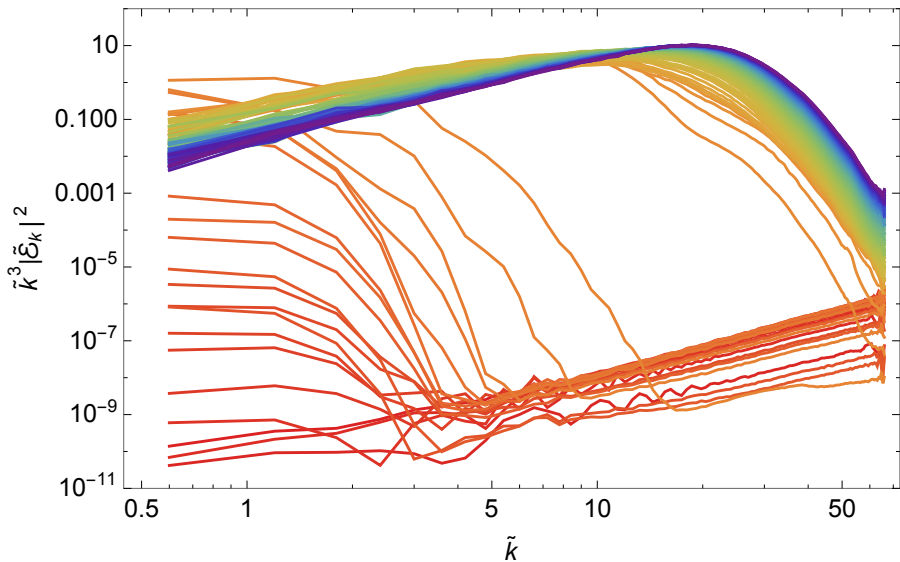
## $U(1)$ magnetic spectrum



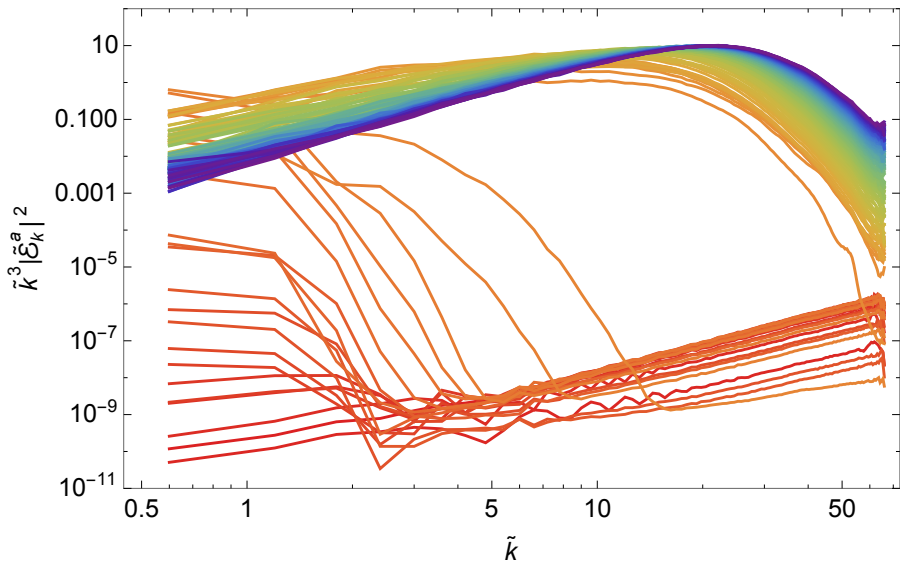
## $SU(2)$ magnetic spectrum



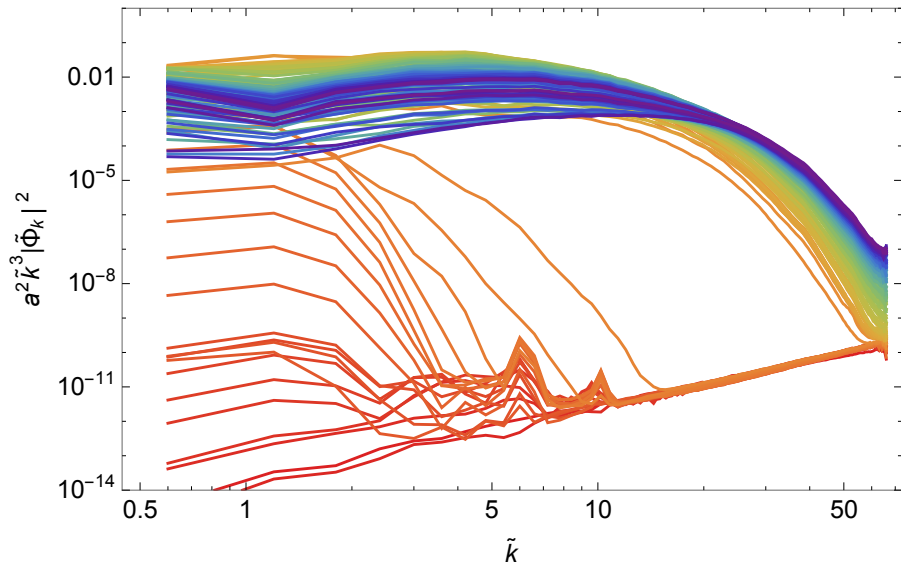
## $U(1)$ electric spectrum



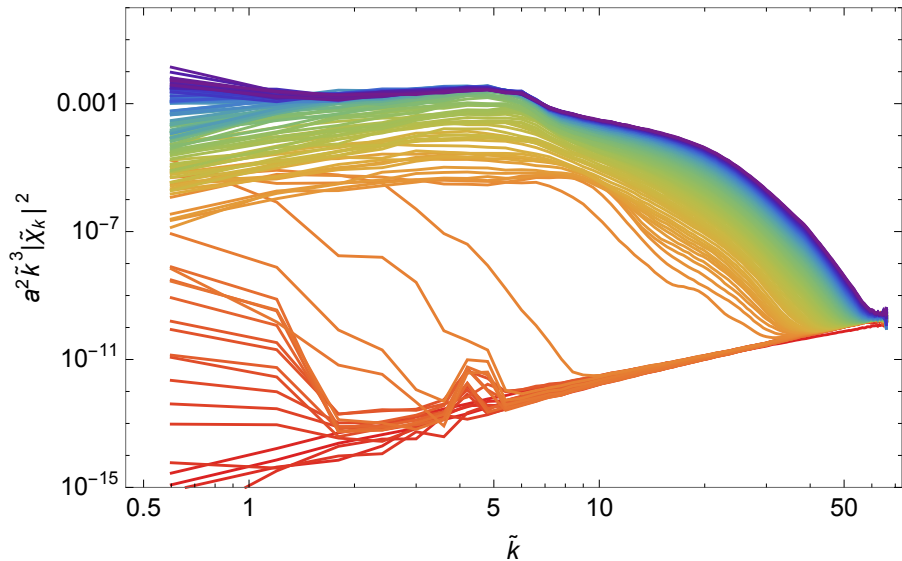
## $SU(2)$ electric spectrum



## Inflaton spectrum



## Daughter field spectrum



**Thanks!**