PHY2026 CH01 Lab Report

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Abstract

This experiment was designed around the principle of Chaos Theory and followed guided experimentation, as well as independently designed and carried out experimentation. Three systems were investigated: A magnetic torsional pendulum, a variable LCR circuit, and a double pendulum. These were set up and carried out and chaotic behaviour was observed. The Feigenbaum constant was estimated in the same order of magnitude as the literature value using the Fourier series of the LCR circuit.

1 Background

Chaos theory is based on the concept that any evolving system that obeys deterministic laws may still behave unpredictably. In popular culture Chaos theory underlies the Butterfly effect by stating that the tiniest change in starting conditions (a gust caused by a butterfly's wings) can cause enormous changes down the path of a chaotic system and completely change the outcome. This was investigated by Edward Lorenz with his Lorenz systems, which are a system of differential equations with chaotic solutions depending on certain initial parameters. In 1975, James A. Yorke coined the term Chaos theory [2] and the field of study was born. Nowadays Chaos theory is used in many fields of study, such and Neuroscience and Economics, and is utilised to describe the variable nature of many systems.

A perfect example of the application of Chaos theory is that of weather patterns. We are able to predict weather in the short term as the number of constricting variables and their ability to change is very limited, however as the weather systems evolve and go further in the future it is shown that the systems converge due to the extending chaos of the system and therefore become harder to predict. As a result, we use supercomputers that can operate up to 1.6×10^{16} floating point calculations per second to determine the most probable outcomes. This is but one way wherein Chaos theory is understood and accounted for in modern life. It is used to explain Ventricular Fibrillation [3], wherein a heart's arrhythmia can lead to inefficient blood transport and therefore death. Some even believe it can lead to the emergent property of consciousness [1].

2 Introduction

In this experiment, there will be two main focuses to observe Chaos theory in action. One method of this will be a torsional pendulum, wherein a lightly damped oscillating pendulum is modified with magnets to create a non-linear restoring force. This non-linearity will introduce chaos to the system which will then be able to be observed using rotary detection software. Once this is observed, graphs can be constructed to show the chaotic nature of the system. Another method used to display chaos will be an LCR circuit with a sinusoidal voltage source (a function generator in this case) and a non-linearly capacitive/resistive capacitor (a diode in this case). As the instantaneous input voltage changes, the resistance and capacitance of the diode will change and lead to chaotic tendencies with the system which can be plotted and measured using an oscilloscope.

2.1 Extension

To extend this experiment, a new chaotic system was constructed and observed using different techniques. A double pendulum was chosen, which was imaged using low exposure and an LED attached to the end of the second node to display an initial recognition of the chaos of the system. Further steps were then taken to utilise a system where further measurements could be taken. Once this was created, a tracking software was used to track different coloured nodes on the pendulum to determine position vectors. This data was then used to make an attempt at making qualitative measurements of chaotic variables in the system and how they evolved.

3 Theory

The Feigenbaum Constant can be obtained using different values of λ where λ_p is the p$^{\text{th}}$ bifurcation

$$\delta_p = \frac{\lambda_p - \lambda_{p-1}}{\lambda_{p+1} - \lambda_p} \tag{1}$$

4 Method

4.1 Torsional Pendulum

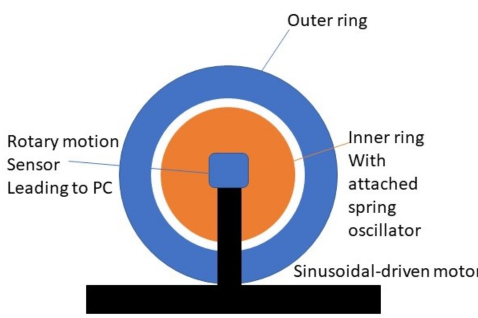


Figure 1: The setup for the torsional pendulum

Set up the system as above, ensuring that the rotary motion sensor is anchored via a retort stand arm and leads via a CASSY interface to the PC. Once this is set up, open the "CASSY Lab 2" application on the PC and select the rotary motion sensor. Set the angle to be 0, and the range to be $\pm 180^o$. Once this is complete, measurements can be taken. First, set the pendulum in free modes of oscillation and measure the resonant frequency of the pendulum. This can be measured loosely by measuring the time taken for 10 complete oscillations, and then dividing that by 10 to get the mean period. The trivial equation $f = \frac{1}{T}$ can then be used to obtain a natural frequency when the system acts a a simple damped harmonic oscillator.

In order to observe and demonstrate chaotic behaviour, amend the previous setup so that the oscillating ring has a set of magnets attached to the equilibrium point (that is, the point that faces directly up when the system is at rest). Also attach two sets of magnets equidistant from the equilibrium point along the outer ring. It is recommended to keep these fairly close so as to operate with lower energies and slower movements. At this point, also use the sinusoidal motor at a 24V driving voltage and adjust the output using the adjustable dial on the motor base. Keep adjusting until resonance can be seen and then measurements can be taken.

The behaviour that this experiment intends to find is an unpredictable change between each end of the bifurcations caused by the magnets as this is an indicator of Chaotic behaviours. The data can then be output via a CSV file and plotted as a θ vs t to show the bifurcations of the pendulum or a $\theta\cdot$ vs θ diagram to view the phase

The model used to obtain this data is to change the location of the magnets which therefore affects the potential energy and the system evolution as a result.

4.2 LCR Circuit

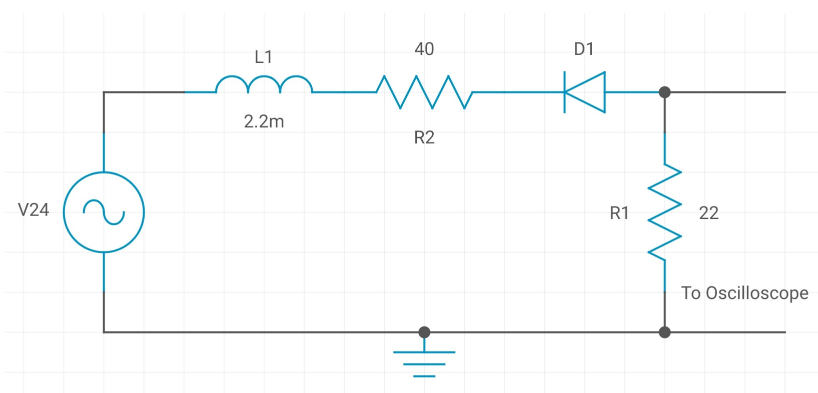


Figure 2: The circuit diagram to set up the apparatus for the LCR circuit

To begin this experiment, connect the pre-made circuit box (or a copy of the above circuit) to a Tektronix signal generator, keeping the input voltage under 100 mV due to the voltage load capacity on the diode, and attach an oscilloscope on the output. As well as this, connect the function generator directly to the oscilloscope on a different channel (1 for data, 2 for comparison). When two waveforms can be seen passing into the oscilloscope, adjust each input to the frequency stated on the signal generator and scale the amplitude and timescale to provide the most relevant data for the voltage input at that time.

Once waveforms can be seen, adjust the frequency of the input source so that the wave provides coherent evidence of bifurcation in the system. When bifurcation can be seen, take note of the frequency output as this is required to better understand the chaos of the experiment. The bifurcations will occur at two frequencies - a low frequency and a high frequency so look out for those.

Bifurcations is used a lot in this experiment description. In the context of a nonlinear LCR circuit, bifurcations are doubling patterns appearing in the peaks and troughs of the waveform

Once accurate waveforms are obtained and saved using the oscilloscope, you are able to Fourier Transform the data with the "FFT" button on the oscilloscope and export an image of the data in k-space. This can then used to try and find the powers of the cosine and sine functions that describe the bifurcations. The exported data also contains the variables with their observed values which can be plotted as a phase plot, or a simple plot of voltage vs time. It is also intended to construct a pitchfork diagram from this setup and therefore obtain a rough estimate for the Feigenbaum constant.

The Feigenbaum constant is obtained by dividing through two gap widths in the pitchfork diagram and is intended to represent the ratios between two states in a bifurcation. If a pitchfork diagram is not obtainable then the use of equation 1 is necessary.

4.3 Extension work - Double Pendulum

This experiment was extended further by the design and construction of multiple Double Pendulum structures that were developed to minimise damping effects and obtain a pendulum that is most alike to the theoretical model. This design started with two rulers attached by a bolt and ended with a set of 3D-printed pendulum units that were attached and utilised to obtain measurements and data.

4.3.1 Method of Extension

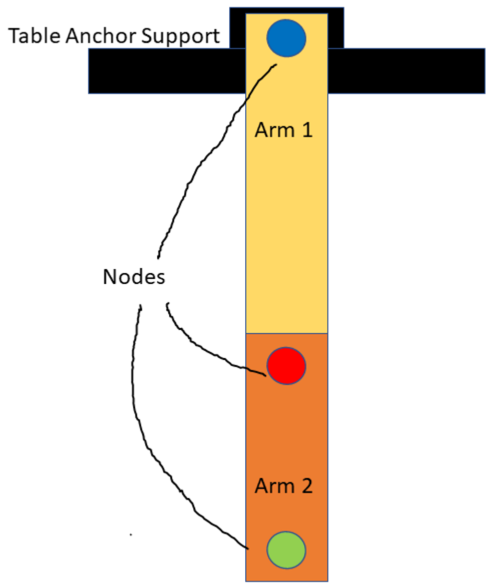


Figure 3: The final setup for the Double Pendulum experiment

In order to obtain data from the double pendulum, 3 nodes were attached to the points as shown in figure 3. The particular measurements used are: Arm 1 with a length of 21 cm, Arm 2 with a length of 10 cm to the node (+0.5 cm after the hole used to affix the node).

To obtain results, set up a device (in this case, a Samsung Galaxy S20 FE was used for the 60fps standard video camera. More concrete results can be obtained by utilising a higher frame rate). Place the pendulum in a consistent position and release it, allowing it to swing until any chaotic effects have been diminished. Repeat this over several releases to obtain a broad spectrum of data to be analysed.

5 Risk Assessment

Each experiment carries a component of risk with them which must be addressed in order to minimize the possibility of injury and harm.

5.1 Torsional Pendulum

This experiment uses an electric signal generator and moving parts. In order to act safely around such a machine, it must be checked for a PAT certification before use. As well as this, the machine should be checked for any visual flaws before operation. Any machine that does not pass these two checks should not be used, as doing so without proper precaution carries the risk of electrocution. Depending on the Amplitude and Voltage operated, this can cause mild to severe electrical burns. Beyond this check, the power pack should not be operated until a completed circuit is created and should operate within the guideline voltage for each appliance. It should be kept away from all liquids in case of a spillage that leads to damage to the power pack or to the operator.

The pendulum itself oscillates, potentially with a high kinetic energy. To reduce the possibility of impact damage, the operator should only perform modifications to the setup when the pendulum is stationary.

5.2 LCR Circuit

This experiment relies entirely on a circuit and electric components attached to it. As such, all components must be PAT certified and have no visual faults. If this is ignored the operator bears the risk of electric burns on their extremities. Each component must be fully connected before operating the power pack, and all liquids must be kept at a safe distance from the electronics. Any failure to follow these guidelines carries the risk of electrical burns and possible severe electrocution.

5.3 Double Pendulum

As this is a mechanical experiment, moving parts are operated during the course of the experiment. The components oscillate with unpredictable motion, often at rather great speeds. As such, impact damage is a prevalent risk. To avoid this, operators should stand clear of the plane of oscillation while the experiment is being completed. The components may also become loose and eject from the setup. If it is vital to stand in the plane of oscillation, the operator should wear protective eyewear to prevent stray components causing visual damage. They should also wear impact-resistant clothing to avoid damage from being hit by the pendulum.

6 Results

6.1 Torsional Pendulum

Throughout the course of the experiment, multiple different measurements were taken of chaotic motion of the torsional pendulum. The phase plot of this data, $x \ vs \ \dot{x}$ has a similar visual property to the y-z plane projection of the Lorenz Attractor.

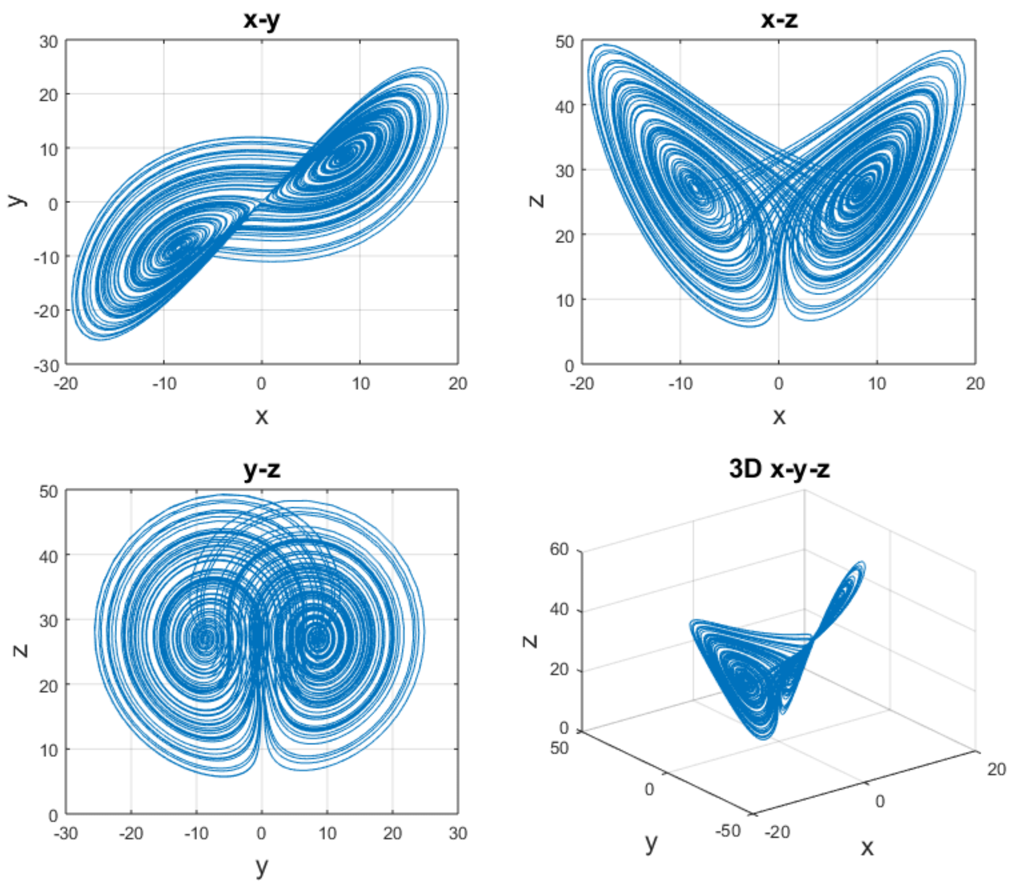
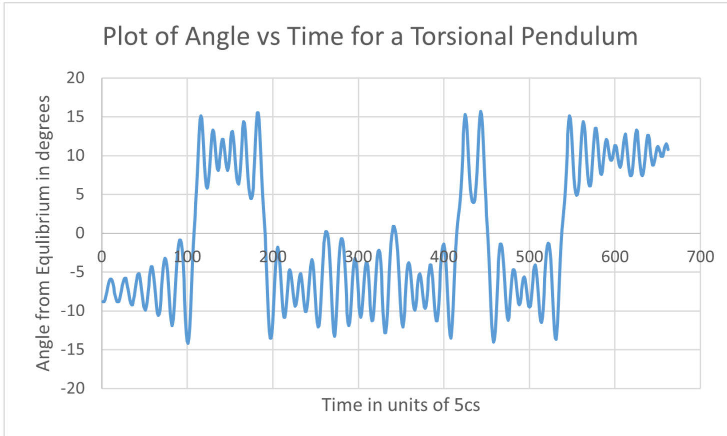
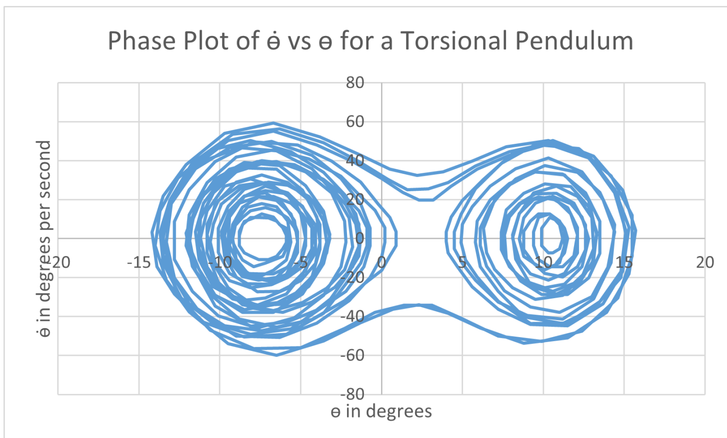


Figure 4: The three different plane projections of the Lorenz Attractor [4]

While multiple data sets were taken, some are particularly interesting. Data for chaotic behaviour was obtained for around 4 instances. Linearity was also observed under the wrong starting conditions that serves to show the difference between the linear and chaotic systems.

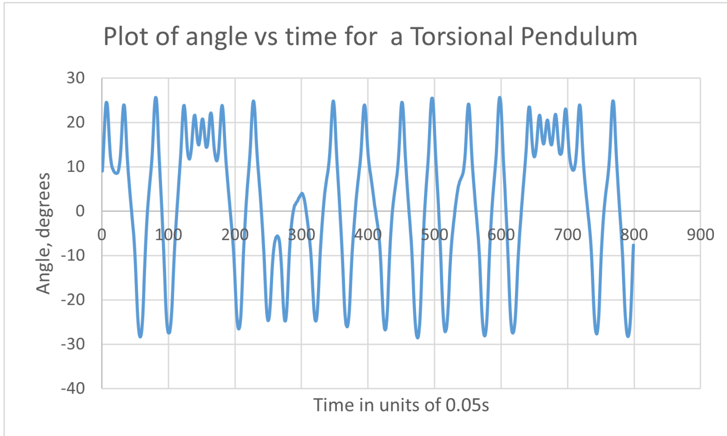


(a) The plot of angle vs time

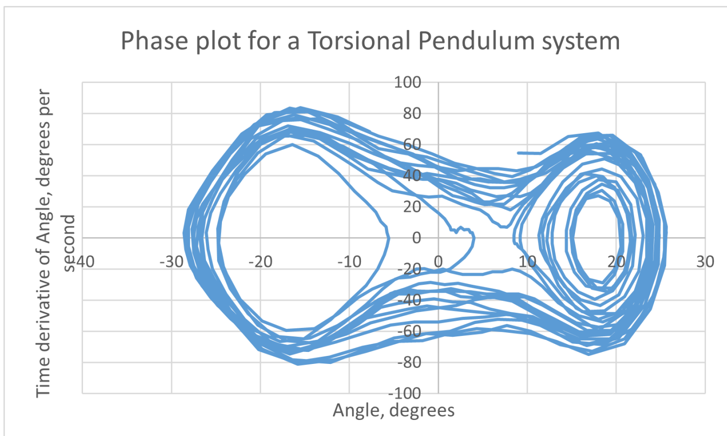


(b) The plot of angular time derivative vs angle

Figure 5: The Plots obtained from one set of data.



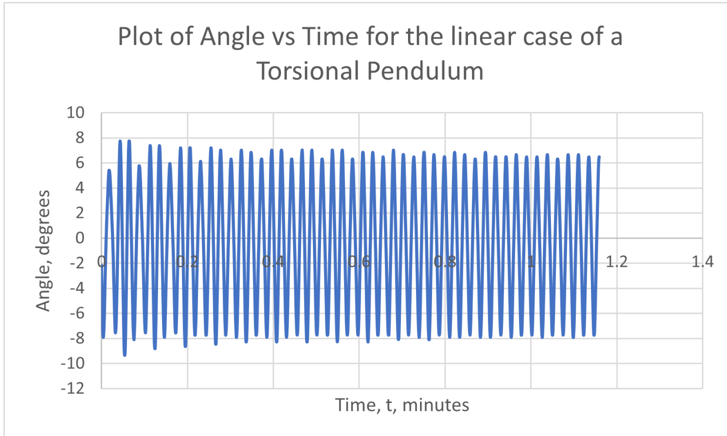
(a) The plot of angle vs time



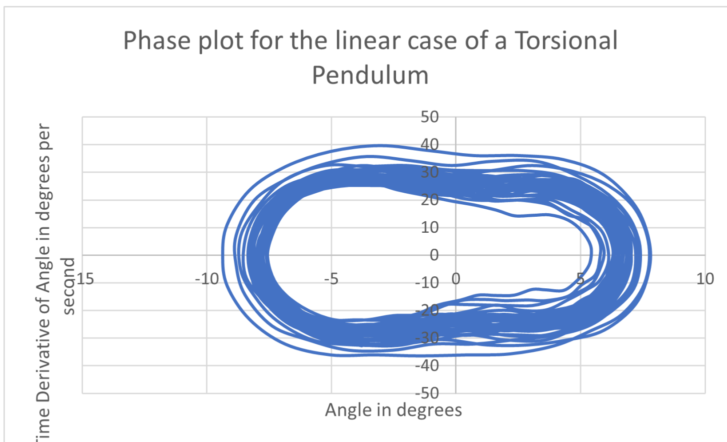
(b) The plot of angular time derivative vs angle

Figure 6: The Plots obtained from another set of data.

For each of the phase plots in figures 5b and 6b there is a notable similarity to the y-z plane projection of the Lorenz attractor in figure 4. This is among some of the best data obtained for this experiment as it shows the chaotic behaviour of the system very clearly. The bifurcations shown do have a tendency to prefer one side to the other - this may be due to imperfections in the experimental setup however it did not hold a serious impact on the behaviour of the system. The fact that the preferred side is different in each result indicates that there may be some other error due to the placement of the magnets or other such variables. Another set of data obtained shows the opposite side, when the energy of the system was too great and chaotic behaviours did not emerge



(a) The plot of angle vs time



(b) The plot of angular time derivative vs angle

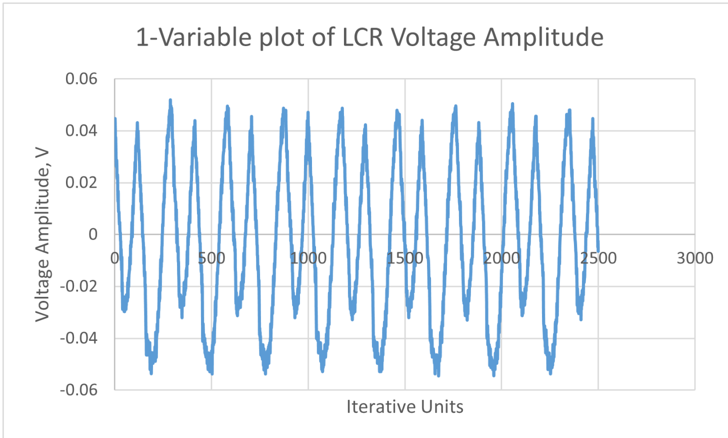
Figure 7: The Plots obtained from the linear data.

Overall it can be shown that chaotic behaviour emerges in a non-linear Torsional Pendulum system by observing and plotting the bifurcations as a phase plot. When this plot resembles a Lorenz attractor projection, the system is approaching a more chaotic behaviour.

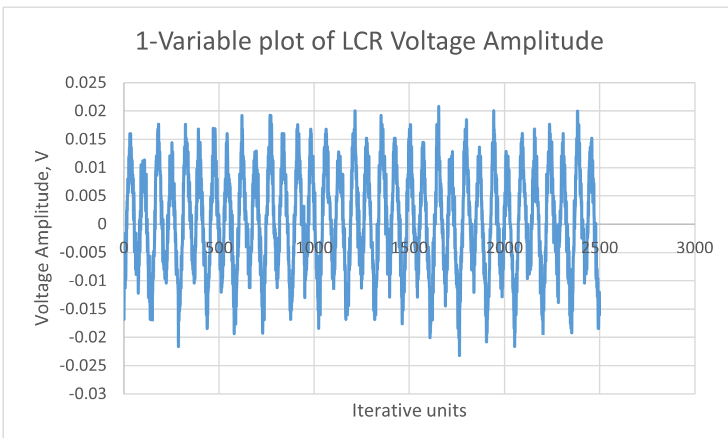
6.2 LCR Circuit

In this portion of the experiment, a lower- and upper- bound frequency was established wherein period doubling occurred. As period doubling is an indicator of chaos, it was possible to investigate this further and use many different techniques to obtain evidence of chaotic behaviour. The lower bound frequency was found to be 340Hz, while the higher bound frequency was found to be 680Hz

The plots of the two voltage amplitudes show the period doubling directly:



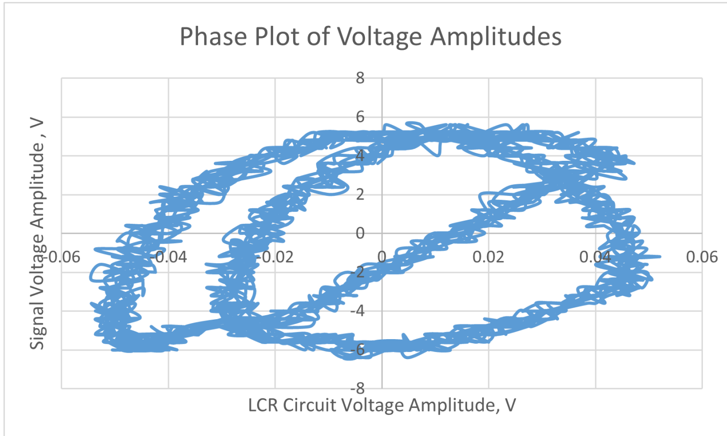
(a) The plot of the voltage amplitude at 340Hz



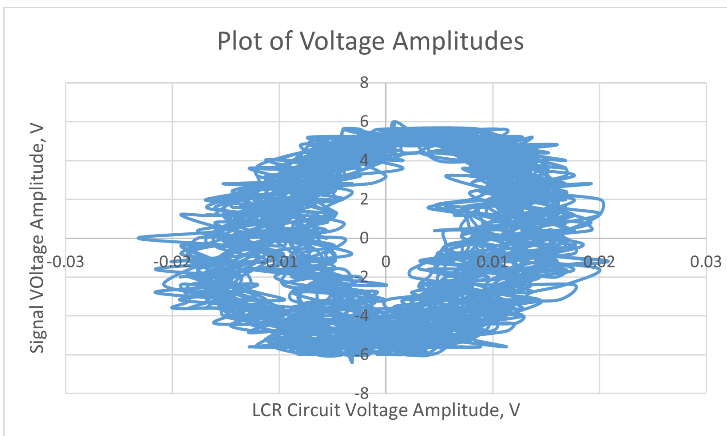
(b) The plot of the voltage amplitude at 680Hz

Figure 8: Plots showing the voltage amplitudes and displaying frequency doubling.

Further to this, chaos could also be seen from the phase plots created when plotting the Signal voltage amplitude vs the LCR-modified voltage amplitude. These phase plots are not directly the same as those in the pendulum experiments as it is based on plotting some different data. The offical name for these plots is "Lissajous Figures".



(a) The phase plot of the voltage amplitudes at 340Hz



(b) The phase plot of the voltage amplitudes at 680Hz

Figure 9: Plots showing the voltage amplitudes and displaying frequency doubling.

When the data was extracted, a fourier transformation of the voltage was available for extraction, which was completed for both the lower and higher frequency. This data was filtered to obtain the powers of the highest peaks and then equation 1 was used to obtain an estimate for the Feigenbaum Constant. For the lower frequency, an estimate obtained was 6.323 which is within 1 order of magnitude of the literature value 4.669. For the higher frequency, a value estimate was obtained as 5.645 which is also within 1 order of magnitude. As well as this, it is closer than the other estimate. This may be due to the greater density of data available as the Feigenbaum constant is obtained on a convergence to infinity.

6.3 Double Pendulum

For this experiment, the intention was to demonstrate chaotic behaviour as qualitatively as possible. For the three datasets analysed, this can be seen in two different means. Firstly, the chaotic behaviour can be seen from observing the angle of the double pendulum vs time. When analysed, frequency doubling can be seen at some peaks which is a hallmark of chaos.

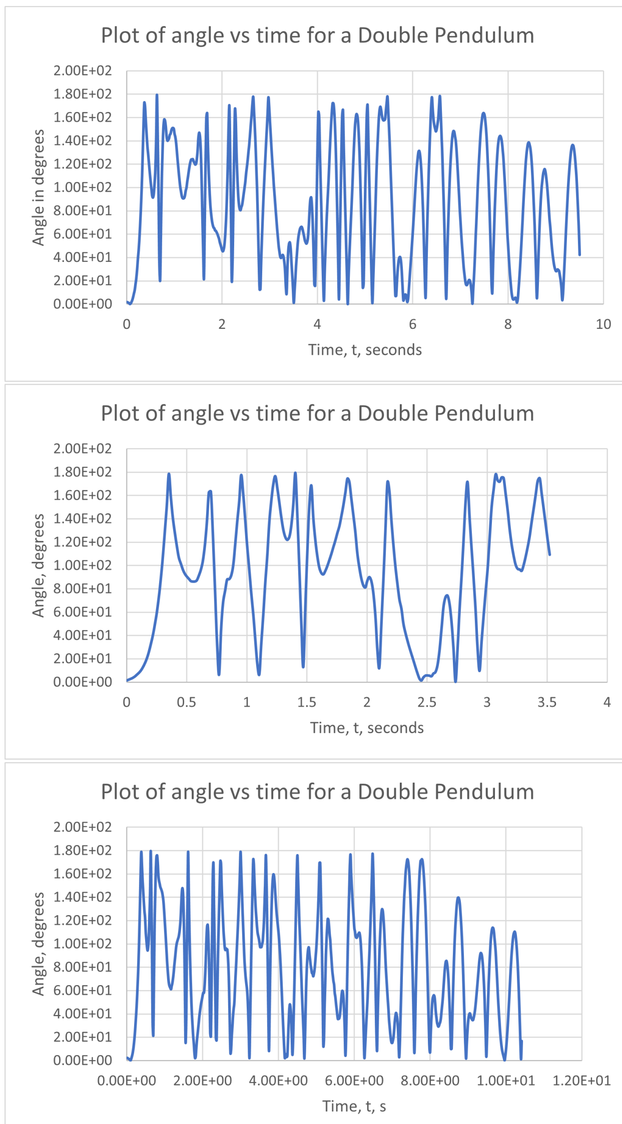


Figure 10: The plots showing the period doubling for 3 different recordings of data for the Double Pendulum.

Further to this, using numerical differentiation a phase plot could be created. These plots resemble the Lorenz Attractor diagram that is often used as a benchmark for chaotic behaviours. This resemblance further solidifies the existence of chaotic behaviour in the systems.

Figure 11: The phase plots showing the chaotic behaviour for 3 different recordings of data for the Double Pendulum.

7 Discussion

This lab session was successful in reaching results that point towards chaos for each system investigated. Each experiment had multiple differences between them - regarding data collection, data analysis and any further assessment.

7.1 Torsional Pendulum

The Torsional Pendulum data was, once refined, notably clean and provided an easy connection between the data and the evidence of chaos. There appeared to have been a bias within the setup throughout the course of the experiment where the pendulum favoured swinging to a certain side. The side chosen seemed to change between measurements. This may have reduced certain chaotic effects as more energy was required to leave that potential well however it did not interfere with results. At worst, an impedance on the power of the results can be expected.

If this experiment were to be repeated, more time should be taken on focusing on resonances. The resonant frequency for the full pendulum was found to be 0.5498Hz however there was little attempt made to investigate the resonance surrounding each point of bifurcation when the magnets were attached. Knowledge of this may have lead to further depth of knowledge for the experiment.

7.2 LCR Circuit

For this experiment, a value of frequency was obtained for the low band and high band of period doubling. This experiment was particularly difficult to work with due to the circuit being very volatile and often it did not carry a signal. Eventually the data was obtained and exported for analysis and an acceptable amount of qualitative data was obtained.

In further experimentation, the LCR circuit would be fully repaired and usable. As well as this, it would be wise to seek evidence for second- or third-order period doubling which would assist in the creating of a pitchfork diagram and the obtaining of a more accurate Feigenbaum diagram.

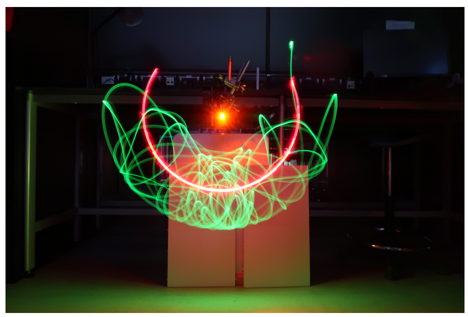
7.3 Double Pendulum

Throughout the trials that this experiment underwent, the design was updated multiple each times. Each instance brought us closer to our desired model - the largest reduction in damping effects possible. Our final design utilised 3D printing techniques to receive a lightweight and smooth unit with which a double pendulum could be constructed.

As can be seen in figure 11, the phase plots created are still not perfect. There is a lot of energy loss as the system is never going to be perfectly conservative. The middle diagram is the easiest to see the similarities to the Lorenz Attractor as it is of a smaller data set and therefore has less noise from the frictional forces acting on the system throughout the operation.

In future repetitions of this experiment, an even more sophisticated design should be used. Notable issues that were faced during this instance was regarding the final analysis of data. The hardware used captured a low framerate and so the nodes were occasionally blurred and an estimation was forced to be used in those cases for position. As well as this, there is some haphazard colour blocking behind the pendulum. In future cases, it would be best to create a pendulum that uses ball bearings to rotate so that friction is reduced there. This pendulum would also attach to a large, plain white board that is well-lit to reduce any need for estimation or issues during analysis.

During the design of this experiment there was a focus on long-exposure shots of LEDs attached to the nodes as they oscillated. The reason for this was to test the amount of chaos our systems were undergoing.



While analytically useless, this does display the evidence and beauty of a well-constructed chaotic system

7.4 Errors

While there was not too much quantitative analysis at any stage, it must be stated that there were errors that may have an effect on the outcome of the plots. In the LCR circuit, the frequencies were manually found for a certain selected voltage. These may have been sub-optimal as the transition to period doubling was so subtle. Chaos was still observed on the parameters set, it is just worth understanding that further steps can be taken to ensure the highest degree of data sanitation.

The Double Pendulum system also suffered at the hands of errors. The source of mechanical error has already been discussed as damping forces acting on the system. During analysis, each node was tracked using an application that logged their data. As this tracking was always an estimate for the position, there is a source of error of about 0.5cm per measurement. As these are all estimates in the first place, the data does not take too much of a hit from this error however it can always do without.

7.5 Chaos

Each experiment displayed chaos in their behaviours. Each experiment showed either bifurcations or could have a phase plot generated. Due to the nature of chaos, these systems are unpredictable and volatile. Each minor change to the systems lead to a large difference in the final result.

Due to this fact, these are by their very nature unable to be replicated. Each result is unique and that comes with both risk and reward; this does display the chaotic behaviours of the system as each measurement is solely unique however it may also show that the data cannot be too deeply understood as there is no apparent coherence. Further research into chaos and the idea of orderly disorder must be conducted in order to understand and potentially model these systems. Humans themselves are inherently chaotic and it would be a great well of knowledge to further deepen the understanding of this world-changing field.

8 Conclusion

Overall, the three experiments were carried out and each one displayed chaotic tendencies. Plots were made for each instance that displayed chaos and the Feigenbaum constant was estimated through the chaotic behaviour of the LCR circuit - the best estimate being 5.645. These plots were comprehensive and showed chaos, providing a direct reference to the Lorenz Attractor that has brought much fame to chaos.

References

[1] Petr Bob. "Chaos, brain and divided consciousness." In: *Acta Universitatis Carolinae. Medica. Monographia* 153 (2007), pp. 9–80.

[2] Tien-Yien Li and James A Yorke. "Period three implies chaos". In: *The theory of chaotic attractors*. Springer, 2004, pp. 77–84.

[3] Zhilin Qu. "Chaos in the genesis and maintenance of cardiac arrhythmias". In: *Progress in biophysics and molecular biology* 105.3 (2011), pp. 247–257.

[4] Lei Zhang. "Artificial neural networks model design of Lorenz chaotic system for EEG pattern recognition and prediction". In: Dec. 2017, pp. 39–42. DOI: 10.1109/LSC.2017.8268138.