Equation for Three Link

Output:-
Torque1

$$\tau_1 = \frac{m_1 \left(2\,\dot{\theta}_1\, l_1^{\,2} \cos(\theta_1)^2 + 2\,\dot{\theta}_1\, l_1^{\,2} \sin(\theta_1)^2\right)}{2} + \frac{m_2 \left(2\,\sigma_{13}\,(\sigma_{10}+\sigma_4-\sigma_6-\sigma_2)+2\,(l_1 \sin(\theta_1)+\sigma_8)\,(\sigma_9+\sigma_5+\sigma_7+\sigma_3)\right)}{2}$$

$$- \frac{m_3 \left(2\,\sigma_{12}\left(l_3\,\sigma_1\,\sigma_{11}-\sigma_{10}-\sigma_4-l_3\,\sigma_{14}\left(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3\right)+\sigma_6+\sigma_2\right)-2\,(l_3\,\sigma_1+l_1 \sin(\theta_1)+\sigma_8)\left(l_3\,\sigma_{14}\,\sigma_{11}+\sigma_9+\sigma_5+\sigma_7+l_3\,\sigma_1\left(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3\right)+\sigma_3\right)\right)}{2} + g\,m_2\,\sigma_{13} + g\,m_3\,\sigma_{12} + g\,l_1\,m_1 \cos(\theta_1)$$

where

$$\sigma_1 = \sin(\theta_1+\theta_2+\theta_3)$$

$$\sigma_2 = l_2 \sin(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)^2$$

$$\sigma_3 = l_2 \cos(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)^2$$

$$\sigma_4 = l_2 \cos(\theta_1+\theta_2)\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2\right)$$

$$\sigma_5 = l_2 \sin(\theta_1+\theta_2)\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2\right)$$

$$\sigma_6 = \dot{\theta}_1^{\,2}\, l_1 \sin(\theta_1)$$

$$\sigma_7 = \dot{\theta}_1^{\,2}\, l_1 \cos(\theta_1)$$

$$\sigma_8 = l_2 \sin(\theta_1+\theta_2)$$

$$\sigma_9 = \dot{\dot{\theta}}_1\, l_1 \sin(\theta_1)$$

$$\sigma_{10} = \dot{\dot{\theta}}_1\, l_1 \cos(\theta_1)$$

$$\sigma_{11} = (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)^2$$

$$\sigma_{12} = l_3\,\sigma_{14} + l_1 \cos(\theta_1) + \sigma_{15}$$

$$\sigma_{13} = l_1 \cos(\theta_1) + \sigma_{15}$$

$$\sigma_{14} = \cos(\theta_1+\theta_2+\theta_3)$$

$$\sigma_{15} = l_2 \cos(\theta_1+\theta_2)$$

After Simplification :-

$$\tau_1 = \ddot{\theta}_1 l_1^2 m_1 + \ddot{\theta}_1 l_1^2 m_2 + \ddot{\theta}_1 l_1^2 m_3 + \ddot{\theta}_1 l_2^2 m_2 + \ddot{\theta}_1 l_2^2 m_3 + \ddot{\theta}_2 l_2^2 m_2 + \ddot{\theta}_1 l_3^2 m_3 + \ddot{\theta}_2 l_2^2 m_3 + \ddot{\theta}_2 l_3^2 m_3 + \ddot{\theta}_3 l_3^2 m_3 + g l_1 m_1 \cos(\theta_1) + g l_1 m_2 \cos(\theta_1) + g l_1 m_3 \cos(\theta_1)$$

$$+ g l_2 m_2 \cos(\theta_1 + \theta_2) + g l_2 m_3 \cos(\theta_1 + \theta_2) + g l_3 m_3 \cos(\theta_1 + \theta_2 + \theta_3) - \dot{\theta}_2^2 l_1 l_3 m_3 \sin(\theta_2 + \theta_3) - \dot{\theta}_3^2 l_1 l_3 m_3 \sin(\theta_2 + \theta_3) + 2 \ddot{\theta}_1 l_1 l_2 m_2 \cos(\theta_2) + 2 \ddot{\theta}_1 l_1 l_2 m_3 \cos(\theta_2)$$

$$+ \ddot{\theta}_2 l_1 l_2 m_2 \cos(\theta_2) + \ddot{\theta}_2 l_1 l_2 m_3 \cos(\theta_2) + 2 \ddot{\theta}_1 l_2 l_3 m_3 \cos(\theta_3) + 2 \ddot{\theta}_2 l_2 l_3 m_3 \cos(\theta_3) + \ddot{\theta}_3 l_2 l_3 m_3 \cos(\theta_3) - \dot{\theta}_2^2 l_1 l_2 m_2 \sin(\theta_2) - \dot{\theta}_2^2 l_1 l_2 m_3 \sin(\theta_2) - \dot{\theta}_3^2 l_2 l_3 m_3 \sin(\theta_3)$$

$$+ 2 \ddot{\theta}_1 l_1 l_3 m_3 \cos(\theta_2 + \theta_3) + \ddot{\theta}_2 l_1 l_3 m_3 \cos(\theta_2 + \theta_3) + \ddot{\theta}_3 l_1 l_3 m_3 \cos(\theta_2 + \theta_3) - 2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_3 m_3 \sin(\theta_2 + \theta_3) - 2 \dot{\theta}_1 \dot{\theta}_3 l_1 l_3 m_3 \sin(\theta_2 + \theta_3) - 2 \dot{\theta}_2 \dot{\theta}_3 l_1 l_3 m_3 \sin(\theta_2 + \theta_3)$$

$$- 2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 m_2 \sin(\theta_2) - 2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 m_3 \sin(\theta_2) - 2 \dot{\theta}_1 \dot{\theta}_3 l_2 l_3 m_3 \sin(\theta_3) - 2 \dot{\theta}_2 \dot{\theta}_3 l_2 l_3 m_3 \sin(\theta_3)$$

Torque2

$$\tau_2 = \frac{m_3\,(\sigma_{11}-\sigma_{10})}{2} - \frac{m_2\,(\sigma_{12}-\sigma_{13})}{2} + \frac{m_3\left(2\,(l_3\,\sigma_{22}+l_2\,\sin(\theta_1+\theta_2))\left(l_3\,\sigma_{21}\,\sigma_9+\sigma_7+\sigma_4+\sigma_6+l_3\,\sigma_{22}\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2+\dot{\dot{\theta}}_3\right)+\sigma_2\right)-\sigma_{11}+\sigma_{10}-2\,\sigma_{14}\left(l_3\,\sigma_{22}\,\sigma_9-\sigma_8-\sigma_3-l_3\,\sigma_{21}\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2+\dot{\dot{\theta}}_3\right)+\sigma_5+\sigma_1\right)\right)}{2}$$

$$+ \frac{m_2\,(2\,l_2\cos(\theta_1+\theta_2)\,(\sigma_8+\sigma_3-\sigma_5-\sigma_1)+2\,l_2\sin(\theta_1+\theta_2)\,(\sigma_7+\sigma_4+\sigma_6+\sigma_2)+\sigma_{12}-\sigma_{13})}{2} + g\,m_3\,\sigma_{14} + g\,l_2\,m_2\cos(\theta_1+\theta_2)$$

where

$$\sigma_1 = l_2\sin(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)^2$$

$$\sigma_2 = l_2\cos(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)^2$$

$$\sigma_3 = l_2\cos(\theta_1+\theta_2)\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2\right)$$

$$\sigma_4 = l_2\sin(\theta_1+\theta_2)\left(\dot{\dot{\theta}}_1+\dot{\dot{\theta}}_2\right)$$

$$\sigma_5 = \dot{\theta}_1^{\,2}\,l_1\sin(\theta_1)$$

$$\sigma_6 = \dot{\theta}_1^{\,2}\,l_1\cos(\theta_1)$$

$$\sigma_7 = \dot{\dot{\theta}}_1\,l_1\sin(\theta_1)$$

$$\sigma_8 = \dot{\dot{\theta}}_1\,l_1\cos(\theta_1)$$

$$\sigma_9 = (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)^2$$

$$\sigma_{10} = 2\,(\sigma_{19}+\sigma_{15})\,(\sigma_{18}+\sigma_{17}+\sigma_{16})$$

$$\sigma_{11} = 2\,(\sigma_{17}+\sigma_{16})\,(\sigma_{20}+\sigma_{19}+\sigma_{15})$$

$$\sigma_{12} = 2\,l_2\cos(\theta_1+\theta_2)\,(\sigma_{18}+\sigma_{17})\,(\dot{\theta}_1+\dot{\theta}_2)$$

$$\sigma_{13} = 2\,l_2\sin(\theta_1+\theta_2)\,(\sigma_{20}+\sigma_{19})\,(\dot{\theta}_1+\dot{\theta}_2)$$

$$\sigma_{14} = l_3\,\sigma_{21}+l_2\cos(\theta_1+\theta_2)$$

$$\sigma_{15} = l_3\,\sigma_{21}\,(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)$$

$$\sigma_{16} = l_3\,\sigma_{22}\,(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)$$

$$\sigma_{17} = l_2\sin(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)$$

$$\sigma_{18} = \dot{\theta}_1\,l_1\sin(\theta_1)$$

$$\sigma_{19} = l_2\cos(\theta_1+\theta_2)\,(\dot{\theta}_1+\dot{\theta}_2)$$

$$\sigma_{20} = \dot{\theta}_1\,l_1\cos(\theta_1)$$

$$\sigma_{21} = \cos(\theta_1+\theta_2+\theta_3)$$

$$\sigma_{22} = \sin(\theta_1+\theta_2+\theta_3)$$

After Simplification:-

$$\tau_2 = \ddot{\theta}_1 l_2^2 m_2 + \ddot{\theta}_1 l_2^2 m_3 + \ddot{\theta}_2 l_2^2 m_2 + \ddot{\theta}_1 l_3^2 m_3 + \ddot{\theta}_2 l_2^2 m_3 + \ddot{\theta}_2 l_3^2 m_3 + \ddot{\theta}_3 l_3^2 m_3 + g l_2 m_2 \cos(\theta_1 + \theta_2) + g l_2 m_3 \cos(\theta_1 + \theta_2) + g l_3 m_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$+ \dot{\theta}_1^2 l_1 l_3 m_3 \sin(\theta_2 + \theta_3) + \ddot{\theta}_1 l_1 l_2 m_2 \cos(\theta_2) + \ddot{\theta}_1 l_1 l_2 m_3 \cos(\theta_2) + 2 \ddot{\theta}_1 l_2 l_3 m_3 \cos(\theta_3) + 2 \ddot{\theta}_2 l_2 l_3 m_3 \cos(\theta_3) + \ddot{\theta}_3 l_2 l_3 m_3 \cos(\theta_3) + \dot{\theta}_1^2 l_1 l_2 m_2 \sin(\theta_2)$$
$$+ \dot{\theta}_1^2 l_1 l_2 m_3 \sin(\theta_2) - \dot{\theta}_3^2 l_2 l_3 m_3 \sin(\theta_3) + \ddot{\theta}_1 l_1 l_3 m_3 \cos(\theta_2 + \theta_3) - 2 \dot{\theta}_1 \dot{\theta}_3 l_2 l_3 m_3 \sin(\theta_3) - 2 \dot{\theta}_2 \dot{\theta}_3 l_2 l_3 m_3 \sin(\theta_3)$$

Torque3

$$\tau_3 = \frac{m_3 (\sigma_3 - \sigma_2)}{2} - \left(m_3 \left(2 l_3 \sigma_4 \left(l_3 \sigma_5 \sigma_1 - \ddot{\theta}_1 l_1 \cos(\theta_1) - l_2 \cos(\theta_1 + \theta_2) \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) - l_3 \sigma_4 \left(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3 \right) + \dot{\theta}_1^2 l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2)^2 \right) \right.\right.$$
$$\left.\left. - 2 l_3 \sigma_5 \left(l_3 \sigma_4 \sigma_1 + \ddot{\theta}_1 l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) + \dot{\theta}_1^2 l_1 \cos(\theta_1) + l_3 \sigma_5 \left(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3 \right) + l_2 \cos(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2)^2 \right) + \sigma_3 - \sigma_2 \right) \right) \Big/ 2 + g l_3 m_3 \sigma_4$$

where

$$\sigma_1 = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2$$

$$\sigma_2 = 2 l_3 \sigma_4 (\dot{\theta}_1 l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) + l_3 \sigma_5 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$$

$$\sigma_3 = 2 l_3 \sigma_5 (\dot{\theta}_1 l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) + l_3 \sigma_4 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$$

$$\sigma_4 = \cos(\theta_1 + \theta_2 + \theta_3)$$

$$\sigma_5 = \sin(\theta_1 + \theta_2 + \theta_3)$$

After Simplification :-

$$\tau_3 = I_3\, m_3 \left(\ddot{\theta}_1\, l_3 + \ddot{\theta}_2\, l_3 + \ddot{\theta}_3\, l_3 + g\cos(\theta_1+\theta_2+\theta_3) + \ddot{\theta}_1\, l_1\cos(\theta_2+\theta_3) + \dot{\theta}_1^{\,2}\, l_1\sin(\theta_2+\theta_3) + \ddot{\theta}_1\, l_2\cos(\theta_3) + \ddot{\theta}_2\, l_2\cos(\theta_3) + \dot{\theta}_1^{\,2}\, l_2\sin(\theta_3) + \dot{\theta}_2^{\,2}\, l_2\sin(\theta_3) + 2\,\dot{\theta}_1\,\dot{\theta}_2\, l_2\sin(\theta_3)\right)$$