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VALIDATION OF GRAVITY ACCELERATION AND TORQUE ALGORITHMS FOR ASTRODYNAMICS

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This paper presents the technique developed to validate the spherical harmonic gravity acceleration and torque algorithms of the Johnson Space Center Engineering Orbital Dynamics (JEOD) simulation software. The technique can be applied to the validation of any general spherical harmonic gravity algorithm used for astrodynamics engineering or space mission operations, and is not restricted to Earth gravity algorithms. A simulated system of finite point masses was developed to represent a large gravitational body on a scale approximating Earth. Normalized spherical harmonic gravity coefficients were generated to represent the total gravitational potential of the point mass system. The coefficients were used as data for the JEOD gravity algorithms to compute acceleration and torque at various test locations external to the point mass system. The acceleration and torque were also computed directly using the basic principles of gravitational force. The difference in acceleration computed by the two methods at all locations was on the order of 10^{-15} m/s^2, and the torque computed at all locations matched to six or more significant figures.

INTRODUCTION

In order to realistically model the orbital motion of a spacecraft, the acceleration due to gravity must be accurately computed. Gravitational torque acting on the spacecraft must also be accurately computed for high precision attitude modeling. The gravitational potential of a large, massive body (i.e. planet) is commonly modeled as a spherical harmonic series from which acceleration and torque can be computed. Several acceleration and torque algorithms exist (e.g. Gottlieb 1993,[1] Pines 1973,[2] Cunningham 1970,[3] Glandorf 1986,[4] Pelivan 2006[5]). Most are quite complex and involve mathematical recursions to efficiently compute high-order Legendre and trigonometric functions.

Any gravity algorithm requires validation to ensure it has been properly implemented. One method of validation is to program a second, independent algorithm for comparison purposes. This method requires a duplication of programming effort. If the results of the two algorithms do not match, it can be difficult to determine which algorithm (if not both) was implemented incorrectly. The logical solution is to program yet a third algorithm, requiring further repetition of effort. Validation techniques exist for simple spherical and oblate planet models.[1,5,6] A technique for validating general, higher-order gravity algorithms was required. This paper presents the technique developed to validate the Gottlieb spherical harmonic gravity acceleration and torque algorithms as implemented in the Johnson Space Center Engineering Orbital Dynamics (JEOD) simulation software.[1,7] The technique validates gravity algorithms, not the specific coefficients of any actual gravitational body, and is therefore not limited to Earth models.

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The layout of this paper is as follows. The technical approach of the newly developed validation technique is presented. A mathematical overview of spherical harmonic gravity models and the contrived point mass system is given. Finally, the validation technique and results for the JEOD acceleration and torque algorithms are presented.

TECHNICAL APPROACH

A fictitious system of point masses was developed to represent a large gravitational body. For familiarity, this so-called "point mass planet" was scaled to approximate the mass of Earth. Low-degree normalized spherical harmonic gravity coefficients were computed to represent the total gravitational potential of the point mass planet. The point masses were configured such that all gravity coefficients above degree two were non-zero, and coefficients of degree five (and higher) were at least several orders of magnitude smaller than the approximate limit of double floating point arithmetic (15 significant figures). The assumption was made that algorithm recursions that worked correctly to degree and order four also worked correctly for higher degree and order. However, to mitigate errors caused by truncation of the infinite spherical harmonic series, gravity coefficients through degree five were included in the model. The coefficients were used as data for the JEOD algorithms to compute acceleration and torque at various test locations external to the point mass system, including points over the north and south poles where mathematical singularities may have occurred.Simultaneously, the acceleration and torque vectors due to each point mass were computed directly from basic gravity principles and summed to give the total acceleration and torque acting at each test location. The total acceleration and torque were compared to those quantities computed using the JEOD algorithms for validation purposes.

SPHERICAL HARMONIC GRAVITY

The spherical harmonic model of gravitational potential commonly used in astrodynamics is[8,9]

$$V(r,\lambda,\phi) = \frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n \bar{P}_{nm}(sin\phi)(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda) \tag{1}$$

where (r,λ,ϕ) are the spherical coordinates radial distance, longitude, and geocentric latitude (figure 1); μ is the gravitational parameter (the product of the universal gravitational constant G and the mass of the central body M, $\mu = GM$); n and m are degree and order indices, a_e is the mean equatorial radius of the central body; $\bar{P}_{nm}$ is a normalized associated Legendre function; and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are normalized unitless gravity coefficients that are related to the mass distribution of the central body. All $\bar{S}_{nm}$ are meaningless when $m = 0$ because the sine term in equation (1) vanishes $(\sin 0 = 0)$. The overhead bar denotes a coefficient that is normalized by the relationship

$$\left\{\begin{array}{c}\bar{C}_{nm}\\ \bar{S}_{nm}\end{array}\right\} = \sqrt{\frac{(n+m)!}{(2-\delta_{0m})(2n+1)(n-1)!}}\left\{\begin{array}{c}C_{nm}\\ S_{nm}\end{array}\right\} \tag{2}$$

where $\delta_{0m} = 1$ when $m = 0$, and $\delta_{0m} = 0$ when $m \neq 0$.

The degree zero term $(n = 0)$ is representative of the spherical surface upon which the higher degree terms are imposed. The associated gravity coefficient is unity by definition $(C_{00} = 1)$. It can be shown that the location of the center of mass of the gravitational body $(\bar{x},\bar{y},\bar{z})$ is related to

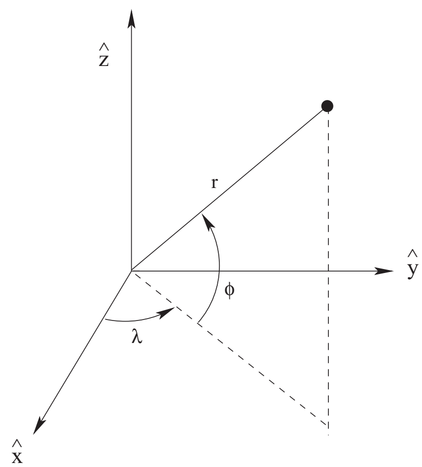

Figure 1 Spherical coordinate system (r, λ, ϕ)**.**

the un-normalized first-degree coefficients by[10,9]

$$\begin{aligned} \bar{x} &= a_e C_{11} \\ \bar{y} &= a_e S_{11} \\ \bar{z} &= a_e C_{10} \end{aligned} \tag{3}$$

The normal practice is to define the spherical coordinate system such that the origin is located at the center of mass ($\bar{x} = \bar{y} = \bar{z} = 0$). Therefore, the first-degree terms C_{10}, C_{11}, and S_{11} all equal zero. With these definitions of the zero-degree and first-degree terms, the spherical harmonic model becomes[11]

$$V(r, \lambda, \phi) = \frac{\mu}{r} \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{a_e}{r} \right)^n \bar{P}_{nm}(sin\phi)(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \right] \tag{4}$$

For a large continuous body of mass the un-normalized unitless gravity coefficients are related to the mass distribution by[10]

$$C_{nm} = \frac{2 - \delta_{0m}}{M} \frac{(n-m)!}{(n+m)!} \int \left(\frac{r'}{a_e} \right)^n P_{nm} \sin(\phi') \cos(m\lambda') \rho(r', \lambda', \phi') d^3vol \tag{5}$$

$$S_{nm} = \frac{2 - \delta_{0m}}{M} \frac{(n-m)!}{(n+m)!} \int \left(\frac{r'}{a_e} \right)^n P_{nm} \sin(\phi') \sin(m\lambda') \rho(r', \lambda', \phi') d^3vol \tag{6}$$

where (r', λ', ϕ') are the spherical coordinates of each infinitesimal particle of mass, $\rho(r', \lambda', \phi')$ is density as a function of position within the body of mass, and d^3vol is the infinitesimal unit of volume which is integrated over the entire body of mass. For an infinitesimal particle of mass dm (i.e. point mass), the volume collapses to a point where the product of density and volume is equal to the mass at that point

$$\rho(r', \lambda', \phi') d^3vol = dm \tag{7}$$

For a system of i discrete point masses (m_i), equations (5) and (6) become

$$C_{nm} = \frac{2-\delta_{0m}}{M}\frac{(n-m)!}{(n+m)!}\sum_i \left(\frac{r_i'}{a_e}\right)^n P_{nm}\sin(\phi_i')\cos(m\lambda_i')m_i \tag{8}$$

$$S_{nm} = \frac{2-\delta_{0m}}{M}\frac{(n-m)!}{(n+m)!}\sum_i \left(\frac{r_i'}{a_e}\right)^n P_{nm}\sin(\phi_i')\sin(m\lambda_i')m_i \tag{9}$$

with the relationship

$$\sum_i m_i = M \tag{10}$$

Using equations (8) and (9), spherical harmonic gravity coefficients of a system of a finite number of point masses can be computed. The coefficients can then be normalized using equation (2).

To model the orbital motion of a spacecraft, the acceleration (force per mass) due to gravity must be computed. Gravitational acceleration is the gradient of the gravitational potential evaluated at the location of the spacecraft

$$\bar{a} = \nabla V \tag{11}$$

Many algorithms exist for computing acceleration using this relationship in cartesian coordinates

$$\bar{a} = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} \tag{12}$$

where the transformation from spherical to cartesian coordinates is

$$\begin{aligned} x &= r\cos\phi\cos\lambda \\ y &= r\cos\phi\sin\lambda \\ z &= r\sin\phi \end{aligned} \tag{13}$$

The gravitational torque acting on a spacecraft can be computed in the spacecraft body-fixed coordinate system from[1]

$$\bar{\tau} = \int \bar{\rho} \times G\bar{\rho}dm \tag{14}$$

where $\bar{\rho}$ is the location of each infinitesimal mass of the spacecraft and, in this case, G is

$$G \equiv B^T \frac{\partial \bar{a}}{\partial \bar{r}} B \tag{15}$$

(not to be confused with the universal gravitational constant). The matrix B relates the spacecraft body-fixed system to the planet-fixed system in which the potential is defined.

The divergence of gravitational acceleration is equal to zero. This is Laplace's equation, which in cartesian coordinates is

$$\nabla\bar{a} = \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{16}$$

This useful property of potential can serve as an additional validation of gravity algorithms.

As shown in this section, a set of gravity coefficients can be computed for a finite system of point masses. The coefficients can be put into a gravity potential model and the resulting acceleration and torque at a location external to the point mass system can be computed. Acceleration and torque can also be computed directly from the point masses using the basic principles of gravitational force in order to verify that the spherical harmonic algorithms are working properly.

POINT MASS SYSTEM

Because the goal of this work was to validate spherical harmonic acceleration and torque algorithms, the gravity coefficients employed did not need to truly represent any real gravitational body. However, for familiarity, a system of point masses was developed that produced accelerations and torques of the same scale encountered by a low-earth orbiting satellite. For the chosen point mass system (or "point mass planet"), the constants of the EGM96 gravity field were used:[12]

$$\begin{aligned} \mu &= 3.986004415 \times 10^{14}\ m^3/s^2 \\ a_e &= 6378136.3\ m \end{aligned} \tag{17}$$

Knowing that $\mu \equiv GM$ and assuming a value for G of

$$G = 6.673 \times 10^{-11}\ \frac{m^3}{kg \cdot s^2} \tag{18}$$

the total mass of the system of point masses was $M = \mu/G$.

It was decided that testing the recursive properties of the spherical harmonic algorithms to degree and order four (4x4) would be sufficient. If the acceleration computed from the spherical harmonic algorithms was equivalent to that computed directly from the point mass planet on the order of 10^{-15} or better (the approximate limit of double-precision arithmetic), it could be assumed that the spherical harmonic algorithms worked correctly with coefficients higher than degree four. However, in order to avoid truncation errors, the gravity coefficients to degree and order five (5x5) were computed and used. The point mass planet was developed such that the degree-five coefficients were several orders of magnitude less than the degree-four coefficients. Test results showed no change in acceleration or torque with the degree-five coefficients omitted. This confirmed the assumption that coefficients higher than degree four were numerically insignificant at all test locations.

A system of twelve point masses was configured by experimentation. The first six point masses were chosen to result in a center of mass at the origin and equal mass in the "northern" and "southern hemispheres." The north/south symmetry of mass caused the odd zonal coefficients $(m = 0)$ to be zero. This was unsuitable for evaluation of the spherical harmonic algorithms, so an additional six point masses were included such that more mass was positioned in the southern hemisphere (similar to Earth). These points were added to the system in pairs such that the center of mass remained at the origin of the coordinate system in spite of the unequal north/south mass distribution. In terms of radial distance from the origin, the center of mass of any two points is

$$c.o.m. = \frac{r_1 m_1 + r_2 m_2}{m_1 + m_2} \tag{19}$$

This assumes the masses are situated in diametrically opposing locations with respect to the origin. Setting equation (19) to zero and solving for r_2 yields

$$r_2 = -r_1 \frac{m_1}{m_2} \tag{20}$$

where the negative sign is dropped because radial distance is always a positive quantity. This is the radial position of the second point mass in each pair that kept the center of mass at the origin.

Table 1 shows the final configuration of the twelve point masses that were selected. This system resulted in extremely small degree-one gravity coefficients on the order of 10^{-20}, indicating that

Table 1 The point mass system ("point mass planet"). The radial distance of points 8, 10, and 12 were computed using equation (20), with the negative sign dropped.

Point	Mass(kg)	Lat.(deg)	Lon.(deg)	Radius(m)
1	$M/12$	45.0	0.0	4000.0
2	$M/12$	45.0	120.0	4000.0
3	$M/12$	45.0	240.0	4000.0
4	$M/12$	-45.0	180.0	4000.0
5	$M/12$	-45.0	300.0	4000.0
6	$M/12$	-45.0	60.0	4000.0
7	$0.8M/12$	23.0	73.0	4000.0
8	$1.2M/12$	-23.0	253.0	(0.8/1.2)4000.0
9	$0.6M/12$	77.0	303.0	4000.0
10	$1.4M/12$	-77.0	123.0	(0.6/1.4)4000.0
11	$0.6M/12$	51.0	12.0	4000.0
12	$1.4M/12$	-51.0	192.0	(0.6/1.4)4000.0

the center of mass of the system was very nearly located at the origin by equation (3). Therefore, the degree-one terms were considered numerically negligible and forced to be exactly zero for all computations. This was necessary because the JEOD Gottlieb spherical harmonic algorithms are based on this assumption and begin computing with the degree-two terms, as many algorithms do. The gravity coefficients of the point mass planet are shown in table 2.

A set of test locations was chosen (quasi-randomly) at which gravity acceleration and torque were to be computed for validation purposes. They include positions directly over the north and south poles, as well as positions nearly over the poles in order to test the singularity-free properties of the JEOD Gottlieb algorithm. A sample of the test locations is shown in table 3. Latitude is geocentric latitude, not geodetic. Additional locations can be used, keeping in mind that locations farther from the point mass system will be less sensitive to the gravity coefficients. Also, the spherical harmonic model breaks down for locations inside the effective radius (a_e) of the gravitational system.[9]

ACCELERATION ALGORITHM VALIDATION

Acceleration was computed at each test location (table 3) using the JEOD algorithm. The acceleration was also computed directly from the point mass system for comparison. The acceleration due to a single point mass can be computed from Newton's second law of motion and his law of universal gravitation[13]

$$\bar{F} = m\bar{a} = G\frac{m_i m}{r_i^2}\frac{\bar{r}_i}{r_i} \tag{21}$$

where, in this case, the vector $\bar{r}_i$ points from the test point location towards the attracting point mass (m_i). The total acceleration due to a system of point masses is the sum of the individual acceleration

Table 2 Normalized gravity coefficients of the point mass planet. The associated gravity constants are shown in equation (17).

n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$
0	0	1.000000000000000E+00	0.000000000000000E+00
1	0	0.000000000000000E+00	0.000000000000000E+00
1	1	0.000000000000000E+00	0.000000000000000E+00
2	0	3.340052631095518E-08	0.000000000000000E+00
2	1	1.656759356170753E-08	9.855566979885049E-09
2	2	-8.176775024351827E-09	9.269248429468812E-09
3	0	1.759117523226707E-12	0.000000000000000E+00
3	1	3.832276836797578E-12	-1.471910512326285E-12
3	2	8.885825377841302E-14	1.828488643978193E-12
3	3	-1.081882767554426E-12	-9.045529521634635E-13
4	0	-9.608634091968108E-15	0.000000000000000E+00
4	1	1.532755741714554E-15	-3.539212041089171E-15
4	2	1.514697168098408E-15	4.836572455402800E-16
4	3	1.212195117185279E-14	-1.135418848925474E-15
4	4	1.098630790621571E-15	-1.950021729407439E-15
5	0	7.689778516079722E-19	0.000000000000000E+00
5	1	5.103903289685401E-19	-1.270409281937703E-18
5	2	1.288578200168745E-18	-3.356781430612713E-19
5	3	4.127243225918551E-19	3.200323268358486E-19
5	4	6.153203543016437E-19	-6.131745252702640E-19
5	5	7.718691941720866E-19	1.485405551920984E-19

Table 3 Test locations. Latitude is geocentric (not geodetic).

Point	Lat. (deg)	Lon. (deg)	Radius (m)
1	90.00	120.00	6800000.0
2	89.99	353.00	6800000.0
3	73.00	9.00	6800000.0
4	13.00	100.00	6800000.0
5	0.00	0.00	6800000.0
6	0.00	180.00	6800000.0
7	-26.00	210.00	6800000.0
8	-52.00	310.00	6800000.0
9	-89.99	10.00	6800000.0
10	-90.00	160.00	6800000.0

vectors

$$\bar{a} = G \sum_i \frac{m_i}{r_i^3} \bar{r}_i \tag{22}$$

Table 4 compares the acceleration vectors computed from the JEOD Gottlieb algorithm and equation (22). Differences are on the order of $10^{-15} m/s^2$, indicating that the acceleration algorithms were correctly implemented. The JEOD gravity acceleration algorithm was therefore considered validated.

Although the goal of this effort was the validation of the acceleration algorithm, a secondary validation was made by comparing the gravitational potential computed by the algorithm with that computed directly from the point mass planet at each test location. The total gravitational potential due to a system of discrete point masses m_i is

$$V = G \sum_i \frac{m_i}{r_i} \tag{23}$$

Table 5 compares the gravitational potential computed from the JEOD Gottlieb algorithm and equation (23). Differences are on the order of $10^{-8} m^2/s^2$, fifteen orders of magnitude smaller than the computed potential, further validating the JEOD Gottlieb algorithm.

An additional validation of the algorithm was to demonstrate that the divergence of the acceleration was numerically zero at each test point location (equation 16). This quantity, as computed by the JEOD Gottlieb, was verified to be on the order of 10^{-19} or smaller for each test location.

TORQUE ALGORITHM VALIDATION

To validate the JEOD Gottlieb gravity torque algorithm, a fictitious spacecraft model consisting of three point masses was developed. To fully test the algorithm, the point masses were chosen such that the corresponding inertia matrix did not have any zero elements and had a mix of positive and negative products of inertia.[14] The spacecraft point masses are defined in table 6. The corresponding inertia matrix is

$$I = \begin{bmatrix} 630.00 & -277.50 & 372.50 \\ -277.50 & 773.75 & 285.00 \\ 372.50 & 285.00 & 573.75 \end{bmatrix} kg \cdot m^2 \tag{24}$$

For validation purposes, a pitch-yaw-roll (2-3-1) Euler sequence was assumed. Attitude angles of pitch = 20.0 deg, yaw = 30.0 deg, and roll = 40.0 deg were used for all test cases. These angles were treated as the attitude of the spacecraft relative to the planet-fixed system because the gravity field is computed in that system. In actual practice, transformation to or from some other system (i.e. planet-centered-inertial) might be needed. Any Euler sequence can be used as long as the sequence is used consistently throughout the validation process.

Torque on the spacecraft was computed at each test point location (table 3) using the JEOD Gottlieb algorithm. The torque was also computed directly from the point mass planet. The total force of gravity acting on each of the three spacecraft point masses was computed in the planet-fixed frame using Newton's law of universal gravitation, equation (21). The individual force vectors were summed to compute the total force on each spacecraft mass and transformed into the spacecraft body-fixed system using the attitude Euler angles. The torque on the spacecraft was computed for each point mass using the familiar equation

$$\bar{\tau} = \bar{r} \times \bar{F} \tag{25}$$

Table 4 Results comparing acceleration vectors computed by the spherical harmonic algorithms vs. the acceleration vectors computed directly from the point mass planet.

Test Point	Sph. Harm. Accel. (m/s^2)	Pt. Mass Accel. (m/s^2)	Difference $\times 10^{-15}$
1	0.000000486802168	0.000000486802168	0
	0.000000289410851	0.000000289410851	0
	-8.620253461893439	-8.620253461893435	4
2	-0.001492817135365	-0.001492817135365	0
	0.000183644115369	0.000183644115369	0
	-8.620253330912043	-8.620253330912043	0
3	-2.489289081277077	-2.489289081277078	1
	-0.394264356254478	-0.394264356254478	0
	-8.243589555959909	-8.243589555959911	2
4	1.458526184452704	1.458526184452703	1
	-8.271710450597217	-8.271710450597213	4
	-1.939134122596513	-1.939134122596512	1
5	-8.620250552438606	-8.620250552438607	1
	0.000000272233971	0.000000272233971	0
	0.000000486579417	0.000000486579417	0
6	8.620250552920716	8.620250552920718	2
	-0.000000272280796	-0.000000272280796	0
	-0.000000486672261	-0.000000486672261	0
7	6.709819312482277	6.709819312482278	1
	3.873915625470781	3.873915625470781	0
	3.778868973925642	3.778868973925643	1
8	-3.411375851766505	-3.411375851766505	0
	4.065518470195100	4.065518470195100	0
	6.792850871083239	6.792850871083238	1
9	-0.001482147809056	-0.001482147809056	0
	-0.000261546360761	-0.000261546360761	0
	8.620253329964955	8.620253329964953	2
10	-0.000000486448833	-0.000000486448833	0
	-0.000000289546561	-0.000000289546561	0
	8.620253461628582	8.620253461628582	0

Table 5 Gravitational potential computed from the spherical harmonic algorithm and directly from the point mass planet. Test locations are listed in table 3.

Test Point	Sph. Harm Potential (m^2/s^2)	Pt. Mass Potential (m^2/s^2)	Difference $\times 10^{-8}$
1	5.861771583708038E+07	5.861771583708036E+07	2
2	5.861771583761174E+07	5.861771583761174E+07	0
3	5.861771630144398E+07	5.861771630144399E+07	1
4	5.861771107877840E+07	5.861771107877839E+07	1
5	5.861770924252718E+07	5.861770924252718E+07	0
6	5.861770924334677E+07	5.861770924334677E+07	0
7	5.861771300445098E+07	5.861771300445100E+07	2
8	5.861771305539908E+07	5.861771305539910E+07	2
9	5.861771583600169E+07	5.861771583600169E+07	0
10	5.861771583663011E+07	5.861771583663011E+07	0

Table 6 Spacecraft point mass system. Coordinates are expressed in the spacecraft body-fixed system.

Point	Mass (kg)	x (m)	y (m)	z (m)
1	50.0	1.625	1.250	-1.750
2	20.0	-2.375	-1.750	3.250
3	10.0	-3.375	-2.750	2.250

where, in this case, the vector $\bar{r}$ is the position of each spacecraft point pass with respect to each mass in the point mass system. The torque vectors were then summed to get the total torque vector. The total torque at each test point was compared to the torque computed from the spherical harmonic algorithm. The results are shown in table 7. The difference in the torque vector components was typically on the order of six (or more) orders of magnitude smaller than the torque magnitude. This was considered acceptable in light of the linearity approximations used in developing the Gottlieb algorithm.[1] The JEOD gravity torque algorithm was therefore considered validated.*

*The torque algorithm will be officially released in a near-future version of JEOD.

Table 7 Results comparing torque vectors computed by the spherical harmonic algorithm vs. torque vectors computed directly from the point mass planet.

Test Point	Sph. Harm. Torque $(N \cdot m)$	Pt. Mass Torque $(N \cdot m)$	Difference
1	-6.576995906413200E-04	-6.576998403033940E-04	2.5E-10
	1.269164209592045E-04	1.269165028503494E-04	8.2E-11
	-4.723482089916854E-04	-4.723484289002045E-04	2.2E-10
2	-6.577475179730652E-04	-6.577477676614763E-04	2.5E-10
	1.270487433086179E-04	1.270488252202995E-04	8.2E-11
	-4.722975646974573E-04	-4.722977844267007E-04	2.2E-10
3	-4.667054619988188E-04	-4.667057505685079E-04	2.9E-10
	2.230918182780193E-04	2.230919879480098E-04	1.7E-10
	-2.308991703765578E-04	-2.308993437338813E-04	1.7E-10
4	9.643230821702463E-04	9.643234462686223E-04	3.6E-10
	3.894826293977236E-04	3.894828155921459E-04	1.9E-10
	1.192559213912787E-03	1.192559729815912E-03	5.2E-10
5	5.947635780445906E-05	5.947628430647001E-05	7.3E-11
	-1.824208387207120E-04	-1.824204566105436E-04	3.8E-10
	-1.311220703920095E-04	-1.311218693729188E-04	2.0E-10
6	5.947635779717441E-05	5.947643117565349E-05	7.3E-11
	-1.824208387253428E-04	-1.824212203018760E-04	3.8E-10
	-1.311220704109403E-04	-1.311222710853599E-04	2.0E-10
7	7.585422706336786E-04	7.585422953582111E-04	2.5E-11
	-1.035277845536193E-03	-1.035277890196085E-03	4.5E-11
	-5.060880799457042E-05	-5.060879752250003E-05	1.0E-11
8	5.494210701133441E-04	5.494212856262948E-04	2.2E-10
	1.890337404179465E-04	1.890338145713599E-04	7.4E-11
	6.827228269632132E-04	6.827230673707163E-04	2.4E-10
9	-6.577823003575999E-04	-6.577820505526688E-04	2.5E-10
	1.268171069274990E-04	1.268170250483536E-04	8.2E-11
	-4.724987427239774E-04	-4.724985225834644E-04	2.2E-10
10	-6.576995906610781E-04	-6.576993405928988E-04	2.5E-10
	1.269164210444634E-04	1.269163389849837E-04	8.2E-11
	-4.723482089531574E-04	-4.723479888752991E-04	2.2E-10

SUMMARY

A fictitious point mass planet was developed (table 1) from which spherical harmonic gravity coefficients were computed and subsequently normalized (table 2). These coefficients were input to the JEOD Gottlieb algorithms, and acceleration and torque were computed at a series of test locations (table 3). Acceleration and torque were also computed from the basic principles of mass, force, acceleration, and torque. Acceleration differences at all test locations were on the order of $10^{-15} m/s^2$ (table 4). Values of torque matched to six or more significant figures (table 7). The slight torque differences were attributed to the linear approximation made in the derivation of the Gottlieb algorithm.[1] Based on the near equivalence of the results between the two methods (within the expected level of numerical precision), the JEOD spherical harmonic gravity acceleration and torque algorithms were considered validated.

The technique presented in this paper can be used validate any general spherical harmonic gravity acceleration and/or torque algorithms for astrodynamics engineering or space mission operations purposes. The tabulated gravity coefficients, associated constants, and test point locations can be used and the results compared directly to those in this paper, or a different set can be created using the presented technique and equations.

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