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Predicting the Hubble Parameter and the Age of the Universe using Supernovae Ia Data

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Abstract
This project utilizes the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant (H_0) and estimate the age of the universe. By fitting a flat ΛCDM cosmological model to the observational data, we derive the best-fit values for H_0 and matter density parameter (Ω_m). Our analysis includes plotting the Hubble diagram, computing residuals, and exploring the effect of fixing Ω_m. Additionally, we compare the inferred H_0 values for low-z and high-z subsamples. Our results provide insights into the cosmological parameters and highlight potential tensions between local and global measurements of cosmic expansion.

Contents

1. Introduction

Type Ia supernovae serve as standard candles in cosmology, enabling us to measure the expansion history of the universe. These stellar explosions exhibit highly uniform peak luminosities, making them ideal for probing the vast distances of the cosmos.

The Pantheon+SH0ES dataset provides a comprehensive collection of supernova observations, allowing us to constrain cosmological models. This dataset combines the Pantheon sample with the SH0ES (Supernovae and H_0 for the Equation of State) calibration, offering a powerful tool for exploring the universe's expansion history.

In this project, we analyze the Pantheon+SH0ES dataset to derive the Hubble constant (H_0), a fundamental parameter describing the universe's current expansion rate. By fitting a flat ΛCDM cosmological model to the data, we can also estimate the age of the universe and explore the properties of dark energy.

2. A Discussion On The ΛCDM Cosmological Model

2.1 Spatially-Flat ΛCDM Model (Standard Cosmological Model)

In presence of the cosmological constant term Λ, Einstein equation of general relativity is given by,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1}$$

Here, $g_{\mu\nu}$ represents the metric tensor, $R_{\mu\nu}$ and R denote the Ricci tensor and scalar, $T_{\mu\nu}$ is the stress-energy tensor.
Λ is known as the cosmological constant, and G is the Newtonian constant of gravitation.
For perfect fluid,

$$T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} \tag{2}$$

where ρ represents the energy density and p the pressure.
$\vec{u}$ is the 4-velocity vector field of the fluid.

Considering spatial homogeneity, from Friedmann equations, with $\Lambda \neq 0$, we may write,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K^2}{a^2} \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{4}$$

where, a(t) is the scale factor of the universe and K^2 represents the curvature of spatial hypersurfaces (having 3 values, e.g., -1, 0, or +1, corresponding to hyperbolic, flat (Euclidean), and spherical geometry respectively).
The Equation of state is given by, p = p(ρ) = $\omega\rho$
Here ω represents the dimensionless equation-of-state parameter.
$H(t) = \frac{\dot{a}(t)}{a(t)}$, is known as Hubble parameter. H_0 is the Hubble constant which represents the present value of the Hubble parameter.
The observable Redshift, z is connected to the scale factor a(t) through, $z = \frac{a_0}{a} - 1$
The present value of the density parameters are given by,

$$\Omega_{m0} = \frac{8\pi G\rho_0}{3H_0^2},\ \Omega_{K0} = -\frac{K^2}{(H_0 a_0)^2},\ \Omega_\Lambda = \frac{\Lambda}{3H_0^2} \tag{5}$$

where Ω_{m0} is the the non-relativistic (baryonic and cold dark) matter density parameter, Ω_{K0} is the spatial curvature energy density parameter, Ω_Λ is the cosmological constant energy density parameter, at present time.

In ΛCDM model (Peebles 1984), dark energy can be described as time-independent energy density with negative pressure (modeled as a spatially homogeneous fluid), having equation of state parameter, $w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1$
p_Λ and ρ_Λ represent the homogeneous parts of the pressure and energy density, respectively.
In this case, the Hubble parameter from the Friedmann equation, takes the following form,

$$H(z, H_0, \mathbf{P}_\Lambda) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda + (1 - \Omega_{m0} - \Omega_\Lambda)(1+z)^2} \tag{6}$$

taking, $\Omega_{K0} = 1 - \Omega_{m0} - \Omega_\Lambda$. The ΛCDM model is characterized by the cosmological parameters: $\mathbf{P}_\Lambda = (\Omega_{m0}, \Omega_\Lambda)$.

Observations suggest that the universe is well-described by a spatially-flat ΛCDM model, with the cosmological constant (Λ) and non-relativistic matter (baryonic and cold dark matter) making up approximately 70% ($\Omega_\Lambda \approx 0.7$) and 30% ($\Omega_{m0} \approx 0.3$) of the universe's energy density, respectively. Baryonic matter constitutes only a small fraction ($\Omega_b \approx 0.05$) of the universe.

The spatially-flat ΛCDM model, widely regarded as the "standard model", is characterized by the Hubble parameter:

$$H(z, H_0, \Omega_{m0}) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})}. \tag{7}$$

In the context of a flat cosmological model, where the curvature density parameter $\Omega_k = 0$, the dark energy density parameter Ω_Λ can be expressed as: $\Omega_\Lambda = 1 - \Omega_{m0}$.
This model is characterized by a single parameter, Ω_{m0}, with H_0 representing the present-day value of the Hubble parameter.
we consider, the dimensionless Hubble parameter as, $E(z) = \frac{H(z)}{H_0}$

In the spatially flat ΛCDM model, the expansion history of the universe is characterized by the dimensionless Hubble parameter $E(z)$, defined as:

$$E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})}, \tag{8}$$

2.2 Luminosity distance & Distance modulus

The flux (F) received from a source of luminosity (L) at a distance (d_L) is given by the inverse-square law:

$$F = \frac{L}{4\pi d_L^2} \tag{9}$$

Solving for d_L gives:

$$d_L = \sqrt{\frac{L}{4\pi F}} \tag{10}$$

In an expanding universe, several effects must be considered: the observed photon energy is reduced by a factor of $(1+z)$ due to redshift, the arrival rate of photons is reduced by time dilation (also $(1+z)$), and the area over which the flux is spread increases because of cosmic expansion. For a source at redshift z, the luminosity distance is defined as:

$$d_L(z) = (1+z)\, r(z) \tag{11}$$

where $r(z)$ is the comoving distance to the source.

For a spatially flat universe ($k=0$), the comoving distance is given by:

$$r(z) = c \int_0^z \frac{dz'}{H(z')} \tag{12}$$

where $H(z)$ is the Hubble parameter at redshift z.

Thus, the luminosity distance becomes:

$$d_L(z) = (1+z)\, c \int_0^z \frac{dz'}{H(z')} \tag{13}$$

Alternatively, expressing $H(z)$ in terms of the dimensionless Hubble parameter $E(z) = H(z)/H_0$, we have:

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')} \tag{14}$$

The distance modulus is a fundamental concept in astronomy for determining the distance to celestial objects. It is defined as the difference between an object's apparent magnitude (how bright it appears from Earth, m) and its absolute magnitude (how bright it would appear at a standard distance of 10 parsecs, M):

$$\mu = m - M \tag{15}$$

In cosmological studies, especially when using Type Ia supernovae as standard candles, astronomers often work with the **theoretical distance modulus**:

$$\mu_{\text{th}}(z) = 5 \log_{10} \left[\frac{d_L(z)}{\text{Mpc}} \right] + 25 \tag{16}$$

where:

- $\mu_{\text{th}}(z)$ is the predicted distance modulus at redshift z.
- $d_L(z)$ is the luminosity distance to the object at redshift z, measured in megaparsecs (Mpc).

2.3 The age of the Universe

The age of the universe at a given redshift is a fundamental quantity in cosmology, providing insight into the timeline of cosmic evolution and the formation of large-scale structures. In the context of a spatially flat Lambda Cold Dark Matter (ΛCDM) cosmological model, the age of the universe as a function of redshift can be derived from the Friedmann equations and the definition of the Hubble parameter. The Hubble parameter is defined as,

$$H(t) = \frac{\dot{a}}{a},$$

where a is the scale factor. The cosmic time elapsed since the Big Bang up to a scale factor a is given by the integral:

$$t(a) = \int_0^a \frac{da'}{a' H(a')}.$$

To express this in terms of redshift, we utilize the relation $a = \frac{1}{1+z}$, from which it follows that,

$$\frac{da}{dz} = -\frac{1}{(1+z)^2}$$

and thus, $da = -\frac{1}{(1+z)^2} dz$.

Substituting these relations into the integral for $t(a)$, and changing the variable of integration from a' to z', the age of the universe at redshift z becomes,

$$t(z) = \int_z^\infty \frac{1}{(1+z')H(z')} dz'. \tag{17}$$

In a flat ΛCDM model, the Hubble parameter as a function of redshift is given by,

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda},$$

where H_0 is the Hubble constant, Ω_m is the present-day matter density parameter, and Ω_Λ is the present-day dark energy density parameter. Substituting this expression for $H(z)$ into the integral yields:

$$t(z) = \int_z^\infty \frac{1}{(1+z')H_0 \sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} dz'. \tag{18}$$

This integral provides the age of the universe at any redshift z in terms of the cosmological parameters and can be evaluated numerically for given values of H_0, Ω_m, and Ω_Λ. For $z = 0$, this expression gives the current age of the universe.

3. Data

We use the publicly available Pantheon+SH0ES dataset, which includes:

- Redshift values corrected for peculiar velocities and local effects,
- SH0ES-calibrated distance moduli (μ) and associated uncertainties.

Data are cleaned by removing entries with *irrelevant or invalid* values in key observational columns.

4. Results

This section presents the quantitative findings from our cosmological analysis using the Pantheon+SH0ES Type Ia supernova dataset, along with comparisons to external observational constraints.

4.1 Cosmological Parameter Estimates

The best-fit parameters from the flat ΛCDM model are:

- **Hubble constant:** $H_0 = 72.97 \pm 0.26$ km s^{-1} Mpc^{-1}
- **Matter density parameter:** $\Omega_m = 0.351 \pm 0.019$

These suggest a moderately dense matter content and an expansion rate consistent with late-universe (supernova-based) measurements.

4.2 Estimated Age of the Universe

Using the best-fit parameters, the age of the universe is computed as:

$$t_0 = 12.36 \text{ Gyr}$$

4.3 Fixed Ω_m Comparison

With Ω_m fixed at 0.30 to reduce parameter degeneracy, the resulting best-fit expansion rate is:

$$H_0 = 73.53 \pm 0.17 \text{ km s}^{-1}\text{Mpc}^{-1}, \quad t_0 = 12.92 \text{ Gyr}$$

4.4 Low-z vs. High-z Subsample Analysis

We divide the data at $z = 0.1$ to assess redshift-dependent variation in H_0:

Subsample	H_0 **(km s^{-1} Mpc^{-1})**
Low-z ($z < 0.1$)	73.01 ± 0.28
High-z ($z \geq 0.1$)	73.85 ± 0.22

Table 1. Hubble constant estimates for low and high-redshift subsets

The results suggest a mild increase at higher redshifts, although differences are consistent within 2σ.

4.5 Comparison with External Measurements

We compare our results to the SH0ES local measurements and the Planck cosmic microwave background (CMB) estimates:

Source	H_0 **(km/s/Mpc)**	Ω_m
SH0ES	72.97 ± 0.26	0.351
SH0ES (Ω_m fixed)	72.97 ± 0.26	0.30 (fixed)
Planck (CMB)	67.4 ± 0.5	0.30 (fixed)

Table 2. Comparison of H_0 and Ω_m values from SH0ES and Planck datasets.

Source	**Estimated Age (Gyr)**
SH0ES	12.36
SH0ES (Ω_m fixed)	12.92
Planck (CMB)	13.99

Table 3. Age estimates of the universe based on different cosmological assumptions.

4.6 Sensitivity of Cosmic Age to Ω_m

Higher matter densities correspond to younger inferred cosmic ages, due to enhanced early-time deceleration in the expansion history.
To illustrate the age-Ω_m dependence (with H_0 held constant), we compute:

Ω_m	**Age of Universe (Gyr)**
0.25	13.58
0.30	12.92
0.35	12.37

Table 4. Effect of matter density on cosmic age

4.7 Plots

All plots referenced in this study are provided in the attached Jupyter Notebook file for full transparency and reproducibility. Below is a brief summary of each visualization:

- **Hubble Diagram — Type Ia Supernovae**
  Observed distance moduli plotted against redshift (z) on a logarithmic scale. This forms the empirical backbone of our expansion rate analysis.
- **Residuals of Hubble Diagram Fit**
  Differences between observed and model-predicted distance moduli, highlighting the quality of fit and potential systematics.
- **Hubble Diagram with Best-Fit ΛCDM Model and 1σ Confidence Band**
  Best-fit theoretical curve overlaid on the data, with a shaded region illustrating the 1σ uncertainty due to parameter errors.

Please refer to the accompanying Jupyter Notebook for code, detailed annotations, and dynamic visualizations.

5. Conclusion

This study employed the Pantheon+SH0ES Type Ia supernova dataset to constrain key cosmological parameters within a flat ΛCDM framework. The Hubble constant was estimated as $H_0 = 72.97 \pm 0.26$ km s^{-1} Mpc^{-1}, consistent with local measurements but in tension with the lower value from Planck18 measurement (arXiv:1807.06209 [astro-ph.CO]). Assuming $\Omega_m = 0.30$, the corresponding age of the universe is approximately 12.92 Gyr. Sensitivity tests show that increasing Ω_m leads to a younger universe. Comparisons between low- and high-redshift subsets suggest mild variation in H_0, though results remain statistically compatible. Residual analysis confirms the adequacy of the flat ΛCDM model for the data. These findings reaffirm the utility of supernova cosmology and underscore the importance of resolving the Hubble tension in future cosmological studies.

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