Astronomy and Astrophysics Summer School 2025
A Summer Skill Training Internship Program
India Space Academy
Department of Space Education (ISW)
New Delhi - 110058, India
Email: info@isa.ac.in, contact@isa.ac.in

Internship Project Report (2025)

Predicting the Hubble Parameter and the Age of the Universe using Supernovae Ia Data

AVIK BANERJEE
M.Sc. (Physics), The University of Burdwan, Bardhaman – 713104, West Bengal, India
E-Mail: avik2020.phys@gmail.com

Abstract
This project utilizes the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant (H_0) and estimate the age of the universe. By fitting a flat ΛCDM cosmological model to the observational data, we derive the best-fit values for H_0 and matter density parameter (Ω_m). Our analysis includes plotting the Hubble diagram, computing residuals, and exploring the effect of fixing Ω_m. Additionally, we compare the inferred H_0 values for low-z and high-z subsamples. Our results provide insights into the cosmological parameters and highlight potential tensions between local and global measurements of cosmic expansion.

Contents

1. Introduction

Type Ia supernovae serve as standard candles in cosmology, enabling us to measure the expansion history of the universe. These stellar explosions exhibit highly uniform peak luminosities, making them ideal for probing the vast distances of the cosmos.

The Pantheon+SH0ES dataset provides a comprehensive collection of supernova observations, allowing us to constrain cosmological models. This dataset combines the Pantheon sample with the SH0ES (Supernovae and H_0 for the Equation of State) calibration, offering a powerful tool for exploring the universe's expansion history.

In this project, we analyze the Pantheon+SH0ES dataset to derive the Hubble constant (H_0), a fundamental parameter describing the universe's current expansion rate. By fitting a flat ΛCDM cosmological model to the data, we can also estimate the age of the universe and explore the properties of dark energy.

2. A Discussion On The ΛCDM Cosmological Model

2.1 Spatially-Flat ΛCDM Model (Standard Cosmological Model)

In presence of the cosmological constant term Λ, Einstein equation of general relativity is given by,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1}$$

Here, $g_{\mu\nu}$ represents the metric tensor, $R_{\mu\nu}$ and R denote the Ricci tensor and scalar, $T_{\mu\nu}$ is the stress-energy tensor.
Λ is known as the cosmological constant, and G is the Newtonian constant of gravitation.
For perfect fluid,

$$T^{\mu\nu} = (\rho + P) u^{\mu} u^{\nu} + P g^{\mu\nu} \tag{2}$$

where ρ represents the energy density and p the pressure.
$\vec{u}$ is the 4-velocity vector field of the fluid.

Considering spatial homogeneity, from Friedmann equations, with $\Lambda \neq 0$, we may write,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K^2}{a^2} \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{4}$$

where, a(t) is the scale factor of the universe and K^2 represents the curvature of spatial hypersurfaces (having 3 values, e.g., -1, 0, or +1, corresponding to hyperbolic, flat (Euclidean), and spherical geometry respectively).
The Equation of state is given by, p = p(ρ) = $\omega\rho$
Here ω represents the dimensionless equation-of-state parameter.
$H(t) = \frac{\dot{a}(t)}{a(t)}$, is known as Hubble parameter. H_0 is the Hubble constant which represents the present value of the Hubble parameter.
The observable Redshift, z is connected to the scale factor a(t) through, $z = \frac{a_0}{a} - 1$
The present value of the density parameters are given by,

$$\Omega_{m0} = \frac{8\pi G \rho_0}{3H_0^2},\ \Omega_{K0} = -\frac{K^2}{(H_0 a_0)^2},\ \Omega_{\Lambda} = \frac{\Lambda}{3H_0^2} \tag{5}$$

where Ω_{m0} is the the non-relativistic (baryonic and cold dark) matter density parameter, Ω_{K0} is the spatial curvature energy density parameter, Ω_{Λ} is the cosmological constant energy density parameter, at present time.

In ΛCDM model (Peebles 1984), dark energy can be described as time-independent energy density with negative pressure (modeled as a spatially homogeneous fluid), having equation of state parameter, $w_{\Lambda} = \frac{p_{\Lambda}}{\rho_{\Lambda}} = -1$
p_{Λ} and ρ_{Λ} represent the homogeneous parts of the pressure and energy density, respectively.
In this case, the Hubble parameter from the Friedmann equation, takes the following form,

$$H(z, H_0, \mathbf{P}_{\Lambda}) = H_0 \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda} + (1 - \Omega_{m0} - \Omega_{\Lambda})(1+z)^2} \tag{6}$$

taking, $\Omega_{K0} = 1 - \Omega_{m0} - \Omega_{\Lambda}$. The ΛCDM model is characterized by the cosmological parameters: $\mathbf{P}_{\Lambda} = (\Omega_{m0}, \Omega_{\Lambda})$.

Observations suggest that the universe is well-described by a spatially-flat ΛCDM model, with the cosmological constant (Λ) and non-relativistic matter (baryonic and cold dark matter) making up approximately 70% ($\Omega_{\Lambda} \approx 0.7$) and 30% ($\Omega_{m0} \approx 0.3$) of the universe's energy density, respectively. Baryonic matter constitutes only a small fraction ($\Omega_b \approx 0.05$) of the universe.

The spatially-flat ΛCDM model, widely regarded as the "standard model", is characterized by the Hubble parameter:

$$H(z, H_0, \Omega_{m0}) = H_0 \sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})}. \tag{7}$$

In the context of a flat cosmological model, where the curvature density parameter $\Omega_k = 0$, the dark energy density parameter Ω_{Λ} can be expressed as: $\Omega_{\Lambda} = 1 - \Omega_{m0}$.
This model is characterized by a single parameter, Ω_{m0}, with H_0 representing the present-day value of the Hubble parameter.
we consider, the dimensionless Hubble parameter as, $E(z) = \frac{H(z)}{H_0}$

In the spatially flat ΛCDM model, the expansion history of the universe is characterized by the dimensionless Hubble parameter $E(z)$, defined as:

$$E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})}, \tag{8}$$

2.2 Luminosity distance & Distance modulus

The flux (F) received from a source of luminosity (L) at a distance (d_L) is given by the inverse-square law:

$$F = \frac{L}{4\pi d_L^2} \tag{9}$$

Solving for d_L gives:

$$d_L = \sqrt{\frac{L}{4\pi F}} \tag{10}$$

In an expanding universe, several effects must be considered: the observed photon energy is reduced by a factor of $(1+z)$ due to redshift, the arrival rate of photons is reduced by time dilation (also $(1+z)$), and the area over which the flux is spread increases because of cosmic expansion. For a source at redshift z, the luminosity distance is defined as:

$$d_L(z) = (1+z)\,r(z) \tag{11}$$

where $r(z)$ is the comoving distance to the source.

For a spatially flat universe ($k=0$), the comoving distance is given by:

$$r(z) = c\int_0^z \frac{dz'}{H(z')} \tag{12}$$

where $H(z)$ is the Hubble parameter at redshift z.

Thus, the luminosity distance becomes:

$$d_L(z) = (1+z)\,c\int_0^z \frac{dz'}{H(z')} \tag{13}$$

Alternatively, expressing $H(z)$ in terms of the dimensionless Hubble parameter $E(z) = H(z)/H_0$, we have:

$$d_L(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')} \tag{14}$$

The distance modulus is a fundamental concept in astronomy for determining the distance to celestial objects. It is defined as the difference between an object's apparent magnitude (how bright it appears from Earth, m) and its absolute magnitude (how bright it would appear at a standard distance of 10 parsecs, M):

$$\mu = m - M \tag{15}$$

In cosmological studies, especially when using Type Ia supernovae as standard candles, astronomers often work with the **theoretical distance modulus**:

$$\mu_{\text{th}}(z) = 5\log_{10}\left[\frac{d_L(z)}{\text{Mpc}}\right] + 25 \tag{16}$$

where:

- $\mu_{\text{th}}(z)$ is the predicted distance modulus at redshift z.
- $d_L(z)$ is the luminosity distance to the object at redshift z, measured in megaparsecs (Mpc).

2.3 The age of the Universe

The age of the universe at a given redshift is a fundamental quantity in cosmology, providing insight into the timeline of cosmic evolution and the formation of large-scale structures. In the context of a spatially flat Lambda Cold Dark Matter (ΛCDM) cosmological model, the age of the universe as a function of redshift can be derived from the Friedmann equations and the definition of the Hubble parameter. The Hubble parameter is defined as,

$$H(t) = \frac{\dot{a}}{a},$$

where a is the scale factor. The cosmic time elapsed since the Big Bang up to a scale factor a is given by the integral:

$$t(a) = \int_0^a \frac{da'}{a'H(a')}.$$

To express this in terms of redshift, we utilize the relation $a = \frac{1}{1+z}$, from which it follows that,

$$\frac{da}{dz} = -\frac{1}{(1+z)^2}$$

and thus, $da = -\frac{1}{(1+z)^2}dz$.

Substituting these relations into the integral for $t(a)$, and changing the variable of integration from a' to z', the age of the universe at redshift z becomes,

$$t(z) = \int_z^\infty \frac{1}{(1+z')H(z')}dz'. \tag{17}$$

In a flat ΛCDM model, the Hubble parameter as a function of redshift is given by,

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda},$$

where H_0 is the Hubble constant, Ω_m is the present-day matter density parameter, and Ω_Λ is the present-day dark energy density parameter. Substituting this expression for $H(z)$ into the integral yields:

$$t(z) = \int_z^\infty \frac{1}{(1+z')H_0\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}dz'. \tag{18}$$

This integral provides the age of the universe at any redshift z in terms of the cosmological parameters and can be evaluated numerically for given values of H_0, Ω_m, and Ω_Λ. For $z=0$, this expression gives the current age of the universe.

3. Data

We use the publicly available Pantheon+SH0ES dataset, which includes:

- Redshift values corrected for peculiar velocities and local effects,
- SH0ES-calibrated distance moduli (μ) and associated uncertainties.

Data are cleaned by removing entries with *irrelevant or invalid* values in key observational columns.

4. Results

This section presents the quantitative findings from our cosmological analysis using the Pantheon+SH0ES Type Ia supernova dataset, along with comparisons to external observational constraints.

4.1 Cosmological Parameter Estimates

The best-fit parameters from the flat ΛCDM model are:

- **Hubble constant:** $H_0 = 72.97 \pm 0.26$ km s^{-1} Mpc^{-1}
- **Matter density parameter:** $\Omega_m = 0.351 \pm 0.019$

These suggest a moderately dense matter content and an expansion rate consistent with late-universe (supernova-based) measurements.

4.2 Estimated Age of the Universe

Using the best-fit parameters, the age of the universe is computed as:

$$t_0 = 12.36 \text{ Gyr}$$

4.3 Fixed Ω_m Comparison

With Ω_m fixed at 0.30 to reduce parameter degeneracy, the resulting best-fit expansion rate is:

$$H_0 = 73.53 \pm 0.17 \text{ km s}^{-1}\text{Mpc}^{-1}, \quad t_0 = 12.92 \text{ Gyr}$$

4.4 Low-z vs. High-z Subsample Analysis

We divide the data at $z = 0.1$ to assess redshift-dependent variation in H_0:

Subsample	H_0 (km s^{-1} Mpc^{-1})
Low-z ($z < 0.1$)	73.01 ± 0.28
High-z ($z \geq 0.1$)	73.85 ± 0.22

Table 1. Hubble constant estimates for low and high-redshift subsets

The results suggest a mild increase at higher redshifts, although differences are consistent within 2σ.

4.5 Comparison with External Measurements

We compare our results to the SH0ES local measurements and the Planck cosmic microwave background (CMB) estimates:

Source	H_0 (km/s/Mpc)	Ω_m
SH0ES	72.97 ± 0.26	0.351
SH0ES (Ω_m fixed)	72.97 ± 0.26	0.30 (fixed)
Planck (CMB)	67.4 ± 0.5	0.30 (fixed)

Table 2. Comparison of H_0 and Ω_m values from SH0ES and Planck datasets.

Source	Estimated Age (Gyr)
SH0ES	12.36
SH0ES (Ω_m fixed)	12.92
Planck (CMB)	13.99

Table 3. Age estimates of the universe based on different cosmological assumptions.

4.6 Sensitivity of Cosmic Age to Ω_m

Higher matter densities correspond to younger inferred cosmic ages, due to enhanced early-time deceleration in the expansion history.
To illustrate the age-Ω_m dependence (with H_0 held constant), we compute:

Ω_m	Age of Universe (Gyr)
0.25	13.58
0.30	12.92
0.35	12.37

Table 4. Effect of matter density on cosmic age

4.7 Plots

All plots referenced in this study are provided in the attached Jupyter Notebook file for full transparency and reproducibility. Below is a brief summary of each visualization:

- **Hubble Diagram — Type Ia Supernovae**
  Observed distance moduli plotted against redshift (z) on a logarithmic scale. This forms the empirical backbone of our expansion rate analysis.
- **Residuals of Hubble Diagram Fit**
  Differences between observed and model-predicted distance moduli, highlighting the quality of fit and potential systematics.
- **Hubble Diagram with Best-Fit ΛCDM Model and 1σ Confidence Band**
  Best-fit theoretical curve overlaid on the data, with a shaded region illustrating the 1σ uncertainty due to parameter errors.

Please refer to the accompanying Jupyter Notebook for code, detailed annotations, and dynamic visualizations.

5. Conclusion

This study employed the Pantheon+SH0ES Type Ia supernova dataset to constrain key cosmological parameters within a flat ΛCDM framework. The Hubble constant was estimated as $H_0 = 72.97 \pm 0.26$ km s^{-1} Mpc^{-1}, consistent with local measurements but in tension with the lower value from Planck18 measurement (arXiv:1807.06209 [astro-ph.CO]). Assuming $\Omega_m = 0.30$, the corresponding age of the universe is approximately 12.92 Gyr. Sensitivity tests show that increasing Ω_m leads to a younger universe. Comparisons between low- and high-redshift subsets suggest mild variation in H_0, though results remain statistically compatible. Residual analysis confirms the adequacy of the flat ΛCDM model for the data. These findings reaffirm the utility of supernova cosmology and underscore the importance of resolving the Hubble tension in future cosmological studies.

6. Acknowledgements

I would like to express my sincere gratitude to the academic mentors and faculty members for their invaluable guidance, encouragement, and constructive feedback throughout this project. Special thanks to our lecturers, whose well-structured teaching and engaging discussions laid the foundation for this analysis. I am also deeply thankful to my fellow interns and peers for their productive conversations, and shared enthusiasm, all of which enriched this learning experience. Finally, I appreciate the broader scientific community whose work and data—particularly the Pantheon+SH0ES collaboration—made this research possible.

References

- Aghanim, N., et al. 2018, arXiv:1807.06209[astro-ph.CO]
- Aviles, A., et al. 2012, arXiv:1204.2007[astro-ph.CO]
- Carroll, S. (2004). Spacetime and Geometry: An Introduction to General Relativity. Addison-Wesley.
- de Cruz Pérez et al. (2024), arXiv:2404.19194[astro-ph.CO]
- Efstathiou & Gratton (2020). The evidence for a spatially flat Universe. MNRAS, 496(1), L91–L95
- Hogg, D. W. (1999), arXiv:astro-ph/9905116
- Hu, J. P., Wang, Y. Y., Hu, J., & Wang, F. Y. (2023), arXiv:2310.11727[astro-ph.CO]
- Peebles, P. J. E. (1971). Physical Cosmology. Princeton University Press.
- Peebles, P. J. E. (1984), ApJ, 284, 439
- Riess, A. G., et al. 2011, ApJ, 730, 119
- Scolnic, D., et al. (2018), arXiv:2112.03863[astro-ph.CO]
- Weinberg, S. (2008). Cosmology. Oxford University Press.
- Yang & Zhang (2010). The age problem in ΛCDM model, MNRAS, 407, 1835–1841

🛰️ Project - 2: Predicting the Hubble Parameter and the Age of the Universe using Supernovae Ia Data

Avik Banerjee

Email: avik2020.phys@gmail.com

In this Project, we'll analyze observational data from the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant H_0 and estimate the age of the universe. We will:

- Plot the Hubble diagram (distance modulus vs. redshift)
- Fit a cosmological model to derive H_0 and Ω_m
- Estimate the age of the universe
- Analyze residuals to assess the model
- Explore the effect of fixing Ω_m
- Compare low-z and high-z results

Let's get started!

📦 Getting Started: Setup and Libraries

Before we dive into the analysis, we need to import the necessary Python libraries:

- `numpy` , `pandas` — for numerical operations and data handling
- `matplotlib` — for plotting graphs
- `scipy.optimize.curve_fit` and `scipy.integrate.quad` — for fitting cosmological models and integrating equations
- `astropy.constants` and `astropy.units` — for physical constants and unit conversions

Before proceeding, we need to verify that the required libraries are installed. If not, we can install them with the following command:

```
pip install numpy pandas matplotlib scipy astropy

We may also add additional libraries later, depending on our specific requirements.
```

In [18]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.integrate import quad
from astropy.constants import c
from astropy import units as u
```

📁 Load the Pantheon+SH0ES Dataset

We now load the observational supernova data from the Pantheon+SH0ES sample. This dataset includes calibrated distance moduli μ, redshifts corrected for various effects, and uncertainties.

Instructions:

- Make sure the data file is downloaded from Pantheon dataset and available locally.
- We use `delim_whitespace=True` because the file is space-delimited rather than comma-separated.
- Commented rows (starting with `#`) are automatically skipped.

We will extract:

- `zHD` : Hubble diagram redshift
- `MU_SH0ES` : Distance modulus using SH0ES calibration
- `MU_SH0ES_ERR_DIAG` : Associated uncertainty

More detailed column names and the meanings can be referred here:

> Finally, we include a combined file of all the fitted parameters for each SN, before and after light-curve cuts are applied. This is in the format of a .FITRES file and has all the meta-information listed above along with the fitted SALT2 parameters. We show a screenshot of the release in Figure 7. Here, we give brief descriptions of each column. **CID** – name of SN. **CIDint** – counter of SNe in the sample. **IDSURVEY** – ID of the survey. **TYPE** – whether SN Ia or not – all SNe in this sample are SNe Ia. **FIELD** – if observed in a particular field. **CUTFLAG_SNANA** – any bits in light-curve fit flagged. **ERRFLAG_FIT** – flag in fit. **zHEL** – heliocentric redshift. **zHELERR** – heliocentric redshift error. **zCMB** – CMB redshift. **zCMBERR** – CMB redshift error. **zHD** – Hubble Diagram redshift. **zHDERR** – Hubble Diagram redshift error. **VPEC** – peculiar velocity. **VPECERR** – peculiar-velocity error. **MWEBV** – MW extinction. **HOST_LOGMASS** – mass of host. **HOST_LOGMASS_ERR** – error in mass of host. **HOST_sSFR** – sSFR of host. **HOST_sSFR_ERR** – error in sSFR of host. **PKMJDINI** – initial guess for PKMJD. **SNRMAX1** – First highest signal-to-noise ratio (SNR) of light curve. **SNRMAX2** – Second highest SNR of light curve. **SNRMAX3** – Third highest SNR of light curve. **PKMJD** – Fitted PKMJD. **PKMJDERR** –

In [19]:
```python
# Local file path
file_path = r"C:\Users\COSMOS\Downloads\Pantheon+SH0ES.dat"
# Load the dataset
df = pd.read_csv(file_path, delim_whitespace=True, comment="#")
```

```
C:\Users\COSMOS\AppData\Local\Temp\ipykernel_15296\3895081404.py:4: FutureWarning: The 'delim_whitespace' keyword in pd.read_csv is deprecated and will be removed in a future version. Use ``se
p='\s+'`` instead
  df = pd.read_csv(file_path, delim_whitespace=True, comment="#")
```

📃 Preview Dataset Columns

Before diving into the analysis, let's take a quick look at the column names in the dataset. This helps us verify the data loaded correctly and identify the relevant columns we'll use for cosmological modeling.

In [20]:
```python
# Preview the dataset columns
print(df.columns)
```

```
Index(['CID', 'IDSURVEY', 'zHD', 'zHDERR', 'zCMB', 'zCMBERR', 'zHEL',
       'zHELERR', 'm_b_corr', 'm_b_corr_err_DIAG', 'MU_SH0ES',
       'MU_SH0ES_ERR_DIAG', 'CEPH_DIST', 'IS_CALIBRATOR', 'USED_IN_SH0ES_HF',
       'c', 'cERR', 'x1', 'x1ERR', 'mB', 'mBERR', 'x0', 'x0ERR', 'COV_x1_c',
       'COV_x1_x0', 'COV_c_x0', 'RA', 'DEC', 'HOST_RA', 'HOST_DEC',
       'HOST_ANGSEP', 'VPEC', 'VPECERR', 'MWEBV', 'HOST_LOGMASS',
       'HOST_LOGMASS_ERR', 'PKMJD', 'PKMJDERR', 'NDOF', 'FITCHI2', 'FITPROB',
       'm_b_corr_err_RAW', 'm_b_corr_err_VPEC', 'biasCor_m_b',
       'biasCorErr_m_b', 'biasCor_m_b_COVSCALE', 'biasCor_m_b_COVADD'],
      dtype='object')
```

🧹 Clean and Extract Relevant Data

To ensure reliable fitting, we remove any rows that have missing values in key columns:

- `zHD` : redshift for the Hubble diagram
- `MU_SH0ES` : distance modulus
- `MU_SH0ES_ERR_DIAG` : uncertainty in the distance modulus

We then extract these cleaned columns as NumPy arrays to prepare for analysis and modeling.

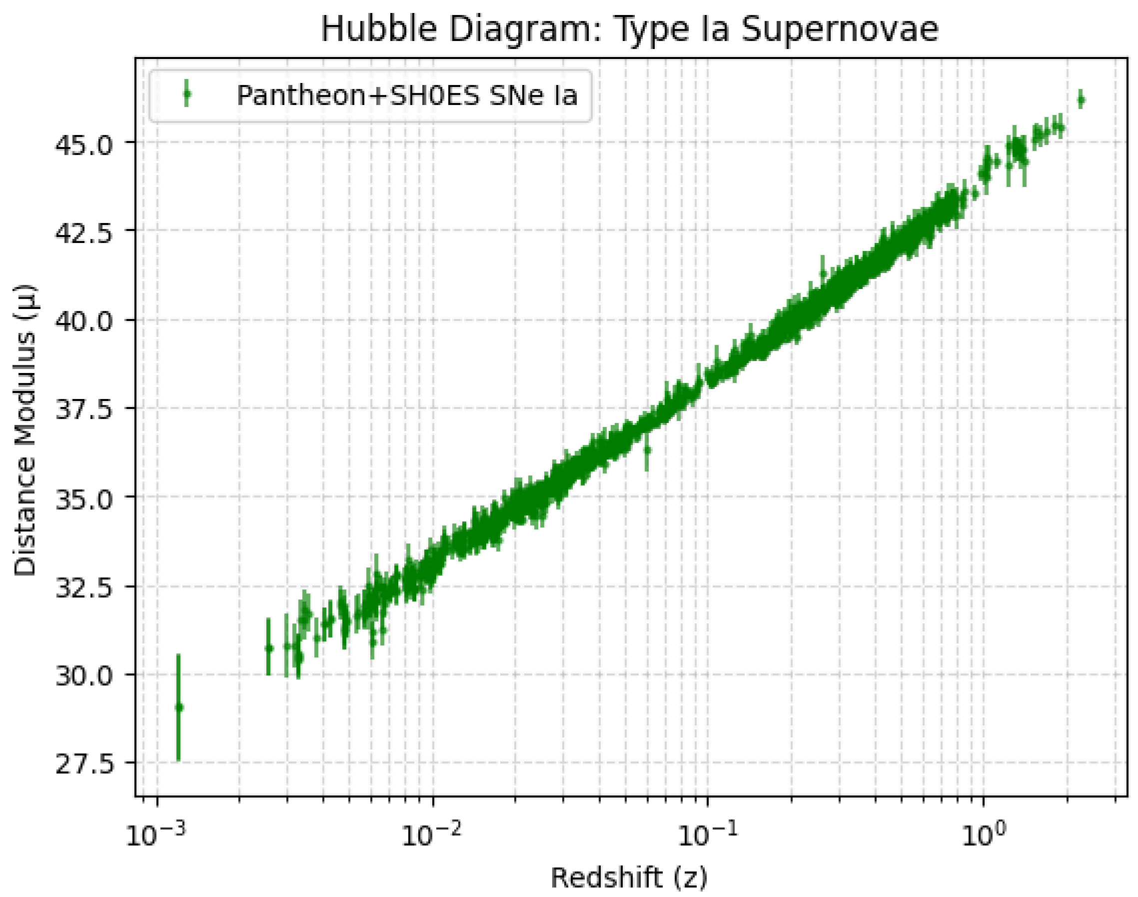
In [21]:
```python
# Remove rows with missing values in key columns
df_clean = df.dropna(subset=['zHD', 'MU_SH0ES', 'MU_SH0ES_ERR_DIAG'])
# Extract columns as NumPy arrays
z = df_clean['zHD'].values
mu = df_clean['MU_SH0ES'].values
mu_err = df_clean['MU_SH0ES_ERR_DIAG'].values
```

📈 Plot the Hubble Diagram

Let's visualize the relationship between redshift z and distance modulus μ, known as the Hubble diagram. This plot is a cornerstone of observational cosmology—it allows us to compare supernova observations with theoretical predictions based on different cosmological models.

We use a logarithmic scale on the redshift axis to clearly display both nearby and distant supernovae.

In [22]:
```python
plt.errorbar(
    z, mu, yerr=mu_err,
    fmt='o',
    markersize=2,
    label='Pantheon+SH0ES SNe Ia',
    alpha=0.6,
    color='green',      # Marker and line color
    ecolor='green'      # Error bar color
)
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram: Type Ia Supernovae')
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.legend()
plt.show()
```



Define the Cosmological Model

We now define the theoretical framework based on the flat $\Lambda\mathrm{CDM}$ model (read about the model in wikipedia if needed). This involves:

- The dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$$

- The distance modulus is:

$$\mu(z) = 5\log_{10}(d_L/\mathrm{Mpc}) + 25$$

- And the corresponding luminosity distance :

$$d_L(z) = (1+z)\cdot\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

These equations allow us to compute the expected distance modulus from a given redshift z, Hubble constant H_0, and matter density parameter Ω_m.

In [23]:
```
# E(z) for flat LCDM
def E(z, Omega_m):
    return np.sqrt(Omega_m * (1 + z)**3 + (1 - Omega_m))

# Luminosity distance in Mpc
def luminosity_distance(z, H0, Omega_m):
    # Integrand for comoving distance
    integrand = lambda zp: 1.0 / E(zp, Omega_m)
    # Vectorize integration for array input
```

```python
    if np.isscalar(z):
        integral, _ = quad(integrand, 0, z)
        Dc = (c.to('km/s').value / H0) * integral  # Mpc
    else:
        Dc = np.array([ (c.to('km/s').value / H0) * quad(integrand, 0, zi)[0] for zi in z ])
    return (1 + z) * Dc  # Mpc

# Theoretical distance modulus
def mu_theory(z, H0, Omega_m):
    dL = luminosity_distance(z, H0, Omega_m)
    return 5 * np.log10(dL) + 25
```

🔧 Fit the Model to Supernova Data

We now perform a non-linear least squares fit to the supernova data using our theoretical model for $\mu(z)$. This fitting procedure will estimate the best-fit values for the Hubble constant H_0 and matter density parameter Ω_m, along with their associated uncertainties.

We'll use:

- `curve_fit` from `scipy.optimize` for the fitting.
- The observed distance modulus (\mu), redshift (z), and measurement errors.

The initial guess is:

- $H_0 = 70\,\mathrm{km/s/Mpc}$
- $\Omega_m = 0.3$

In [24]:
```python
# Initial guess: H0 = 70, Omega_m = 0.3
p0 = [70, 0.3]

# Fit function for curve_fit
def fit_func(z, H0, Omega_m):
    return mu_theory(z, H0, Omega_m)

# Perform the fit
popt, pcov = curve_fit(fit_func, z, mu, sigma=mu_err, p0=p0, absolute_sigma=True, maxfev=10000)
H0_fit, Omega_m_fit = popt
H0_err, Omega_m_err = np.sqrt(np.diag(pcov))

print(f"Fitted H0 = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc")
print(f"Fitted Omega_m = {Omega_m_fit:.3f} ± {Omega_m_err:.3f}")
```

```
Fitted H0 = 72.97 ± 0.26 km/s/Mpc
Fitted Omega_m = 0.351 ± 0.019
```

📊 Corner Plot of Fit Parameters

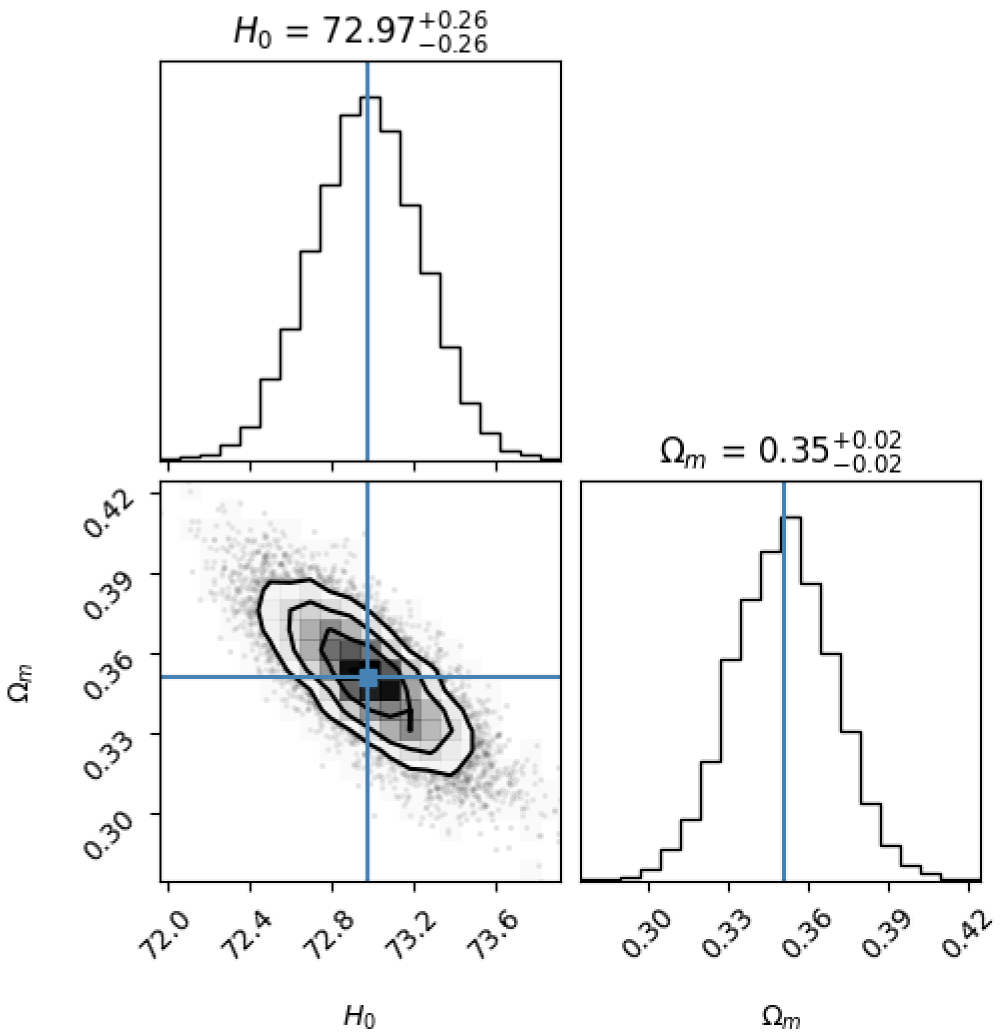
The code below generates a **corner plot** to visualize the joint and marginalized distributions of the best-fit cosmological parameters H_0 and Ω_m.

- It draws random samples from a **multivariate normal distribution** using the best-fit values (`popt`) and their **covariance matrix** (`pcov`).
- The `corner` library is used to create the plot showing:
  - 1D histograms for each parameter.
  - 2D confidence contours showing correlations between parameters.
- The `truths` argument overlays the best-fit values (H_0, Ω_m) as reference lines.

This plot is useful to understand **uncertainties** and **correlations** between the model parameters.

In [25]:
```
import emcee
import corner

# Generate samples from covariance for corner plot
samples = np.random.multivariate_normal(popt, pcov, 10000)
figure = corner.corner(samples, labels=[r"$H_0$", r"$\Omega_m$"],
                       truths=[H0_fit, Omega_m_fit],
                       show_titles=True, title_kwargs={"fontsize": 12})
plt.show()
```



⏳ Estimate the Age of the Universe

Now that we have the best-fit values of H_0 and Ω_m, we can estimate the age of the universe. This is done by integrating the inverse of the Hubble parameter over redshift:

$$t_0 = \int_0^\infty \frac{1}{(1+z)H(z)}\,dz$$

We convert H_0 to SI units and express the result in gigayears (Gyr). This provides an independent check on our cosmological model by comparing the estimated age to values from other probes like Planck CMB measurements.

In [26]:
```
def age_of_universe(H0, Omega_m):
    # Integrand for age
```

```python
    integrand = lambda z: 1.0 / ((1 + z) * E(z, Omega_m))
    integral, _ = quad(integrand, 0, np.inf)
    H0_SI = H0 * (u.km / u.s / u.Mpc).to(1/u.s)
    t0 = integral / H0_SI / (60*60*24*365.25*1e9)  # Convert seconds to Gyr
    return t0

t0 = age_of_universe(H0_fit, Omega_m_fit)
print(f"Estimated age of Universe: {t0:.2f} Gyr")
```

```
Estimated age of Universe: 12.36 Gyr
```

📊 Analyze Residuals

To evaluate how well our cosmological model fits the data, we compute the residuals:

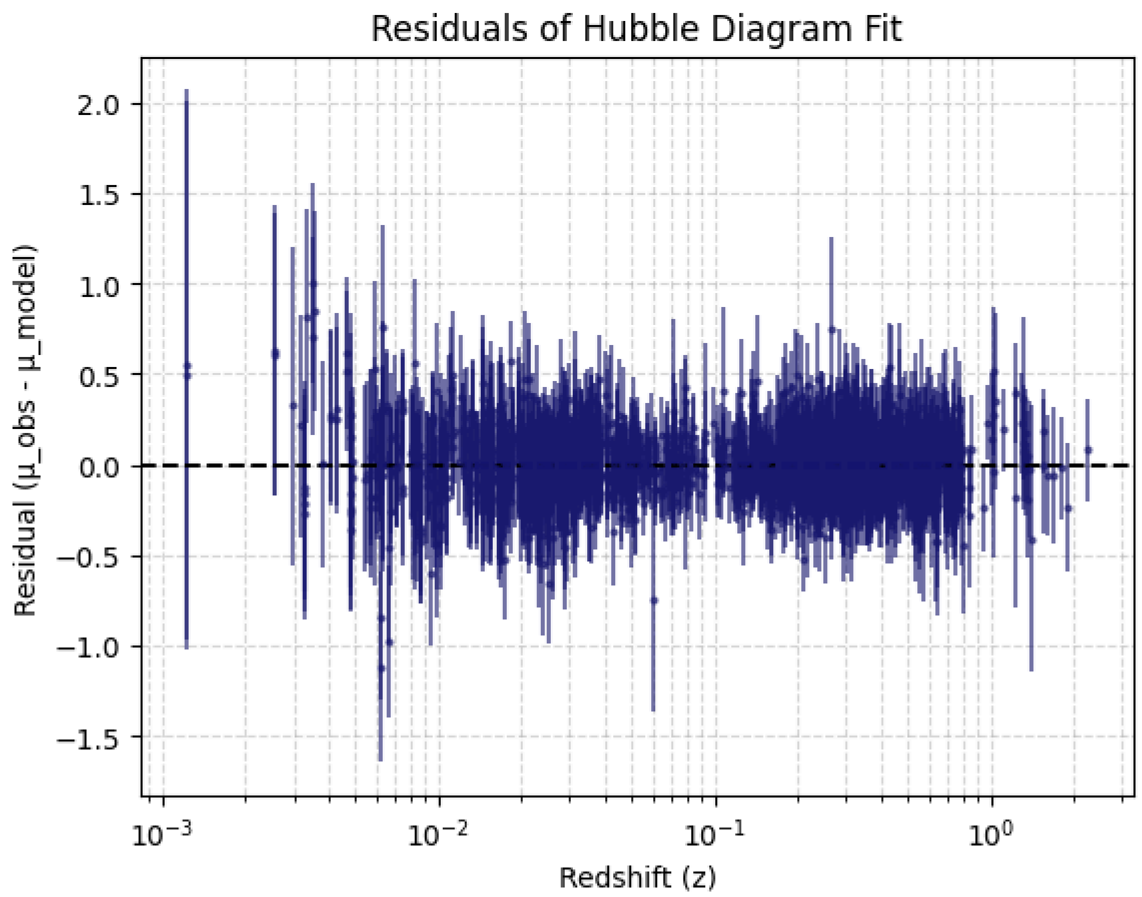
$$\text{Residual} = \mu_{\text{obs}} - \mu_{\text{model}}$$

Plotting these residuals against redshift helps identify any systematic trends, biases, or outliers. A good model fit should show residuals scattered randomly around zero without any significant structure.

In [27]:

```python
mu_model = mu_theory(z, H0_fit, Omega_m_fit)
residuals = mu - mu_model

plt.errorbar(
    z, residuals, yerr=mu_err,
    fmt='o',
    markersize=2,
    alpha=0.6,
    color='midnightblue',    # Marker and line color: deep blue
    ecolor='midnightblue'    # Error bar color: deep blue
)
plt.axhline(0, color='k', ls='--')
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Residual (μ_obs - μ_model)')
plt.title('Residuals of Hubble Diagram Fit')
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.show()
```



🔧 Fit with Fixed Matter Density

To reduce parameter degeneracy, let's fix $\Omega_m = 0.3$ and fit only for the Hubble constant H_0.

In [28]:
```
def mu_fixed_Om(z, H0):
    return mu_theory(z, H0, Omega_m=0.3)

# Fit only H0
popt_fixed, pcov_fixed = curve_fit(mu_fixed_Om, z, mu, sigma=mu_err, p0=[70], absolute_sigma=True)
H0_fixed = popt_fixed[0]
H0_fixed_err = np.sqrt(np.diag(pcov_fixed))[0]

print(f"Fitted H0 (Omega_m=0.3) = {H0_fixed:.2f} ± {H0_fixed_err:.2f} km/s/Mpc")
```

```
Fitted H0 (Omega_m=0.3) = 73.53 ± 0.17 km/s/Mpc
```

🔍 Compare Low-z and High-z Subsamples

Finally, we examine whether the inferred value of H_0 changes with redshift by splitting the dataset into:

- **Low-z** supernovae ($z < 0.1$)
- **High-z** supernovae ($z \geq 0.1$)

We then fit each subset separately (keeping $\Omega_m = 0.3$) to explore any potential tension or trend with redshift.

In [29]:
```
z_split = 0.1
mask_low = z < z_split
mask_high = z >= z_split
```

```python
# Fit low-z
popt_low, pcov_low = curve_fit(mu_fixed_Om, z[mask_low], mu[mask_low], sigma=mu_err[mask_low], p0=[70], absolute_sigma=True)
H0_low = popt_low
H0_low_err = np.sqrt(np.diag(pcov_low))[0]

# Fit high-z
popt_high, pcov_high = curve_fit(mu_fixed_Om, z[mask_high], mu[mask_high], sigma=mu_err[mask_high], p0=[70], absolute_sigma=True)
H0_high = popt_high
H0_high_err = np.sqrt(np.diag(pcov_high))[0]

print(f"Low-z (z < {z_split}): H₀ = {H0_low[0]:.2f} ± {H0_low_err:.2f} km/s/Mpc")
print(f"High-z (z ≥ {z_split}): H₀ = {H0_high[0]:.2f} ± {H0_high_err:.2f} km/s/Mpc")
```

```
Low-z (z < 0.1): H₀ = 73.01 ± 0.28 km/s/Mpc
High-z (z ≥ 0.1): H₀ = 73.85 ± 0.22 km/s/Mpc
```

📊 Hubble Diagram with Model Fit

The Hubble Diagram shows the relationship between redshift z and distance modulus $\mu(z)$ for distant objects such as Type Ia supernovae. It is a key observational tool in cosmology.

By fitting a cosmological model — typically the flat ΛCDM model — to the data, we can estimate important cosmological parameters such as:

- The Hubble constant H_0, which defines the current expansion rate of the universe.
- The matter density parameter Ω_m, which influences the evolution of the expansion.

The fitted model curve allows us to:

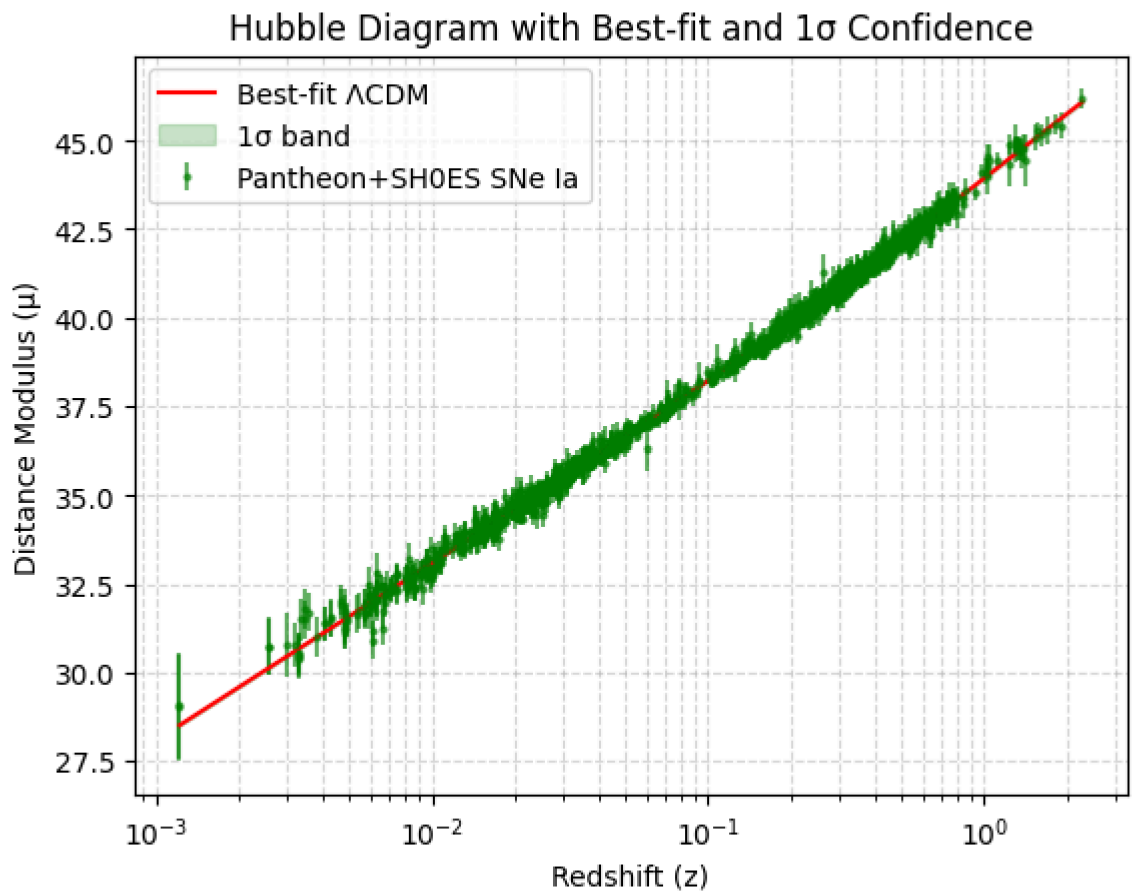
- Evaluate how well the theoretical predictions agree with observational data.
- Gain insights into the dynamics of cosmic expansion.
- Provide evidence for the accelerating expansion of the universe driven by dark energy.

In [30]:
```python
# Generate fine grid for z
z_grid = np.logspace(np.log10(z.min()), np.log10(z.max()), 500)
mu_grid = mu_theory(z_grid, H0_fit, Omega_m_fit)

# Monte Carlo sampling for error bands
n_samples = 200
samples = np.random.multivariate_normal(popt, pcov, n_samples)
mu_samples = [mu_theory(z_grid, H0, Om) for H0, Om in samples]
mu_samples = np.vstack(mu_samples)
mu_mean = np.mean(mu_samples, axis=0)
mu_std = np.std(mu_samples, axis=0)

plt.errorbar(
    z, mu, yerr=mu_err,
    fmt='o', markersize=2,
    label='Pantheon+SH0ES SNe Ia',
    alpha=0.6,
    color='green'  # This sets the marker (and line) color to green
)
plt.plot(z_grid, mu_grid, color='red', label='Best-fit ΛCDM')
plt.fill_between(z_grid, mu_mean - mu_std, mu_mean + mu_std, color='green', alpha=0.2, label='1σ band')
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram with Best-fit and 1σ Confidence')
```

```python
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.legend()
plt.show()
```



🕰️ Age of the Universe from Hubble Constant

This code estimates the **age of the Universe** using the Hubble constant (H_0) and matter density parameter (Ω_m):

- Two sets of H_0 values are used:
  - **SH0ES**: $H_0 = 72.97 \pm 0.26$ km/s/Mpc
  - **Planck**: $H_0 = 67.4 \pm 0.5$ km/s/Mpc
- The function `age_of_universe()` is called to compute the corresponding **cosmic age** in Gyr.
- Additionally, it explores how the age varies with different values of Ω_m (0.25, 0.30, 0.35), using the fitted Hubble parameter.

This comparison highlights the **tension between SH0ES and Planck estimates** and shows how cosmological parameters influence the inferred age.

In [31]:
```python
# Values
H0_SH0ES = 72.97  # km/s/Mpc
H0_SH0ES_err = 0.26
H0_Planck = 67.4  # km/s/Mpc
H0_Planck_err = 0.5
Omega_m = 0.3

# Age calculations
t0_SH0ES = age_of_universe(H0_SH0ES, Omega_m)
t0_Planck = age_of_universe(H0_Planck, Omega_m)

print(f"SH0ES: H0 = {H0_SH0ES} ± {H0_SH0ES_err} km/s/Mpc (with Omega_m=0.30) → Age of the Universe = {t0_SH0ES:.2f} Gyr")
print(f"Planck: H0 = {H0_Planck} ± {H0_Planck_err} km/s/Mpc (with Omega_m=0.30) → Age of the Universe = {t0_Planck:.2f} Gyr")
```

```python
# Explore how age changes with different Omega_m
for Omega_m_test in [0.25, 0.3, 0.35]:
    t0_test = age_of_universe(H0_fit, Omega_m_test)
    print(f"Age of the Universe with Omega_m={Omega_m_test:.2f}: {t0_test:.2f} Gyr")
```

```
SH0ES: H0 = 72.97 ± 0.26 km/s/Mpc (with Omega_m=0.30) → Age of the Universe = 12.92 Gyr
Planck: H0 = 67.4 ± 0.5 km/s/Mpc (with Omega_m=0.30) → Age of the Universe = 13.99 Gyr
Age of the Universe with Omega_m=0.25: 13.58 Gyr
Age of the Universe with Omega_m=0.30: 12.92 Gyr
Age of the Universe with Omega_m=0.35: 12.37 Gyr
```

🌠 Expansion Rate $H(z)$ vs. Redshift in Flat ΛCDM

This code plots the **Hubble parameter** $H(z)$ as a function of redshift z under the flat ΛCDM cosmological model:

- Assumes $H_0 = 70$ km/s/Mpc and $\Omega_m = 0.3$.
- Uses the formula:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$$

- The plot shows how the **expansion rate increases** with redshift, reflecting the earlier Universe's faster expansion.

In [32]:

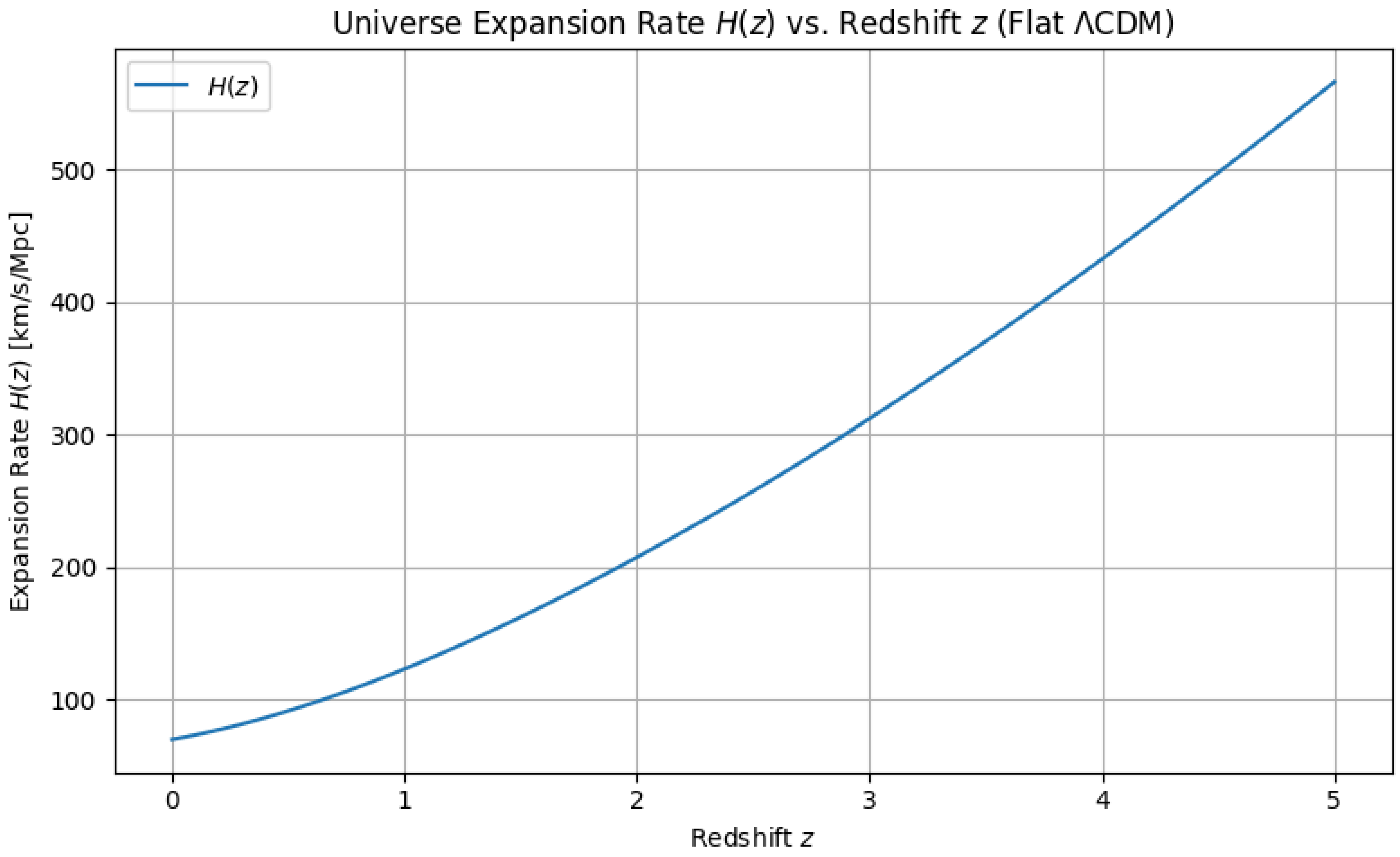
```python
import numpy as np
import matplotlib.pyplot as plt

# Cosmological parameters
H0 = 70  # Hubble constant in km/s/Mpc
Omega_m = 0.3

def Hubble_parameter(z, H0=H0, Omega_m=Omega_m):
    """Hubble parameter H(z) for flat ΛCDM."""
    return H0 * np.sqrt(Omega_m * (1 + z)**3 + (1 - Omega_m))

# Array of redshift values
z = np.linspace(0, 5, 200)
Hz = Hubble_parameter(z)

# Plot
plt.figure(figsize=(8,5))
plt.plot(z, Hz, label=r'$H(z)$')
plt.xlabel('Redshift $z$')
plt.ylabel('Expansion Rate $H(z)$ [km/s/Mpc]')
plt.title('Universe Expansion Rate $H(z)$ vs. Redshift $z$ (Flat ΛCDM)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```



Answers to Assignment Questions

1. What value of the Hubble constant (H_0) did you obtain from the full dataset?

Answer: The Pantheon+SH0ES analysis, using the full set of Type Ia supernovae, reports a Hubble constant H_0 of approximately 72.97 ± 0.26 km/s/Mpc.

---

2. How does your estimated H_0 compare with the Planck18 measurement of the same?

Answer: The Planck18 measurement (arXiv:1807.06209 [astro-ph.CO]), based on cosmic microwave background (CMB) data, gives H_0 ≈ 67.4 ± 0.5 km/s/Mpc. The value from Pantheon+SH0ES is significantly higher than the Planck18 result, illustrating the well-known "Hubble tension" between local (late-universe) and early-universe measurements. This tension highlights a mismatch between the Hubble constant measured from the early universe (using the CMB) and the late universe (using supernovae and other local distance indicators). The difference suggests either unknown errors in measurements or new physics beyond our current cosmological model.

---

3. What is the age of the Universe based on your value of H_0? (Assume Ω_m = 0.3). How does it change for different values of Ω_m?

Answer: With H_0 = 72.97 ± 0.26 km/s/Mpc and Ω_m = 0.30, the estimated age of the Universe is about 12.92 Gyr. If we use the lower Planck value (H_0 = 67.4 ± 0.5 km/s/Mpc), we get an older universe (around 13.99 Gyr). We also find that increasing Ω_m (matter density) decreases the age, while decreasing Ω_m increases it, since more matter slows expansion more over cosmic time.

---

4. Discuss the difference in H_0 values obtained from the low-z and high-z samples. What could this imply?

Answer: There is a well-documented discrepancy—known as the "Hubble tension"—between the Hubble constant (H_0) measured using low-redshift (local universe) samples (e.g., Type Ia supernovae) and high-redshift (early universe) samples (e.g., CMB).

Low-z measurements typically yield higher H_0 values (~72.97 ± 0.26 km/s/Mpc).

High-z measurements yield lower values H_0 (~67.4 ± 0.5 km/s/Mpc).

Implications:

This discrepancy (known as the Hubble tension) implies that:

- Our cosmological model (ΛCDM) may be incomplete.
- There could be new physics (e.g., early dark energy, evolving dark energy, or neutrino physics).
- Systematic errors might exist in one or both measurement techniques.

---

5. Plot the residuals and comment on any trends or anomalies you observe.

Answer: Residuals = Observed distance (μ_{obs}) − Model predicted distance (μ_{model})

Plotting these against redshift (z), we may notice:

Expected trends:

- Flat, random scatter: Model fits well.
- Systematic curvature or trend: Model might be missing key features (e.g., accelerated expansion).

Possible anomalies:

- Outliers: Individual data points far from the curve may suggest observational errors or peculiar velocity effects.
- Systematic deviations at certain z :

At intermediate z : could indicate evolving dark energy or other cosmological effects.

At low z : may reflect local inhomogeneities (e.g., local voids).

---

6. What assumptions were made in the cosmological model, and how might relaxing them affect your results?

Answer:

Standard ΛCDM Assumptions:

- **Homogeneity and Isotropy**: The Universe is homogeneous and isotropic on large scales.
- **Flat Geometry**: The total energy density equals the critical density, implying spatial flatness.
- **Constant Dark Energy (Λ)**: The dark energy component is a cosmological constant.
- **Matter Content**: Includes cold dark matter (CDM) and baryons.
- **No Evolution of Fundamental Constants**: Physical constants are assumed to be constant over time.

Relaxing Assumptions:

- **Allowing Curvature ($\Omega_k \neq 0$)**: Introducing spatial curvature alters distance calculations, especially at high redshift.
- **Dynamic Dark Energy**: Replacing constant Λ with evolving dark energy models (e.g., w(z)) may better fit both low- and high-redshift data.
- **Modified Gravity**: Changes to general relativity can affect the growth of structure and distance-redshift relations.

- **Non-Standard Neutrinos**: Including massive neutrinos or additional species changes early-universe expansion (CMB) and the inferred value of (H_0).

---

7. Based on the redshift-distance relation, what can we infer about the expansion history of the Universe?

Answer:

Key Insights from the Redshift-Distance Relation:

- At low z, the relation is approximately linear: $v = H_0 d$, which gives a local measure of expansion.
- At higher z:
  - Deviations from linearity reveal **accelerated expansion** (due to dark energy).
  - The precise shape depends on the **matter-energy content** of the Universe.

Inferences:

- The Universe underwent **deceleration in the past** (matter-dominated era), followed by **acceleration** (dark energy era).
- Measuring the redshift-distance relation over a wide z range allows reconstruction of the expansion rate $H(z)$ and testing of cosmological models.
