$$\left\{ \begin{matrix} n+1 \\ k+1 \end{matrix} \right\} = (k+1) \left\{ \begin{matrix} n \\ k+1 \end{matrix} \right\} + \left\{ \begin{matrix} n \\ k \end{matrix} \right\} \tag{Eq[0]}$$

$$\left\{ \begin{matrix} n+1 \\ k+1 \end{matrix} \right\} = \left| \left\{ \{*x_{0:k+1}\} \middle| \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \right\} \right| \tag{Eq.stirling1}$$

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \left| \left\{ \{*x_{0:k}\} \middle| \forall_{0\le i< k} |x_i| > 0 \wedge \sum_{i=0}^{k-1} |x_i| = n \wedge \bigcup_{i=0}^{k-1} x_i = [0;n) \right\} \right| \tag{Eq.stirling2}$$

$$\left\{ \begin{matrix} n \\ k+1 \end{matrix} \right\} = \left| \left\{ \{*x_{0:k+1}\} \middle| \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n \wedge \bigcup_{i=0}^{k} x_i = [0;n) \right\} \right| \tag{Eq.stirling3}$$

$$s_1 = \left\{ \{*x_{0:k+1}\} \middle| \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \right\} \tag{Eq[1]}$$

$$\left\{ \begin{matrix} n+1 \\ k+1 \end{matrix} \right\} = |s_1| \tag{Eq.stirling1}$$

$$s_1' = \left\{ x_{0:k+1} \mid \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \right\} \tag{Eq[2]}$$

$$s_1 = \left\{ \{*x_{0:k+1}\} \middle| x_{0:k+1} \in s_1' \right\} \tag{Eq.s1\_definition}$$

$$s_2 = \left\{ \{*x_{0:k}\} \middle| \forall_{0\le i< k} |x_i| > 0 \wedge \sum_{i=0}^{k-1} |x_i| = n \wedge \bigcup_{i=0}^{k-1} x_i = [0;n) \right\} \tag{Eq[3]}$$

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = |s_2| \tag{Eq.stirling2}$$

$$s_2' = \left\{ x_{0:k} \mid \forall_{0\le i< k} |x_i| > 0 \wedge \sum_{i=0}^{k-1} |x_i| = n \wedge \bigcup_{i=0}^{k-1} x_i = [0;n) \right\} \tag{Eq[4]}$$

$$s_2 = \left\{ \{*x_{0:k}\} \middle| x_{0:k} \in s_2' \right\} \tag{Eq[5]}$$

$$s_3 = \left\{ \{*x_{0:k+1}\} \middle| \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n \wedge \bigcup_{i=0}^{k} x_i = [0;n) \right\} \tag{Eq[6]}$$

$$\left\{ \begin{matrix} n \\ k+1 \end{matrix} \right\} = |s_3| \tag{Eq.stirling3}$$

$$s_3' = \left\{ x_{0:k+1} \mid \forall_{0\le i\le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n \wedge \bigcup_{i=0}^{k} x_i = [0;n) \right\} \tag{Eq[7]}$$

$$s_3 = \left\{ \{*x_{0:k+1}\} \middle| x_{0:k+1} \in s_3' \right\} \tag{Eq[8]}$$

$$\{ e \cup \{\{n\}\} | e \in s_2 \} \subset s_1 \tag{Eq[9]}$$

$$\forall_{e \in s_2} e \cup \{\{n\}\} \in s_1 \tag{Eq[10]}$$

$$\forall_{x_{0:k} \in s_2'} \{\{n\}\} \cup \{*x_{0:k}\} \in s_1 \tag{Eq[11]}$$

$$\forall_{x_{0:k} \in s_2'} \{\{n\}\} \cup \{*x_{0:k}\} \in \left\{ \{*x_{0:k+1}\} \middle| x_{0:k+1} \in s_1' \right\} \tag{Eq[12]}$$

$$\forall_{x0:k+\in s_1'} \forall_{0 \le i \le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \tag{Eq.s1\_quote\_definition}$$

$$\forall_{x0:k\in s_2'} \forall_{0 \le i < k} |x_i| > 0 \wedge \sum_{i=0}^{k-1} |x_i| = n \wedge \bigcup_{i=0}^{k-1} x_i = [0;n) \tag{Eq[13]}$$

$$\forall_{x0:k+\in s_3'} \forall_{0 \le i \le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n \wedge \bigcup_{i=0}^{k} x_i = [0;n) \tag{Eq[14]}$$

$$\forall_{\substack{0 \le i < k \\ x0:k\in s_2'}} |x_i| > 0 \tag{Eq[15]}$$

$$\forall_{x0:k\in s_2'} \sum_{i=0}^{k-1} |x_i| = n \tag{Eq[16]}$$

$$\forall_{x0:k\in s_2'} \bigcup_{i=0}^{k-1} x_i = [0;n) \tag{Eq[17]}$$

$$\exists_{x_k} x_k = \{n\} \tag{Eq[18]}$$

$$\exists_{x_k} \forall_{x0:k\in s_2'} \bigcup_{i=0}^{k} x_i = [0;n] \tag{Eq[19]}$$

$$\exists_{x_k} \{x_k\} = \{\{n\}\} \tag{Eq[20]}$$

$$\exists_{x_k} \{*x_{0:k+1}\} = \{\{n\}\} \cup \{*x_{0:k}\} \tag{Eq[21]}$$

$$\exists_{x_k} \forall_{x0:k\in s_2'} \{*x_{0:k+1}\} \in \left\{\{*x_{0:k+1}\} \middle| \, x_{0:k+1} \in s_1'\right\} \tag{Eq[22]}$$

$$\exists_{x_k} \forall_{x0:k\in s_2'} \forall_{0 \le i \le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \tag{Eq[23]}$$

$$\exists_{x_k} |x_k| = 1 \tag{Eq[24]}$$

$$\exists_{x_k} |x_k| > 0 \tag{Eq[25]}$$

$$\exists_{x_k} \forall_{x0:k\in s_2'} \sum_{i=0}^{k} |x_i| = n+1 \tag{Eq[26]}$$

$$\exists_{x_k} \forall_{\substack{0 \le i \le k \\ x0:k\in s_2'}} |x_i| > 0 \tag{Eq[27]}$$

$$\exists_{x_k} \forall_{x0:k\in s_2'} \forall_{0 \le i \le k} |x_i| > 0 \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \tag{Eq[23]}$$

$$\forall_{\substack{0 \le i \le k \\ x0:k+\in s_3'}} |x_i| > 0 \tag{Eq.x\_abs\_positive\_s3}$$

$$\forall_{x0:k+\in s_3'} \sum_{i=0}^{k} |x_i| = n \tag{Eq.x\_abs\_sum\_s3}$$

$$\forall_{x0:k+\in s_3'} \bigcup_{i=0}^{k} x_i = [0;n) \tag{Eq.x\_union\_s3}$$

$$x'_i = \begin{cases} \{n\} \cup x_i & \text{if } i = j \\ x_i & \text{else} \end{cases} \tag{Eq.x\_quote\_definition}$$

$$\left\{\left\{*x'\right\} \middle| x_{0:k+1} \in s'_3\right\} \subset s_1 \qquad \text{Eq.x\_quote\_set\_in\_s1}$$

$$\forall_{x_{0:k+} \in s'_3} \left\{*x'\right\} \in s_1 \qquad \text{Eq[28]}$$

$$\forall_{x_{0:k+} \in s'_3} \left\{*x'\right\} \in \left\{\left\{*x_{0:k+1}\right\} \middle| x_{0:k+1} \in s'_1\right\} \qquad \text{Eq[29]}$$

$$\forall_{x_{0:k+} \in s'_3} \forall_{0 \le i \le k} \left|x'_i\right| > 0 \wedge \sum_{i=0}^{k} \left|x'_i\right| = n+1 \wedge \bigcup_{i=0}^{k} x'_i = [0;n] \qquad \text{Eq[30]}$$

$$\bigcup_{i=0}^{k} x'_i = \{n\} \cup \bigcup_{i=0}^{k} x_i \qquad \text{Eq[31]}$$

$$\forall_{x_{0:k+} \in s'_3} \bigcup_{i=0}^{k} x'_i = [0;n] \qquad \text{Eq[32]}$$

$$\left|x'_i\right| = \begin{cases} |\{n\} \cup x_i| & \text{if } i = j \\ |x_i| & \text{else} \end{cases} \qquad \text{Eq[33]}$$

$$\sum_{i=0}^{k} \left|x'_i\right| = -\left|x_j\right| + \left|\{n\} \cup x_j\right| + \sum_{i=0}^{k} \left|x_i\right| \qquad \text{Eq[34]}$$

$$\left|\{n\} \cup x_j\right| \le \left|x_j\right| + 1 \qquad \text{Eq[35]}$$

$$\sum_{i=0}^{k} \left|x'_i\right| \le \sum_{i=0}^{k} \left|x_i\right| + 1 \qquad \text{Eq[36]}$$

$$\forall_{x_{0:k+} \in s'_3} \sum_{i=0}^{k} \left|x'_i\right| \le n+1 \qquad \text{Eq[37]}$$

$$\forall_{x_{0:k+} \in s'_3} \left|\bigcup_{i=0}^{k} x'_i\right| = n+1 \qquad \text{Eq[38]}$$

$$\left|\bigcup_{i=0}^{k} x'_i\right| \le \sum_{i=0}^{k} \left|x'_i\right| \qquad \text{Eq[39]}$$

$$\forall_{x_{0:k+} \in s'_3} \sum_{i=0}^{k} \left|x'_i\right| \ge n+1 \qquad \text{Eq[40]}$$

$$\forall_{x_{0:k+} \in s'_3} \sum_{i=0}^{k} \left|x'_i\right| = n+1 \qquad \text{Eq[41]}$$

$$\left|x'_j\right| = \left|\{n\} \cup x_j\right| \qquad \text{Eq[42]}$$

$$\forall_{i | i \ne j} \left|x'_i\right| = \left|x_i\right| \qquad \text{Eq[43]}$$

$$\left|\{n\} \cup x_j\right| \ge \left|x_j\right| \qquad \text{Eq[44]}$$

$$\forall_{x_{0:k+} \in s'_3} \left|\{n\} \cup x_j\right| > 0 \qquad \text{Eq[45]}$$

$$\forall_{x_{0:k+} \in s'_3} \left|x'_j\right| > 0 \qquad \text{Eq[46]}$$

$$\forall_{\substack{i \in [0;k] \setminus \{j\} \\ x_{0:k+} \in s'_3}} \left|x'_i\right| > 0 \qquad \text{Eq[47]}$$

$$\forall_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_3}} \left|x'_i\right| > 0 \qquad \text{Eq[48]}$$

$$\forall_{x_{0:k+} \in s'_3} \forall_{0 \le i \le k} \left|x'_i\right| > 0 \wedge \sum_{i=0}^{k} \left|x'_i\right| = n + 1 \wedge \bigcup_{i=0}^{k} x'_i = [0; n] \qquad \text{Eq[30]}$$

$$x' = [i] \begin{cases} \{n\} \cup x_i & \text{if } i = j \\ x_i & \text{else} \end{cases} \qquad \text{Eq.x\_quote\_definition}$$

$$A_j = \left\{ \left\{ *x' \right\} \middle| \, x_{0:k+1} \in s'_3 \right\} \qquad \text{Eq.A\_definition}$$

$$A_j \subset s_1 \qquad \text{Eq[49]}$$

$$\bigcup_j A_j \subset \bigcup_j s_1 \qquad \text{Eq[50]}$$

$$B = \{ e \cup \{\{n\}\} | \, e \in s_2 \} \qquad \text{Eq.B\_definition}$$

$$B \subset s_1 \qquad \text{Eq[51]}$$

$$B \cup A_j \subset s_1 \qquad \text{Eq[52]}$$

$$s_1 = \{ e \in s_1 \mid \{n\} \in e \} \cup \{ e \in s_1 \mid \{n\} \notin e \} \qquad \text{Eq[53]}$$

$$\{ e \in s_1 \mid \{n\} \in e \} \subset B \qquad \text{Eq.subset\_B}$$

$$\{ e \in s_1 \mid \{n\} \in e \} \supset B \qquad \text{Eq.supset\_B}$$

$$\{ e \in s_1 \mid \{n\} \notin e \} \subset \bigcup_j A_j \qquad \text{Eq.subset\_A}$$

$$\{ e \in s_1 \mid \{n\} \notin e \} \supset A_j \qquad \text{Eq.supset\_A}$$

$$|s_1| = |\{ e \in s_1 \mid \{n\} \in e \}| + |\{ e \in s_1 \mid \{n\} \notin e \}| \qquad \text{Eq.s1\_abs}$$

$$\{ e \in s_1 \mid \{n\} \notin e \} \supset \left\{ \left\{ *x' \right\} \middle| \, x_{0:k+1} \in s'_3 \right\} \qquad \text{Eq[54]}$$

$$\forall_{x_{0:k+} \in s'_3} \left\{ *x' \right\} \in s_1 \wedge \{n\} \notin \left\{ *x' \right\} \qquad \text{Eq[55]}$$

$$\forall_{x_{0:k+} \in s'_3} \left\{ *x' \right\} \in s_1 \qquad \text{Eq[28]}$$

$$\forall_{x_{0:k+} \in s'_3} \{n\} \notin \left\{ *x' \right\} \qquad \text{Eq[56]}$$

$$\exists_{x_{0:k+} \in s'_3} \{n\} \in \left\{ *x' \right\} \qquad \text{Eq[57]}$$

$$\exists_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_3}} \{n\} = x'_i \qquad \text{Eq[58]}$$

$$x'_j = \{n\} \cup x_j \qquad \text{Eq[59]}$$

$$\exists_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_3}} x'_i \cap x'_j = \{n\} \qquad \text{Eq[60]}$$

$$\left|x'_i \cup x'_j\right| = \left|x'_i\right| + \left|x'_j\right| - \left|x'_i \cap x'_j\right| \qquad \text{Eq[61]}$$

$$\exists_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_3}} \left|x'_i \cup x'_j\right| = \left|x'_i\right| + \left|x'_j\right| - 1 \qquad \text{Eq[62]}$$

$$\exists_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_3}} \left|x'_i \cup x'_j\right| < \left|x'_i\right| + \left|x'_j\right| \qquad \text{Eq.set\_size\_inequality}$$

$$\forall_{x_{0:k+} \in s'_3} \bigcup_{i \in [0;k] \setminus \{i,j\}} x'_i \cup \left(x'_j \cup \bigcup_{i \in [0;k] \cap \{i\}} x'_i \right) = [0; n] \qquad \text{Eq[63]}$$

$$\left| \bigcup_{i=0}^{k} x'_i \right| \le \left| x'_j \cup \bigcup_{i \in [0;k] \cap \{i\}} x'_i \right| + \left| \bigcup_{i \in [0;k] \setminus \{i,j\}} x'_i \right| \qquad \text{Eq[64]}$$

$$\left|\bigcup_{i\in[0;k]\setminus\{i,j\}} x'_i\right| \leq \sum_{i\in[0;k]\setminus\{i,j\}} \left|x'_i\right| \qquad \text{Eq[65]}$$

$$\exists_{\substack{0\leq i\leq k\\ x_{0:k+}\in s'_3}} \left|x'_i \cup x'_j\right| + \left|\bigcup_{i=0}^{k} x'_i\right| < \left|x'_i\right| + \left|x'_j\right| + \left|x'_j \cup \bigcup_{i\in[0;k]\cap\{i\}} x'_i\right| + \sum_{i\in[0;k]\setminus\{i,j\}} \left|x'_i\right| \qquad \text{Eq[66]}$$

$$\exists_{x_{0:k+}\in s'_3} \left|\bigcup_{i=0}^{k} x'_i\right| < \sum_{i=0}^{k} \left|x'_i\right| \qquad \text{Eq[67]}$$

$$\exists_{x_{0:k+}\in s'_3} n+1 < \sum_{i=0}^{k} \left|x'_i\right| \qquad \text{Eq[68]}$$

False

$$\forall_{x_{0:k+}\in s'_3} \left\{*x'\right\} \in s_1 \wedge \{n\} \notin \left\{*x'\right\} \qquad \text{Eq[55]}$$

$$\{e \in s_1 \mid \{n\} \in e\} \supset \{e \cup \{\{n\}\} | \, e \in s_2\} \qquad \text{Eq[69]}$$

$$\forall_{e\in s_2} e \cup \{\{n\}\} \in s_1 \qquad \text{Eq[10]}$$

$$\{e \in s_1 \mid \{n\} \in e\} \subset \{e \cup \{\{n\}\} | \, e \in s_2\} \qquad \text{Eq[70]}$$

$$\forall_{o\in\{e\in s_1|\{n\}\in e\}} \exists_{e\in s_2} o = e \cup \{\{n\}\} \qquad \text{Eq[71]}$$

$$\forall_{o\in\{e\in s_1|\{n\}\in e\}} \exists_{e\in s_2} o \setminus \{\{n\}\} = e \setminus \{\{n\}\} \qquad \text{Eq.subset\_B\_definition}$$

$$\forall_{e\in s_2} \{n\} \notin e \qquad \text{Eq.plausible\_notcontains}$$

$$\forall_{x_{0:k}\in s'_2} \{n\} \notin \{*x_{0:k}\} \qquad \text{Eq[72]}$$

$$\exists_{x_{0:k}\in s'_2} \{n\} \in \{*x_{0:k}\} \qquad \text{Eq[73]}$$

$$\exists_{\substack{0\leq i<k\\ x_{0:k}\in s'_2}} \{n\} = x_i \qquad \text{Eq[74]}$$

$$\exists_{x_{0:k}\in s'_2} \bigcup_{i=0}^{k-1} x_i = [0;n] \qquad \text{Eq[75]}$$

False

$$\forall_{e\in s_2} e \cap \{\{n\}\} = \emptyset \qquad \text{Eq[76]}$$

$$\forall_{e\in s_2} e \setminus \{\{n\}\} = e \qquad \text{Eq.s2\_complement\_n}$$

$$\forall_{o\in\{e\in s_1|\{n\}\in e\}} o \setminus \{\{n\}\} \in s_2 \qquad \text{Eq[77]}$$

$$s_{1,n} = \{e \in s_1 \mid \{n\} \in e\} \qquad \text{Eq.s1\_n\_definition}$$

$$\forall_{e\in s_{1,n}} e \in s_1 \wedge \{n\} \in e \qquad \text{Eq[78]}$$

$$\forall_{e\in s_{1,n}} e \in s_1 \qquad \text{Eq[79]}$$

$$\forall_{e\in s_{1,n}} \{n\} \in e \qquad \text{Eq[80]}$$

$$\forall_{e\in s_{1,n}} e \in \left\{\{*x_{0:k+1}\} | \, x_{0:k+1} \in s'_1\right\} \qquad \text{Eq[81]}$$

$$\forall_{e\in s_{1,n}} \exists_{x_{0:k+}\in s'_1} e = \{*x_{0:k+1}\} \qquad \text{Eq.s1\_n\_assertion}$$

$$\exists_{x_{0:k+}\in s'_1} \{n\} \in \{*x_{0:k+1}\} \qquad \text{Eq[82]}$$

$$\exists_{\substack{0\leq i\leq k\\ x_{0:k+}\in s'_1}} \{n\} = x_i \qquad \text{Eq[83]}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} x_j = \{n\} \quad \text{Eq.x\_j\_definition}$$

$$\forall_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_1}} |x_i| > 0 \quad \text{Eq.x\_abs\_positive\_s1}$$

$$\forall_{x_{0:k+} \in s'_1} \sum_{i=0}^{k} |x_i| = n + 1 \quad \text{Eq.x\_abs\_sum\_s1}$$

$$\forall_{x_{0:k+} \in s'_1} \bigcup_{i=0}^{k} x_i = [0; n] \quad \text{Eq.x\_union\_s1}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} \bigcup_{i=0}^{k} x_i \setminus x_j = [0; n) \quad \text{Eq[84]}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} \bigcup_{i \in [0;k] \setminus \{j\}} x_i = [0; n) \quad \text{Eq[85]}$$

$$\tilde{x}_i = \begin{cases} x_i & \text{if } i < j \\ x_{i+1} & \text{else} \end{cases} \quad \text{Eq.x\_tilde\_definition}$$

$$\bigcup_{i=0}^{k-1} \tilde{x}_i = \bigcup_{i=0}^{j-1} x_i \cup \bigcup_{i=j}^{k-1} x_{i+1} \quad \text{Eq[86]}$$

$$\bigcup_{i=0}^{k-1} \tilde{x}_i = \bigcup_{i \in [0;k] \setminus \{j\}} x_i \quad \text{Eq[87]}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} \bigcup_{i=0}^{k-1} \tilde{x}_i = [0; n) \quad \text{Eq.x\_tilde\_union}$$

$$|\tilde{x}_i| = \begin{cases} |x_i| & \text{if } i < j \\ |x_{i+1}| & \text{else} \end{cases} \quad \text{Eq.x\_tilde\_abs}$$

$$\sum_{i=0}^{k-1} |\tilde{x}_i| = \sum_{i=j}^{k-1} |x_{i+1}| + \sum_{i=0}^{j-1} |x_i| \quad \text{Eq[88]}$$

$$\sum_{i=0}^{k-1} |\tilde{x}_i| = -|x_j| + \sum_{i=0}^{k} |x_i| \quad \text{Eq[89]}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} \sum_{i=0}^{k-1} |\tilde{x}_i| = n \quad \text{Eq.x\_tilde\_abs\_sum}$$

$$\forall_{i | i < j} |\tilde{x}_i| = |x_i| \quad \text{Eq[90]}$$

$$\forall_{i | i \ge j} |\tilde{x}_i| = |x_{i+1}| \quad \text{Eq[91]}$$

$$\forall_{\substack{0 \le i < j \\ x_{0:k+} \in s'_1}} |\tilde{x}_i| > 0 \quad \text{Eq[92]}$$

$$\forall_{\substack{j \le i < k \\ x_{0:k+} \in s'_1}} |\tilde{x}_i| > 0 \quad \text{Eq[93]}$$

$$\forall_{\substack{0 \le i < k \\ x_{0:k+} \in s'_1}} |\tilde{x}_i| > 0 \quad \text{Eq[94]}$$

$$\exists_{\substack{j \\ x_{0:k+} \in s'_1}} \forall_{0 \le i < k} |\tilde{x}_i| > 0 \wedge \sum_{i=0}^{k-1} |\tilde{x}_i| = n \wedge \bigcup_{i=0}^{k-1} \tilde{x}_i = [0; n) \quad \text{Eq[95]}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \tilde{x} \in s'_2 \quad \text{Eq[96]}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \forall_{0\le i<k} |\tilde{x}_i| > 0 \wedge \sum_{i=0}^{k-1} |\tilde{x}_i| = n \wedge \bigcup_{i=0}^{k-1} \tilde{x}_i = [0;n) \quad \text{Eq[95]}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \{*\tilde{x}\} \in s_2 \quad \text{Eq.x\_tilde\_set\_in\_s2}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \{*\tilde{x}\} \in \left\{\{*x_{0:k}\} \middle| x_{0:k} \in s'_2\right\} \quad \text{Eq[97]}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \tilde{x} \in s'_2 \quad \text{Eq[96]}$$

$$\{*\tilde{x}\} = \{*x_{0:k+1}\} \setminus \left\{x_j\right\} \quad \text{Eq[98]}$$

$$\exists_{\substack{j\\ x_{0:k+}\in s'_1}} \{*\tilde{x}\} = \{*x_{0:k+1}\} \setminus \{\{n\}\} \quad \text{Eq[99]}$$

$$\exists_j \forall_{e\in s_{1,n}} \exists_{x_{0:k+}\in s'_1} \{*\tilde{x}\} = e \setminus \{\{n\}\} \quad \text{Eq[100]}$$

$$\forall_{e\in s_{1,n}} e \setminus \{\{n\}\} \in s_2 \quad \text{Eq[101]}$$

$$\forall_{e\in\{e\in s_1 | \{n\}\in e\}} e \setminus \{\{n\}\} \in s_2 \quad \text{Eq[77]}$$

$$\{e \in s_1 \mid \{n\} \notin e\} \subset \bigcup_j \left\{\left\{*x'\right\} \middle| x_{0:k+1} \in s'_3\right\} \quad \text{Eq[102]}$$

$$\forall_{l\in\{e\in s_1 | \{n\}\notin e\}} \exists_{\substack{x_{0:k+}\in s'_3\\ j}} l = \left\{*x'\right\} \quad \text{Eq[103]}$$

$$\hat{s}_{1,n} = \{e \in s_1 \mid \{n\} \notin e\} \quad \text{Eq[104]}$$

$$\forall_{l\in\hat{s}_{1,n}} \exists_{\substack{x_{0:k+}\in s'_3\\ j}} l = \left\{*x'\right\} \quad \text{Eq.s1\_hat\_n\_assertion}$$

$$\hat{s}_{1,n} = \left\{ \{*x_{0:k+1}\} \middle| \forall_{0\le i\le k} |x_i| > 0 \wedge \{n\} \notin \{*x_{0:k+1}\} \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \right\} \quad \text{Eq[105]}$$

$$s'_{1,n} = \left\{ x_{0:k+1} \mid \forall_{0\le i\le k} |x_i| > 0 \wedge \{n\} \notin \{*x_{0:k+1}\} \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \right\} \quad \text{Eq[106]}$$

$$\hat{s}_{1,n} = \left\{\{*x_{0:k+1}\} \middle| x_{0:k+1} \in s'_{1,n}\right\} \quad \text{Eq[107]}$$

$$\forall_{a\in s'_{1,n}} \exists_{\substack{x_{0:k+}\in s'_3\\ j}} \{*a\} = \left\{*x'\right\} \quad \text{Eq.s1\_hat\_n\_hypothesis}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \forall_{0\le i\le k} |x_i| > 0 \wedge \{n\} \notin \{*x_{0:k+1}\} \wedge \sum_{i=0}^{k} |x_i| = n+1 \wedge \bigcup_{i=0}^{k} x_i = [0;n] \quad \text{Eq[108]}$$

$$\forall_{\substack{0\le i\le k\\ x_{0:k+}\in s'_{1,n}}} |x_i| > 0 \quad \text{Eq.x\_abs\_positive\_s1\_n}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \{n\} \notin \{*x_{0:k+1}\} \quad \text{Eq.n\_not\_in\_x}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \sum_{i=0}^{k} |x_i| = n+1 \quad \text{Eq.x\_abs\_sum\_s1\_n}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \bigcup_{i=0}^{k} x_i = [0;n] \quad \text{Eq.x\_union\_s1\_n}$$

$$\forall_{\substack{0 \le i \le k \\ x_{0:k+} \in s'_{1,n}}} \{n\} \ne x_i \tag{Eq[109]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \{n\} \ne x_j \tag{Eq.x_j_inequality}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} n \in \bigcup_{i=0}^{k} x_i \tag{Eq[110]}$$

True

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_{0 \le i \le k} n \in x_i \tag{Eq[111]}$$

$$\hat{x}_i = \begin{cases} x_i \setminus \{n\} & \text{if } i = j \\ x_i & \text{else} \end{cases} \tag{Eq.x_hat_definition}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j n \in x_j \tag{Eq[112]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \{n\} \subset x_j \tag{Eq.x_j_subset}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j x_j \setminus \{n\} \ne \emptyset \tag{Eq[113]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \left|x_j \setminus \{n\}\right| > 0 \tag{Eq[114]}$$

$$|\hat{x}_i| = \begin{cases} |x_i \setminus \{n\}| & \text{if } i = j \\ |x_i| & \text{else} \end{cases} \tag{Eq.x_hat_abs}$$

$$\left|\hat{x}_j\right| = \left|x_j \setminus \{n\}\right| \tag{Eq[115]}$$

$$\forall_{i | i \ne j} |\hat{x}_i| = |x_i| \tag{Eq[116]}$$

$$\forall_{\substack{i \in [0;k] \setminus \{j\} \\ x_{0:k+} \in s'_{1,n}}} |\hat{x}_i| > 0 \tag{Eq[117]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \left|\hat{x}_j\right| > 0 \tag{Eq[118]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \forall_{0 \le i \le k} |\hat{x}_i| > 0 \tag{Eq.x_hat_abs_positive}$$

$$\bigcup_{i=0}^{k} \hat{x}_i = \left(x_j \setminus \{n\}\right) \cup \bigcup_{i \in [0;k] \setminus \{j\}} x_i \tag{Eq.x_hat_union}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \bigcup_{i=0}^{k} x_i \setminus \{n\} = [0; n) \tag{Eq.x_union_complement}$$

$$\forall_{\substack{j \in [0;k] \setminus \{i\} \\ 0 \le i \le k \\ x_{0:k+} \in s'_{1,n}}} x_i \cap x_j = \emptyset \tag{Eq[119]}$$

$$\forall_{\substack{i \in [0;k] \setminus \{j\} \\ x_{0:k+} \in s'_{1,n}}} x_i \cap x_j = \emptyset \tag{Eq[120]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \forall_{i \in [0;k] \setminus \{j\}} x_i \setminus \{n\} = x_i \tag{Eq.x_complement_n}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \bigcup_{i \in [0;k] \setminus \{j\}} x_i \setminus \{n\} = \bigcup_{i \in [0;k] \setminus \{j\}} x_i \tag{Eq[121]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \bigcup_{i=0}^{k} \hat{x}_i = \bigcup_{i=0}^{k} x_i \setminus \{n\} \tag{Eq[122]}$$

$$\forall_{x_{0:k+} \in s'_{1,n}} \exists_j \bigcup_{i=0}^{k} \hat{x}_i = [0; n) \tag{Eq.x_hat_union}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \sum_{i=0}^{k} |\hat{x}_i| = n + |x_j \setminus \{n\}| - |x_j| + 1 \qquad \text{Eq[123]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j |x_j \setminus \{n\}| = |x_j| - 1 \qquad \text{Eq[124]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \sum_{i=0}^{k} |\hat{x}_i| = n \qquad \text{Eq[125]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \forall_{0\le i\le k} |\hat{x}_i| > 0 \wedge \sum_{i=0}^{k} |\hat{x}_i| = n \wedge \bigcup_{i=0}^{k} \hat{x}_i = [0;n) \qquad \text{Eq[126]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \hat{x}_{0:k+1} \in s'_3 \qquad \text{Eq.x\_hat\_in\_s3}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \forall_{0\le i\le k} |\hat{x}_i| > 0 \wedge \sum_{i=0}^{k} |\hat{x}_i| = n \wedge \bigcup_{i=0}^{k} \hat{x}_i = [0;n) \qquad \text{Eq[126]}$$

$$\hat{x}_j = x_j \setminus \{n\} \qquad \text{Eq[127]}$$

$$\forall_{i|i\neq j} \hat{x}_i = x_i \qquad \text{Eq[128]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \forall_{i\in[0;k]\setminus\{j\}} \hat{x}_i = x_i \setminus \{n\} \qquad \text{Eq[129]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \forall_{0\le i\le k} \hat{x}_i = x_i \setminus \{n\} \qquad \text{Eq[130]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_j \hat{x} = [i : k+1] x_i \setminus \{n\} \qquad \text{Eq[131]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} [i : k+1] x_i \setminus \{n\} \in s'_3 \qquad \text{Eq[132]}$$

$$\forall_{a\in s'_{1,n}} \exists_{\substack{x_{0:k+}\in s'_3\\ j}} \forall_{0\le i\le k} a_i = x'_i \qquad \text{Eq[133]}$$

$$\forall_{a\in s'_{1,n}} \exists_{\substack{x_{0:k+}\in s'_3\\ j}} \forall_{0\le i\le k} a_i = \begin{cases} \{n\} \cup x_i & \text{if } i = j \\ x_i & \text{else} \end{cases} \qquad \text{Eq[134]}$$

$$\forall_{a\in s'_{1,n}} \exists_{x_{0:k+}\in s'_3} \forall_{0\le i\le k} a_i \setminus \{n\} = x_i \setminus \{n\} \qquad \text{Eq.equation}$$

$$\forall_{x_{0:k+}\in s'_3} \{n\} \cap \bigcup_{i=0}^{k} x_i = \emptyset \qquad \text{Eq[135]}$$

$$\forall_{\substack{0\le i\le k\\ x_{0:k+}\in s'_3}} \{n\} \cap x_i = \emptyset \qquad \text{Eq.nonoverlapping\_s3\_quote}$$

$$\forall_{\substack{0\le i\le k\\ x_{0:k+}\in s'_3}} x_i \setminus \{n\} = x_i \qquad \text{Eq[136]}$$

$$\forall_{a\in s'_{1,n}} \exists_{x_{0:k+}\in s'_3} \forall_{0\le i\le k} a_i \setminus \{n\} = x_i \qquad \text{Eq[137]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} \exists_{a\in s'_3} \forall_{0\le i\le k} x_i \setminus \{n\} = a_i \qquad \text{Eq[137]}$$

$$\forall_{x_{0:k+}\in s'_{1,n}} [i : k+1] x_i \setminus \{n\} \in s'_3 \qquad \text{Eq[132]}$$

$$|s_1| = (k+1)\,|s_3| + |s_2| \qquad \text{Eq.s1\_abs\_plausible}$$

$$\{e \in s_1 \mid \{n\} \notin e\} \supset \bigcup_{j} A_j \qquad \text{Eq.supset\_A}$$

$$\{e \in s_1 \mid \{n\} \notin e\} = \bigcup_{j} A_j \qquad \text{Eq[138]}$$

$$\{e \in s_1 \mid \{n\} \in e\} = B \qquad \text{Eq[139]}$$

$$|s_1| = |B| + \left|\bigcup_{j} A_j\right| \tag{Eq.s1\_abs}$$

$$\forall_{a \in B} \exists_{e \in s_2} e \cup \{\{n\}\} = a \tag{Eq.B\_assertion}$$

$$\forall_{a \in B} \exists_{e \in s_2} e \setminus \{\{n\}\} = a \setminus \{\{n\}\} \tag{Eq[140]}$$

$$\forall_{e \in B} e \setminus \{\{n\}\} \in s_2 \tag{Eq[141]}$$

$$s_2 \supset \{e \setminus \{\{n\}\} | e \in B\} \tag{Eq.s2\_supset}$$

$$\forall_{e \in B} e \setminus \{\{n\}\} \in s_2 \tag{Eq[141]}$$

$$s_2 \subset \{e \setminus \{\{n\}\} | e \in B\} \tag{Eq.s2\_subset}$$

$$\forall_{a \in s_2} \exists_{e \in B} a = e \setminus \{\{n\}\} \tag{Eq[142]}$$

$$\forall_{a \in s_2} \exists_{e \in B} a \cup \{\{n\}\} = e \cup \{\{n\}\} \tag{Eq[143]}$$

$$\forall_{e \in s_2} e \cup \{\{n\}\} \in B \tag{Eq[144]}$$

$$\forall_{e \in B} \{\{n\}\} = e \cap \{\{n\}\} \tag{Eq[145]}$$

$$\forall_{e \in B} e \cup \{\{n\}\} = e \tag{Eq[146]}$$

$$\forall_{a \in s_2} a \cup \{\{n\}\} \in B \tag{Eq[144]}$$

$$\forall_{a \in s_2} \exists_{e \in B} a \setminus \{\{n\}\} = e \setminus \{\{n\}\} \tag{Eq[147]}$$

$$\forall_{a \in s_2} a \setminus \{\{n\}\} = a \tag{Eq.s2\_complement\_n}$$

$$\forall_{a \in s_2} \exists_{e \in B} a = e \setminus \{\{n\}\} \tag{Eq[142]}$$

$$s_2 = \{e \setminus \{\{n\}\} | e \in B\} \tag{Eq[148]}$$

$$|\{e \setminus \{\{n\}\} | e \in B\}| \le |B| \tag{Eq[149]}$$

$$|s_2| \le |B| \tag{Eq[150]}$$

$$|\{e \cup \{\{n\}\} | e \in s_2\}| \le |s_2| \tag{Eq[151]}$$

$$|B| \le |s_2| \tag{Eq[152]}$$

$$|s_2| = |B| \tag{Eq[153]}$$

$$|s_1| = |s_2| + \left|\bigcup_{j} A_j\right| \tag{Eq[154]}$$

$$\left|\bigcup_{j} A_j\right| = (k+1)\,|s_3| \tag{Eq.A\_union\_abs}$$

$$A' = [j]\left\{\left\{*x'\right\}\right\} \tag{Eq.A\_quote\_definition}$$

$$A_j = \bigcup_{x_{0:k+1} \in s'_3} A'_j \tag{Eq.A\_definition\_simplified}$$

$$\forall_{\substack{j' \in [0;k] \setminus \{j\} \\ x_{0:k+1} \in s'_3}} A'_{j'} \cap A'_j = \emptyset \tag{Eq.nonoverlapping}$$

$$\exists_{\substack{j' \in [0;k] \setminus \{j\} \\ x_{0:k+1} \in s'_3}} A'_{j'} \cap A'_j \ne \emptyset \tag{Eq[155]}$$

$$\exists_{\substack{k\in A'_j \\ j'\in[0;k]\setminus\{j\} \\ x_{0:k+}\in s'_3}} k \in A'_{j'} \quad \text{Eq[156]}$$

$$\exists_{\substack{j'\in[0;k]\setminus\{j\} \\ x_{0:k+}\in s'_3}} \left\{*x'\right\} \in A'_{j'} \quad \text{Eq[157]}$$

$$\exists_{\substack{j'\in[0;k]\setminus\{j\} \\ x_{0:k+}\in s'_3}} \left\{*x'\right\} = \bigcup_{i=0}^{k} \left\{ \begin{cases} \{n\} \cup x_i & \text{if } i = j' \\ x_i & \text{else} \end{cases} \right\} \quad \text{Eq[158]}$$

$$\exists_{\substack{j'\in[0;k]\setminus\{j\} \\ x_{0:k+}\in s'_3}} \forall_{0\le i\le k} \begin{cases} \{n\} \cup x_i & \text{if } i = j' \\ x_i & \text{else} \end{cases} \in \left\{*x'\right\} \quad \text{Eq[159]}$$

$$\exists_{\substack{x_{0:k+}\in s'_3 \\ j'\in[0;k]\setminus\{j\}}} \{n\} \cup x_{j'} \in \left\{*x'\right\} \quad \text{Eq[160]}$$

$$\exists_{\substack{0\le i\le k \\ x_{0:k+}\in s'_3 \\ j'\in[0;k]\setminus\{j\}}} \{n\} \cup x_{j'} = x'_i \quad \text{Eq[161]}$$

$$\exists_{\substack{0\le i\le k \\ x_{0:k+}\in s'_3 \\ j'\in[0;k]\setminus\{j\}}} \{n\} \cup x_{j'} = \begin{cases} \{n\} \cup x_i & \text{if } i = j \\ x_i & \text{else} \end{cases} \quad \text{Eq[162]}$$

$$\exists_{\substack{x_{0:k+}\in s'_3 \\ j'\in[0;k]\setminus\{j\}}} \{n\} \cup x_{j'} = \{n\} \cup x_j \quad \text{Eq[163]}$$

$$\exists_{\substack{x_{0:k+}\in s'_3 \\ j'\in[0;k]\setminus\{j\}}} \forall_{i|i\neq j} \{n\} \cup x_{j'} = x_i \quad \text{Eq[164]}$$

$$\exists_{x_{0:k+}\in s'_3} \forall_{i|i\neq j} \{n\} = \{n\} \cap x_i \quad \text{Eq[165]}$$

False

$$\forall_{j'\in[0;k]\setminus\{j\}} \bigcup_{x_{0:k+}\in s'_3} A'_{j'} \cap A'_j = \emptyset \quad \text{Eq[166]}$$

$$\forall_{j'\in[0;k]\setminus\{j\}} \bigcup_{x_{0:k+}\in s'_3} A'_{j'} \cap \bigcup_{x_{0:k+}\in s'_3} A'_j = \emptyset \quad \text{Eq[167]}$$

$$A_{j'} = \bigcup_{x_{0:k+}\in s'_3} A'_{j'} \quad \text{Eq[168]}$$

$$\forall_{j'\in[0;k]\setminus\{j\}} A_{j'} \cap A_j = \emptyset \quad \text{Eq[169]}$$

$$\left|\bigcup_j A_j\right| = \sum_j |A_j| \quad \text{Eq[170]}$$

$$\forall_{B\in[0;k]\setminus\{j\}} A_B \cap A_j = \emptyset \quad \text{Eq[169]}$$

$$\sum_j |A_j| = (k+1)\,|s_3| \quad \text{Eq[171]}$$

$$\hat{A}_j = \bigcup_{B\in A_j} \left\{[i : |B|][*B]_{a_i} \middle| \{*a\} = [0;|B|)\right\} \quad \text{Eq[172]}$$