MODERN PHYSICS

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[COSMOS]

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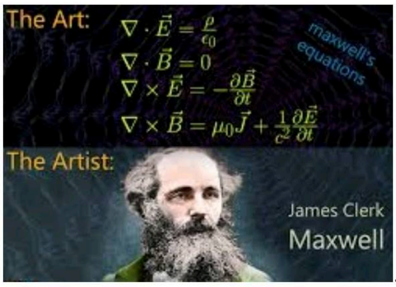
Part 1. NATURE OF LIGHT

Understanding the nature of light has played the most crucial role for the development of the modern physics. The journey was started by Newton . In 1675 Newton proposed the "Corpuscular theory of light". According to it light consists of a stream of tiny particles moving with a great speed.Though this model could explain many facts of geometrical optics but failed to explain phenomena like interference, diffraction,polarisation, etc. Next came the mechanical model of light , proposed by Huygens. Huygens's theory of longitudinal wave failed to explain the phenomena of polarisation.
A completely different point of view about nature of light was put forwarded by Maxwell.



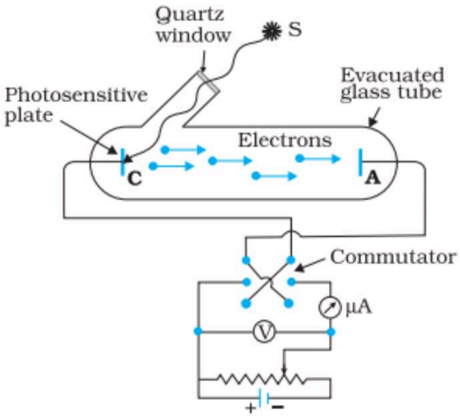
According to him light is transverse electromagnetic wave. A changing magnetic field produces electric field and changing electric field in turn produces a changing magnetic field. The existent of such theory was verified by Hertz. The model was in safe zone until the discovery of "Photoelectric Effect" in the end of nineteenth century. Maxwell's wave theory of light miserably failed to explain this newly discovered phenomena.Then Einstein came (he is everywhere) with his "Photon theory of Light". According to this theory light consists of packet of energy or quanta called photons.

Well you are totally confused now and thinking about what is real nature of light is. For this moment I will not recover you from this beautiful confusion.

Part 2. PHOTOELECTRIC EFFECT

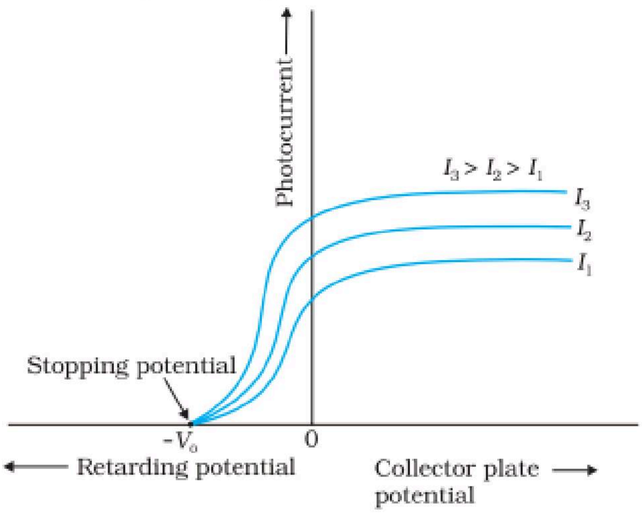
Definition. When light of sufficiently small wavelength is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called the photoelectric effect. The electrons ejected from the metal are called photoelectrons.

1. EXPERIMENT



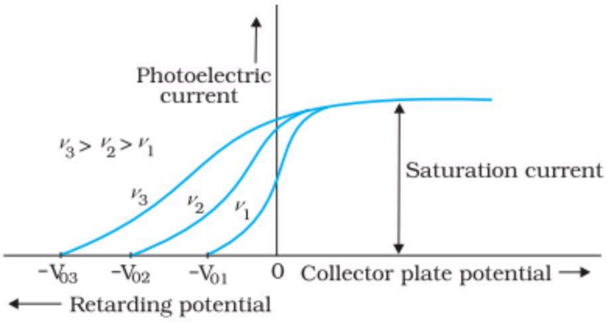
2. RESULTS FROM THE EXPERIMENT

1. The photoelectric current depends on the intensity of the light used. It is



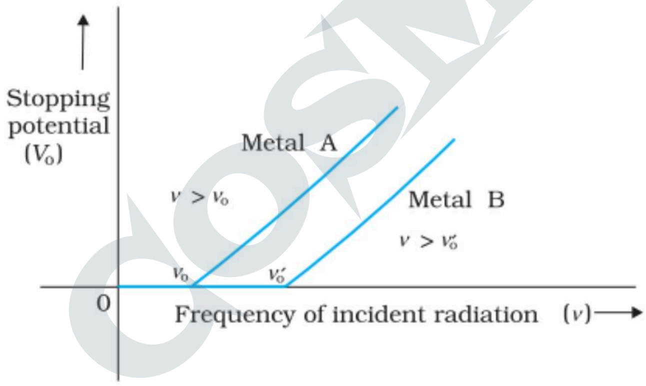
independent of the wavelength of light.

Definition. If the potential of A is made negative w.r.t C , the current decreases and ultimately becomes zero at some definite negative potential $-V_0$ known as stopping potential which is found to be independent of intensity of the light used, but depends only on the wavelength of the light for a given cathode material.



2.The photoelectrons are emitted with all possible velocities from 0 upto a maximum v_m which is independent of the light used. Maximum K.E of the photoelectrons increases linearly with the frequency of the light used.
For a potential $-V_0$ the photoelectrons of all possible velocities upto the maximum v_m are prevented from reaching A.obviously at this point ,

$$\frac{1}{2}mv_m^2 = eV_0 \tag{2.1}$$



.

3.Photoelctron emission is an instantaneous effect .There is no time gap between incidence of light and emission of photoelectron.
4. No photoelectron emission takes place if the frequency $\nu < \nu_0$.This minimum frequency $\nu_0$is known as "THRESHOLD FREQUENCY"..

3. FAILURE OF ELECTROMAGNETIC THEORY OF LIGHT

The above experimental facts can not be explained on the basis of classical EM theory of light according to which light consists of mutually perpendicular oscillating electric field and magnetic field as a transverse wave with a definite velocity in the medium. The intensity of the light is determined by the amplitudes of these oscillation.When light falls on an electron bound in an atom, it is acted upon by em field and gains the energy from from the latter. Larger the amplitude of the wave , larger is the quantity of energy

gained by the electron. So , according to this theory, the energy of the emitted electron should depend on the intensity of the incident light which is contrary to the observed fact (point 2).
According to EM theory , the velocity of the emitted electron should not depend on frequency of the light. Whatever be the frequency of the incident light , electron would be emitted if it gets sufficient time to collect energy for emission. As the EM waves pass by an electron , the latter gets a some amount of energy from each passing wave. Ultimately the when it is able to collect sufficient energy to break loose bondage in the atom, it is emitted. The time necessary for this purpose may be some order of seconds. So these two conclusions are contrary to the observed facts(point 3 and 4).
The incident EM wave acts equally on all the electrons of the metal surface. There is no reason why only some electrons will be able to collect necessary energy for emission. Given sufficient time , all electrons should be able to collect the energy necessary for the emission. So, there is no reason why photoelectric current should depend upon intensity of the incident light. However , this is again contrary to the observed facts.(point 1).

4. EINSTEIN'S LIGHT QUANTUM HYPOTHESIS

Einstein postulated that light is emitted from a source in the form of bundles of energy by of energy of the amount $h\nu$ known as light quantum or photon. When a photon of energy $h\nu$ falls on an electron bound inside an atom , the electron absorbs the energy $h\nu$ and is emitted from the atom provided that the $h\nu$ is greater than the binding energy of electron in metal which is equal to Work function , W_0 of the metal. The surplus energy $h\nu - W_0$is carried away by the electron as its kinetic energy.

$$h\nu - W_0 = \frac{1}{2}mv_m^2 = eV_0 \tag{4.1}$$

.
Let at $\nu = \nu_0$there is no emission of electron . So,

$$h\nu_0 = W_0 \tag{4.2}$$

.

$$h\nu - h\nu_0 = \frac{1}{2}mv_m^2$$

Obviously if $h\nu < W_0$i.e., $\nu < \nu_0$ no photoelectron emission can take place. This explains existence of threshold frequency (point 4,2).
According to this theory , larger the number of photon falling on the metal , greater is the probability of their encounter with atomic electron and hence greater is photoelectric current. So the increase of photoelectric current with the increasing light intensity finds an easy explanation in this theory (point 1).
As soon as a photon falls on an electron , the latter absorbs it and is emitted instantaneously. Thus instantaneous emission is also easily explained in this theory .(point 3).

Part 3. WAVE- PARTICLE DUALITY

DE BROGLIE'S HYPOTHESIS

We have seen that photoelectric effect is the evidance of corpuscular nature of light. On the other hand the phenomena of interference , diffraction and polarisation reveal the wave nature of light.In 1923 De- Broglie took things even further by suggesting that - all material particles should display a dual wave-particle behaviour. According to him-
Each material particle of of momentum $\vec{p}$behaves as a group of waves with whose waveleangth is

$$\lambda = \frac{h}{p} \tag{4.3}$$

. De Broglie's idea was confirmed experimentally in 1927 by Davisson and Germer and later by Thomson , who obtained interference patterns with electrons.

Classically a wave of definite wavelength and frequency is of infinite extent in space. On the other hand a particle is localized at a definite point in space. So, the wave and corpuscular charecters of an entity are mutually exclusive.

The theory of Quantum Mechanics , however provides a proper framework for reconciling the particle and wave aspect of matter. Quantum mechanics can simultaneously make statement about particle behaviour and wave behaviour of a microscopic system. The true reality of quantum system is that it is neither a pure particle nor a pure wave. It is "particle wave".

WAVELENGTH VS VOLTAGE

Let us now consider an electron accelerated through a potential difference of V volt .

The kinetic energy of the electron of mass m and charge e is

$$E_k = \frac{1}{2}mv^2 = eV \tag{4.4}$$

$$mv = p = \sqrt{2meV} \tag{4.5}$$

So , the de Broglie wavelength ,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \tag{4.6}$$

On substituting the values of h, m, e,

$$\lambda = \frac{12.26}{\sqrt{V}} A^0 \tag{4.7}$$

Here,

$$\lambda \propto \frac{1}{\sqrt{V}}, \lambda \propto \frac{1}{\sqrt{q}}, \lambda \propto \frac{1}{\sqrt{m}} \tag{4.8}$$

PROBLEMS

Problem 1. If an electron is accelerated through a potential difference 54V , then what will be it's de Broglie waveleangth? Now if a particle of charge 4e is accelerated through the same potential difference , then what will be it's de Broglie wavelength?

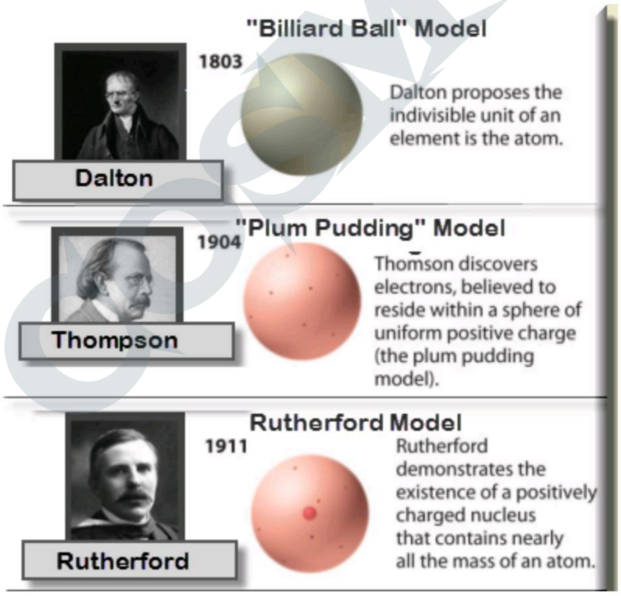
Problem 2. Calculate the de Broglie wavelength for a 100g bullet moving at 900 m/s.

Exercise 3. Calculate de Broglie wavelength of a proton of kinetic energy 700 MeV

Exercise 4. Consider a cylinder filled with a gas at tempertaure T . If the mass of a constitutent particle is m. What will be de Broglie wavelength of a gas particle?

Part 4. ATOMIC MODELS

BEFORE BOHR MODEL

- 

BOHR MODEL

Bohr postulated that -

(1) Instead of a continuum oribit of electrons, only a discrete set of circular stable orbits, called stationary orbits are allowed.
(2) The allowed orbits correspond to those for which orbital angular momentum of the electron is integar multiple of $\frac{h}{2\pi}$,

$$L = mv_n r_n = \frac{nh}{2\pi} \qquad (4.9)$$

where $n = 1, 2, 3, ...$ This relation is known as Bohr quantization principle.

3. As long as an electron remains in a stationary orbit , it does not radiate any electromagnetic energy. Emission or absorption of radiation can take place only when an electron jumps from one allowed orbit to another. The radiation corresponding to the electron's transition from one orbit of energy $E_n$to another E_m is carried out by a photon of energy ,

$$h\nu = E_n - E_m \tag{4.10}$$

So an atom may emit (or absorb) radiation by having the electron jump to a lower (or higher) orbit.

CALCULATION OF RADII OF ORBITS

If the mass of the electron is m, its charge is-e and the nuclear charge is +ze , then assuming the nuclear mass M , to be infintely large w.r.t mass of electron, we can equate electrostatic attraction between nulceus and the electron to the centripetal force.

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n^2} \tag{4.11}$$

$$v_n^2 = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{mr_n} \tag{4.12}$$

Where r_n =radius of nth orbit , $v_n = velocity$ of electron in nth orbit.

Now from bohr quaintization principle

$$mv_nr_n = \frac{nh}{2\pi} \tag{4.13}$$

$$v_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^2} \tag{4.14}$$

So comparing (4.12) and (4.14) we get

$$r_n = \frac{\epsilon_0 n^2h^2}{\pi mZe^2} \tag{4.15}$$

Remark. $r_n \propto n^2$, $r_n \propto \frac{1}{Z}$

For H-atom $Z = 1$, then

$$r_n = \frac{\epsilon_0 n^2h^2}{\pi me^2} \tag{4.16}$$

The radius of first bohr orbit of H-atom i.e., for n=1, putting the values of constants

$$r_1 = 0.53 * 10^{-10}m \tag{4.17}$$

Remark.

$$r_1 = a_0 = 0.53A^0 \tag{4.18}$$

Problem 5. In a H-atom an electron is moving around nucleus in 3rd bohr orbit .What will be the angular momentum of the electron?

Problem 6. Find the ratio of radius of 2nd and 4th bohr orbit of H-atom.

CALCULATION OF VELOCITY OF ELECTRON IN NTH ORBIT

For H-atom , from equation (4.12)

$$v_n^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr_n} \tag{4.19}$$

$$v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mv_nr_n} \tag{4.20}$$

$$v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\frac{nh}{2\pi}} \tag{4.21}$$

$$v_n = \frac{c}{n}[\frac{e^2}{4\pi\epsilon_0 c(\frac{h}{2\pi})}] \tag{4.22}$$

Let $[\frac{\boldsymbol{e^2}}{\boldsymbol{4\pi\epsilon_0 c(\frac{h}{2\pi})}}] = \boldsymbol{\alpha}$then

$$v_n = \frac{c}{n}\alpha \tag{4.23}$$

Here, $\alpha = \frac{1}{137}$, $\alpha$is called fine structure constant.

Remark. $v_n \propto \frac{1}{n}$

Problem 7. What will be the number of complete revolution in 1s of an electron in first bohr orbit of H-atom.

Problem 8. What will be ratio of frequencies of electron in first and 2nd bohr orbit of H-atom?

ENERGY LEVELS

kinetic energy of an electron in nth orbit ,

$$E_k = \frac{1}{2}mv_n^2 \tag{4.24}$$

$$E_k = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n} \tag{4.25}$$

potential energy ,

$$E_p = \frac{1}{4\pi\epsilon_0}\frac{Ze*(-e)}{r_n} \tag{4.26}$$

Total energy ,

$$E_n = E_k + E_p = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n}(\frac{1}{2} - 1) \tag{4.27}$$

$$E_n = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r_n} \tag{4.28}$$

Now, we know that $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$, So,

$$E_n = (-\frac{me^4 Z^2}{8\epsilon_0^2 n^2 h^2}) \tag{4.29}$$

Remark. $E \propto \frac{1}{n^2}$

For H-atom,

$$E_n = (-\frac{me^4}{8\epsilon_0^2 n^2 h^2}) \tag{4.30}$$

The energy of electron in first bohr orbit of H-atom

$$E_n = -13.6ev \tag{4.31}$$

So,

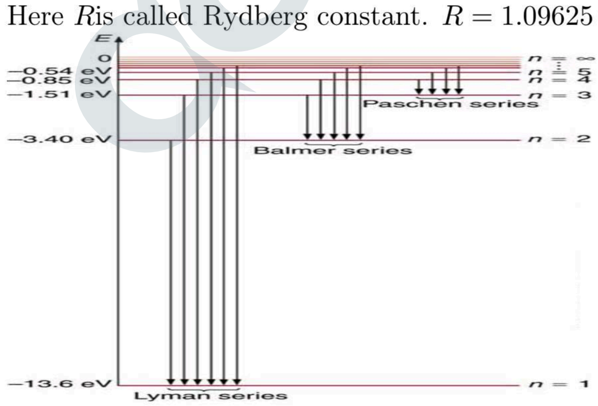
$$E_n = -\frac{13.6}{n^2}ev \tag{4.32}$$

Now ,

$$E_n = -(\frac{me^4}{8\epsilon_0^2 ch^3})(\frac{ch}{n^2}) \tag{4.33}$$

$$E_n = -\frac{Rch}{n^2} \tag{4.34}$$

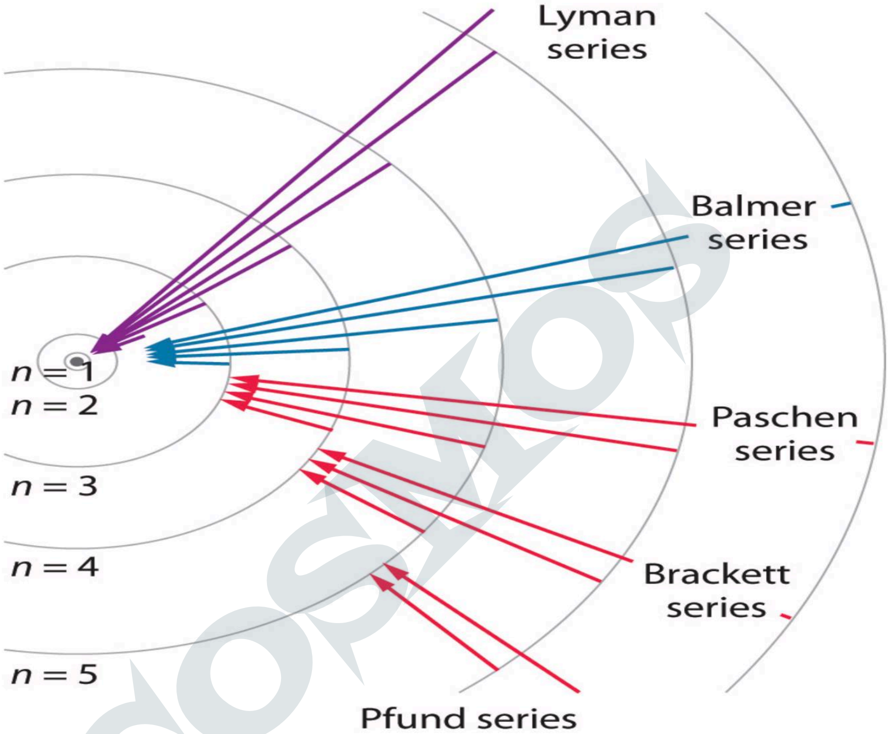
Here $R$is called Rydberg constant. $R = 1.09625 * 10^7 m^{-1}$



Remark. Look here $E_n \propto \frac{1}{n^2}$. So, the energy of electron is quanitized here i.e., the electrons can not have any arbitary amount of energy whereas in classical mechanics there is no such constraints.This is fundamental difference between classical and quantum mechanics.

When the quantum number n is very large , the radius of electronic orbit becomes large and energy spectrum becomes continious i.e., we are going towards classical physics.So, we may state that results deduced from the new atomic mechanics should be identical with the results of classical mechanics in the very large quantum number n. This is called Bohr's correspondence principle.

TRANSITION OF ELECTRON



Let an electron makes a transition from an i th orbit to f th orbit (i>f).If the frequency of the emitted photon is νthen,

$$E_i - E_f = h\nu \tag{4.35}$$

$$Rch(\frac{1}{f^2} - \frac{1}{i^2}) = \frac{hc}{\lambda} \tag{4.36}$$

$$\frac{1}{\lambda} = R(\frac{1}{f^2} - \frac{1}{i^2}) \tag{4.37}$$

Problem 9. Calculate the energy of He^+ion in its first excited state.

Problem 10. How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?