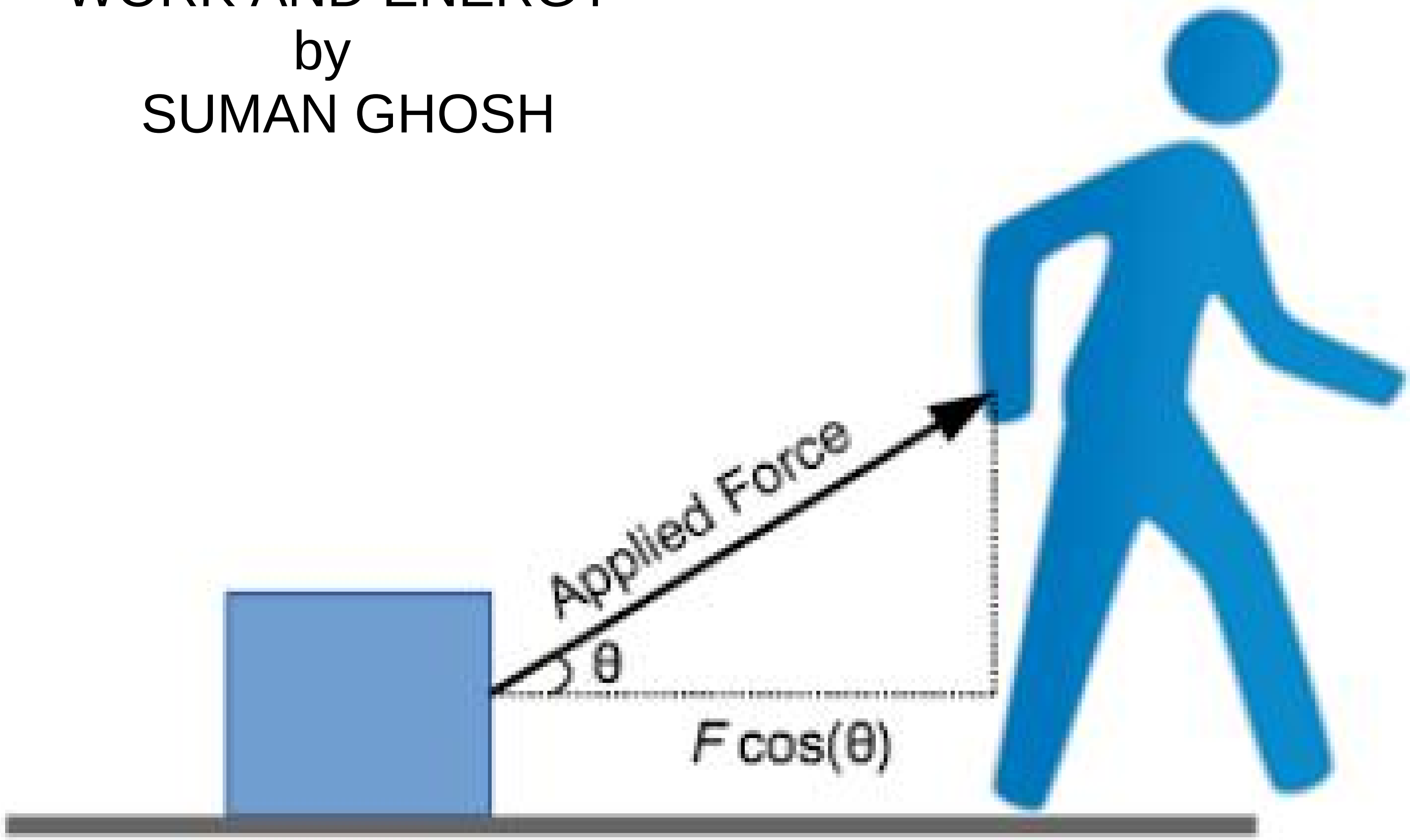
WORK AND ENERGY

by

SUMAN GHOSH



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Contents

Definition. Under the action of a force $\overrightarrow{F}$if small displacement of a particle is $d\overrightarrow{r}$, then the quantity $\overrightarrow{F}.d\overrightarrow{r}$is called work done by the force during this displacement.

The total work done if the particle is displced form position $\overrightarrow{r}_i$ to $\overrightarrow{r}_f$.Then the work done,

$$W = \int_{\overrightarrow{r}_i}^{\overrightarrow{r}_f} \overrightarrow{F}.d\overrightarrow{r}$$

$$W = \int_{\overrightarrow{r}_i}^{\overrightarrow{r}_f} m\overrightarrow{a}.d\overrightarrow{r}$$

$$W = \int_{\overrightarrow{r}_i}^{\overrightarrow{r}_f} m\frac{d\overrightarrow{v}}{dt}.d\overrightarrow{r}$$

$$W = \int_{v_i}^{v_f} md\overrightarrow{v}.\frac{d\overrightarrow{r}}{dt}$$

$$W = \int_{v_i}^{v_f} md\overrightarrow{v}.\overrightarrow{v}$$

Now, $\overrightarrow{v}.\overrightarrow{v} = v^2$
Taking derivative on both side

$$d\overrightarrow{v}.\overrightarrow{v} = \frac{1}{2}d(v^2)$$

So substituting the above relation

$$W = \int_{v_i}^{v_f} m\frac{1}{2}d(v^2)$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

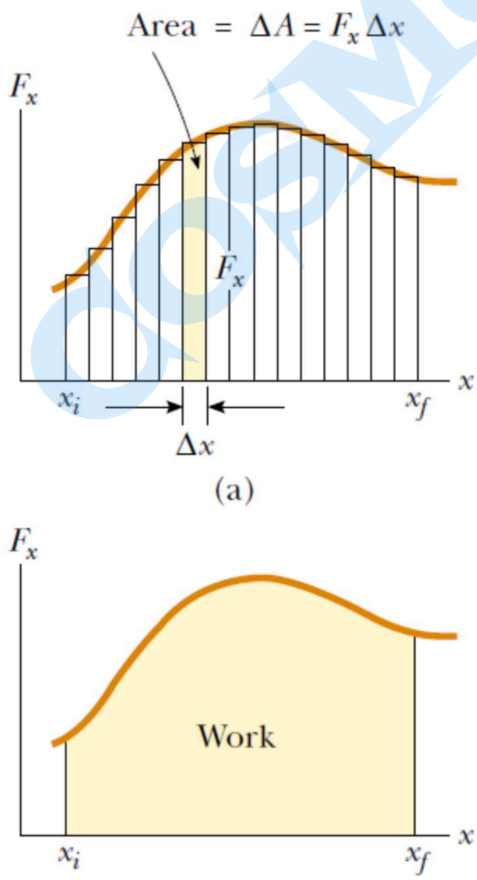
So here the term $\frac{1}{2}mv^2$ is called kinetic energy of the particle

$$W = (E_k)_f - (E_k)_i$$

This is called Work-Energy theorem.

Theorem. *The net work done on a particle equals the change in its kinetic energy.*

CALCULATION OF WORK DONE USING FORCE DISPLACEMENT GRAPH



Here consider a particle is dispalced of particle along x -axis from $x_i$to $x_f$under the action of a force F_x. Now for the small displacement Δx the work done by the force is

$$\Delta W = F_x \Delta x$$

If we imagine that the F_x versus x curve is divided into a large number of such intervals, then the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms,

$$\sum \Delta W \approx \sum F_x \Delta x$$

The work done by the force component $F_x$for the small displacement F_x is $F_x \Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles

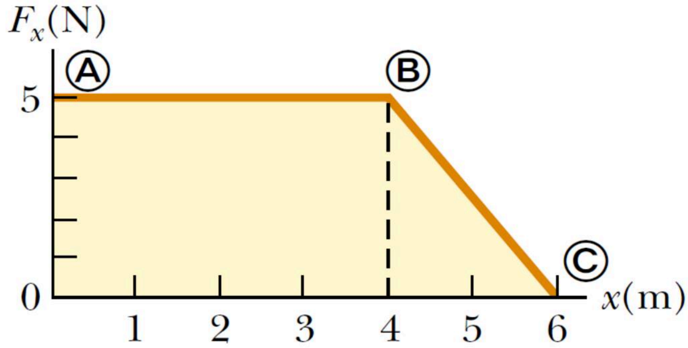
If the small displacements are infinitesimally small, then

$$\lim_{\Delta x \to 0} \sum F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

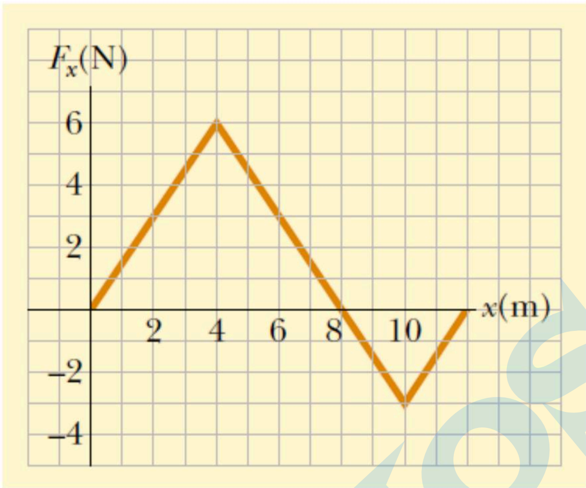
$$W = \int_{x_i}^{x_f} F_x dx$$

So, Work done=Area under force-displacement curve.

Problem. A force acting on a particle varies with x, as shown in Figure. Calculate the work done by the force as the particle moves from x =0 to x = 6.0 m.

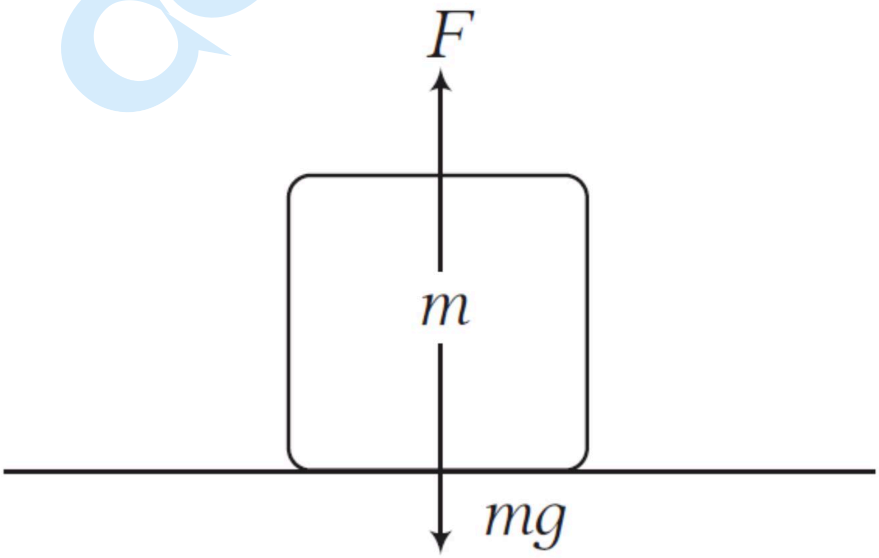


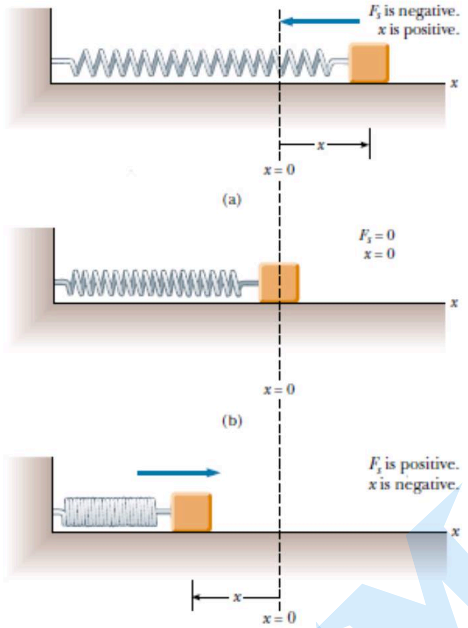
Problem. Find the work done.



Remark. Work done by force is positive and work done aginst the force is negative.

Problem. A block of mass m = 2 kg is pulled by a force F = 40 N upwards through a height h = 2 m. Find the work done on the block by the applied force F and its weight mg. (Take, g = 10 m/s^2)





We know that $F_s = -kx$. Now the spring is compressed to $-x_{max}$and released.Let us calculate the work Ws done by the spring force as the block moves from $x_i = x_{max}$ to $x_f = 0$.

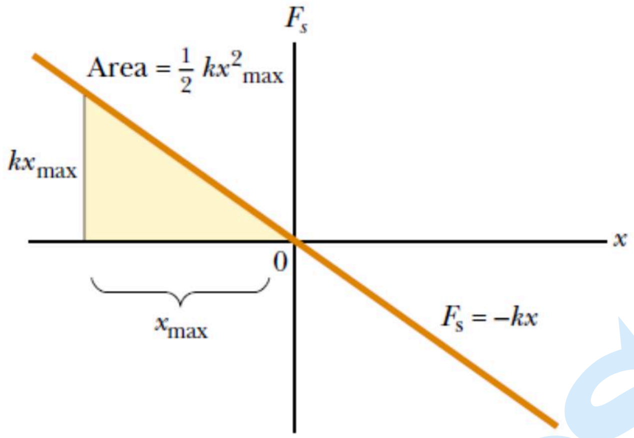
$$W_s = \int_{x_i}^{x_f} (-kx)dx$$

$$W_s = \frac{1}{2}kx_{max}^2$$

When we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{max}$, work done

$$W_s = -\frac{1}{2}kx_m^2$$

Therefore, the net work done by the spring force as the block moves from $x_i = -x_{max}$ to $x_f = x_{max}$ is zero.

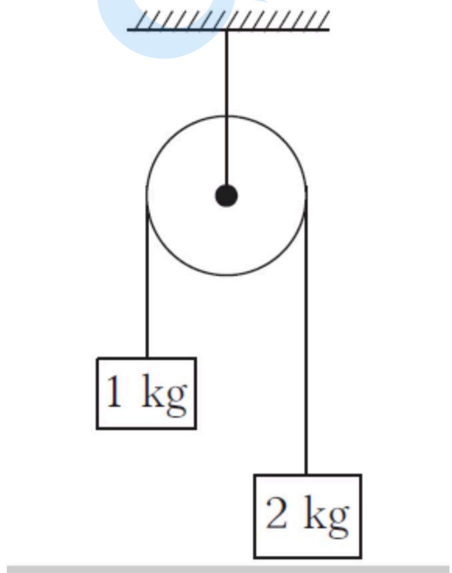


If the block undergoes an arbitrary displacement from x_i to x_f , the work done by the spring force is,

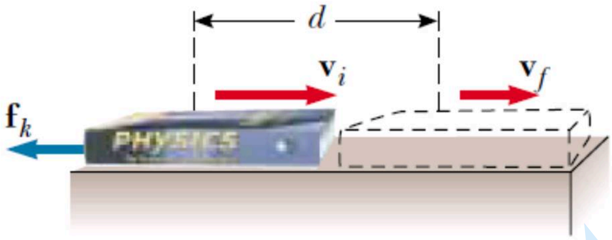
$$W_s = \int_{x_i}^{x_f} (-kx)dx$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Problem. Two unequal masses of 1 kg and 2 kg are attached at the two ends of a light inextensible string passing over a smooth pulley as shown in figure. If the system is released from rest, find the work done by string on both the blocks in 1 s. (Take, g = 10 m/s^2)



SITUATIONS INVOLVING KINETIC FRICTION



Suppose a book moving on a horizontal surface is given an initial horizontal velocity v_i and slides a distance d before reaching a final velocity v_f as shown in Figure.

Work done by the friction

$$W_f = -f_k d$$

So there will be loss of kinetic energy of the block of amount

$$\Delta K = -f_k d$$

When friction—as well as other forces—acts on an object, the work–kinetic energy theorem reads-

$$\sum W_{other} - f_k d = K_f - K_i$$

Problem. A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

Find the final speed of the block if the surface is not frictionless but instead has a coefficient of kinetic friction of 0.15.