Problem:

$$\min_{\{x_i\}} F\left(\vec{x}\right)$$

where

$$F\left(\vec{x}\right) = \sum_t b_t \ln Z_t\left(\vec{x}\right) - \sum_i a_i x_i$$

$$Z_t\left(\vec{x}\right) = \sum_s z_{st}\left(\vec{x}\right), \qquad z_{st}\left(\vec{x}\right) = \mathrm{e}^{c_{st} + \sum_i p_{si} x_i}$$

and a_i, b_t, c_{st}, p_{si} are given parameters. We assume that $a_i, b_t, p_{si} \geq 0$. The indices ranges are $1 \leq i \leq N$, $1 \leq t \leq T$ and $1 \leq s \leq S$. Note that $F\left(\vec{x}\right)$ is a convex function of $\vec{x}$. Its gradient is given by:

$$\frac{\partial F}{\partial x_i} = \sum_{s,t} z_{st} \frac{b_t}{Z_t} p_{si} - a_i = \sum_s g_s p_{si} - a_i$$

where

$$g_s = \sum_t \frac{z_{st}}{Z_t} b_t$$