I thought I knew the simplest of all RBC models

Carlos Eduardo Goncalves, IMF and Sao Paulo School of Economics*

July 5, 2023

1 Introduction

But alas, after these many years, it came as shock (pun intended) that my intuition for a standard TFP shock hitting a bare bones closed-economy representative agent model with absolute no bell and certainly no whistle was faulty.

Let us start with what at first-blush I thought was the response of consumption C_t to a Z_t shock (level productivity, see model below). Because of the frictionless nature of the set-up, my knee-jerk reaction was: consumption will jump and then return to its steady-state level gradually, as the TFP bonanza dissipates. However, the path is not monotonic and consumption is hump-shaped. But not because GDP grows over time![1]

Second, what happens to real interest rates following the same shock? How does its IRF look like? Common answer without opening the computer: real interest rates will go up because demand for capital increases as it becomes more productive and then converge monotonically back to its steady state level of $\frac{1}{\beta}$. Initial part is correct. Not the second: real interest rates fall below steady-sate and then rise again towards it. Let's go thru a quick description of the model before trying to dissect some of these dynamics.

2 The model

Let's consider a standard RBC model with the following equilibrium conditions. Beginning with the euler equation describing the intertemporal pattern of private consumption.

$$C_t^{-\sigma} - \beta(1 + r_{t+1})C_{t+1}^{-\sigma} = 0 \tag{1}$$

*Thanks Zione, DS and Yvan for the comments.

[1]By the way, if you are biased towards the study of open economy models you risk making an even more basic mistake, replying that consumption will jump by just a tiny bit and then stay at this new level forever, with the new discounted value a bit larger than before. Not in this set-up: remember it is a closed economy and therefore there is no way to smooth it out.

Second is the labor supply schedule: the higher the real wage, the more hours people are willing to work (at diminishing rates, of course).

$$w_t - \varphi L_t^{\nu} C_t^{\sigma} = 0 \tag{2}$$

When people invest more than the necessary to maitain the stock of capital, there are more machines available for workers tomorrow.

$$K_t - (1-\delta)K_{t-1} - I_t = 0 \tag{3}$$

Behold the most basic Cobb-Douglas production function: things are produced with labor and capital and these factors are repaid the shares α and $1-\alpha$ of national income.

$$Y_t - Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = 0 \tag{4}$$

Next, the total return on capital which increases with αZ_t is given by:

$$r_t + \delta - \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1} = 0 \tag{5}$$

Real wages are higher the higher are the capital stock and the overall productivity (firms demand more labor if the latter churns out more output).

$$w_t - (1-\alpha) Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha} = 0 \tag{6}$$

Finally, the national account identity with no government and no foreign countries to interact with.

$$Y_t - C_t - I_t = 0 \tag{7}$$

3 Consumption is hump-shaped!

If you read some papers in what people call monetary macro, chances are you already bumped into an utility function featuring a dislike for changing consumption levels abruptly. The justification is: people value not only the absolute level of consumption today; in addition they feel miserable changing their consumption habits. Under this assumption, changes in C will be more gradual. The response of consumption to, say a monetary policy shock, will have that beautiful hump-shaped format that we see when running SVARs (beware of the identification police!). Absent this feature, and given consumption is a jump variable, no hump-shaped response is to be expected, correct? C goes straight up and then converges down as the TFP shock subsides, right?

No, that is not how things unfold.

During the period TFP is above 1 (its steady state level), the economy builds up some extra capital. Investment responds strongly and immediately to a productivity shock.[2] Intuition is clear: k is temporarily more productive and you do not want to squander the opportunity of getting a larger kick for the invested buck.[3] Since $Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$, and labor also goes up with the higher wages[4], overall GDP edges up by more than the size of the shock (1 per cent in our case, see Figure 1). The cake will eventually fall back to its normal size. How quickly, depends on the ρ parameter in the equation describing the shock dynamics: $Z_t = \rho Z_t - 1 + \epsilon_t$.

What about the consumption path? Remember, because of risk aversion agents strive to smooth consumption out of a non-smooth income pattern. Usually, someone different, a foreigner or a government, is embedded in the model to provide people with an asset in which the extra (but temporary) income can be saved in. In the future, the representative agent then consumes a perpetuity out of this extra saving accumulated during good times, the times of extraordinarily high z. Again, not in the bare-bones RBC model. Here the agent has to eat the factory she built herself, brick by brick (no b, no b^*).

Right after the shock, the super productive capital generates a lot of output. Where does it go? To consumption and investment, sure, but relatively less to consumption at $t = 0$ than at $t = 1$. This is key and little appreciated. The smart decision is to hold back on consumption for a while, in order to build up a larger factory, which is going to be eaten down to steady state later on. The mirror image of this decision: consumption grows over time for some periods - hump shaped. For how long? It depends on the persistence of the productivity shock. But not only that and here comes the second overlooked detail: the rate of depreciation δ plays a major role. Curbing consumption growth just after the TFP shock has no benefits for future consumption if the damn bricks rot out quickly! The hump-shaped consumption response in the RBC whistleless model is a function of δ.

Calibration of the simple model's steady-state is as follows: $L, Z = 1$, real rate is set $r = 0.01$, elasticity of intertemporal substitution σ is set to 1, as is ν, the Fristch labor elasticity. Finally, quarterly depreciation of capital, $\delta = 0.025$ and α, the income share accruing to capital is set to 0.3. In steady state, the model delivers private consumption of 78 per cent of GDP, the rest of GDP being allocated to investment. Again, no government, closed-economy.

[2] In the main calibration, I grows more than 10 times more than consumption, relative to their respective steady states.

[3] A trick to hump shape investment is to add capital adjustment costs that are very high if you want to increase the capital stock very fast but decrease as the firm builds up more and more capital (not here, though, model here is the ultimate! bare-bones).

[4] Firms demand more labor to operate their more productive machines, but to elicit people to put in more hours they have to pay more.

3.1 The case of $\rho = 0.9$

Let us start with a persistent shock to productivity.

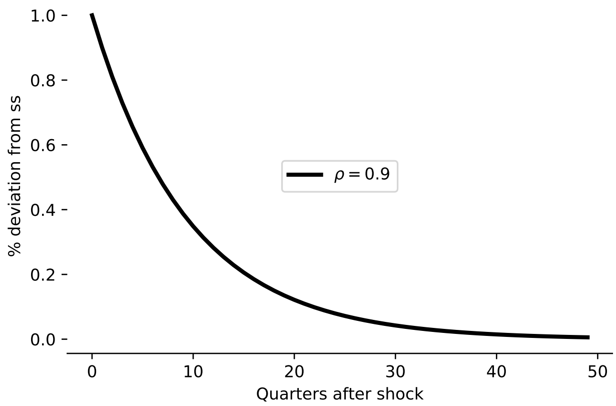

Figure 1: TFP shock

As can be seen in Figure 2, investment increases sharply, more than ten times the increase in consumption. The latter does not peak instantaneously, whereas the former does. But both are unconstrained by adjustment costs.

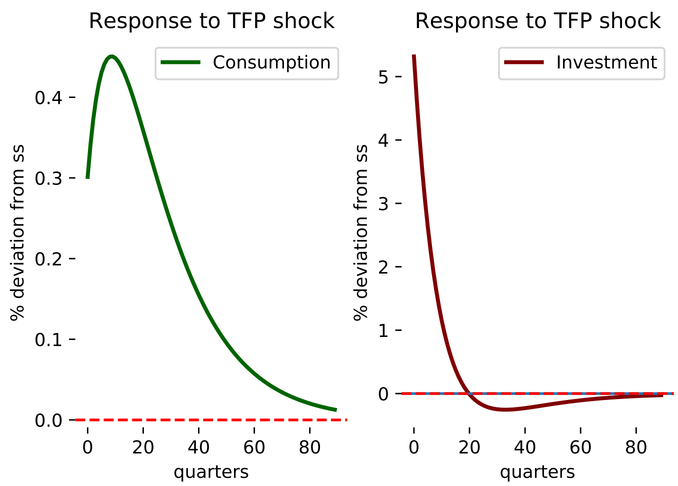

Figure 2: C has a hump! When I turns negative, C starts to fall, but beware of scale

The representative agent who owns the firm loses no time to invest because tomorrow capital will already be less productive than today. Consumption rises for 10 quarters before peaking. Importantly, this is not because a gradual accumulation of capital allows for a gradually larger GDP pie from which to consume: GDP peaks at $t = 0$!

I plotted C and I with different scales and upon reflection maybe that was not exactly fair. People do strive to smooth consumption because of concavity. In Figure 3 one can see both together with a single scale for better visualization. Note the very prolonged period of negative investment. Negative investment = people are eating the bricks, K is going back to steady state.

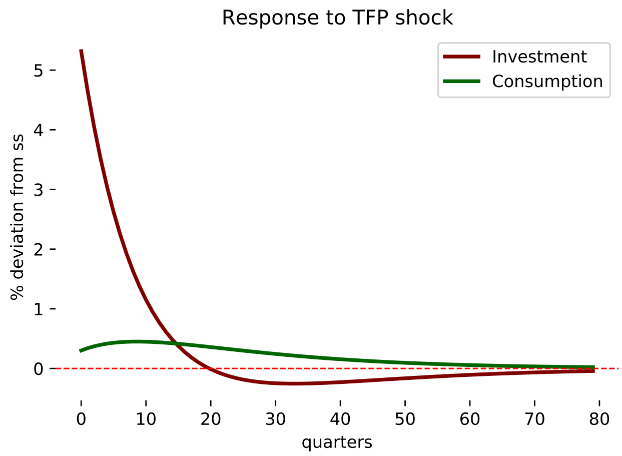

Figure 3: C has a hump ... but it is sort of small, really

Finally, in Figure 4 we can see the dynamics of output and our friend K itself. It drives home the point that consumption is not increasing from 0 to 10 because output is rising and thus more resources are becoming gradually available over time (though that was my immediate response when asked...). Output peaks with investment, at $t = 0$. The intuition is: use the resources to build more bricks together while this process is ultra efficacious; as Z comes down from the skies, then devote more of the extra Y to consumption (hence C rises for 9 periods).

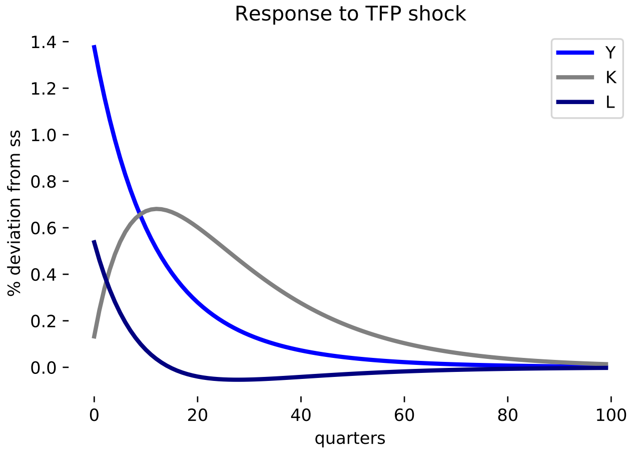

Figure 4: Y is not hump-shaped!

Now, to highlight the importance of the rate of capital depreciation for the hump-shapedness of consumption, let us re-calibrate the model to destroy capital at a super fast pace by setting $\delta = 0.3$. In each quarter, 30 per cent of what had been hitherto accumulated vanishes into thin air. Not nice.

First, investment has become a less interesting proposition now: what it creates goes back to dust so rapidly.[5] Yes, TFP is equally persistent, since we have not changed that for this exercise. However, there is not much of a point in building capital for the sake of future consumption anymore. Crucially, by weakening this channel, we considerably shorten the period during which consumption initially rises: from 10 to 3 quarters. Third, now investment converges monotonically towards its steady-state level of δK.

[5]Careful how you read this, though: without capital there is no output, and without output there is no consumption and without consumption you die.

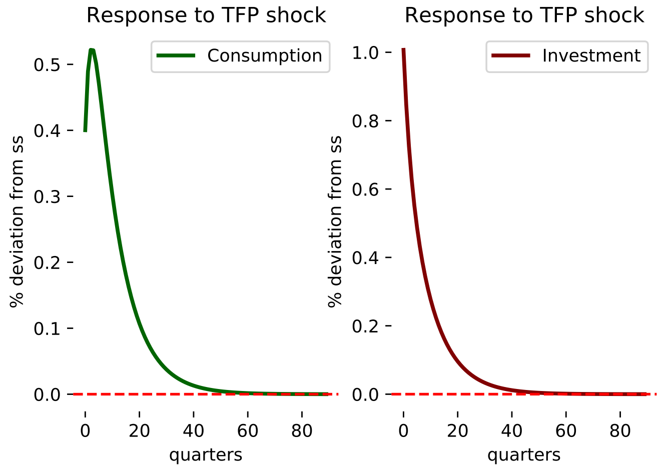

Figure 5: Vanishing hump-shapedness when capital dies out quickly

To further test the mechanism discussed let us now decrease the rate of depreciation of capital. Make K immortal, or almost, by setting δ to 0.0001 per quarter (warning: the y-axis below is not for the faint of the heart). Investment's extra capacity to generate output is as temporary as before, but since time does not deplete its oeuvre, one can truly smooth consumption for decades by saving more today. Bricks are forever! In this case, the hump-shapedness is more prominent. Consumption now increases for 17 quarters. Beware: I dramatically increased the extent of the x-axis below from 80 to 200 quarters.

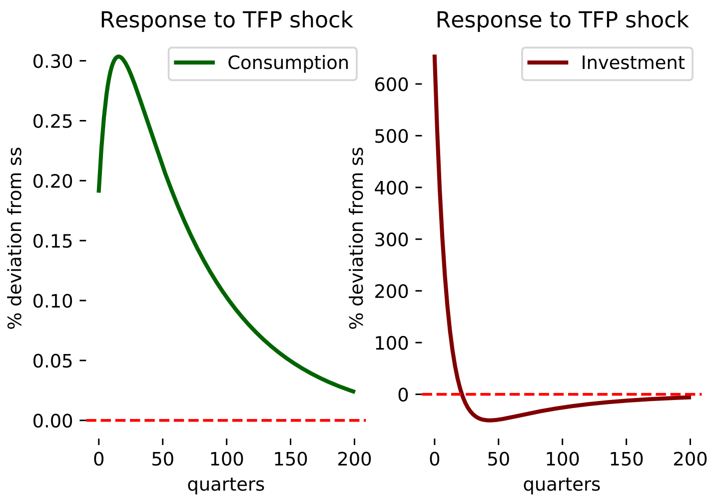

Figure 6: Bricks are forever!

Needless to say (so why say it?), if capital vanishes infinitely slowly, people are better off. The money you would have to spend just to replace the ageing bricks can now be diverted to consumption. Figure 7 is not intended to show these level differences, since it depicts deviations from (very different) steady-states. Its aim is to illustrate the degree to which consumption smoothing varies when capital depreciation varies, and how this affects the hump-shapednesses of the IRF.

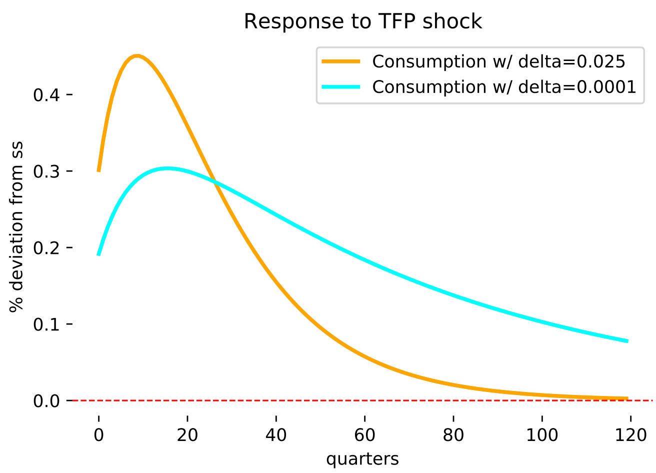

Figure 7: When bricks are forever you get more smoothing and a more prolonged hump

3.2 The case of $\rho = 0.1$

Now let us strongly reduce the persistence of positive productivity shock.

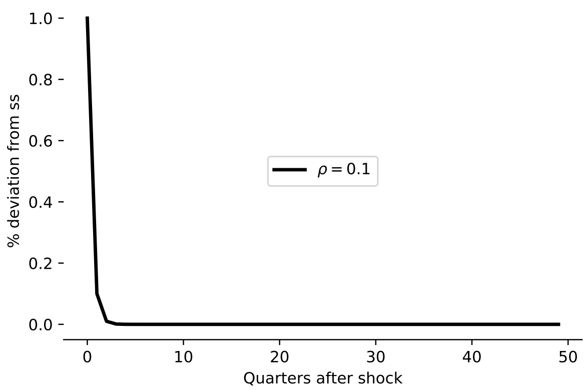

Figure 8: TFP shock

Only 10 percent of it lives to see another quarter. With this new shock dynamics, consumption is not hump-shaped anymore. No more ascending path for a couple of quarters. Why? Because capital is exceptional only today. So fleeting...

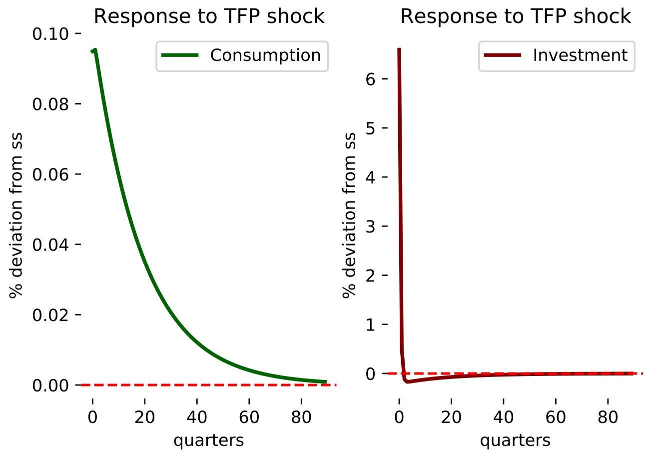

Figure 9: Where is the hump?

Before, as capital gradually lost its brilliance, agents incrementally gave up investing. Now the logic emphlet us consume a bit less during these initial periods so that firms can recycle the extraordinarily higher output into bricks with extraordinary powers that will guarantee our future consumption is broken. Not much use in postponing the rise in consumption, at $t = 1$ investment has already ben deprived of its magical $t = 0$ powers. Additionally, because the productivity bonanza lasts so little the overall rise in consumption is much meager.

But that, I guess, is no surprise for no one. The really interesting feature is hump-shapednessLESS of C for the same calibration, same model, only not-persistent-anymore TFP shock.

Adding insult to injury, Figure 10 cranks up the depreciation rate to 0.30.

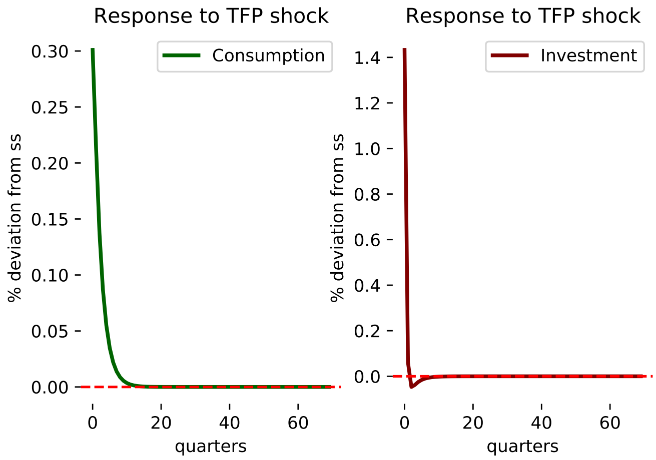

Figure 10: Insult to Injury and the End of Hump

4 Interest rates dive below SS

Interest rates, as all else here of course, are real (another intended pun). There is no money in the model and no central bank adjusting nominal interest rates to combat inflation. It is in a sense much less interesting.

Interest rates rise above steady state levels when demand for capital increases (which, in turn, happens when capital is more productive after the positive TFP shock). That much is intuitive. Using our standard calibration, real interest rates rise by a modest 0.05 percent before falling again. But what perhaps is not immediately obvious is this: the real interest rate dives below its steady-state level even if the TFP shock that perturbs it never goes into negative territory.

Why interest rates do not converge to its pre-shock level as smoothly as the shock that upsets it itself (beware the deviations are real small here)?

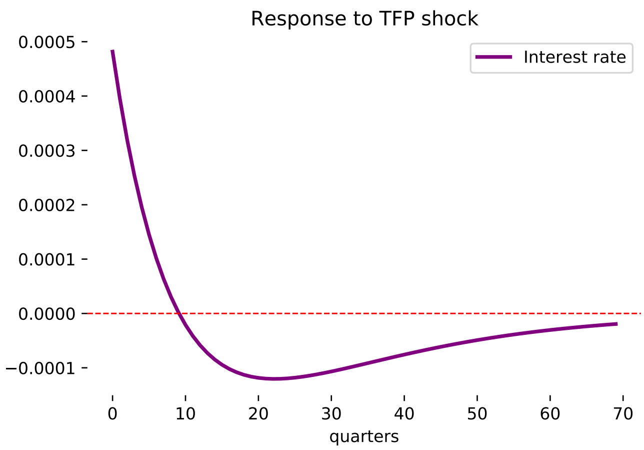

Figure 11: Not monotonically back

It might spend a lot of time above the steady state, as in the case where consumption inches up more slowly as a response to the shock, or fall and rise very quickly, as in the case with $\rho = 0.1$.

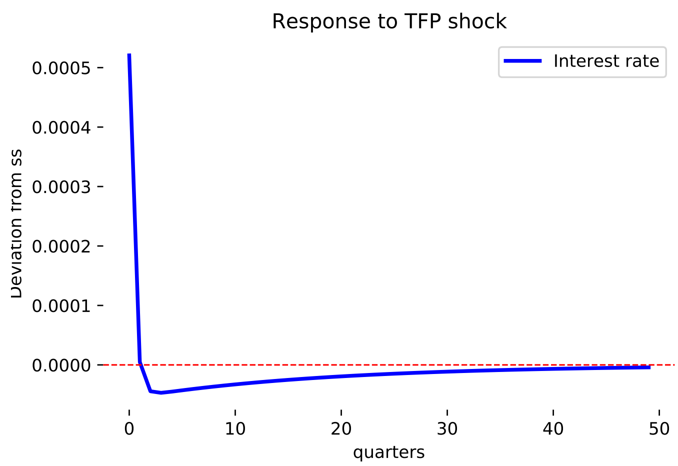

Figure 12: Not monotonically back even when $rho = 0.1$

This non-monotonicity in the path for interest rates is directly connected to the path of consumption. Whenever $C_{t+1} > C_t$, it is necessarily the case that $r < r^* = \frac{1}{\beta}$ as well. It is 100% euler equation explained. If c spends no more than 1 period going up (the initial one in the case with tremendously rapid decay in Z), r dives almost immediately, at $t = 1$, below r^* and remains there until C stops moving again (Figure 12).

5 Concluding thoughts

All has been said already, dynamics clarified.
Sometimes we think our intuitions are sharp, when they are not really. Even in the case of a very basic model, which scared me and prompted me to write this note. Do not rest, delve into the problem, dissect it, go to the computer simulate the damn thing and think again, looping over the chart-equation-chart-equation-chart until you get it right.

6 References

All those papers I read and apparently had not fully comprehended.