

Econometria Aplicada

Modelos ARIMA

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"The most important questions of life are, for the most part, really only problems in probability."

Laplace (1812)

"In God we trust. All others must bring data."

William Edwards Deming

Projeções

*If we could first know where we are and whither we are tending, we
could better judge what to do and how to do it*

Abraham Lincoln

Motivação (tudo começa com uma pergunta)

Qual será o crescimento trimestral do PIB nos próximos quatro trimestres?

Modelos

Seguindo Hyndman and Athanasopoulos (2021), para um AR(p) temos:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \quad (1)$$

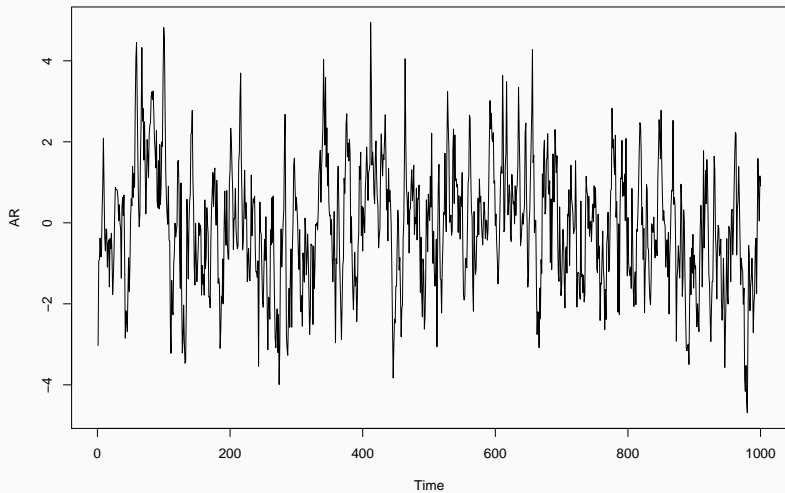
Assuma um AR(1):

- $\phi_1 = 0$ e $c = 0$: ruído branco.
- $\phi_1 = 1$ e $c = 0$: passeio aleatório.
- $\phi_1 = 0$ e $c \neq 0$: passeio aleatório com “drift”
- $\phi_1 < 0$: a série oscila em torno de uma média.

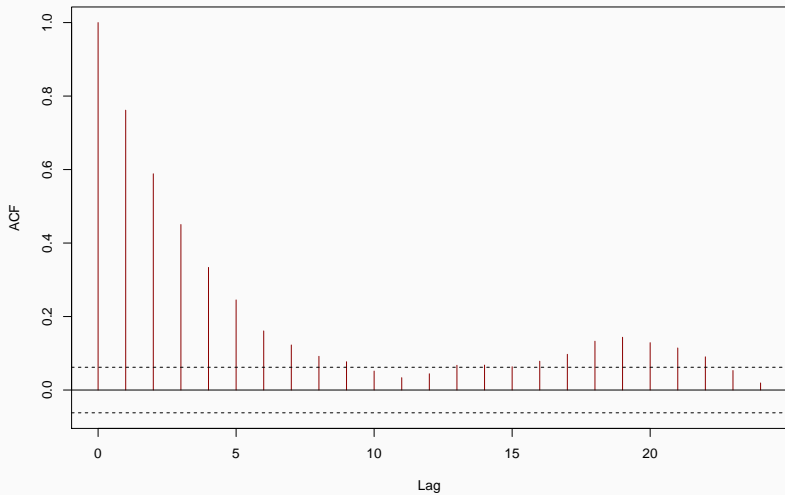
Estacionariedade:

- AR(1): $-1 < \phi_1 < 1$
- AR(2): $-1 < \phi_2 < 1, \phi_1 + \phi_2 < 1, \phi_2 - \phi_1 < 1$

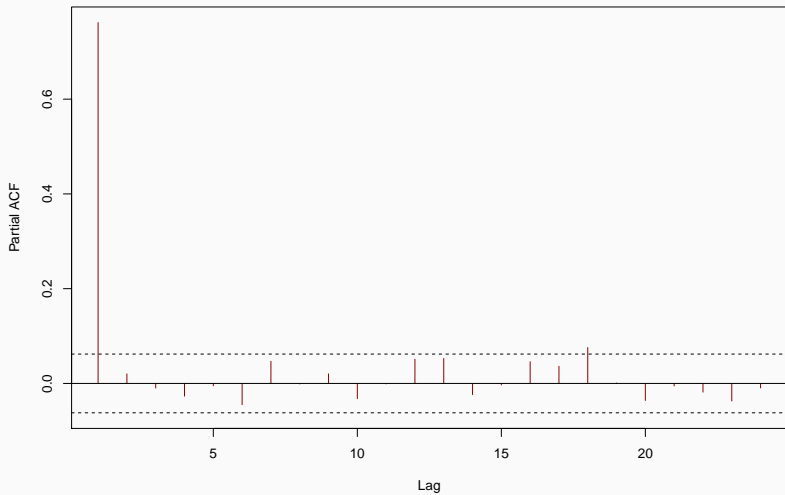
$$\text{AR}(1) - \phi_1 = 0,8$$



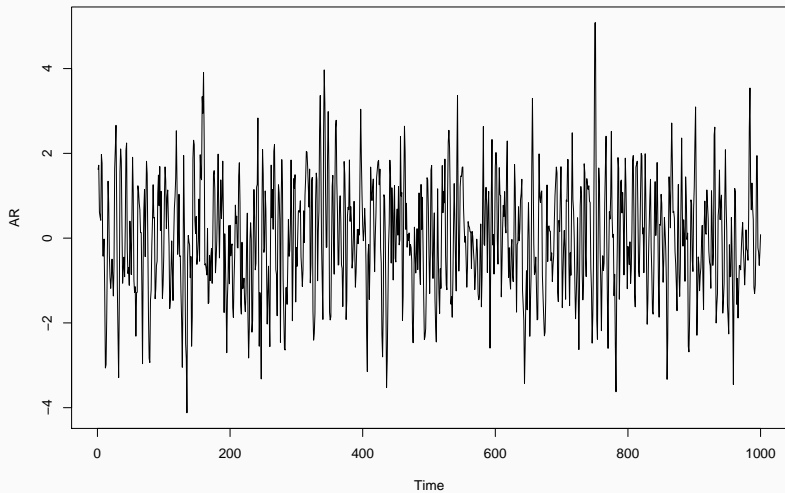
AR(1) - ACF



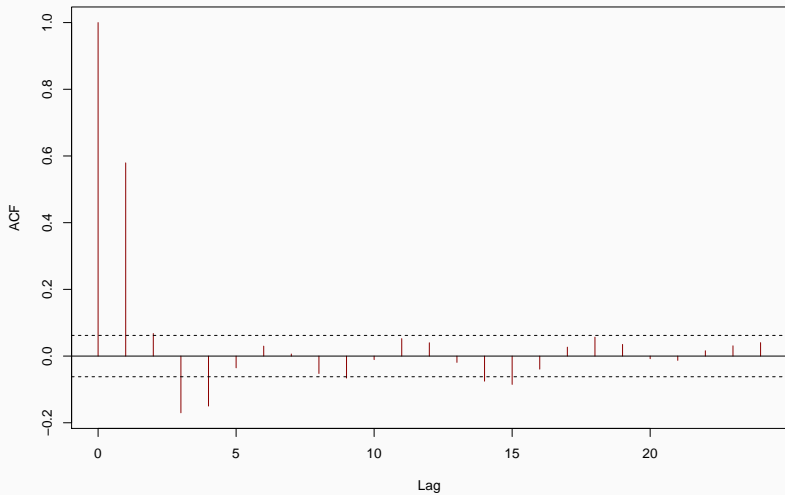
AR(1) - PACF



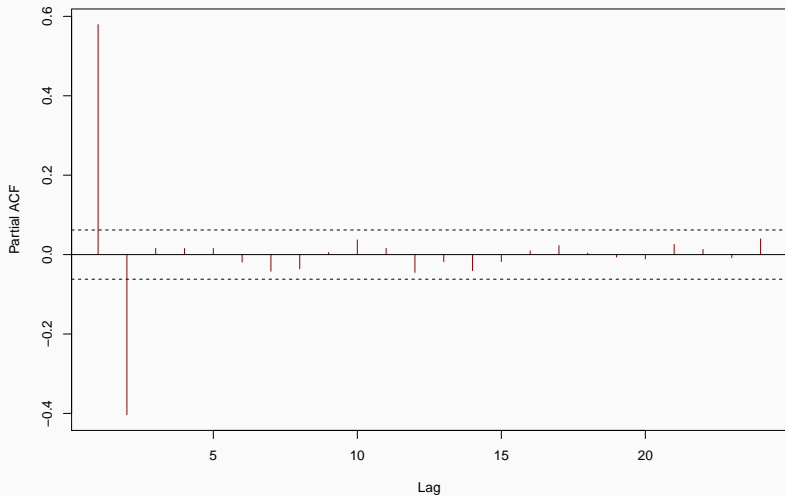
$$\text{AR}(2) - \phi_1 = 0,8 \text{ e } \phi_2 = -0,4$$



AR(2) - ACF



AR(2) - PACF



Média Móvel (MA)

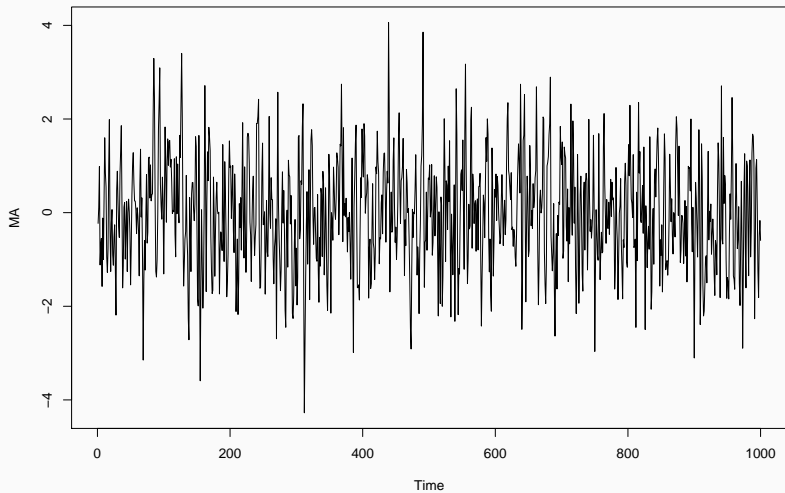
Seguindo Hyndman and Athanasopoulos (2021), para um MA(q) temos:

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad (2)$$

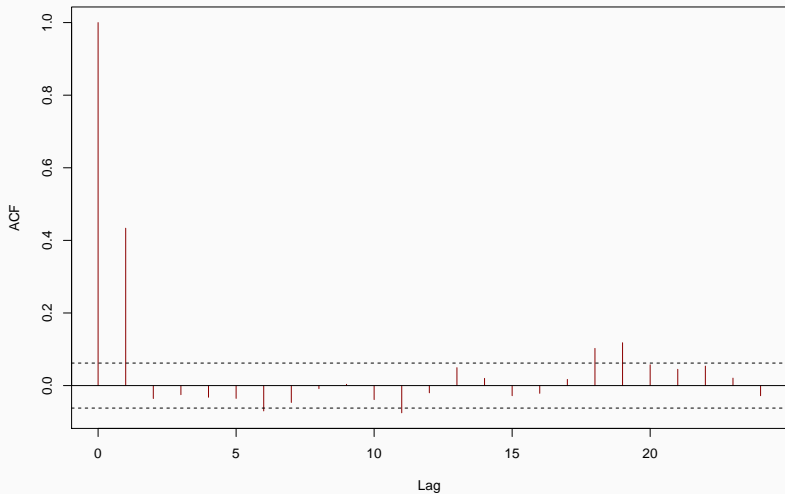
Estacionariedade:

- MA(1): $-1 < \theta_1 < 1$
- MA(2): $-1 < \theta_2 < 1, \theta_1 + \theta_2 > -1, \theta_1 - \theta_2 < 1$

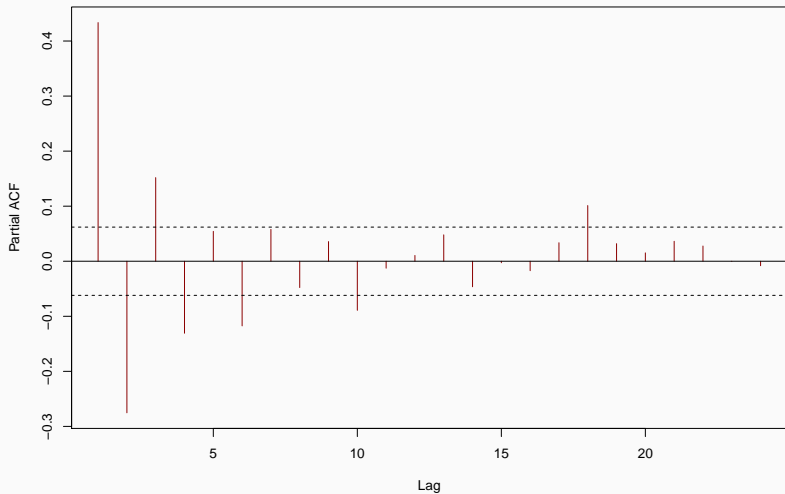
$$\text{MA}(1) - \theta_1 = 0,7$$



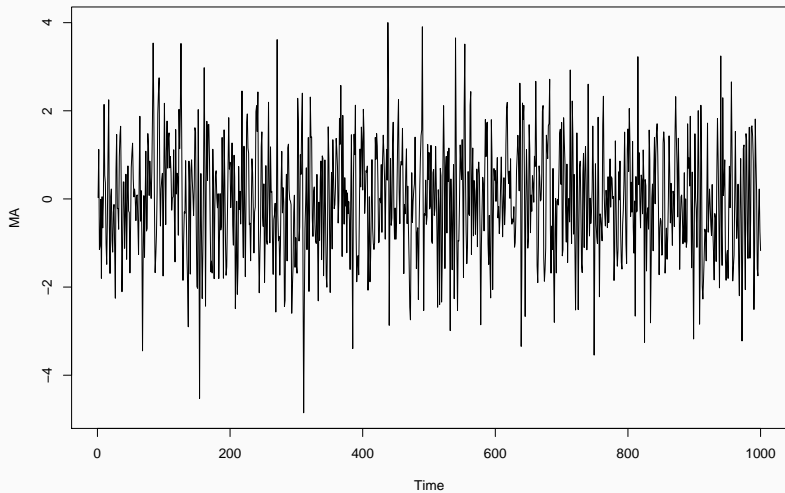
MA(1) - ACF



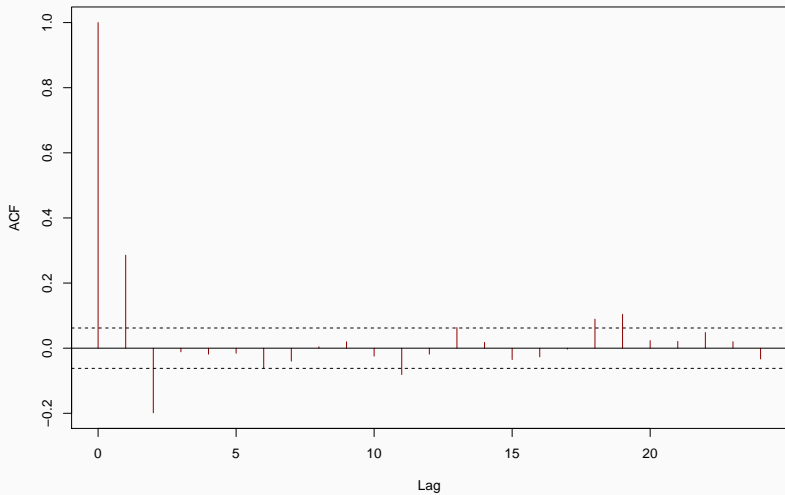
MA(1) - PACF



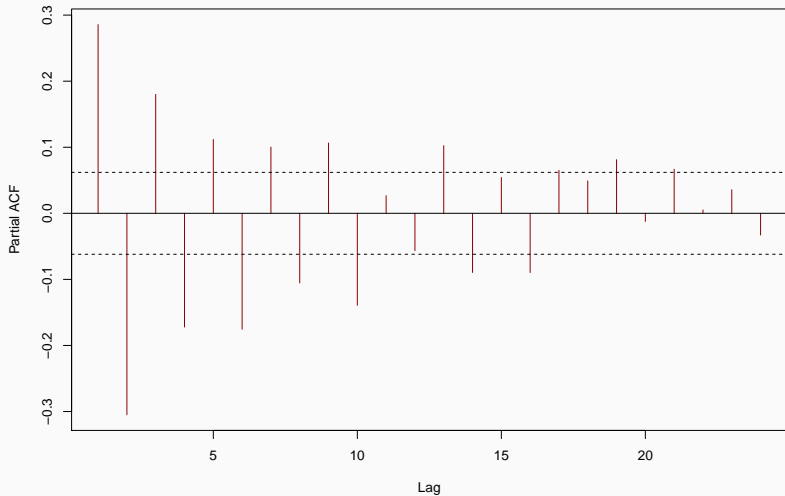
MA(2) - $\theta_1 = 0,7$ e $\theta_2 = -0,3$



MA(2) - ACF



MA(2) - PACF



Seguindo Hyndman and Athanasopoulos (2021), para um ARIMA(p,d,q) temos:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t, \quad (3)$$

- p : ordem do componente autorregressivo.
- q : grau de diferenciação envolvido.
- q : ordem do componente de média móvel.

Alguns casos especiais (Hyndman and Athanasopoulos 2021):

- Ruído branco: $\text{ARIMA}(0,0,0)$, sem constante
- Random walk: $\text{ARIMA}(0,1,0)$, sem constante
- Random walk with drift: $\text{ARIMA}(0,1,0)$, com constante
- $\text{AR}(p)$: $\text{ARIMA}(p,0,0)$
- $\text{MA}(q)$: $\text{ARIMA}(0,0,q)$

Qual ARIMA?

Como seleccionar a ordem de um ARIMA?

- Critérios de informação: AIC, BIC, SC, HQ, etc. . .

$$\text{ARIMA} \underbrace{(p, d, q)}_{\text{(I)}} \underbrace{(P, D, Q)_m}_{\text{(II)}}$$

(I): Componente não-sazonal

(II): Componente sazonal

m : período sazonal (e.g. $m = 4$ para dados trimestrais)

E.g. $\text{ARIMA}(1, 1, 1)(1, 1, 1)_4$

Hyndman, Rob J, and George Athanasopoulos. 2021. *Forecasting: Principles and Practice*. Third. OTexts.