# Cálculo Essencial

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**MÓDULO 1**
**Superando Limites**

## Lista de Exercícios - Aula 07

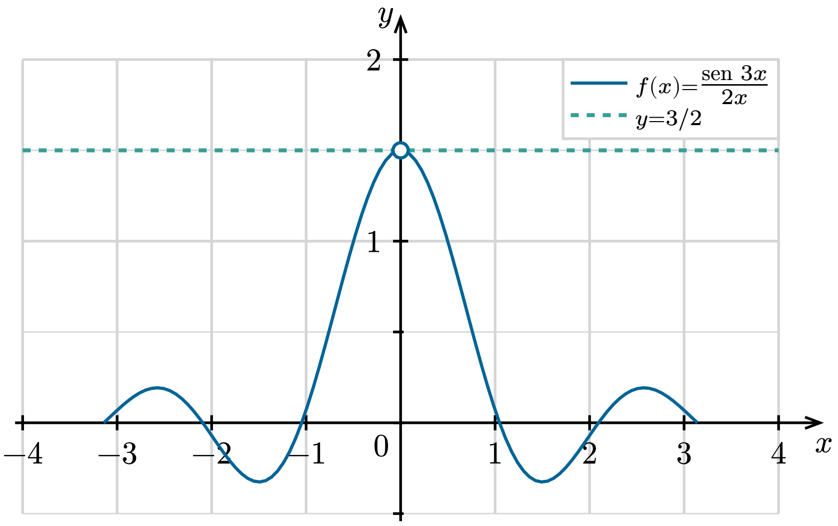
1. Calcule os limites abaixo:

   (a) $\displaystyle\lim_{x\to 0} \frac{\text{sen } 3x}{2x}$

   **Solução:**

   Seja $u = 3x$. Observe que $\lim_{x\to 0} u = \lim_{x\to 0} 3x = 0$. Então,

$$\lim_{x\to 0} \frac{\text{sen } 3x}{2x} = \lim_{u\to 0} \frac{\text{sen } u}{2 \cdot \dfrac{u}{3}} = \frac{3}{2} \cdot \lim_{u\to 0} \frac{\text{sen } u}{u} = \frac{3}{2} \cdot 1 = \frac{3}{2}$$



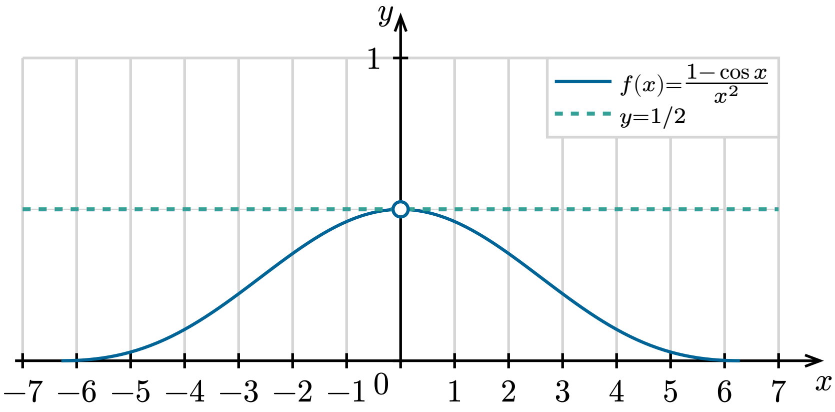
(b) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

**Solução:**

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\left[\left(\frac{1-\cos x}{x^2}\right)\left(\frac{1+\cos x}{1+\cos x}\right)\right] = \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$= \lim_{x\to 0}\frac{\text{sen}^2 x}{x^2(1+\cos x)} = \left(\lim_{x\to 0}\frac{\text{sen} x}{x}\right)^2\left(\lim_{x\to 0}\frac{1}{1+\cos x}\right)$$

$$= 1^2\cdot\frac{1}{1+1} = \frac{1}{2}$$




(c) $\lim\limits_{x\to 0} \dfrac{\tan 2x}{\text{sen } 5x}$

**Solução:**

$$\lim_{x\to 0}\frac{\tan 2x}{\text{sen } 5x} = \lim_{x\to 0}\frac{\frac{\text{sen } 2x}{\cos 2x}}{\text{sen } 5x} = \left(\lim_{x\to 0}\frac{1}{\cos 2x}\right)\cdot\left(\lim_{x\to 0}\frac{\text{sen } 2x}{\text{sen } 5x}\right)$$

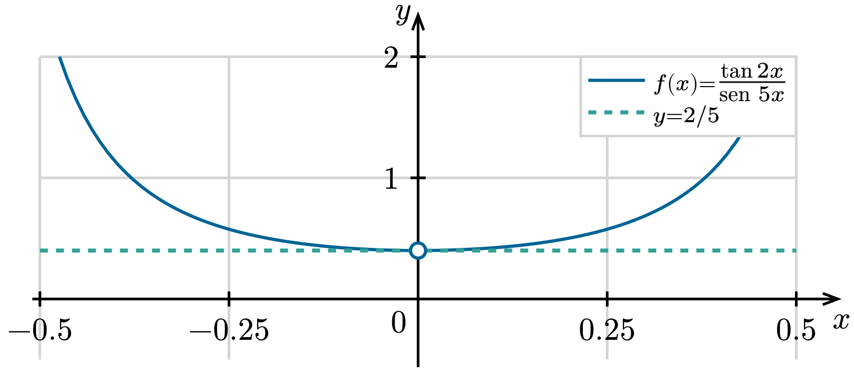
$$= 1\cdot\left(\lim_{x\to 0}\frac{2\cdot\frac{\text{sen } 2x}{2x}}{5\cdot\frac{\text{sen } 5x}{5x}}\right) = \frac{2}{5}\cdot\left(\frac{\lim\limits_{x\to 0}\frac{\text{sen } 2x}{2x}}{\lim\limits_{x\to 0}\frac{\text{sen } 5x}{5x}}\right)$$

Sejam as novas variáveis: $u = 2x$ e $v = 5x$. Note que:

$$\begin{cases} \lim\limits_{x\to 0} u = \lim\limits_{x\to 0} 2x = 0 \Rightarrow u \to 0 \quad \text{quando} \quad x \to 0 \\ \lim\limits_{x\to 0} v = \lim\limits_{x\to 0} 5x = 0 \Rightarrow v \to 0 \quad \text{quando} \quad x \to 0 \end{cases}$$

Substituindo as novas variáveis no limite dado, temos:

$$\lim_{x\to 0}\frac{\tan 2x}{\text{sen } 5x}=\frac{2}{5}\cdot\left(\frac{\displaystyle\lim_{u\to 0}\frac{\text{sen } u}{u}}{\displaystyle\lim_{v\to 0}\frac{\text{sen } v}{v}}\right)=\frac{2}{5}\cdot\frac{1}{1}=\frac{2}{5}$$




(d) $\displaystyle\lim_{x\to 0}\frac{x\text{ sen } x}{1-\cos x}$

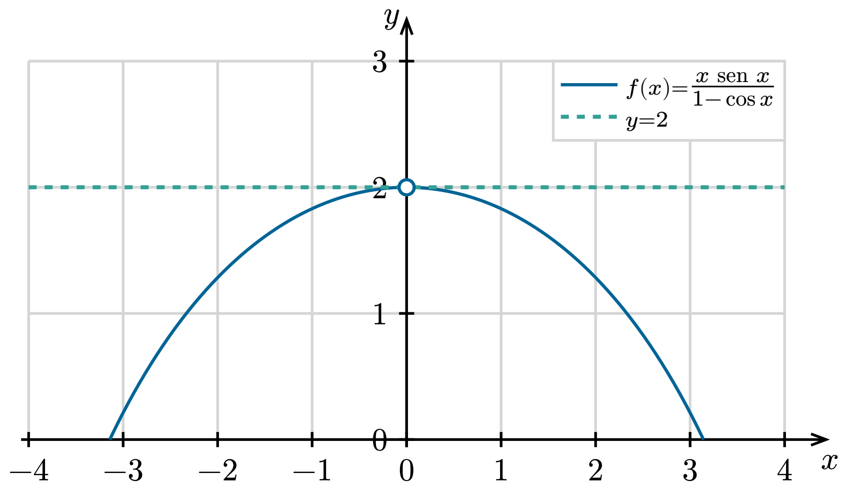
**Solução:**

$$\lim_{x\to 0}\frac{x\text{ sen } x}{1-\cos x}=\lim_{x\to 0}\left[\left(\frac{x\text{ sen } x}{1-\cos x}\right)\cdot\left(\frac{1+\cos x}{1+\cos x}\right)\right]=\lim_{x\to 0}\frac{x\text{ sen } x(1+\cos x)}{1-\cos^2 x}$$

$$=\left[\lim_{x\to 0}(1+\cos x)\right]\cdot\left(\lim_{x\to 0}\frac{x\text{ sen } x}{\text{sen}^2 x}\right)=2\cdot\lim_{x\to 0}\frac{x}{\text{sen } x}$$

$$=\frac{2}{\displaystyle\lim_{x\to 0}\frac{\text{sen } x}{x}}=\frac{2}{1}$$

$$=2$$



(e) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\text{sen } x - 1}{x - \frac{\pi}{2}}$

**Solução:**

Seja a nova variável $t = x - \frac{\pi}{2}$. Note que $t \to 0$ quando $x \to \pi/2$. Então,

$$\begin{aligned}
\lim_{x \to \frac{\pi}{2}} \frac{\text{sen } x - 1}{x - \frac{\pi}{2}} &= \lim_{t \to 0} \frac{\text{sen}\left(t + \frac{\pi}{2}\right) - 1}{t} = \lim_{t \to 0} \frac{\left(\text{sen } t \cdot \cos \frac{\pi}{2} + \cos t \cdot \text{sen } \frac{\pi}{2}\right) - 1}{t} \\
&= \lim_{t \to 0} \frac{(\text{sen } t \cdot 0 + \cos t \cdot 1) - 1}{1} = \lim_{t \to 0} \frac{\cos t - 1}{t} \\
&= \lim_{t \to 0} \left[\left(\frac{\cos t - 1}{t}\right) \cdot \left(\frac{\cos t + 1}{\cos t + 1}\right)\right] = \lim_{t \to 0} \frac{\cos^2 t - 1}{t(\cos t + 1)} \\
&= \lim_{t \to 0} \frac{\text{sen}^2 t}{t(\cos t + 1)} = \left(\lim_{t \to 0} \frac{\text{sen } t}{t}\right) \cdot \left(\lim_{t \to 0} \frac{\text{sen } t}{\cos t + 1}\right) \\
&= 1 \cdot \frac{0}{1} = 0
\end{aligned}$$