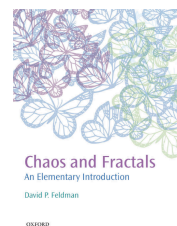# Chaos and Fractals

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Resoluções por

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Capítulo 2: Interacting Functions

Exercises

1. Let g the doubling function. Determine the first five numbers in the orbit for the following seeds:

   (a) $x_0 = -2$

   (b) $x_0 = -0.5$

   (c) $x_0 = 0$

   (d) $x_0 = 0.5$

   (e) $x_0 = 2$

   Solução:

	(a)	(b)	(c)	(d)	(e)
x_0	-2,0000	-0,5000	0,0000	0,5000	2,0000
x_1	-4,0000	-1,0000	0,0000	1,0000	4,0000
x_2	-8,0000	-2,0000	0,0000	2,0000	8,0000
x_3	-16,0000	-4,0000	0,0000	4,0000	16,0000
x_4	-32,0000	-8,0000	0,0000	8,0000	32,0000
x_5	-64,0000	-16,0000	0,0000	16,0000	64,0000

2. Let $f(x) = \sqrt{x}$. Determine the first five numbers in the orbit for the followng seeds:

   1. $x_0 = 0$

   2. $x_0 = \frac{1}{2}$

3. $x_0 = 1$
4. $x_0 = 2$
5. $x_0 = 4$

Solução:

	(a)	(b)	(c)	(d)	(e)
x_0	0,0000	0,5000	1,0000	2,0000	4,0000
x_1	0,0000	0,7071	1,0000	1,4142	2,0000
x_2	0,0000	0,8409	1,0000	1,1892	1,4142
x_3	0,0000	0,9170	1,0000	1,0905	1,1892
x_4	0,0000	0,9576	1,0000	1,0443	1,0905
x_5	0,0000	0,9786	1,0000	1,0219	1,0443

3. Considere the function $f(x) = \sqrt{x}$.

   (a) Complete the following table for f.

x_0	9
x_1	
x_2	
x_3	
x_4	

Solução:

x_0	9,0000
x_1	3,0000
x_2	1,7321
x_3	1,3161
x_4	1,1472
x_5	1,0711

(b) Determine a formula for $f^{(2)}(x)$.

> **Solução:**

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \Rightarrow f^{(2)}(x) = f(f(x)) = f\left(x^{\frac{1}{2}}\right)$$

$$f^{(2)}(x) = \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{1}{2} \cdot \frac{1}{2}}$$

$$f^{(2)}(x) = x^{\frac{1}{4}}$$

(c) Determine a formula for $f^{(3)}(x)$.

> **Solução:**

$$f^{(3)} = f\left(f^{(2)}(x)\right) = \left(x^{\frac{1}{4}}\right)^{\frac{1}{2}} = x^{\frac{1}{4} \cdot \frac{1}{2}} = x^{\frac{1}{8}}$$

(d) Determine a formula for $f^{(4)}(x)$.

> **Solução:**

$$f^{(4)} = f\left(f^{(3)}(x)\right) = \left(x^{\frac{1}{8}}\right)^{\frac{1}{2}} = x^{\frac{1}{8} \cdot \frac{1}{2}} = x^{\frac{1}{16}}$$

(e) Determine a formula for $f^{(n)}(x)$, the nth iterate of x.

> **Solução:**

As fórmulas obtidas para $f^{(2)}(x)$, $f^{(3)}(x)$ e $f^{(4)}(x)$ obtidas nos itens anteriores sugerem a seguinte fórmula geral para o n-ésimo termo $f^{(n)}(x)$ da órbita de x:

$$f^{(n)}(x) = x^{\frac{1}{2^n}}$$

Referências

FELDMAN, D. P. **Chaos and Fractals**. : #book.subtitle. Oxford: Oxford University Press, 2012.