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Bachelor Project

Barycentric Data Visualization for Triangle Meshes

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Abstract

Abstract goes here (?)

Dedication

to somebody (?)

Declaration

I declare that.. (?)

Acknowledgements

I want to thank... (?)

Contents

Chapter 1

Introduction

1.1 Barycentric coordinates

Barycentric coordinates, discovered by Möbius in 1827, consist of one of the most progessive area of research in computer graphics and mathematics thanks to the numerous applications in image and geometry processing. The position of any point in a triangle can be expressed using a linear combination with three scalars using barycentric coordinates:

$$p = \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3$$

where p_1, p_2 and p_3 are the vertices of a triangle and λ_1, λ_2 and λ_3 (the barycentric coordinates) are three scalars such that respect the following barycentric coordinates properties.

- partition of unity: $\sum_{i=1}^{3} \lambda_i(p) = 1$
- reproduction: $\sum_{i=1}^{3} \lambda_i(p) p_i = p$
- Lagrange-property: $\lambda_i(p_j) = \delta_{i,j}$
- linearity: $\lambda_i \in \prod_1$
- non-negativity: $\lambda_i(p) \geq 0$ for $p \in [p_1, p_2, p_3]$

The point is inside the triangle if $0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1$. If a barycentric coordinates is less than zero or greater than one, the point is outside the triangle. Barycentric coordinates allow to interpolate values from a set of control points over the interior of a domain, using weighted combinations of values associated with the control points (Fig. 1.1).

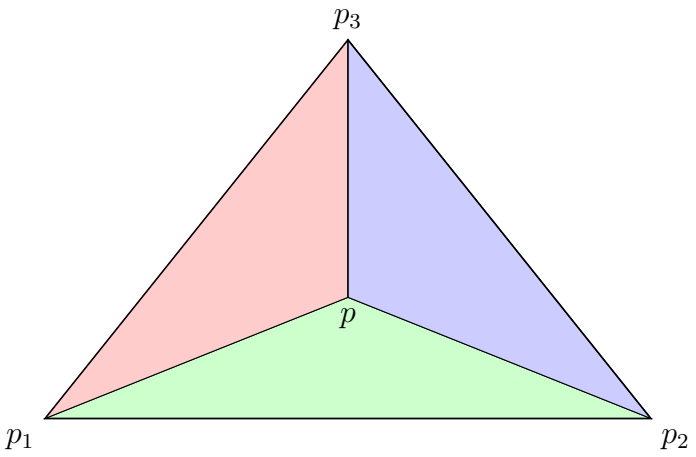

Figure 1.1: Let w_1 be the red area, w_2 the green one and w_3 the blue one. Normalizing each of them by the area of the triangle, we will get three values $(\lambda_1, \lambda_2, \lambda_3)$ that are the barycentric coordinates of p with respect to the triangle $[p_1, p_2, p_3]$.

1.2 Triangle mesh

See book

1.3 Lighting

formula + draw and explanation of angle (like at 90 degree some light..e.tc.)

1.4 Linear Interpolation

The standard linear interpolated visualisation is made passing three attributes (colors) for each vertex of a triangle. OpenGL will interpolate linearly the colors. That is possible thanks to the barycentric coordinates that will tell how much of each color is being mixed at any position.

1.5 Flat Shading

1.6 Gouraud Shading

Gouraud Shading can be calculated in the *vertex shader*. The main idea is to compute a normal at the vertex and an intensity for each vertex.

1.7 Curvature

1.7.1 Gaussian Curvature

Gaussian Curvature works like a logical `AND`, it will check if there is a curvature along both directions.

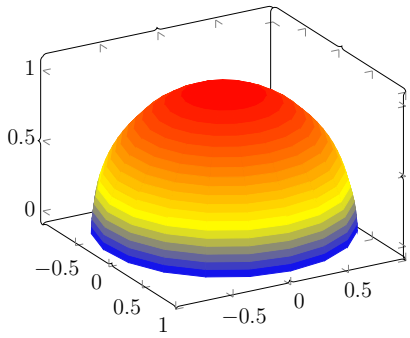

Figure 1.2: Positive gaussian curvature

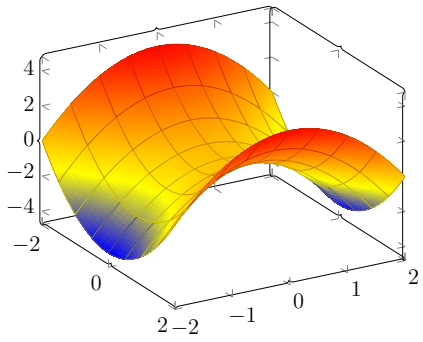

Figure 1.3: Negative gaussian curvature

Surfaces that have a zero gaussian curvature are called *developable surfaces* because they can be flattened out into the plane without any stretching. Gaussian curvature should be zero inside each mesh triangle and the same along edges since it can be flattened symmetrically into the plane.

1.7.2 Mean Curvature

Chapter 2

GPU program

A program that runs on GPU is called *shader*. Shaders are principally used to modify the representation and the behaviour of 3D objects. They are also used to create lighting effects. Shaders can perform tasks efficiently thanks to the GPU. That guarantee faster results than CPU since GPU is designed to work in parallel. These shader programs are written in *GLSL*.

2.1 GPU pipeline

Using a program will allow us to control the rendering pipeline, by default there is no pipeline set in OpenGL. We pass a set of vertices as input, then the *vertex shader* transforms them (translation, rotation, projection...) and passes the transformed vertices to the *geometry shader*. This shader takes vertices to create primitive shapes and then it rasterizes them. These rasterized flat images are then passed as input to the *fragment shader* that adds the lighting, apply textures and color these images (Fig. 2.1).

2.2 Vertex Shader

The program that perform vertex operations is called *vertex shader*. It receives one vertex at a time and then it passes the output to a *fragment shader* or to a *geometry shader*, if any.

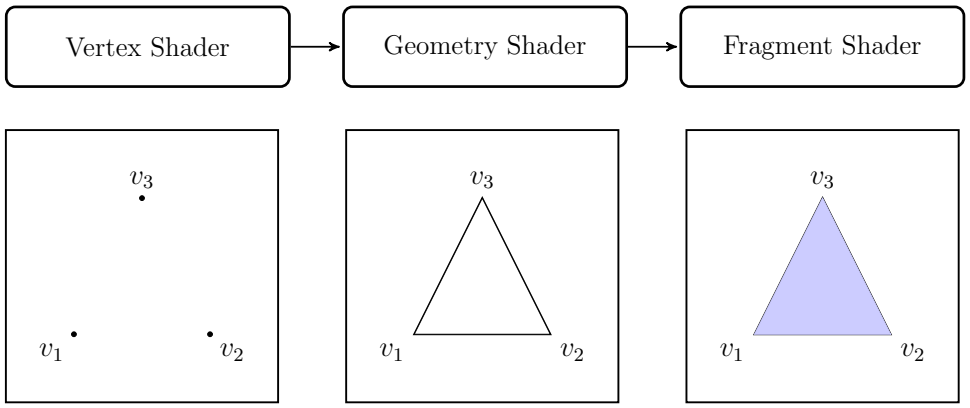

Figure 2.1: GPU pipeline

2.3 Fragment Shader

Fragment shader performs color computation for every visible pixel of the rasterized object. It works on a fragment at a time, but thanks to the power of GPU it can work in parallel for all vertices (*vertex shader*) and fragments (*fragment shader*).

2.4 Geometry Shader

Geometry shader is used for layered rendering. It takes as input a set of vertices (single primitive, example: triangle or a point) and it transforms them before sending to the next shader stage. In this way, we can obtain different primitives.

Each time we call the function `EmitVertex()` the vector currently set to `gl_Position` is added to the primitive. All emitted vertices are combined for the primitive and output when we call the function `EndPrimitive()`.

Chapter 3

Vertex area based

Alternative data visualization techniques can be found using the power of barycentric coordinates and GPU programming.

The usual way to visualize data for a triangle mesh is to associate data to vertices and then interpolating over the mesh triangles, that does not work in case of edges and triangles.

3.1 Region around a vertex

We can split the surface of triangle meshes into regions around vertex (Fig. 3.2) and color them.

These regions can be determined using barycentric coordinates and GPU fragment program. Visualizing data given at the vertices or edges of the mesh in a piecewise constant simulates the classical triangle flat shading. An example of this vertex data is the discrete Gaussian curvature.

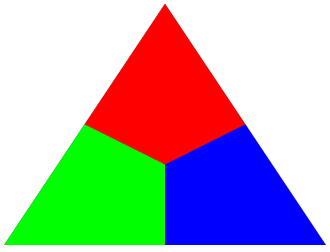

Figure 3.1: Vertex based area

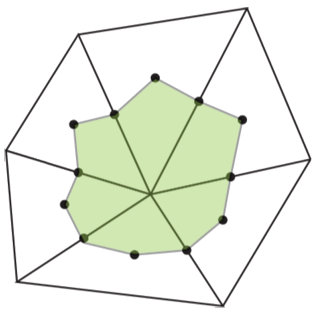

Figure 3.2: Region around a vertex

3.1.1 Max diagram - Vertex based area

Passing barycentric coordinates to the fragment shader will clearly demonstrate that we can get results different from the classic color interpolation.

There are different approaches to color interpolation focusing on the distance from vertices. For each point in a triangle, we can easily determine its closest vertex, which we use as a cue for coloring.

A different approach from interpolating, can be found coloring vertex areas based on the minimum barycentric coordinate. The color is given by the region farthest from a vertex (Fig. 3.1, Pseudocode A.1).

3.2 Flat shading extension

Suppose now that you want each vertex area to be in one constant color. This color can be taken from shading interpolation using the normal at the vertex and the vertex position. Then you can compute the color has it be done in *Gouraud Shading*.

The idea is to compute the color per vertex but instead of linearly interpolated it in each triangle (as *Gouraud shading* does) we color regions around a vertex with that constant color.

To implement that we need to pass the barycentric coordinates, the vertex color, the normal at the vertex and the lighting calculations to the *fragment shader*.

We want to avoid an automatic interpolation of colors, in order to return the resulting color using the *max diagram*, to do that we have used a *Geometry shader* in order to access to all three vertex colors in fragment shader. (Pseudocodes: A.2, A.3, A.4)

3.2.1 Comparison

Gouraud shading vs extension flat shading.

3.3 Discrete Gaussian Curvature

We want to compute the *Gaussian curvature* per vertex. We want to sum up, for each vertex, angles at this vertex with adjacent triangles and then we subtract this value to 2π. After having obtain this value, called *angle defect* (Fig. 3.3), we map linearly this value to a color range.

Then we can pass this color to the *vertex shader* to see the color vertex flat shading visualisation of Gaussian curvature.

$$G = 2\pi - \sum_j \theta_j$$

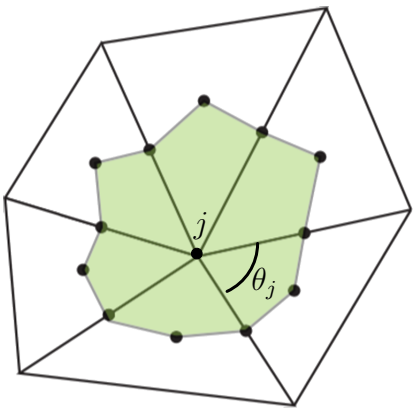

Figure 3.3: Angle defect

Gaussian curvature should return a constant color around each vertex (Fig. 3.4, Pseudocode A.5).

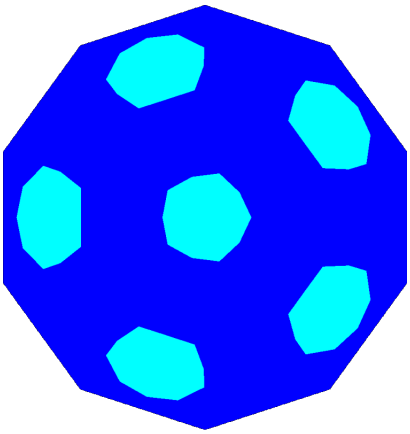

Figure 3.4: Gaussian curvature on icosahedron

3.3.1 Comparison

We want now to compare the classic linear interpolation visualisation (Fig. 3.5) with two possible way to extend the flat shading. In Fig.

3.6 and Fig. 3.7 we have color each vertex area applying the method *max diagram* described in the above subsection 3.1.1. As we can see in Fig. 3.6 and Fig. 3.7, the horse get a more realistic shape thanks to the lighting calculation for Fig. 3.6 and to the *Gaussian curvature* for Fig. 3.7.

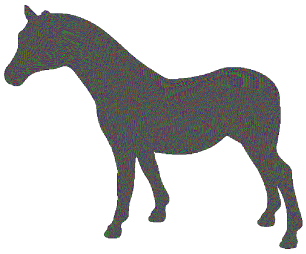

Figure 3.5: Interpolation

Figure 3.6: Lighting calculation

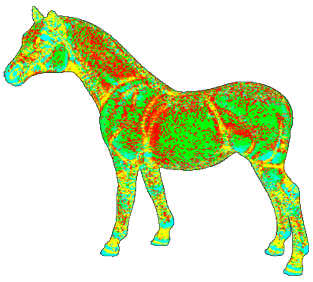

Figure 3.7: Gaussian Curvature

Visualization of the principal curvatures of the model as colors from blue (highest values of curvature) to red (lower values of curvature), Fig. 3.7, better highlighs the geometry of the horse. These changes of curvature, positive (blue), flat (green) and negative regions (red), better emphasises the 3-dimensionality of the model. *Gaussian curvature* better shows the muscle constrasts given a more realistic character to the horse than the model obtained using the lighting calculation.

Bibliography

Appendix A

Pseudocodes

A.0.1 Chapter 3

CHECK AND UPDATE CODES

Listing A.1: Max diagram - Vertex based area (Section: 3.1.1)

```
1  if (Coords.x > Coords.y && Coords.x > Coords.z)
       {
2      vec3 blue = vec3(0.0f, 0.0f, 1.0f);
3      FragColor = vec4(blue, 1.0f);
4  } else if(Coords.y > Coords.x && Coords.y >
       Coords.z) {
5      vec3 green = vec3(0.0f, 1.0f, 0.0f);
6      FragColor = vec4(green, 1.0f);
7  } else {
8      vec3 red = vec3(1.0f, 0.0f, 0.0f);
9      FragColor = vec4(red, 1.0f);
10 }
```

Listing A.2: Vertex Shader for flat shading extension using lighting (Section: 3.2)

```
1  #version 330 core
2  layout (location = 0) in vec3 aPos;
3  layout (location = 1) in vec3 aNormal;
4  layout (location = 2) in vec3 aColor;
5
6  struct Light {
7      // ...
8  };
```

```
9
10 out vec4 vertex_color;
11
12 uniform mat4 model;
13 uniform mat4 view;
14 uniform mat4 projection;
15
16 void main() {
17     vec3 world_position = vec3(model * vec4(aPos
          , 1.0));
18     vec3 world_normal = mat3(transpose(inverse(
          model))) * aNormal;
19
20     // color obtained with lighting calculations
21     vertex_color = get_result_color_lighting
          (...);
22
23     gl_Position = projection * view * model *
          vec4(aPos, 1.0);
24     }
```

Listing A.3: Geometry Shader for flat shading extension (Section: 3.2)

```
1  #version 330 core
2  layout (triangles) in;
3  layout (triangle_strip, max_vertices = 3) out;
4
5  in vec4 vertex_color[3];
6  out vec3 coords;
7  out vec4 wedge_color[3];
8
9  void main() {
10     wedge_color[0] = vertex_color[0];
11     wedge_color[1] = vertex_color[1];
12     wedge_color[2] = vertex_color[2];
13
14     coords = vec3(1.0, 0.0, 0.0);
15     gl_Position = gl_in[0].gl_Position;
16     EmitVertex();
17
18     coords = vec3(0.0, 1.0, 0.0);
19     gl_Position = gl_in[1].gl_Position;
```

```
20      EmitVertex();
21
22      coords = vec3(0.0, 0.0, 1.0);
23      gl_Position = gl_in[2].gl_Position;
24      EmitVertex();
25
26      EndPrimitive();
27  }
```

Listing A.4: Fragment Shader for flat shading extension (Section: 3.2)

```
1  #version 330 core
2  in vec3 coords;
3  in vec4 wedge_color[3];
4  out vec4 fragColor;
5
6  void main() {
7      // max diagram
8      if (coords[0] > coords[1]) {
9          if (coords[0] > coords[2]) {
10             fragColor = wedge_color[0];
11         } else {
12             fragColor = wedge_color[2];
13         }
14     } else {
15         if (coords[1] > coords[2]) {
16             fragColor = wedge_color[1];
17         } else {
18             fragColor = wedge_color[2];
19         }
20     }
21 }
```

Listing A.5: Vertex Shader for flat shading extension using gaussian curvature (Section: 3.3)

```
1  #version 330 core
2  layout (location = 0) in vec3 aPos;
3  layout (location = 2) in vec3 gaussian_curvature
       ;
4
5  out vec4 vertex_color;
```

```
6
7  uniform mat4 model;
8  uniform mat4 view;
9  uniform mat4 projection;
10
11 uniform float min_gc;
12 uniform float max_gc;
13 uniform float mean_negative_gc;
14 uniform float mean_positive_gc;
15
16
17 vec3 interpolation(vec3 v0, vec3 v1, float t) {
18     return (1 - t) * v0 + t * v1;
19 }
20
21 vec4 get_result_color_gc() {
22     float val = gaussian_curvature[0];
23     vec3 red = vec3(1.0, 0.0, 0.0);
24     vec3 green = vec3(0.0, 1.0, 0.0);
25     vec3 blue = vec3(0.0, 0.0, 1.0);
26
27     //negative numbers until 0 -> map from red
          to green
28     if (val < 0) {
29         return vec4(interpolation(red, green,
              val/min_gc)/(mean_negative_gc/5),
              1.0);
30     } else {
31         //map from green to blue, from 0 to
              positive
32         return vec4(interpolation(green, blue,
              val/max_gc)/(mean_positive_gc/5),
              1.0);
33     }
34 }
35
36 void main() {
37     vec3 pos = vec3(model * vec4(aPos, 1.0));
38
39     vertex_color = get_result_color_gc();
40
41     gl_Position = projection * view * model *
```

```
        vec4(aPos, 1.0);
42 }
```