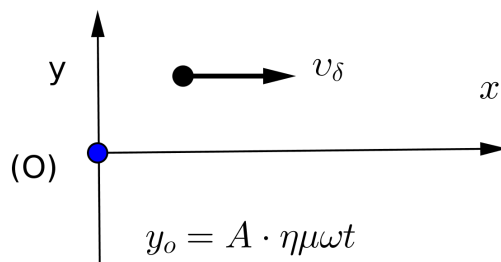


$$f_B = \frac{v_{\eta\chi} + v_2}{v_{\eta\chi} + v_1} f_s = \frac{v_{\eta\chi} + \frac{10}{5}}{v_{\eta\chi} + \frac{10}{5}} f_s = \frac{\frac{11v_{\eta\chi}}{10}}{\frac{6v_{\eta\chi}}{10}} f_s = \frac{11 \cdot 5}{10 \cdot 6} f_s = \frac{11}{12} f_s$$

άρα σωστό το (ii)

Θέμα Γ



Γ1.

$$-A \rightarrow +A : \Delta t = \frac{T}{2} = 0,4s \implies T = 0,8s$$

σε Δt η διαταραχή διαδόθηκε σε απόσταση $\Delta x = 4cm = 0,04m$.

$$v_\delta = \frac{\Delta x}{\Delta t} = \frac{0,04}{0,4} = 0,1m/s$$

$$v_\delta = \frac{\lambda}{T} \implies \lambda = 0,08m$$

Το Δm εκτελεί α.α.τ.:

$$D = \Delta m \cdot \omega^2 = \Delta m \frac{4\pi^2}{T^2} = 10^{-6} \frac{4\pi^2}{0,64} = \frac{\pi^2}{16} 10^{-4} \frac{N}{m}$$

$$E_T = \frac{1}{2} D \cdot A^2 \implies 5\pi^2 10^{-7} = \frac{1}{2} \cdot \frac{\pi^2}{16} 10^{-4} A^2 \implies A^2 = 0,16 \implies A = 0,4m \text{ (πλάτος)}$$

Γ2. Η εξίσωση του κύματος είναι:

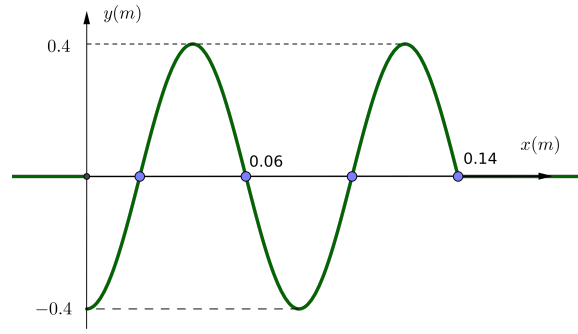
$$y_{(x,t)} = A\eta\mu\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) = 0,4\eta\mu\left(\frac{5\pi t}{2} - 25\pi x\right) \text{ (SI)}$$

Στιγμιότυπο την $t_1 = 1,4s$:

$$\text{Την } t_1 \text{ η διαταραχή έφτασε στη θέση } \begin{cases} x_1 = v_\delta t_1 = 0,1 \cdot 1,4 = 0,14m, N_1 = \frac{x_1}{\lambda} = \frac{0,14}{0,08} = \frac{14}{8} = \frac{7}{4} \text{ μήκη κύματος} \\ \text{ή} \\ \varphi_{t_1} = 0 \implies \frac{5\pi \cdot 1,4}{2} - 25\pi x_1 = 0 \implies x_1 = 0,14m \end{cases}$$

$$y_{t_1} = \begin{cases} 0,4\eta\mu(3,5\pi - 25\pi x), & 0 \leq x \leq 0,14m \\ 0, & 0,14m < x \end{cases}$$

$$x = 0, y_0 = 0,4\eta\mu 3,5\pi = -0,4m$$



Γ3. $A\Delta E_{\tau\alpha\lambda}$ για Δm

Για

$$y = 0, 2m = \frac{A}{2}$$

$$E_T = K + U \Rightarrow E_T = K + \frac{1}{2}Dy^2 \xrightarrow{y=\frac{A}{2}} E_T = K + \frac{1}{2}D\frac{A^2}{4} \Rightarrow$$

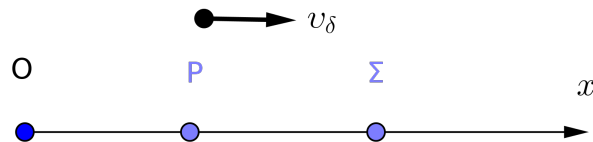
$$E_T = K + \frac{1}{4}E_T \Rightarrow K = \frac{3}{4}E_T = \frac{3}{4}5\pi^2 10^{-7} = \frac{3\pi^2}{8} 10^{-6} J$$

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$$y = A\eta\mu\varphi = \frac{A}{2} \Rightarrow \eta\mu\varphi = \frac{1}{2} \Rightarrow \varphi = \begin{cases} 2k\pi + \frac{\pi}{6}, & k = 0, 1, 2, \dots (1) \\ 2k\pi + \frac{5\pi}{6}, & k = 0, 1, 2, \dots (2) \end{cases}$$

$$v = \omega A \sigma v \nu \varphi = \begin{cases} = \omega A \sigma v \nu (2k\pi + \frac{\pi}{6}) = \frac{\omega A \sqrt{3}}{2} \\ \omega A \sigma v \nu (2k\pi + \frac{5\pi}{6}) = -\frac{\omega A \sqrt{3}}{2} \end{cases}$$

$$K = \frac{1}{2}\Delta m \cdot v^2 = \frac{1}{2}\Delta m \left(\pm \frac{\omega A \sqrt{3}}{2} \right)^2 = \frac{1}{2}\Delta m \cdot \frac{\omega^2 A^2 3}{4} = \frac{3}{4} \cdot \frac{1}{2}D \cdot A^2 = \frac{3}{4}E_T$$



Γ4.

$$\varphi_P - \varphi_\Sigma = \frac{3\pi}{2} rad, (\varphi_P > \varphi_\Sigma)$$

$$\left. \begin{array}{l} y_P = 0, 4m = A \\ y_P = A\eta\mu\varphi_P \end{array} \right\} \eta\mu\varphi_P = 1 \Rightarrow \varphi_P = 2k\pi + \frac{\pi}{2}, k = 0, 1, 2, \dots$$

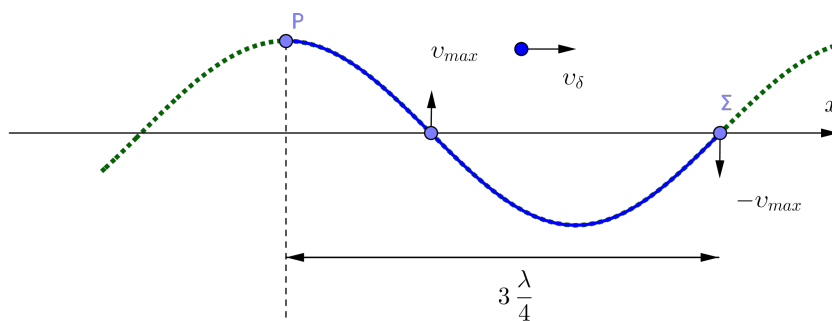
$$2k\pi + \frac{\pi}{2} - \varphi_\Sigma = \frac{3\pi}{2} \Rightarrow \varphi_\Sigma = 2k\pi - \pi$$

Άρα

$$v_{\Sigma} = \frac{5\pi}{2} \cdot 0,4 \cdot \sigma \nu \nu (2k\pi - \pi) \Rightarrow v_{\Sigma} = -\pi \frac{m}{s}$$

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$$\varphi_P - \varphi_{\Sigma} = \frac{3\pi}{2} \Rightarrow \left. \begin{aligned} \frac{5\pi t}{2} - 25\pi x_P - \left(\frac{5\pi t}{2} - 25\pi x_{\Sigma} \right) &= \frac{3\pi}{2} \Rightarrow x_{\Sigma} - x_P = \frac{3}{50} \\ \frac{2\pi t}{T} - \frac{2\pi x_P}{\lambda} - \left(\frac{2\pi t}{T} - \frac{2\pi x_{\Sigma}}{\lambda} \right) &= \frac{3\pi}{2} \Rightarrow x_{\Sigma} - x_P = \frac{3\lambda}{4} \end{aligned} \right\} \Rightarrow \frac{3\lambda}{4} = \frac{3}{50} \Rightarrow \lambda = 0,08 = \frac{4}{50} m$$



όταν $y_P = A$,

$$v_{\Sigma} = -\omega A = -\pi \frac{m}{s}$$

Θέμα Δ

Δ1.

$$\left. \begin{aligned} \Sigma F &= m \cdot a_{cm} \Rightarrow W - T_1 = m \cdot a_{cm} \\ \Sigma \tau &= I \alpha_{\gamma \omega \nu} \Rightarrow T_1 \cdot R = \frac{1}{2} m R^2 \cdot \alpha_{\gamma \omega \nu} \end{aligned} \right\}$$

$$T_1 = \frac{m \cdot a_{cm}}{2}, W = \frac{3m \cdot a_{cm}}{2}$$

$$0 = v_{\Gamma} = v_Z = v_{cm} - v_{\gamma \rho Z} \Rightarrow v_{cm} = \omega R \Rightarrow a_{cm} = a_{\gamma} R \Rightarrow a_{cm} = \frac{2}{3} g = \frac{20}{3} \frac{m}{s}$$

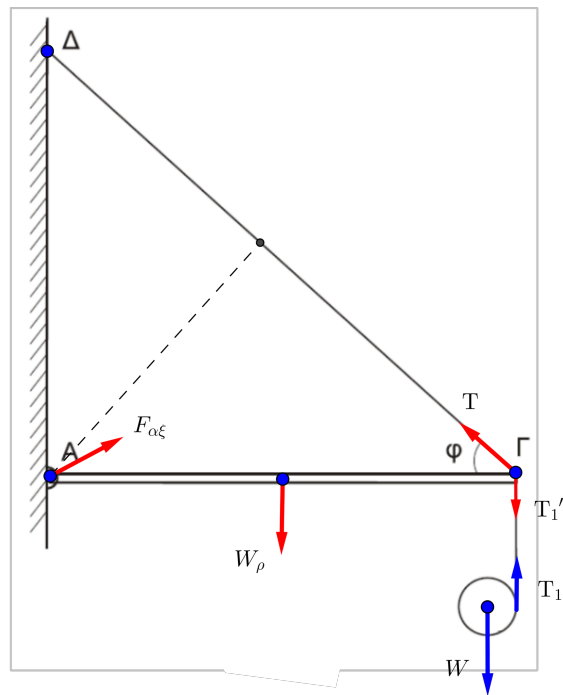
Δ2.

$$T'_1 = T_1 = \frac{2 \cdot 20}{2 \cdot 3} = \frac{20}{3} N$$

3ος Νόμος του Νεύτωνα και νήμα αβαρές μη εκτατό

Για τη ράβδο που ισορροπεί:

$$\Sigma \tau_{(\Lambda)} = 0 \Rightarrow w_{\rho} \cdot \frac{l}{2} + T'_1 \cdot l - T \cdot l \cdot \eta \mu \varphi = 0 \Rightarrow T = \frac{100}{3} N$$



Δ3. Την t_1 , $h_1 = 0,3m$

$$h_1 = \frac{1}{2}a_{cm}t_1^2 \Rightarrow 0,3 = \frac{1}{2} \cdot \frac{20}{3}t_1^2 \Rightarrow t_1 = 0,3s$$

$$a_{\gamma\omega\nu} = \frac{a_{cm}}{R} = \frac{200}{3}r/s^2$$

$$w_1 = a_{\gamma\omega\nu}t_1 = 20r/s$$

$t \rightarrow t_1 + \Delta t$:

$$\tau_{w_{cm}} = 0 = \frac{\Delta L}{\Delta t} \Rightarrow L_{t_1} = L_{t_1+\Delta t}$$

Η μόνη δύναμη που ασκείται στο δίσκο είναι το βάρος W .

Όπου

$$L_{t_1} = I \cdot \omega_1$$

$$I = \frac{1}{2}mR^2 = \frac{1}{2} \cdot 2 \cdot 0,1^2 = 0,01kgm^2$$

Άρα

$$L_{t_1} = 0,2kg \frac{m^2}{s} = L_{t_1+\Delta t}$$

ή ΘΜΚΕ_(0→h₁) :

$$\begin{aligned} K_{(0)} + U_{(0)} &= K_{(t_1)} + U_{(t_1)} \\ 0 + mgh_1 &= \left(\frac{1}{2}mv_{cm1}^2 + \frac{1}{2}I\omega_1^2 \right) + 0 \\ mgh_1 &= \frac{1}{2}m\omega_1^2 R^2 + \frac{1}{2}I\omega_1^2 \\ mgh_1 &= I\omega_1^2 + \frac{1}{2}I\omega_1^2 = 3 \cdot \frac{1}{2}I\omega_1^2 \\ \left. \begin{aligned} K_{\sigma\tau\rho} &= \frac{1}{2}I\omega_1^2 \\ L &= I\omega_1 \\ mgh_1 &= 3K_{(\sigma\tau\rho)} \end{aligned} \right\} K_{\sigma\tau\rho} = \frac{L^2}{2I} \Bigg\} \implies mgh_1 = \frac{3L^2}{2I} \\ 9 \cdot 10 \cdot 0,3 &= \frac{3L_{t_1}^2}{2 \cdot 0,01} \implies L_{t_1} = 0,2kg \frac{m^2}{s} \end{aligned}$$

$$\Delta 4. \; t_2 = t_1 + \Delta t':$$

$$\frac{K_{\sigma\tau\rho(t_2)}}{K_{\mu\varepsilon\tau(t_2)}} = \frac{\frac{1}{2}I\omega_2^2}{\frac{1}{2}mv_{cm_2}^2}$$

$$t_1 = t_2:$$

$$\begin{aligned} \Sigma \tau_{cm} = 0 &\implies \omega_2 = \omega_1 = 20r/s \\ \Sigma F = m \cdot a_{cm} &\implies W = m \cdot a_{cm} \implies a_{cm} = g = 10m/s^2 \\ v_{cm_2} &= v_{cm_1} + g \cdot \Delta t' = 2 + 10 \cdot 0,1 = 3m/s \end{aligned}$$

$$\begin{aligned} \frac{K_{\pi\varepsilon\rho}}{K_{\mu\varepsilon\tau}} &= \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}mR^2\omega_2^2}{mv_2^2} = \frac{2}{9} \\ \frac{K_{\pi\varepsilon\rho}}{K_{\mu\varepsilon\tau}} &= \frac{2}{9} \end{aligned}$$