$$
\begin{aligned}
A_G(x) &= \sum_{i=1}^{n} a_i x^i \\
A_G(x) &= a_1x + a_2x^2 + \sum_{i=3}^{n} a_i x^i \\
A_G(x) &= a_1x + a_2x^2 + \sum_{i=3}^{n} (a_{i-1} + a_{i-2}) x^i \\
A_G(x) &= a_1x + a_2x^2 + \sum_{i=3}^{n} a_{i-1} x^i + \sum_{i=3}^{n} a_{i-2} x^i \\
A_G(x) &= a_1x + a_2x^2 + x\sum_{i=3}^{n} a_{i-1} x^{i-1} + x^2\sum_{i=3}^{n} a_{i-2} x^{i-2} \\
A_G(x) &= a_1x + a_2x^2 + x\left(A_G(x) - a_1x\right) + x^2 A_G(x) \\
A_G(x) - xA_G(x) - x^2A_G(x) &= a_1x + (a_2 - a_1)x^2 \\
A_G(x)(1 - x - x^2) &= a_1x + (a_2 - a_1)x^2 \\
A_G(x) &= \frac{a_1x + (a_2 - a_1)x^2}{1 - x - x^2}
\end{aligned}
$$

Given that the values of f must be natural, we solve for x

$$
\begin{aligned}
k - kx - kx^2 = a_1x + (a_2 - a_1)x^2 \\
(a_2 - a_1 + k)x^2 + (a_1 + k)x + k = 0
\end{aligned}
$$

For the given sequence $a_1 = 1$ and $a_2 = 4$, so

$$
(3 + k)x^2 + (1 + k)x - k = 0
$$

The discriminant of the quadratic equation is

$$
\begin{aligned}
\Delta &= b^2 - 4ac \\
&= (1 + k)^2 + 4(3 + k)(k) \\
&= k^2 + 2k + 1 + 12k + 4k^2 \\
&= 5k^2 + 14k + 1
\end{aligned}
$$

It must be a square of a natural number, so

$$
\begin{aligned}
5k^2 + 14k + 1 &= m^2 \\
25k^2 + 70k + 5 &= 5m^2 \\
25k^2 + 70k + 49 &= 5m^2 + 44 \\
(5k + 7)^2 &= 5m^2 + 44 \\
k(5k + 14) &= (m - 1)(m + 1)
\end{aligned}
$$

Mathematica code to find the sum of the first 30 values of k

```
Solve[5 k^2 + 14 k + 1 == m^2 && k > 0 && m > 0, Integers];
Sort[Flatten[Table[k /. % /. { C[1] -> a}, {a, 0, 10}] // FullSimplif
% // Total
```