First of all, the ratio shouldn't be one! Let k be the ratio.
Let r, p and q be the consecutive integers of geometric progression.
The permutations of (r, d, q) are the following:

1. (r, d, q)
2. (r, q, d)
3. (d, r, q)
4. (d, q, r)
5. (q, d, r)
6. (q, r, d)

1 with 5 are the same regarding $k < 1$ or $k > 1$. The same for 2 with 4 and 3 with 6. We are left with

1. (r, d, q)
2. (r, q, d)
3. (d, r, q)

The first 2 are the same if the q and d are unknown. So we are left with

1. (r, d, q)
2. (d, r, q)

case (r, d, q)

k is rational, so let

$$k = \frac{a}{b}$$

So it will be r, $d = kr$ and $q = k^2r$ or r, $d = \frac{a}{b}r$, $q = \frac{a^2}{b^2}r$. They are all integers, so $b^2|r$. This means that $r = cb^2$, $d = abc$ and $q = a^2b$ for some integers a, b and c.
So

$$m^2 = a^3b^2c + cb^2$$

or

$$k^3x^2 + x - m^2 = 0$$

$$(kx)^4 + kx - km^2 = 0$$

$$(ax)^4 + b^3ax - ab^3m^2 = 0$$

Let q, p and r be consecutive integers of geometric progression (in that order). Then $q = x$, $p = kx$ and $r = k^2x$ for some x and k.

So $m^2 = kx^2 + k^2x = kx(k + x)$

$$m^2 = \frac{ax(a + bx)}{b^2}$$