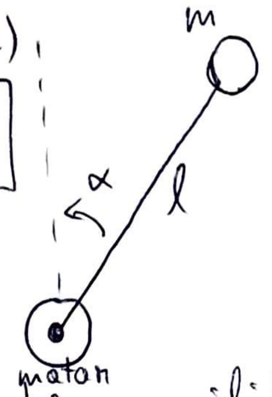
CHITIC
COSTIN.

Simulation, Control and Estimation for an Inverted Pendulum

$$\ddot{\alpha} = \frac{1}{J}\cdot\left[m\cdot g\cdot l\cdot \sin(\alpha) - b\cdot\dot{\alpha} - \frac{K^2\cdot\dot{\alpha}}{R} + \frac{K\cdot u}{R}\right] \quad (1)$$



$$x = [\alpha, v]^T \qquad u \in [-10, 10]\,V$$

Goal: Stabilize pendulum in the unstable equilibrium $x_{eq} = [0,0]^T$ (pointing up).

Liniarizare: $x_{eq} = [0, 0]^T$

$u_{eq} = 0$

First-Order Taylor Expansion: $L(x) = f(a) + f'(a)(x-a)$.

! Singurul termen neliniar al ecuației (1) este „$\sin(\alpha)$".

$$\Rightarrow \sin(\alpha) \approx \sin(\alpha_{eq}) + \cos(\alpha_{eq})(\alpha - \alpha_{eq}) = \underbrace{\sin(0)}_{0} + \underbrace{\cos(0)}_{1}\underbrace{(\alpha - 0)}_{\alpha}$$

$$\Rightarrow \boxed{\sin(\alpha) \approx \alpha.}$$

$$\Rightarrow \ddot{\alpha} \approx \frac{1}{J}\left[m\cdot g\cdot l\cdot \alpha - b\cdot\dot{\alpha} - \frac{K^2}{R}\cdot\dot{\alpha} + \frac{K}{R}\cdot u\right]\Bigg|_{\alpha_{eq}} \quad (2)$$

Reprezentarea în spațiul stărilor:

$$\begin{cases} \dot{x}_1 = \dot{\alpha} = x_2 \\ \dot{x}_2 = \ddot{\alpha} = \frac{1}{J}\left[m\cdot g\cdot l\cdot x_1 - \left(b + \frac{K^2}{R}\right)\cdot x_2 + \frac{K}{R}\cdot u\right] \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{m \cdot g \cdot l}{J} & -\frac{b+\frac{K^2}{R}}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{RJ} \end{bmatrix} u$$

$$\Rightarrow A_C = \begin{bmatrix} 0 & 1 \\ \frac{m \cdot g \cdot l}{J} & -\frac{b+\frac{K^2}{R}}{J} \end{bmatrix} ; \quad B_C = \begin{bmatrix} 0 \\ \frac{K}{RJ} \end{bmatrix}$$

= Euler's Forward Method =

$$x(t+\Delta t) \approx x(t) + \frac{dx}{dt} \cdot \Delta t.$$

$$\Rightarrow \boxed{x_{K+1} \approx x_K + T_S \cdot \dot{x}_K} \quad (3)$$

* Discretization by using Euler with $T_S = 0{,}01 [s]$

$\dot{x}_K = A_C \cdot x(t) + B_C \cdot x(t)$.

Inlocuind în (3), obținem $x_{K+1} = x_K + T_S(A_C \cdot x_K + B_C \cdot u_K)$

$$\Rightarrow x_{K+1} = \underbrace{(I_2 + T_S \cdot A_C)}_{Ad} x_K + \underbrace{T_S \cdot B_C}_{Bd} \cdot u_K.$$

$$Ad = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 0{,}01 \begin{bmatrix} 0 & 1 \\ \frac{mgl}{J} & -\frac{b+\frac{K^2}{R}}{J} \end{bmatrix} \Rightarrow Ad = \begin{bmatrix} 1 & 0{,}01 \\ 0{,}01 \cdot \frac{mgl}{J} & 1 - 0{,}01 \frac{b+\frac{K^2}{R}}{J} \end{bmatrix}$$

$$Bd = 0{,}01 \cdot \begin{bmatrix} 0 \\ \frac{K}{RJ} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0{,}01 \cdot K}{RJ} \end{bmatrix}$$

$$\Rightarrow x_{K+1} = \begin{bmatrix} 1 & 0{,}01 \\ 0{,}01 \frac{mgl}{J} & 1 - 0{,}01 \frac{b+\frac{K^2}{R}}{J} \end{bmatrix} x_K + \begin{bmatrix} 0 \\ \frac{0{,}01 \cdot K}{RJ} \end{bmatrix} \cdot u_K.$$