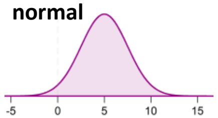
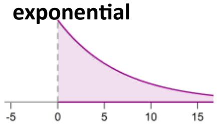
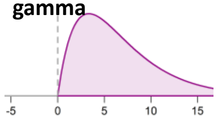
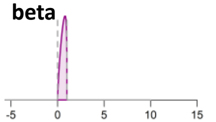
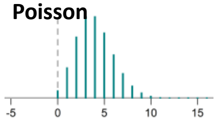
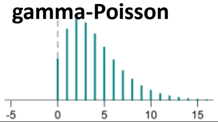
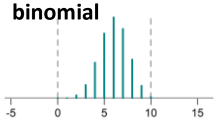
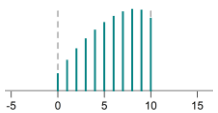
distribution	example	parameters	probability density | mass function for x	mean	standard deviation	supported x	generate 10 numbers in R estimate the likelihood of x in R
normal 	SAT score	μ = mean * σ = SD	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ	continuum $(-\infty,\infty)$	`rnorm(n=10,mean=μ,sd=σ)` `dnorm(x,mean=μ,sd=σ)`
log-normal 	species area	m = logarithm of median * $= \ln\left(\frac{\mu^2}{\sqrt{\mu^2+\sigma^2}}\right)$ s = spread $= \sqrt{\ln\left(1+\frac{\sigma^2}{\mu^2}\right)}$ $= \sqrt{\ln\left(\frac{e^{2m}+\sqrt{e^{4m}+4e^{2m}\sigma^2}}{2e^{2m}}\right)}$	$\frac{1}{xs\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln(x)-m}{s}\right)^2}$	$e^{\left(m+\frac{s^2}{2}\right)}$	$\sqrt{\left(e^{s^2}-1\right)e^{2m+s^2}}$	continuum $(0,\infty)$	`rlnorm(n=10,meanlog=m,sdlog=s)` `dlnorm(x,meanlog=m,sdlog=s)`
exponential 	time between two earthquakes	λ = rate	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	continuum $[0,\infty)$	`rexp(n=10,rate=λ)` `dexp(x,rate=λ)`
gamma 	time it takes to finish PhD	$\mathbb{E}$ = shape (how many events) λ = rate or ϑ = scale $= \frac{1}{\lambda}$	$\frac{\lambda^{\mathbb{E}}}{\Gamma(\mathbb{E})}x^{\mathbb{E}-1}e^{-\lambda x}$	$\mathbb{E}\frac{1}{\lambda}$	$\sqrt{\mathbb{E}}\frac{1}{\lambda}$	continuum $[0,\infty)$	`rgamma(n=10,shape=𝔼,rate=λ)` `dgamma(x,shape=𝔼,rate=λ)`

beta

	proportion of skin covered by clothing	μ = mean
		θ = shape (concentration)
		or
		$\alpha = \mu\theta$
		$\beta = (1-\mu)\theta$
			$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	μ or $\frac{\alpha}{\alpha+\beta}$	$\sqrt{\frac{\mu-\mu^2}{\theta+1}}$ or $\frac{\sqrt{\alpha\beta}}{(\alpha+\beta)\sqrt{\alpha+\beta+1}}$	continuum
						$[0,1]$
							`rbeta(n=10,shape1=μ*θ,`
							`shape2=(1-μ)*θ)`
							`dbeta(x,shape1=μ*θ,shape2=(1-μ)*θ)`

Poisson 	customers per day (same shop)	λ = rate	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	$\sqrt{\lambda}$	counts $\{0,1,\ldots\}$	`rpois(n=10,lambda=λ)` `dpois(x,lambda=λ)`

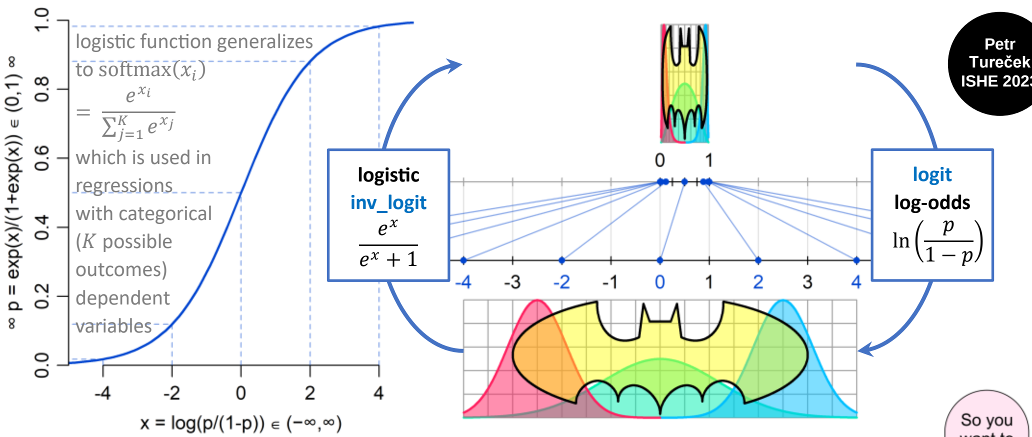
gamma-Poisson

	customers per day (different shops)	μ = mean
		θ = shape (concentration)
			$\frac{\Gamma(x+\theta)}{x!\,\Gamma(\theta)}\left(\frac{\theta}{\theta+\mu}\right)^{\theta}\left(\frac{\mu}{\theta+\mu}\right)^{x}$	μ	$\sqrt{\mu+\frac{\mu^2}{\theta}}$	counts
						$\{0,1,\ldots\}$
							`rpois(n=10,lambda=`
							`rgamma(n=10,shape=θ,rate=θ/μ))`
							`dnbinom(x,size=θ,prob=θ/(μ+θ))`

binomial 	number of dates out of n invitations for coffee	p = probability of $+1$ n = total attempts	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$\sqrt{np-np^2}$	counts $\{0,\ldots,n\}$	`rbinom(n=10,size=n,prob=p)` `dbinom(x,size=n,prob=p)`

beta-binomial

	number of dates out of n invitations for coffee (different people)	μ = mean of beta
		θ = shape (concentration)
		or
		$\alpha = \mu\theta$
		$\beta = (1-\mu)\theta$
		n = total attempts
			$\binom{n}{x}\frac{\Gamma(x+\alpha)\Gamma(n-x+\beta)\Gamma(\alpha+\beta)}{\Gamma(n+\alpha+\beta)\Gamma(\alpha)\Gamma(\beta)}$	$n\mu$ or $\frac{n\alpha}{\alpha+\beta}$	$\sqrt{\frac{n(\theta+n)(\mu-\mu^2)}{\theta+1}}$ or $\frac{\sqrt{n\alpha\beta(\alpha+\beta+n)}}{(\alpha+\beta)\sqrt{\alpha+\beta+1}}$	counts
						$\{0,\ldots,n\}$
							`rbinom(n=10,size=n,prob=rbeta(n=10,`
							`shape1=μ*θ,shape2=(1-μ)*θ))`
							`dbetabinom<-function(x,mup,shape,size){`
							`a<-mup*shape;  b<-(1-mup)*shape`
							`choose(size,x)*`
							`(gamma(x+a)*gamma(size-x+b)*gamma(a+b))/`
							`(gamma(size+a+b)*gamma(a)*gamma(b))}`
							`dbetabinom(x,mup=μ,shape=θ,size=n)`

*parameters with asterisk can assume both positive and negative values, other parameters are always positive, for the explanation of the gamma function Γ and intuition about α, β parameters of beta, flip the paper

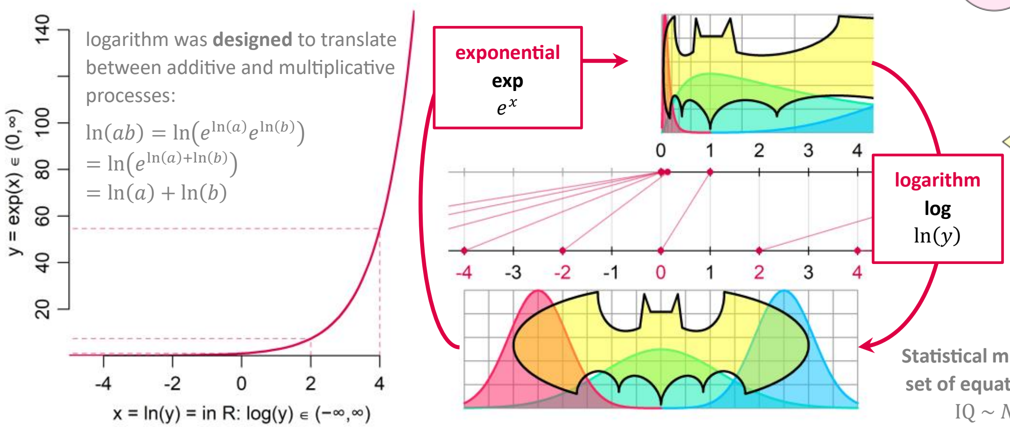
Mapping between real number $(-\infty,\infty)$ and probability $(0,1)$ domain with logit and logistic functions



Petr Tureček
ISHE 2023

realm of additive processes realm of multiplicative processes

Mapping between real $(-\infty,\infty)$ and positive $(0,\infty)$ domain with logarithm and exponential f.

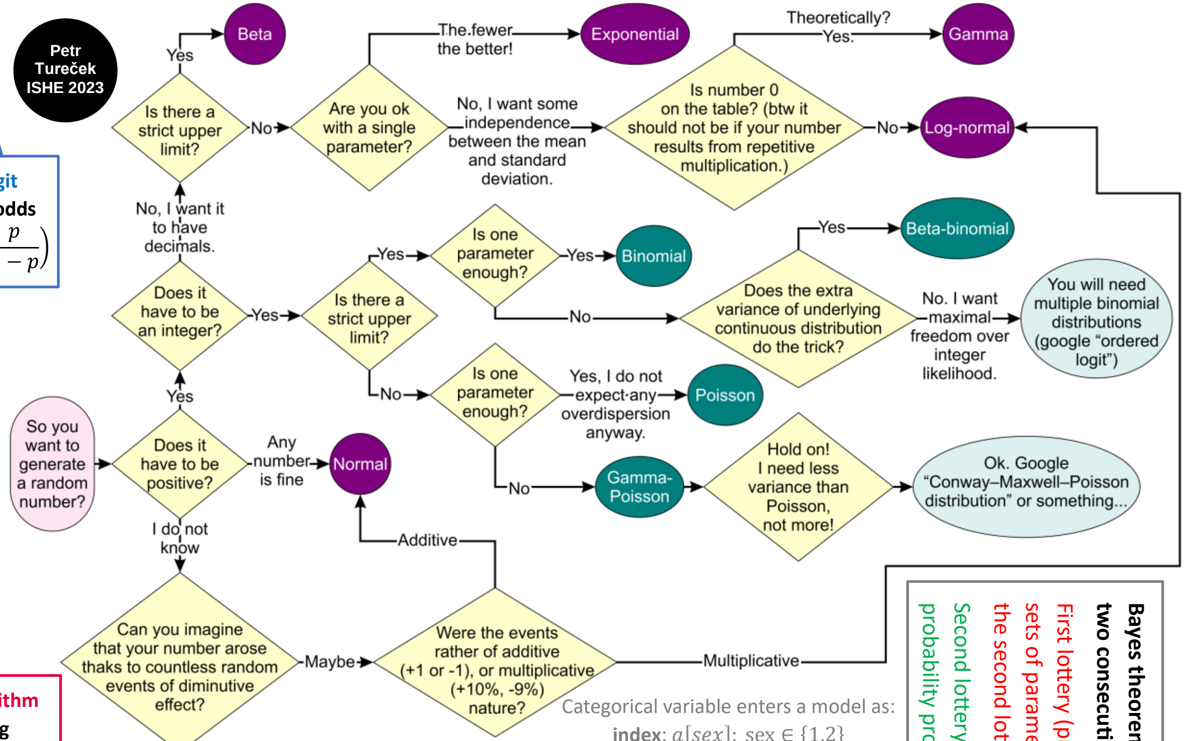


Statistical model is just a set of equations such as

$IQ \sim N(\mu,\sigma)$

$\mu = a[\text{sex}] + b \times age$

Maximum entropy inspired flowchart of distribution selection

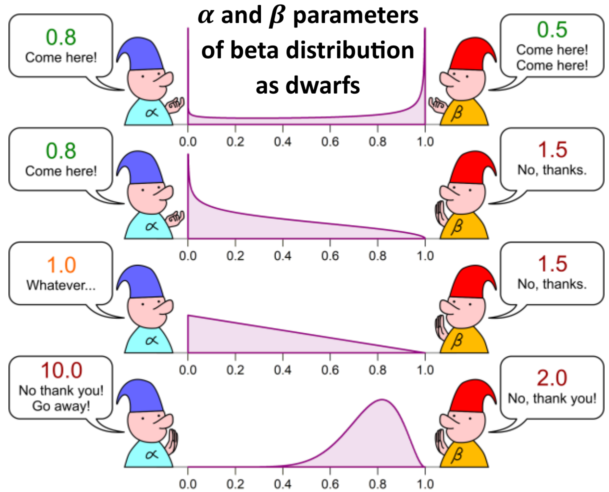


Categorical variable enters a model as:

index: $a[sex]$; sex $\in \{1,2\}$

indicator: $b \times$ sex; sex $\in \{0,1\}$

contrast: $c \times$ sex; sex $\in \{-0.5, 0.5\}$

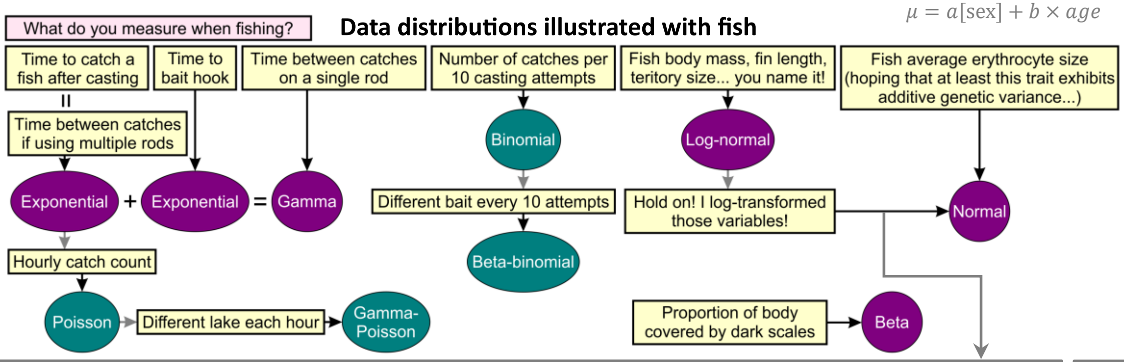


Bayes theorem (assessing hypothesis probability) can be viewed as two consecutive lotteries: $P(H|d) \propto P(d|H)P(H)$

First lottery (prior) works with probabilities that random hypotheses, sets of parameter values H, are even considered for participation in the second lottery.

Second lottery draws each hypothesis from the first drawth with probability proportional to likelihood that data d occur under H.

Data distributions illustrated with fish



Some clever(er) log-transformations

```
lx<-log(x,1.1)-log(median(x),1.1) #each +1 represents 10% increase, 0 is median
lx<-log(x,2)-log(median(x),2)     #each +1 represents doubling, 0 is median

#Back to the original scale (works with any base, just replace 1.1)
1.1^(lx+log(median(x),1.1))
```

gamma function $\Gamma(n) = (n-1)!$

is a generalization of factorial ($3! = 3 \times 2 \times 1$) to all complex numbers except non-positive integers. Because it generalizes to all positive real numbers, it extends many distribution functions beyond integer parameter values ($\mathbb{E}$ is an example of parameter worthy of extension).