MSc. in HPC

Trinity College Dublin

Quantitative Finance (565b)

Programming Exercises

(To be handed in by March 23^{rd}, 2015)

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1 Notes

- **To complete these exercises, submit a tarball with a document containing results for each question and the code used to obtain the results**
- Use any of C, C++, python, ruby, fortran, R, matlab, mathematica to do these exercises. If you want to use another package or language ask me first
- if the language you are using has any implementations of the models referred to in the exercises below (e.g., matlab) then you may **not** use those implementations to carry out the exercises. Obviously you can use them to check your answers
- You **may** use any random number generators that are provided by the package or language
- In all simulations below, it is acceptable to use equal time length intervals when partitioning any interval on the real line

2 Exercises

1. Brownian Motion.
   a) Simulate a Brownian Motion process up to time $t = 2$. Create a plot displaying multiple brownian motion paths
   b) For times $t = 1$ and $t = 2$ simulate multiple values. Create a histogram of the distributions of the values for both time points (i.e., two histograms). Calculate sample means and variances at both time points.

      At both times, use the relevant histogram and the sample moments to indicate that the distribution of the brownian motion values is as expected.
   c) If possible graphically overlay the (normalised) histogram with the expected theoretical distribution at each time

2. Simulate the *analytic* solution of the SDE

$$dX(t) = aX(t)dt + bX(t)dW(t)$$

   where $X(0) = 1$, $a = 1.5$ and $b = 1$. Display multiple simulated paths graphically. Simulate paths at more than one time interval (e.g., $2^{-2}, 2^{-3}, 2^{-4}, \cdots$).

3. Write software to solve the ordinary differential equation

$$dx = -5xdt, \qquad x(0) = 1\,,$$

using the Euler method. Compare your numerical solution to the analytic solution graphically. Run simulations for multiple step sizes (e.g., $2^{-2}, 2^{-3}, 2^{-4}, \cdots$), plot the errors on a log-log plot and estimate the order of convergence.

4. Use the Euler-Maruyama method to solve the SDE

$$dX(t) = aX(t)dt + bX(t)dW(t)$$

   where $X(0) = 1$, $a = 1.5$ and $b = 1$.

   Using a log-log plot and a number of different interval sizes estimate the order of strong and weak convergence for the Euler-Maruyama method for this SDE. If possible fit the line using a linear fit. However visual fits will be accepted

5. Analytically solve the SDE

$$dX(t) = -aX(t)dt + \sigma dW(t)$$

   where a and σ are constant and $X(0) = X_0$.

   For $X_0 = 2, \sigma = 1.1$ and $b = 1.5$, solve the equation using the Euler-Maruyama method. Obtain estimates of the order of strong and weak convergence for this SDE