CS381 Homework 2 Problem 1

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1 Exercise 5.2.1

`longest-common-sub(`i`, `k`)`: Given $A[1, ..., m]$ and $B[1, ..., n]$ as globals where parameter $i \in [m]$ and $k \in [n]$, returns the length of the longest common subsequence of $A[1, ..., i]$ and $B[1, ..., k]$.

1. If $i = 0$ or $k = 0$ Return 0
2. If $A[i] = A[k]$
   A. Return $Max($`longest-common-sub(`$i - 1$`, `k`)`$,$`longest-common-sub(`i`, `$k - 1$`)`$, 2+$ `longest-common-sub(`$i - 1$`, `$k - 1$`)`$,)$
3. Return $Max($`longest-common-sub(`$i - 1$`, `k`)`$,$`longest-common-sub(`i`, `$k - 1$`)`$,)$

Given arrays A with length m and B with length n, find the longest common subsequence with the call `longest-common-sub(`m`, `n`)`. The number of possible parameter combinations is $m \cdot n$, thus the runtime is $O(mn)$ when the function is memoized based on parameters i and k

2 Exercise 5.2.2

`shortest-common-super(`i`, `k`)`: Given $A[1, ..., m]$ and $B[1, ..., n]$ as globals where parameter $i \in [m]$ and $k \in [n]$, returns the length of the shortest common supersequence of $A[1, ..., i]$ and $B[1, ..., k]$.

1. If $i = 0$ Return n // |B|
2. If $k = 0$ Return m // |A|
3. If $A[i] = B[k]$
   A. Return $1 +$ `shortest-common-super(`$i - 1$`, `$k - 1$`)`
4. Return $Min(1+$`shortest-common-super(`$i - 1$`, `k`)`$, 1+$`shortest-common-super(`i`, `$k - 1$`)`$)$

Given arrays A with length m and B with length n, find the shortest common supersequence with the call `shortest-common-super(`m`, `n`)`. The number of possible parameter combinations is $m \cdot n$, thus the runtime is $O(mn)$ when the function is memoized based on parameters i and k.

3 Exercise 5.2.3

palindrome(i, j, k, l): Given $A[1, ..., m]$ and $B[1, ..., n]$ as globals where parameter $i, j \in [m]$ and $k, l \in [n]$, returns the length of the longest common subsequence of $A[1, ..., i]$ and $B[1, ..., k]$ that is a palindrome.

1. If $i > j$ or $k > l$ Return 0
2. If $i = j$
   A. If $A[i] = B[k]$ or $A[i] = B[l]$
      1. Return 1
   B. Else
      1. Return 0
3. If $k = l$
   A. If $B[k] = A[i]$ or $B[k] = A[j]$
      1. Return 1
   B. Else
      1. Return 0
4. If $A[i] = A[j] = B[k] = B[l]$
   A. Return $Max($ $2 +$ palindrome($i + 1$, $j - 1$, $k + 1$, $l - 1$), palindrome($i + 1$, j, k, l), palindrome(i, $j - 1$, k, l), palindrome(i, j, $k + 1$, l), palindrome(i, j, k, $l - 1$))
5. Else
   A. $Max($ palindrome($i + 1$, j, k, l), palindrome(i, $j - 1$, k, l), palindrome(i, j, $k + 1$, l), palindrome(i, j, k, $l - 1$))

Given arrays A with length m and B with length n, find the longest common subsequence that is a palindrome with the call palindrome(0, m, 0, n). The number of possible parameter combinations is $m \cdot m \cdot n \cdot n$, thus the runtime is $O(m^2n^2)$ when the function is memoized based on parameters i, j, k and l.