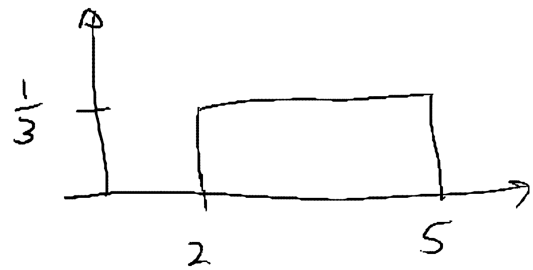
Chapter 8 – System Reliability

8.1



a) $\mu_X = \int_{-\infty}^{\infty} x\, p_X(x)\, dx = \int_2^5 \frac{1}{3} x\, dx$

$= \frac{1}{3} \frac{x^2}{2} \Big|_2^5 = \frac{1}{6}\left[25 - 4\right]$

$$\boxed{\mu_X = \frac{21}{6} = 3.5}$$

$\sigma_X^2 = E[X^2] - \mu_X^2$

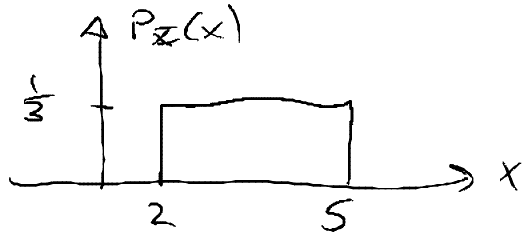
$= \frac{1}{3}\int_2^5 x^2\, dx - \mu_X^2 = \frac{x^3}{9}\Big|_2^5 - \mu_X^2$

$\sigma_X^2 = \frac{125 - 8}{9} - (3.5)^2 = \boxed{\frac{3}{4}}$

b) $P(2 \le X \le 3) = \int_2^3 p_X(x)\, dx = \int_2^3 \frac{1}{3}\, dx$

$= \frac{1}{3} x \Big|_2^3 = \boxed{\frac{1}{3}}$

c)



8.2

$$F(t) = \int_{-\infty}^{t} p_{\underline{x}}(\tau)\, d\tau$$

$$\frac{d}{dt} F(t) = p_{\underline{x}}(t)$$

$$\frac{d}{dt} F(t) \geq 0$$

Since the slope is always ≥ 0 the function must be monotonically increasing.

8.3

Failure Rate = average # of failures of a device per unit time. Can be determined by experimental observation of device under test. Often shape of bathtub curve.

Failure Function $(F(t))$ = Probability that a device has failed at time t.

Reliability, $R(t)$: Complement of $F(t)$. This is the probability a device is working at time t.

8.4

$$\lambda = \frac{50}{10^6 \text{ hrs}}$$

a) $$MTTF = \frac{1}{\lambda} = \frac{10^6 \text{ hrs}}{50} = \boxed{2.3 \text{ years}}$$

b) $$R(t) = e^{-\lambda t}$$

$$R(5\text{ years}) = \exp\left(-\frac{1}{2.3}5\right) = 0.11 = \boxed{11\%}$$

$$R(15\text{ years}) = \exp\left(-\frac{15}{2.3}\right) = \boxed{0.15\%}$$

$$R(20\text{ years}) = \exp\left(-\frac{20}{2.3}\right) = \boxed{0.017\%}$$

$$R(10\text{ years}) = \exp\left(\frac{-10}{2.3}\right) = \boxed{1.3\%}$$

8.5

Problem 5

CD4001C

2 Input Quad NOR

CMOS, Glass sealed, 15 years, $\Theta_{JA} = 70°C/W$, B-1 Quality

$T_A = 25°C$, $P_D = 10\,mW$

$$\lambda_p = (C_1 \pi_T + C_2 \pi_E)\,\pi_Q \pi_L \ \frac{\text{failures}}{10^6 \text{ hours}}$$

$C_1 = \underline{0.00085}$ Gates = 1→500 (MOS)

$C_2 = 9.0\times10^{-5}\,(N_p)^{1.51} = 9.0\times10^{-5}\,(14)^{1.51}$

$C_2 = 4.84\times10^{-3}$

$$\pi_T = 0.1\exp\left[-\frac{E_A}{8.617\times10^{-5}}\left(\frac{1}{T_J+273} - \frac{1}{296}\right)\right]$$

$E_A = 0.35$

$T_J = T_A + P_D\,\Theta_{JA} = 25°C + \left(70\frac{°C}{W}\right)10\,mW = 25.7°C$

$$\pi_T = 0.1\exp\left[-\frac{0.35}{8.617\times10^{-5}}\left(\frac{1}{25.7°C+273} - \frac{1}{296}\right)\right]$$

$\underline{\pi_T = 0.113}$

$\pi_L = \underline{1.0}$ $\quad\quad \pi_Q = 2.0 \quad\quad \pi_E = \underline{0.50}$

$$\lambda = \left[(.00085)(.113) + (4.84\times10^{-3})\,0.5\right](2)(1)\ /10^6\,hrs$$

$\lambda = 5.03\times10^{-3}\ /10^6\,hrs$

$$\boxed{MTTF = \frac{1}{\lambda} = 1.98\times10^{8}\,hrs}$$

$R(t) = e^{-\lambda t}$

$R(25\text{ years}) = \exp\left[-\frac{6.59\times10^{-3}}{10^6\,hrs}\times\frac{24h}{day}\times\frac{365\,day}{year}\times25\,year\right]$

$\boxed{= 99.8\%}$

8.6

Given

- 32 Bit CMOS
- Missile Launcher
- $T_A = 120° C$
- $N_P = 64$ (pins)
- 6 months of production
- B-1 Quality

$$\lambda = (C_1 \pi_T + C_2 \pi_E)\pi_Q \pi_L \ \frac{\text{failures}}{10^6 \text{ hours}}$$

$C_1 = \underline{0.56}$ (Direct Read from 3rd table)

$$C_2 = 3.0 \times 10^{-5} (N_P)^{1.08} = 3.0 \times 10^{-5} (64)^{1.08}$$

$$= \underline{2.68 \times 10^{-3}}$$

$$\pi_T = 0.1 \exp\left[\frac{-E_a}{8.617 \times 10^{-5}}\left(\frac{1}{T_j + 273} - \frac{1}{296}\right)\right]$$

$E_a = 0.35$

$$= 0.1 \exp\left[\frac{-0.35}{8.617 \times 10^{-5}}\left[\frac{1}{120 + 273} - \frac{1}{296}\right]\right]$$

$$= \underline{2.95}$$

$\pi_Q = \underline{2.0}$ $\qquad$ $\pi_E = \underline{12}$

$\pi_L = \underline{1.8}$

$$\lambda = (C_1 \pi_T + C_2 \pi_E)\pi_Q \pi_L \ \frac{\text{failures}}{10^6 \text{ hours}}$$

$$= \left[(0.56)(2.95) + (2.68 \times 10^{-3})12\right] 2.0\,(1.8)$$

$$\lambda = 6.1 \ \frac{\text{failures}}{10^6 \text{ hours}}$$

$$MTTF = \frac{1}{\lambda} = \frac{10^6 \text{ hours}}{6.1} \times \frac{\text{day}}{24 \text{ hrs}} \times \frac{\text{year}}{365 \text{ days}}$$

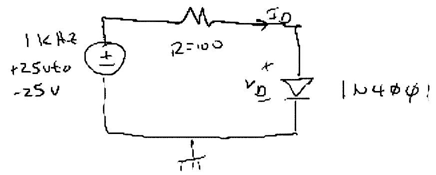
$$\boxed{MTTF = 18.7 \text{ years}}$$

$$R(t) = e^{-\lambda t}$$

$$R(20 \text{ years}) = \exp\left(-\frac{1}{18.7 \text{ years}} \times 20 \text{ years}\right)$$

$$\boxed{R(20 \text{ years}) = 34.3\%}$$

8.7



Low Frequency Diode

$\lambda = \lambda_b \pi_T \pi_S \pi_C \pi_Q \pi_E \ \frac{\text{failures}}{10^6 \text{hrs.}}$

$\lambda_b = 0.001$ (switching app)

$\pi_T = \exp\left(-3091\left(\frac{1}{T_j+273} - \frac{1}{298}\right)\right)$

$T_j = T_A + \theta_{JA} P_D$

$\theta_{JA} = 50° \text{C}/\text{W}$ (See 1N4001 data sheet)

P_D -> need to solve for

ON VOLTAGE

$P_D = V_D I_D$ $\quad I_D = \frac{25-0.7}{100} = 243\text{ mA}$

(forward bias)

$P_D = \frac{(0.7)(0.243)}{2}$ ← only on $\frac{1}{2}$ time

$P_D = 85\text{ mW}$

$T_j = 50°\text{C} + (50°\text{C}/\text{W})(0.085\text{ W})$

$= 54°\text{C}$

Now $\pi_T = \exp\left[-3091\left(\frac{1}{54+273} - \frac{1}{298}\right)\right]$

$= 1.001$

$\pi_S = ?$ Need to find V_S

$V_S = \frac{25\text{V}}{50\text{V}}$ ← Rated from Data sheet

$= \frac{1}{2}$

$\pi_S = V_S^{2.43} = (.5)^{2.43} = \underline{0.19}$

$\pi_C = 1.0$ $\quad \pi_E = 13$

$\pi_Q = 5.0$

$\lambda = \lambda_b \pi_T \pi_S \pi_C \pi_Q \pi_E$

$= (0.001)(1.001)(0.19)(1.0)(5.0)(13)$

$\lambda = 0.012 \frac{\text{failures}}{10^6 \text{ hrs}}$

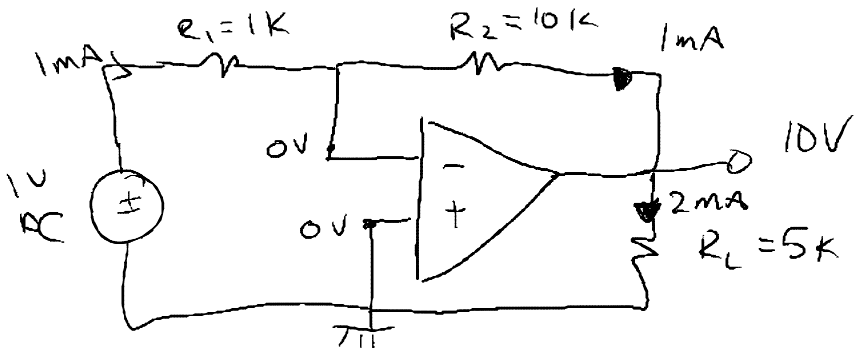
$\text{MTTF} = \frac{1}{\lambda} = \frac{10^6 \text{ hrs}}{0.012} = \boxed{9512 \text{ years}}$

$R(t) = e^{-\lambda t}$

$R(25 \text{ years}) = \exp\left(-\frac{1}{9512 \text{ yrs}} \times 25 \text{ yrs}\right)$

$\boxed{R(25 \text{ years}) = 99.7\%}$

8.8



In this problem, need to consider Reliability of all components.

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

Resistor R_1 operate power $= VI = (1V)(1mA) = 1mW$

$$\lambda = \lambda_b \pi_R \pi_Q \pi_E \ \frac{\text{failures}}{10^6 \text{ hrs}}$$

$$\lambda_b = 4.5\times10^{-9} \exp\left[12\left(\frac{T+273}{343}\right)\right] \exp\left[\frac{S}{0.6}\left(\frac{T+273}{273}\right)\right]$$

$$= 4.9\times10^{-9} \exp\left[12\left(\frac{80+273}{273}\right)\right] \exp\left[\frac{\frac{1}{250}}{.6}\left(\frac{80+273}{273}\right)\right]$$

$$= 4.9\times10^{-9} \exp(15.5) \exp(8.62\times10^{-3})$$

$$= 0.027$$

$\pi_R = 1.0$ (Table Read)

$\pi_Q = 0.03$ $\pi_E = 8$

$$\lambda_{R_1} = \lambda_b \pi_R \pi_Q \pi_E \ \frac{\text{failures}}{10^6 \text{ hrs}}$$

$$\lambda_{R_1} = (0.027)(1.0)(.03)(8) = 6.48\times10^{-3} \frac{\text{failures}}{10^6 \text{ hours}}$$

Pretty Reliable

Resistor R_2

$$P = VI = (10)(1\,mA) = 10\,mW$$

Produces virtually same λ as R_1

$$\lambda_{R_2} \cong 6.5 \times 10^{-3}\ \text{failures}/10^6\ \text{hours}$$

Resistor R_L

$$P = VI = (10)(2\,mA) = 20\,mW$$

$$\vdots$$

$$\lambda_{R_L} = 7.4 \times 10^{-3}\ \text{failures}/10^6\ \text{hrs}$$

Op Amp

$$\lambda = (C_1 \pi_T + C_2 \pi_E)\pi_Q \pi_L$$

$C_1 = 0.01$ (Bipolar Device)

$C_2 = 9.0 \times 10^{-5} (N_p)^{1.51}$ (DIP package)

$$= (9.0 \times 10^{-5})(8)^{1.51} = 2.08 \times 10^{-3}$$

$$\pi_T = 0.1 \exp\left[-\frac{E_a}{8.617 \times 10^{-5}}\left(\frac{1}{T_j + 273} - \frac{1}{296}\right)\right]$$

$E_a = 0.65$ (Linear, Bipolar)

$T_j = T_A + \Theta_{JA} P_D$

$\Theta_{JA} = 100°\ \text{C/W}$ (741 Data sheet)

$P_D = ?$ $P_{IN} = (1V)(1mA)$ source in

$= 1mW$

$P_{out} = (10V)(2mA)$ Load Resistor out

$= 20mW$

$\Rightarrow$ op Amp $P_D = 19\ mW$

$T_J = T_A + \Theta_{JA} P_D$

$= 80°C + (100°\ \text{C/W})(0.019\ W)$

$= 82°C$

now $\pi_T = 0.1 \exp\left[-\frac{E_a}{8.617\times10^{-5}}\left(\frac{1}{T_j+273} - \frac{1}{296}\right)\right]$

$= 0.1 \exp\left[\frac{-0.65}{8.617\times10^{-5}}\left(\frac{1}{82+273} - \frac{1}{296}\right)\right]$

$\pi_T = 6.91$

$\pi_L = 1.0$ $\pi_Q = 0.25$ $\pi_E = 4$

$\lambda = (C_1 \pi_T + C_2 \pi_E)\pi_Q \pi_L$

$= [(0.01)(6.91) + (2.08\times10^{-3})(4.0)](0.25)(1.0)$

$\lambda_{op\ Amp} = 0.02$ failures/10^6 hrs

Let's finish!

$$\lambda = \lambda_{Res1} + \lambda_{res2} + \lambda_{R_L} + \lambda_{op\ amp}$$

$$= 6.48\times10^{-3} + 6.5\times10^{-3} + 7.4\times10^{-3} + 0.02$$

$$\lambda = 0.04 \text{ failures}/10^6 \text{ hrs}$$

$$R(t) = e^{-\lambda t}$$

$$R\left(\begin{smallmatrix}25\\ \text{years}\end{smallmatrix}\right) = \exp\left(-\frac{0.04}{10^6 \text{ hrs}} \times \frac{24 \text{ hrs}}{\text{day}} \times \frac{365 \text{ days}}{\text{year}} \times 15 \text{ years}\right)$$

$$\boxed{= 99.4\%}$$

8.9

Add heat sink

$\Theta_{CS} = 10^\circ C/W \qquad \Theta_{SA} = 8^\circ C/W$

$T_A = 130^\circ C$

a) $P_{D\,max} = ?$

$$P_{D,max} = \frac{T_{J,max} - T_A}{\Theta_{JA}} = \frac{T_{J,max} - T_A}{\Theta_{JC} + \Theta_{CS} + \Theta_{SA}}$$

$$= \frac{150 - 130}{(83.3 + 10 + 8)^\circ C/W}$$

Datasheet of 2N3904

$$\boxed{P_{D,max} = 197\,mW}$$

b) Reliability in 50 years

Use values from Example 8.4 where possible

$$R(t) = e^{-\lambda t}$$

$$\lambda = \lambda_b \pi_T \pi_A \pi_R \pi_S \pi_Q \pi_E \,/10^6\,hrs$$

$$\pi_T = \exp\left[-2114\left(\frac{1}{T_j + 273} - \frac{1}{298}\right)\right]$$

$$T_j = T_A + \Theta_{JA} P_D$$

$$= 130^\circ C + (101.3^\circ C/W)(.125W) = 142.7^\circ C$$

$$\pi_T = \exp\left[-2114\left(\frac{1}{143 + 273} - \frac{1}{298}\right)\right] = 7.48$$

All else is same from Example 8.4

$$\Rightarrow \lambda = 6.87 \times 10^{-3}/10^6\,hours$$

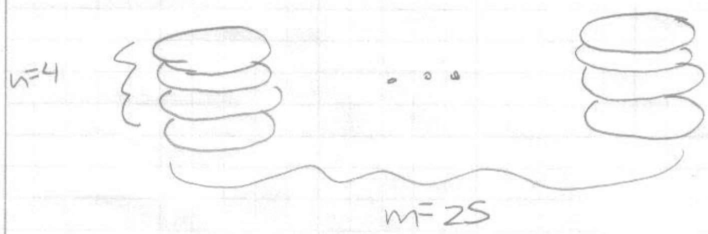
$$R(50\,years) = \exp\left[-\frac{6.87 \times 10^{-3}}{10^6\,hrs} \times \frac{24\,hrs}{day} \times \frac{365\,days}{year} \times 50\,years\right]$$

$$\boxed{R(50\,years) = 99.6\%}$$

8.10

<u>Problem 10</u> RAID Design +4 Round. concept

$R_s(10\text{ years}) = 0.95$



This is a combination series-parallel system. 25 banks in series, each w/ a redundancy (parallel) of 4.

$\Rightarrow$ Problem is to determine $R(10\text{yrs})$ for the disks

$$R_S(t) = \prod_{i=1}^{m} \left[1 - \prod_{j=1}^{n_i} \left(1 - R_{ij}(t)\right) \right]$$

Could actually DO 4 banks of 25 too, but worse reliability

but $R_{ij}(t) = R_i(t)$

$$= \prod_{i=1}^{m} \left[1 - \left(1 - R(t)\right)^n \right]$$

$$= \left[1 - \left(1 - R(t)\right)^n \right]^m$$

$$0.95 = \left[1 - \left(1 - R(10\text{yrs})\right)^4 \right]^{25}$$

$$0.99795 = 1 - \left(1 - R(10\text{yrs})\right)^4$$

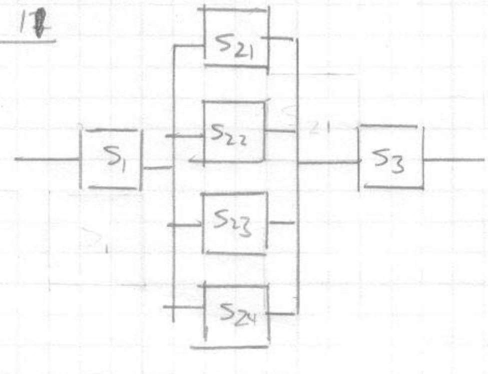
$$2.05 \times 10^{-3} = \left[1 - R(10\text{yrs})\right]^4$$

$$0.213 = 1 - R(10\text{yrs})$$

$$\boxed{R(10\text{years}) = .787 = 78.7\%}$$

8.11

Problem 1



$F_1 = 2\%$

$F_3 = 3\%$

Reliability of $S_{21} = S_{22} = S_{23} = S_{24} = S_2$

$R_1 = R_3 = 1 - 3\% =$

$$R_S = R_1 \left[1 - (1 - R_2)^4\right] R_3$$

$$0.94 = (0.98)\left[1 - (1 - R_2)^4\right] 0.97$$

$$0.988 = 1 - (1 - R_2)^4$$

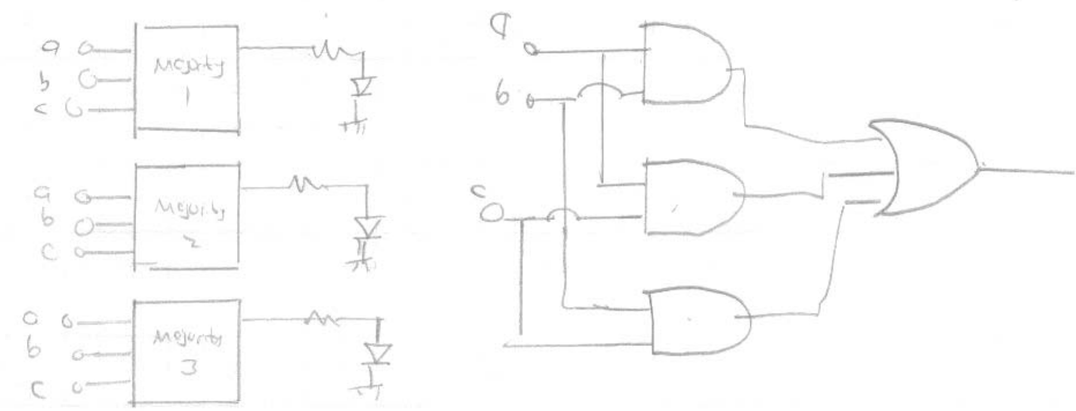
$$0.012 = (1 - R_2)^4$$

$$0.33 = 1 - R_2$$

$$\Rightarrow \boxed{R_2 = 0.67}$$

8.12

Problem 12



Vote, based on 2 LEDs.

• Each component is 90% reliable

What is prob. get a false reading?

Solution

Need to find the reliability of each circuit

$R_M = (0.9)^6 = 0.53$ Reliability of majority + resistor + LED (6 components)

$F_M = 1 - R_M = 0.47$ Prob. has failed

Now need to examine probability that it is working properly

$\text{Prob}\left(\begin{smallmatrix}\text{2 or more have}\\ \text{failed}\end{smallmatrix}\right) = \text{Prob}(\text{2 have failed}) + P(\text{3 have failed})$

3 possibilities

$= 3\,(0.53)(0.47)(0.47) + (0.47)(0.47)(0.47)$

$= 0.455$

$\boxed{\text{Prob}\left(\begin{smallmatrix}\text{2 or more}\\ \text{have failed}\end{smallmatrix}\right) = 45.6\%}$