

$$2''x + 4x + 3x = 1, \quad x(0) = 2, \quad \dot{x}(0) = 1$$

Taking Laplace Transform and applying initial conditions

$$2[s^2 X(s) - 2s - 1] + 4[sX(s) - 2] + 3X(s) = \frac{1}{s}$$

$$X(s)[2s^2 + 4s + 3] = 4s + 10 + \frac{1}{s} = \frac{1}{s}(4s^2 + 10s + 1)$$

$$X(s) = \frac{4s^2 + 10s + 1}{s(2s^2 + 4s + 3)}$$

Taking partial fraction decomposition step to obtain inverse Laplace transform

$$\frac{4s^2 + 10s + 1}{s(2s^2 + 4s + 3)} = \frac{A}{s} + \frac{Bs + C}{2s^2 + 4s + 3}$$

$$4s^2 + 10s + 1 = A(2s^2 + 4s + 3) + (Bs + C)s$$

$$\text{at } s=0; \quad 1 = 3A$$

$$\boxed{A = \frac{1}{3}}$$

Comparing powers of s terms -
 $s^2: \quad 4 = 2A + B$

$$B = 4 - 2A = 4 - \frac{2}{3}$$

$$\boxed{B = \frac{10}{3}}$$

$$s: \quad 10 = 4A + C$$

$$C = 10 - 4A = 10 - \frac{4}{3} = \frac{26}{3}$$

$$\boxed{C = \frac{26}{3}}$$

$$X(s) = \frac{1}{3s} + \frac{\frac{10}{3}s + \frac{26}{3}}{2s^2 + 4s + 3} = \frac{1}{3s} + \frac{\frac{10}{3}s + (\frac{10}{3} + \frac{16}{3})}{2s^2 + 4s + 3}$$

$$= \frac{1}{3s} + \frac{\frac{10}{3}(s+1)}{2s^2 + 4s + 3} + \frac{\frac{16}{3}}{2s^2 + 4s + 3}$$

$$= \frac{1}{3s} + \frac{s}{3} \frac{(s+1)}{(s+1)^2 + (\sqrt{0.5})^2} + \frac{8}{3} \cdot \frac{1}{\sqrt{0.5}} \cdot \frac{\sqrt{0.5}}{(s+1)^2 + (\sqrt{0.5})^2}$$