$$2\ddot{x} + 4\dot{x} + 3x = 1 \qquad x(0) = 2, \ \dot{x}(0) = 1$$

$$2\left[s^2 X(s) - s\,x(0) - \dot{x}(0)\right] + 4\left[s\,X(s) - x(0)\right] + 3X(s) = \frac{1}{s}$$

$$2\left(s^2 X(s) - 2s - 1\right) + 4\left(s\,X(s) - 2\right) + 3X(s) = \frac{1}{s}$$

$$X(s)\left(2s^2 + 4s + 3\right) = \frac{1}{s} + 4s + 2 + 8 = \frac{4s^2 + 10s + 1}{s}$$

$$X(s) = \frac{4s^2 + 10s + 1}{(2s^2 + 4s + 3)s} = \frac{2s^2 + 5s + \frac{1}{2}}{s\left(s^2 + 2s + \frac{3}{2}\right)} = \frac{2s^2 + 5s + \frac{1}{2}}{s\left((s+1)^2 + \frac{1}{2}\right)}$$

$$= \frac{A}{s} + \frac{Bs + C}{(s+1)^2 + \frac{1}{2}}$$

$$2s^2 + 5s + \frac{1}{2} = A\left(s^2 + 2s + \frac{3}{2}\right) + Bs^2 + Cs$$

MATCH COEFF'S:

$$s^2: \ 2 = A + B \qquad\qquad \Rightarrow B = \frac{5}{3}$$

$$s^1: \ 5 = 2A + C \qquad\qquad \Rightarrow C = \frac{13}{3}$$

$$s^0: \ \frac{1}{2} = \frac{3}{2}A \ \Rightarrow A = \frac{1}{3}$$

$$X(s) = \frac{\frac{1}{3}}{s} + \frac{\frac{5}{3}s + \frac{13}{3}}{(s+1)^2 + \left(\sqrt{\frac{1}{2}}\right)^2} \qquad \Rightarrow \omega = \sqrt{\frac{1}{2}}$$

$$= \frac{\frac{1}{3}}{s} + \frac{\frac{5}{3}(s+1)}{(s+1)^2 + \left(\sqrt{\frac{1}{2}}\right)^2} + \frac{\frac{8}{3}(\sqrt{2})\left(\sqrt{\frac{1}{2}}\right)}{(s+1)^2 + \left(\sqrt{\frac{1}{2}}\right)^2}$$

$$\therefore \ x(t) = \left(\frac{1}{3} + \frac{5}{3}e^{-t}\cos\sqrt{\tfrac{1}{2}}\,t + \frac{8\sqrt{2}}{3}e^{-t}\sin\sqrt{\tfrac{1}{2}}\,t\right)u(t)$$