$$\mathcal{L}\left\{ \frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} = e^{-t} \right\}$$ (take Laplace transform of both sides.)

$$X(s)\,s^3 + X(s)\,s^2 = \frac{1}{s+1}$$

$$\mathcal{L}^{-1}\left\{ X(s) = \frac{1}{s^2(s+1)^2} \right\}$$ (take inverse Laplace transform of both sides)

First, do partial fraction expansion:

$$\frac{1}{s^2(s+1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{(s+1)^2}$$