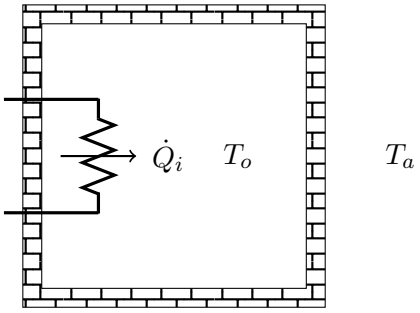
EENG307: Intro to Feedback Control

Fall 2020

Homework Assignment #4

Quiz #8

Quiz Question Wednesday: Consider the following oven with a controllable heating element that produces heat flux $\dot{Q}_i$. Suppose the oven has thermal capacity C and conductive heat transfer to the outside characterized by thermal resistance R. The oven temperature is T_o and the external temperature is T_a.



(a) Draw the equivalent impedance network for this system.

(b) Find the transfer function $\frac{T_o(s)}{\dot{Q}_i(s)}$.

(c) If $\dot{Q}_i = 0$ and T_a is constant, then the oven will come to equilibrium with the ambient, so that $T_o = T_a$. The transfer function $G(s) = \frac{T_o(s)}{\dot{Q}_i(s)}$ will tell us how the temperature of the oven will vary away from this equilibrium. That is, in the Laplace domain, we will have

$$T_o(s) = G(s)\dot{Q}_i(s) + \frac{T_a}{s}$$

Suppose the oven is initially at ambient, and then a step input of $\dot{Q}_i(t) = 1$ J/s is applied. Find $T_o(t)$ for $t > 0$. (It will be a function of R and C).

(d) If this step experiment was actually applied, and it took 20 minutes for the oven to reach a temperature of 20 °C above ambient, with a final steady state temperature of 40 °C above ambient, what is R and C?

Which of the following is the correct answer for part (c) above?

(a) $T_o(t) = \left(1 - e^{-t/RC}\right) - T_a \quad t \geq 0$

(b) $T_o(t) = R\left(1 - e^{-t/RC}\right) - T_a \quad t \geq 0$

(c) $T_o(t) = R\left(1 - e^{-t/RC}\right) + T_a \quad t \geq 0$

(d) $T_o(t) = R * e^{-t/RC} + T_a \quad t \geq 0$

(e) $T_o(t) = -\left(1 - e^{-t/RC}\right) + T_a \quad t \geq 0$