EENG307: Lecture 12 Handout

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1 Using Pole (and Zero) Locations to Predict Response Characteristics

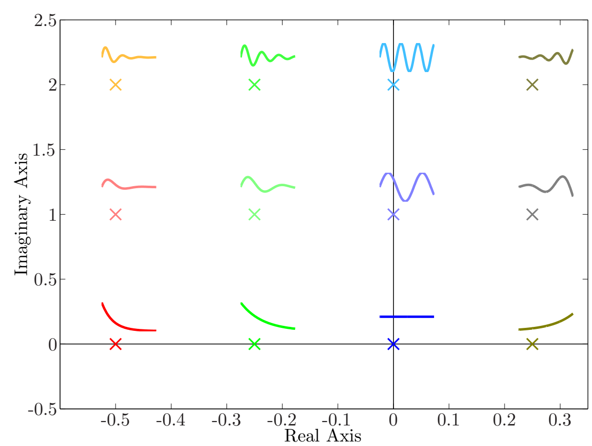
In Lectures 10-11, we derived the following step response transient characteristics for first and second order systems:

System	System Parameters	Rise Time	Settling Time	% Overshoot
$\frac{K\sigma}{s+\sigma}$	σ - pole magnitude	$t_r = \frac{2.2}{\sigma}$	$t_s = \frac{4.6}{\sigma}$	0
$\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ζ - damping ratio ω_n - natural frequency $\sigma = \zeta\omega_n$ - real part of poles	$t_r = \frac{2.2}{\omega_n}$	$t_s = \frac{4.6}{\zeta\omega_n}$	$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$

Using the partial Laplace Transform Table

$e^{-\sigma t}u(t)$	$\frac{1}{s+\sigma}$
$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

we can visualize the types of responses that occur from $e^{p_i t}$ when $p_i = -\sigma$ is real, or $e^{p_j t} + e^{p_k t}$ when p_j and p_k are complex conjugates $-a \pm j\omega$ for these first and second order systems (shown at right).



(Conjugate poles not shown)

Now let's look at the following questions:

1. What happens if there are additional poles or zeros?

2. Under what circumstances can we approximate a higher-order system or one with zeros by a first- or second-order system?

3. What are the steps to make this approximation?

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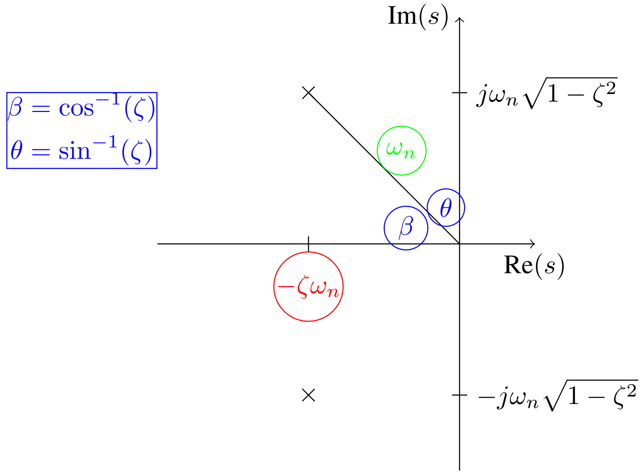
2 Using Design Specifications to Determine Desired Poles

Although being able to predict behavior without needing to solve a differential equation is useful, the real value of the theory in these time response lectures (10-12) is to *use the relationships we have developed to design controllers*.

Recall the pole map for under-damped poles (second-order systems):



4. If you are provided with specifications on rise time, settling time, and percent overshoot, what are the corresponding requirements on ω_n, $\zeta\omega_n$, and ζ, respectively?

5. Sketch the boundaries and shade the appropriate regions corresponding to the specifications you found in question 4 in three complex planes below.