

EENG307: Stability and Routh Hurwitz Criterion¹

Lecture 16

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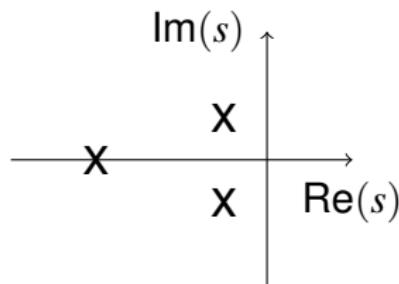
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²Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kuplik, University of Alaska, Anchorage

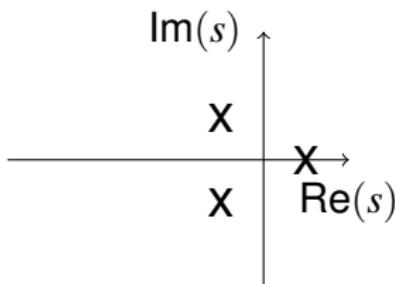


BIBO Stability Examples

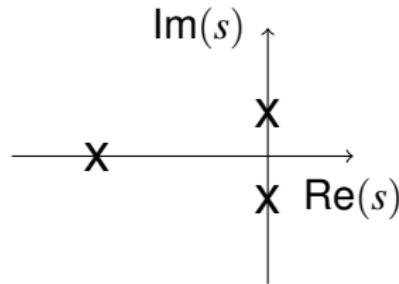
BIBO stable



not BIBO stable



not BIBO stable



Start of Routh Array

$s^n : \begin{matrix} a_0 & a_2 & a_4 & \cdots \end{matrix}$ } polynomial coefficients
 $s^{n-1} : \begin{matrix} a_1 & \cancel{a_3} & a_5 & \cdots \end{matrix}$ } calculated from previous rows
 $s^{n-2} : \begin{matrix} b_1 & b_2 & b_3 & \cdots \end{matrix}$ } calculated from previous rows

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{-1}{a_1} \begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \quad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

Routh Array Continued

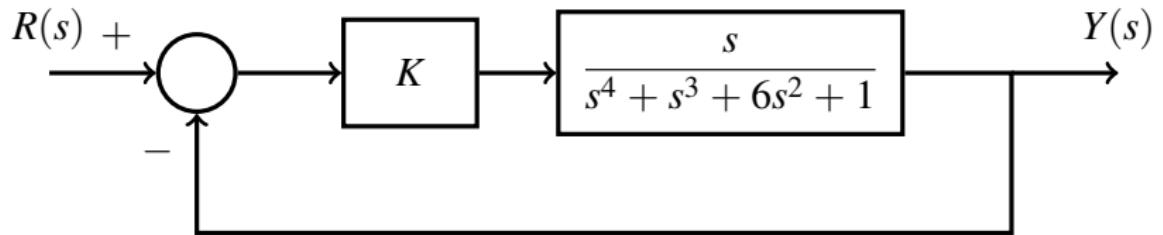
$s^n :$	a_0	a_2	a_4	\cdots
$s^{n-1} :$	a_1	a_3	a_5	\cdots
$s^{n-2} :$	b_1	b_2	b_3	\cdots
$s^{n-3} :$	c_1	c_2	c_3	\cdots
	\vdots	\vdots		
$s^0 :$	z_1			

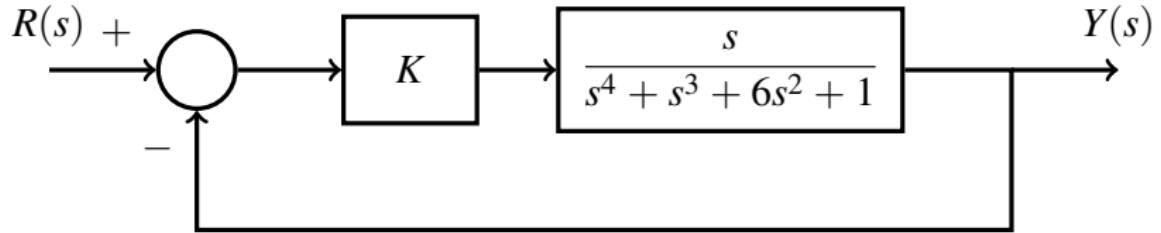
$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \quad \dots$$

Routh-Hurwitz Test:

Last Step

$s^n :$	a_0	a_2	a_4	\cdots
$s^{n-1} :$	a_1	a_3	a_5	\cdots
$s^{n-2} :$	b_1	b_2	b_3	\cdots
$s^{n-3} :$	c_1	c_2	c_3	\cdots
:	:			
$s^0 :$	z_1			





$$\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{K \frac{s}{s^4 + s^3 + 6s^2 + 1}}{1 + K \frac{s}{s^4 + s^3 + 6s^2 + 1}} \\ &= \frac{Ks}{s^4 + s^3 + 6s^2 + Ks + 1}\end{aligned}$$

$$\begin{array}{rcccl}s^4 & : & 1 & 6 & 1 \\ s^3 & : & 1 & K & \\ s^2 & : & 6 - K & 1\end{array}$$

$$\begin{aligned}s^4 &: & 1 && 6 && 1 \\ s^3 &: & 1 && K && \\ s^2 &: & 6 - K && 1 && \\ s^1 &: & K - \frac{1}{6-K} && && \end{aligned}$$

$$\begin{array}{rccccc}s^4 & : & 1 & 6 & 1 \\ s^3 & : & 1 & K & \\ s^2 & : & 6 - K & & 1 \\ s^1 & : & K - \frac{1}{6-K} & & \\ s^0 & : & & 1 & \end{array}$$

$$-K^2 + 6K - 1 > 0$$

