EEENG307: Modeling Mechanical Systems[1]

Lecture 2

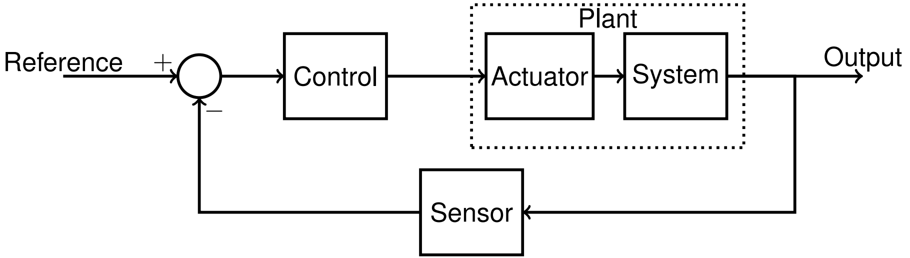
Elenya Grant, Kathryn Johnson, and Hisham Sager[2]

Department of Electrical Engineering
Colorado School of Mines

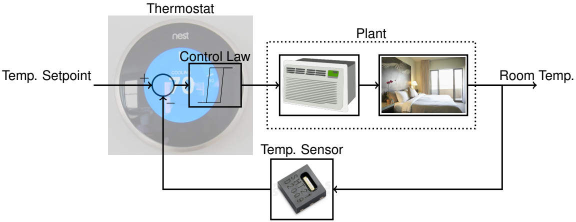
Fall 2022

[2] Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kupilik, University of Alaska, Anchorage

Feedback Control System

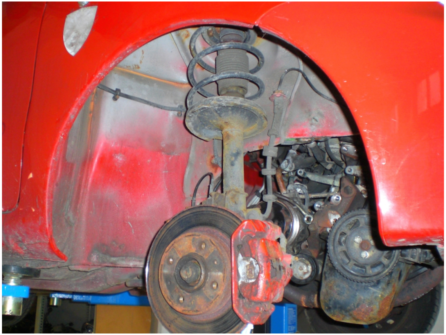


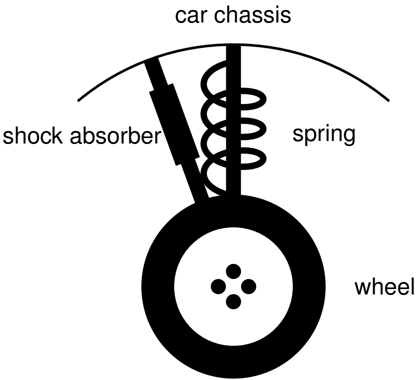
Thermostat Feedback Control System[3]



[3] Images: Amanitamano (Own work) [CC-BY-SA-3.0], Paul Robinson (New Image) [CC-BY-SA-3.0], via Wikimedia Commons

Components of an automobile suspension[4]





[4] Images: By Bukk (Own work) [Public domain], via Wikimedia Commons

Modeling Domains and Idealized Elements

System Class	Idealized Elements
Mechanical Systems, Translational Motion	Mass, Spring, Damper
Mechanical Systems, Rotational Motion	Inertia, Spring, Damper
Electrical Systems	Resistor, Capacitor, Inductor
Fluid Systems	Tank, Valve
Thermal Systems	Thermal Capacitance, Thermal Resistance

Modeling variables for translational motion

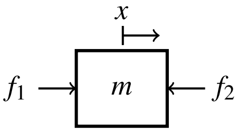
Modeling variables for translational motion

- force, which has units of Newtons [N],
- position, which has units of meters [m].

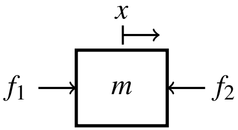
Definition

The laws that describe the relationship between position (or velocity, or acceleration) and force on an ideal element are called the *component model*, or constitutive relationship.

Idealized Mass



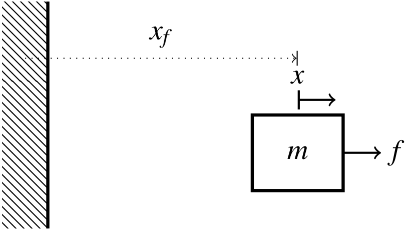
Idealized Mass



$$m\ddot{x} = f_1 - f_2$$

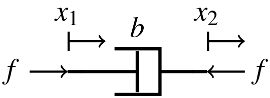
$$\dot{x} \equiv \frac{dx}{dt}, \quad \ddot{x} \equiv \frac{d^2x}{dt^2}, \quad \ldots .$$

Positions are measured with respect to fixed point

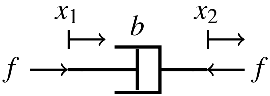


$$\frac{d^2(x_f + x)}{dt^2} = \frac{d^2x}{dt^2}$$

Idealized Damper

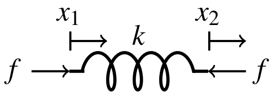


Idealized Damper

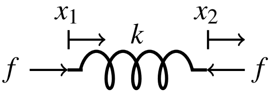


$$f = b(\dot{x}_1 - \dot{x}_2)$$

Idealized Spring

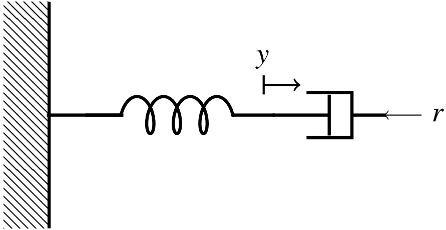


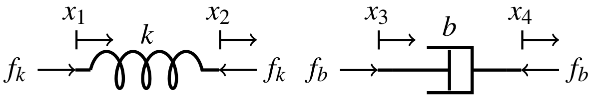
Idealized Spring

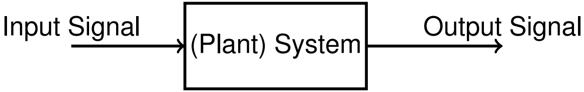


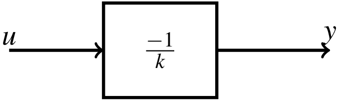
$$f = k(x_1 - x_2)$$

Connection Law Example: Spring-Damper System with Input

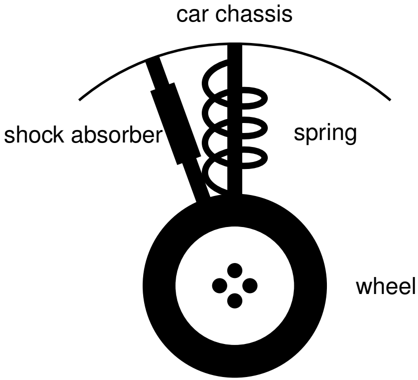


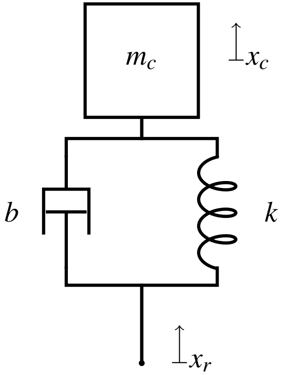

x_1
k
x_2
f_k
f_k
x_3
b
x_4
f_b
f_b


Input Signal
(Plant) System
Output Signal

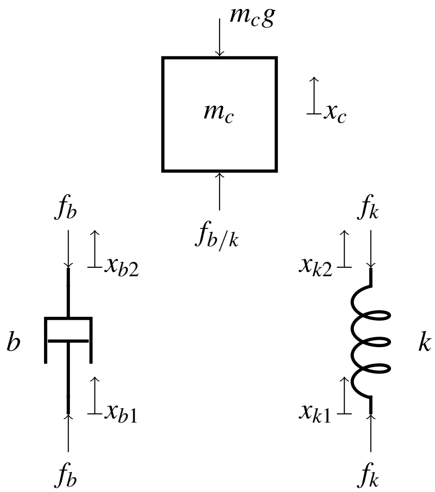

u
$\frac{-1}{k}$
y

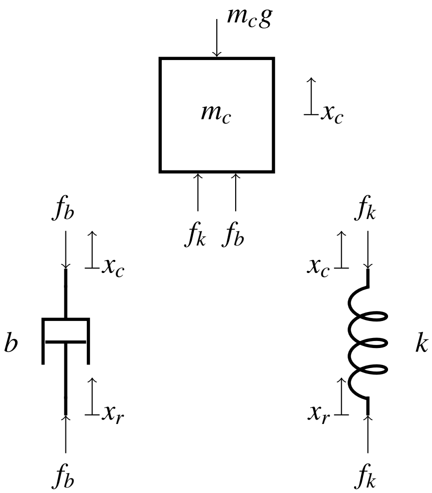
Suspension system



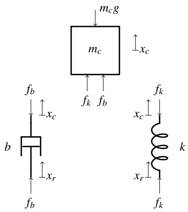


Free Body Diagram for Suspension System





$m_c g$
m_c
x_c
f_k f_b
f_b
x_c
b
x_r
f_b
f_k
x_c
k
x_r
f_k



$$m_c\ddot{x}_c = f_k + f_b - m_c g$$

$$f_b = b(\dot{x}_r - \dot{x}_c)$$
$$f_k = k(x_r - x_c)$$

$$m_c\ddot{x}_c = k(x_r - x_c) + b(\dot{x}_r - \dot{x}_c) - m_c g$$

or

$$m_c\ddot{x}_c + b\dot{x}_c + kx_c = kx_r + b\dot{x}_r - m_c g$$