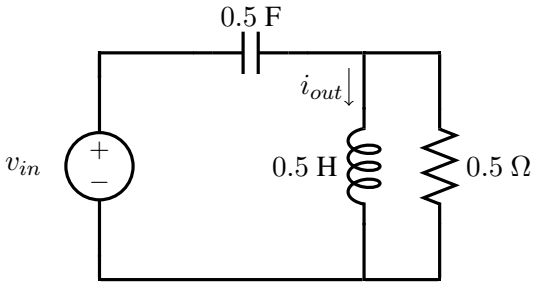
EENG307: Intro to Feedback Control

Fall 2020

Homework Assignment #9

Due: Monday, Nov 9th, 11:59pm

1. The following is a lumped model for an antenna. The input is v_{in} and we are interested in the current through the inductor, i_{out}.



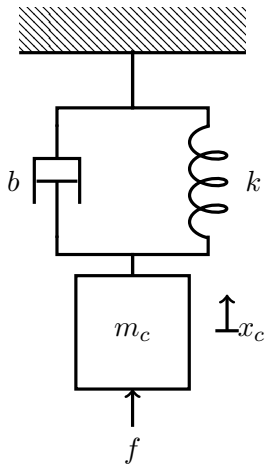
   (a) Find the transfer function $G(s) = \frac{I_{out}(s)}{V_{in}(s)}$.

   (b) If the driving voltage is $v_{in}(t) = \cos(t)$, what is the steady state output current?

   (c) If the driving voltage is $v_{in}(t) = \cos(10t)$, what is the steady state output current?

   (d) When using a sinusoidal input, for what frequency is the steady state output current the largest?

2. You are sizing a motor for a paint shaker, that will mix paint in a can by shaking it. The paint can is placed in a container that is connected by a spring to the paint shaker case. The damper models air resistance. The motor will apply the input force f as a sinusoid.



   (a) Let $m_c = 2$kg, $b = 40$ Ns/m, $k = 1000$ N/m. Find the transfer function $\frac{X_c(s)}{F(s)}$.

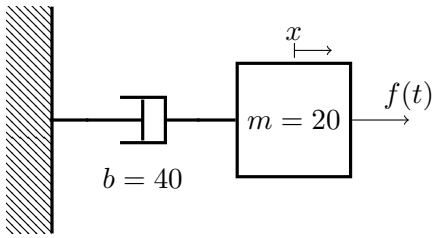
   (b) Suppose the shaking frequency will be 50 Hz, which is 100π rad/s. If the amplitude of the force input is 200 N, what is the amplitude of the steady state response for $x_c(t)$?

3. (a) Use Matlab to create the Bode plot of

$$G(s) = \frac{8(s+1)(s+20)}{s(s^2+5s+16)(s+10)}$$

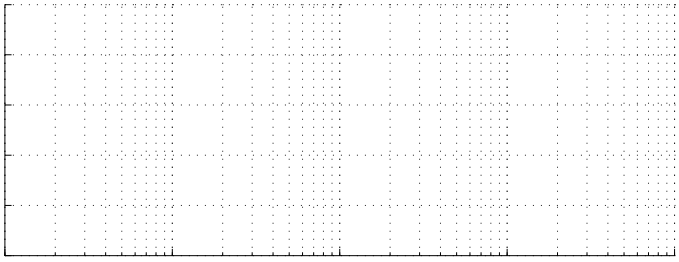
(b) If $Y(s) = G(s)U(s)$ and $u(t) = 3\cos(2t + 20°)$, what is the steady-state output $y_{ss}(t)$?

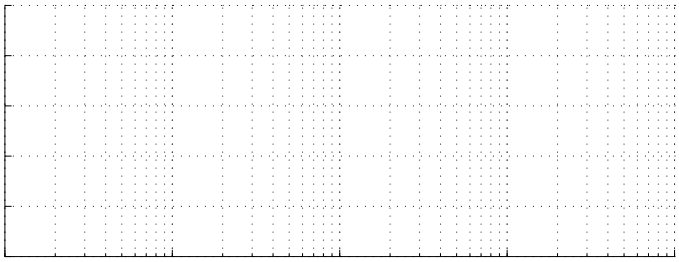
4. The following set of ideal elements models a motorcycle of mass 20 kg with input force f from the motor, and drag 40 Nsm^{-1} due to wind resistance.



(a) Find the transfer function from input force to **velocity** $\frac{V(s)}{F(s)}$, where $v = \dot{x}$.

(b) Sketch the Bode plot for this transfer function.



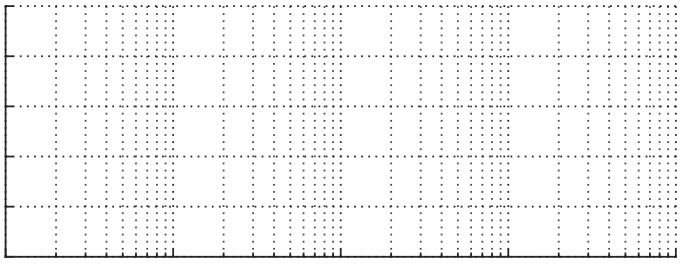


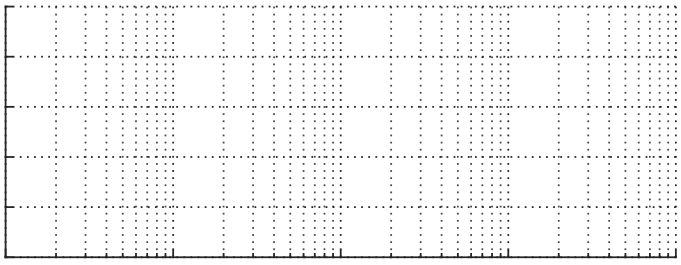
(c) Suppose the input force is $f(t) = \cos(0.5t)$. Using your approximate sketch, find the steady state response for velocity.

5. A second order system has transfer function

$$G(s) = \frac{200}{s^2 + 10s + 100}$$

(a) Using the linear approximation rules, sketch the Bode plot using the provided log scale





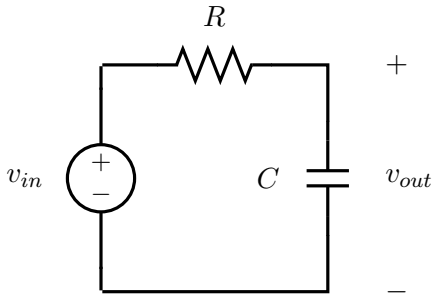
(b) According to your sketch, if the input to the system is $\cos(20t)$ what is the steady state output?

(c) According to your sketch, if the input to the system is $\cos(10t)$ what is the steady state output?

(d) Using the transfer function, if the input to the system is $\cos(10t)$ what is the steady state output?

6. Quiz Question Friday: Find the steady-state output $y_{ss}(t)$ of the transfer function $G(s) = \frac{Y(s)}{R(s)} = \frac{1}{s+1}$ to the input $r(t) = 4\cos(\sqrt{3}t + 30°), t \geq 0$. Note, $\sqrt{3} \approx 1.7$

7. Quiz Question Monday: The following circuit implements a low-pass filter, in that the output v_{out} is reduced when the input is a high frequency sinusoid.



(a) Let $R = 100\ \Omega$ and $C = .01$ F. Find the transfer function $\frac{V_{out}(s)}{V_{in}(s)}$.

(b) Sketch the Bode plot for this system

(c) According to your sketch, at what frequency is the magnitude of the output 10 times smaller than the magnitude of the input?

Solutions:

1. (a) $\frac{I_{out}(s)}{V_{in}(s)} = \frac{2s}{s^2+4s+4}$
   (b) $i_{out}(t) = \frac{2}{5}\cos(t + 37°)$
   (c) $i_{out}(t) = 0.19\cos(10t - 67.4°)$
   (d) $\omega = 2$
2. (a) $\frac{X_c(s)}{F(s)} = \frac{0.5}{s^2+20*s+500}$
   (b) .00102 m
3. (b) $y_{ss}(t) = 3.4\cos(2t - 50°)$
4. (a) $\frac{V(s)}{F(s)} = \frac{1}{20s+40}$
   (b) No partial solution
   (c) $v_{ss}(t) = \frac{1}{40}\cos(0.5t - 18°)$
5. (a) no partial solution
   (b) $\frac{1}{2}\cos(20t - 117°)$
   (c) $2\cos(10t - 90°)$
   (d) $2\cos(10t - 90°)$