EEENG307: Rotational and Fluid Systems[1]

Lecture 14

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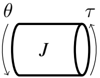
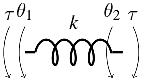
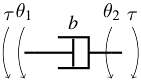
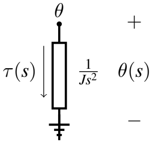
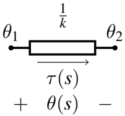
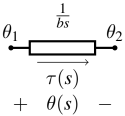
[2] Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kupilik, University of Alaska, Anchorage

Across and Through Variables

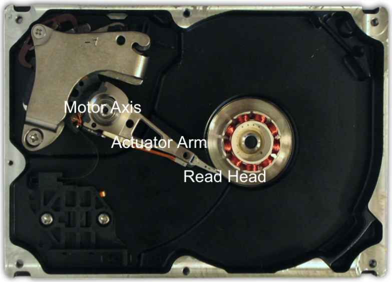
Domain	Across Variable	Through Variable
Electrical	Voltage	Current
Translational Mechanical	Position	Force
Fluid	Pressure	Flow
Rotational Mechanical	Angular Position	Torque

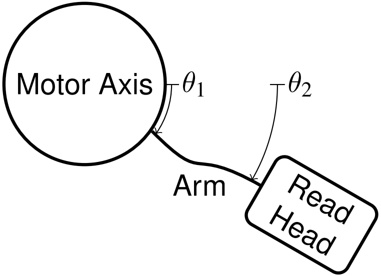
Rotational Impedance

	mass	spring	damper
Component		 $\theta = \theta_1 - \theta_2$	 $\dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2$
Laplace Transform	$\theta(s) = \frac{1}{Js^2}\tau(s)$	$\theta(s) = \frac{1}{k}\tau(s)$	$\theta(s) = \frac{1}{bs}\tau(s)$
Impedance Component (force direction agrees with positive direction)			

Hard Disk Drive Read Head

In order to move the read head to the correct track and hold it there, we need to be able to predict the relationship between the motor torque τ and the angular position of the read head θ_2. First, find the equivalent impedance model. Then, find the transfer function $\frac{\theta_2}{\tau}$.





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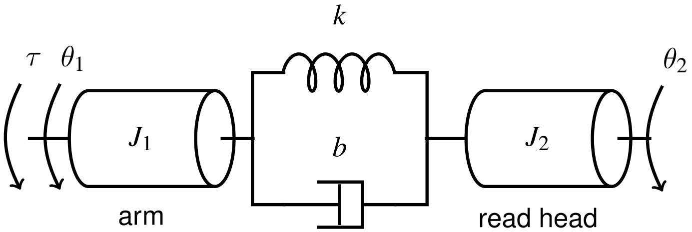
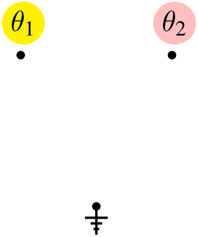
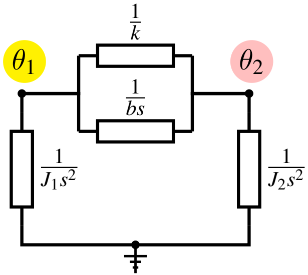
Hard Disk Drive Ideal Elements



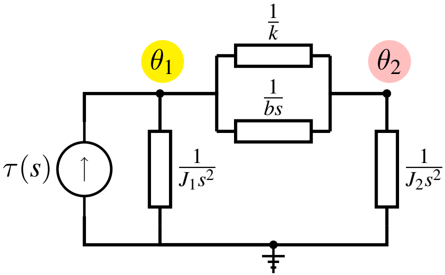
Nodes



Disk Drive Impedance Network

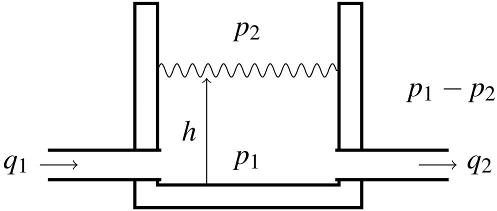


Disk Drive Complete Circuit



Tank

The change in the volume V of fluid inside the tank is equal to the difference between the input and output volumetric flow rates, q_1 and q_2, respectively, in this diagram:

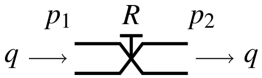


$$p_1 - p_2 = \rho g h$$

Tank Area: A

Linear Valve

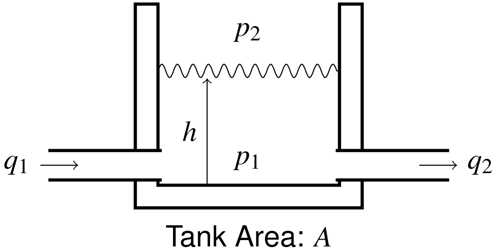
A valve causes a restriction that causes the pressure on one end of the value to be higher than the other end. When this pressure drop is proportional to the flow, the valve is linear with valve constant R. (Most valves are nonlinear, however.) By unit analysis, the units of valve resistance are [N s m^{-5}] or equivalently [kg m^{-4}s^{-1}].



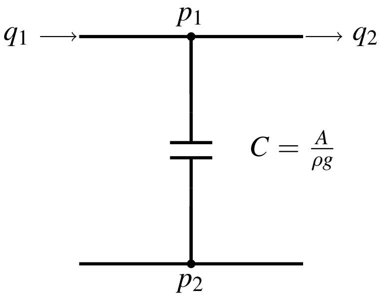
$$p_1 - p_2 = Rq$$

Electrical Analogy for Fluid Elements:

Tank



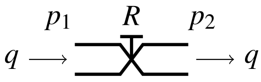
$$\frac{A}{\rho g}\frac{d(p_1-p_2)}{dt} = q_1 - q_2$$



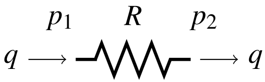
$$C\frac{d(p_1-p_2)}{dt} = q_1 - q_2$$

Electrical Analogy for Fluid Elements:

Valve

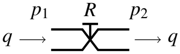
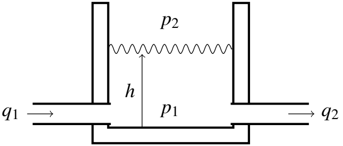
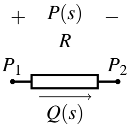
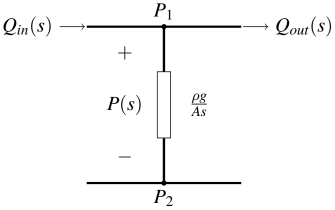


$$p_1 - p_2 = Rq$$



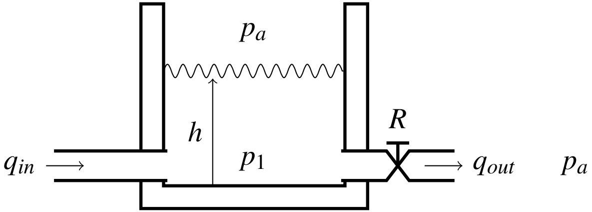
$$p_1 - p_2 = Rq$$

Fluid Impedances

	valve	tank
Component	$p = p_1 - p_2$ 	 Tank Area: A
Component law	$p = Rq$	$\frac{A}{\rho g}\frac{dp}{dt} = q_{in} - q_{out}$
Laplace Transform	$P(s) = RQ(s)$	$\frac{A}{\rho g}sP(s) = Q_{in}(s) - Q_{out}(s)$
Impedance Component		

Tank and Valve Problem

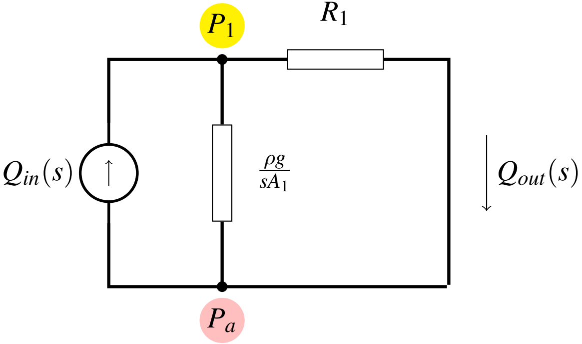
Example: Find the equivalent impedance model of the tank and valve system below with input flow q_{in} and output flow q_{out}.



Tank Area: A

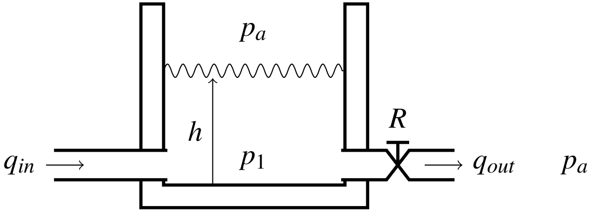
Tank system nodes

Tank system impedance model



Tank and Valve Example

Assume the valve is linear with valve constant R and that the density of the fluid is ρ. The valve empties to atmospheric pressure, p_a, which is the same as the pressure at the top of the tank.



Tank Area: A