EEENG307: Understanding Bode Plots Using Matlab[1]

Lecture 27

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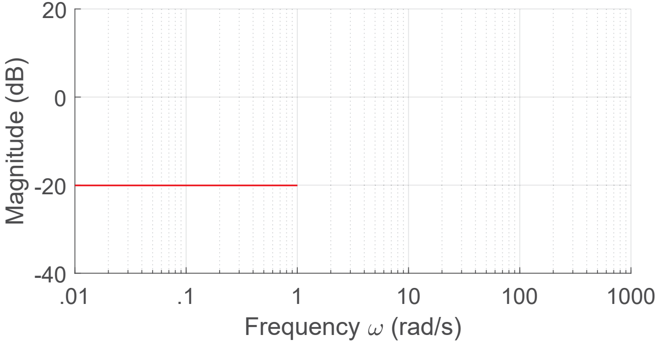
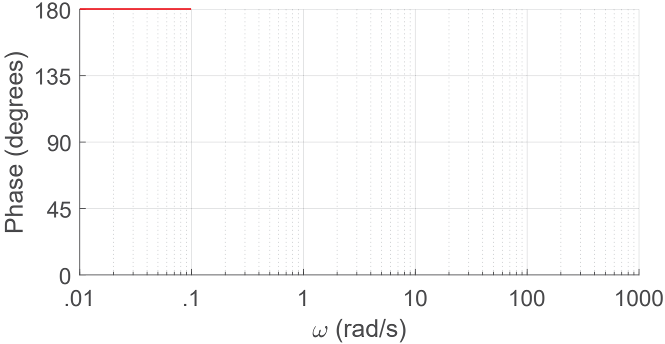
Questions for Example Bode Plot

1. What is the magnitude in dB of the DC gain of $G(s)$? In other words, what is $20\log_{10}(K)$?
2. What is the low frequency phase of $G(s)$, i.e., $\angle K$?
3. Will the magnitude at high frequencies be smaller or larger than the magnitude at low frequencies (near DC)?
4. What will be the total phase change of the phase Bode plot given the RHP zero and LHP pole?
5. What are the asymptotic phase slopes at medium frequencies (near the zero σ_z and the pole σ_p)?

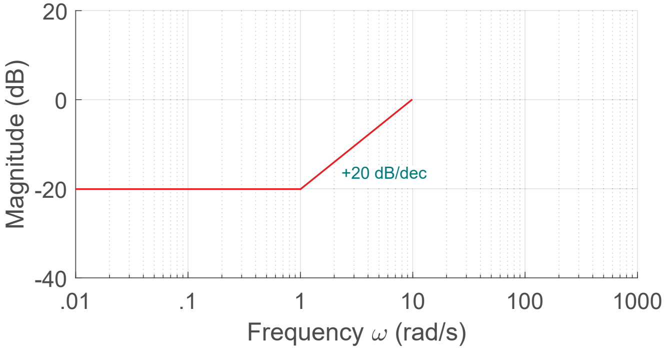
Magnitude at Low Frequencies



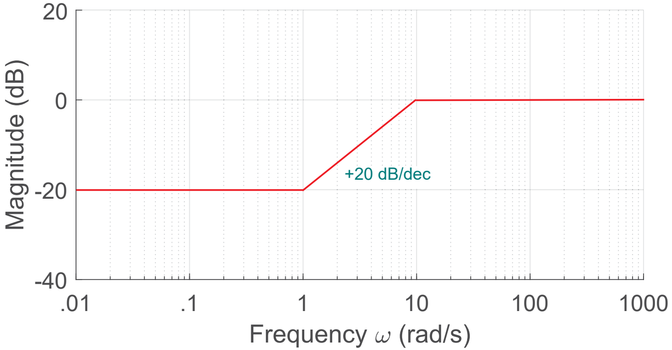
Phase at Low Frequencies



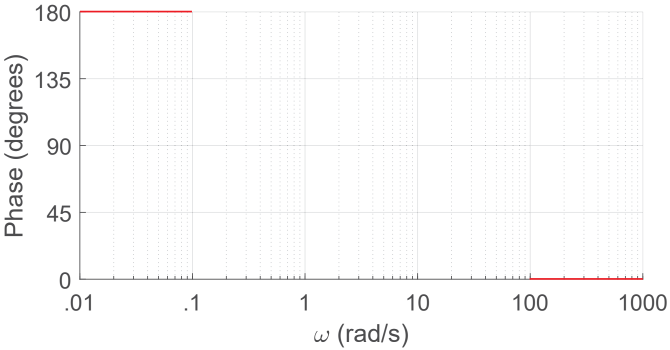
Magnitude at Medium Frequencies



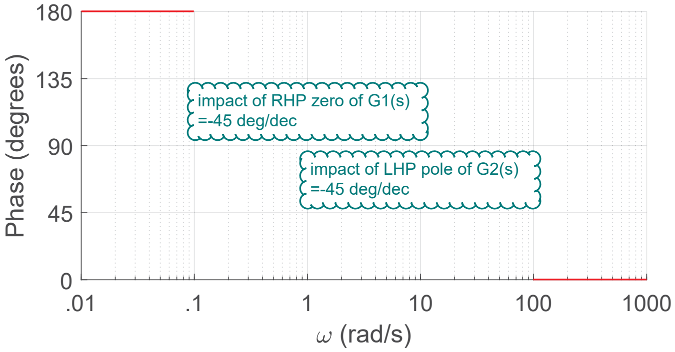
Magnitude at High Frequencies



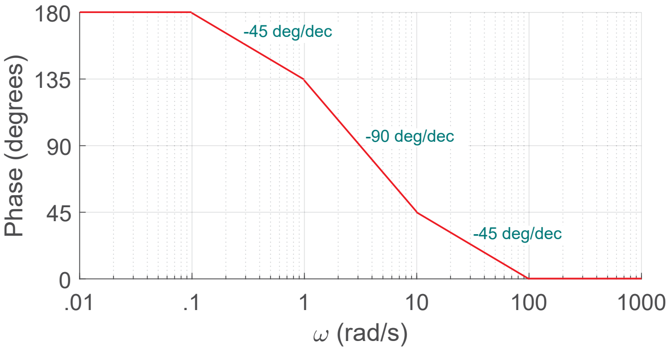
Phase at High Frequencies



Phase Ranges for Medium Frequencies (Near Zero and Pole)



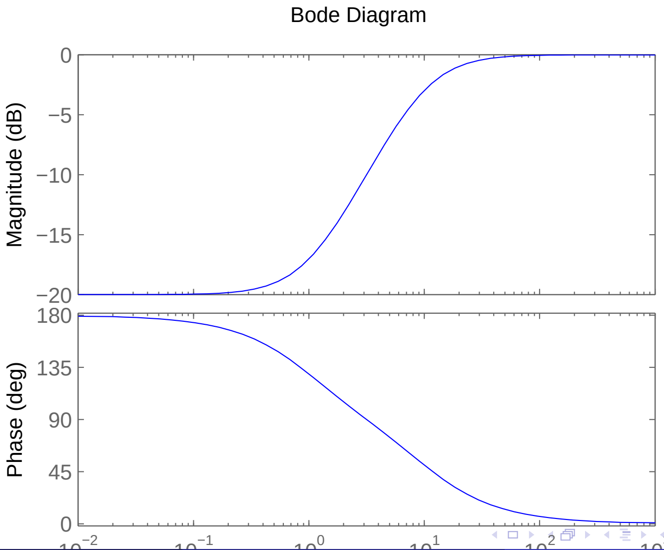
Phase Slopes Near Zero and Pole



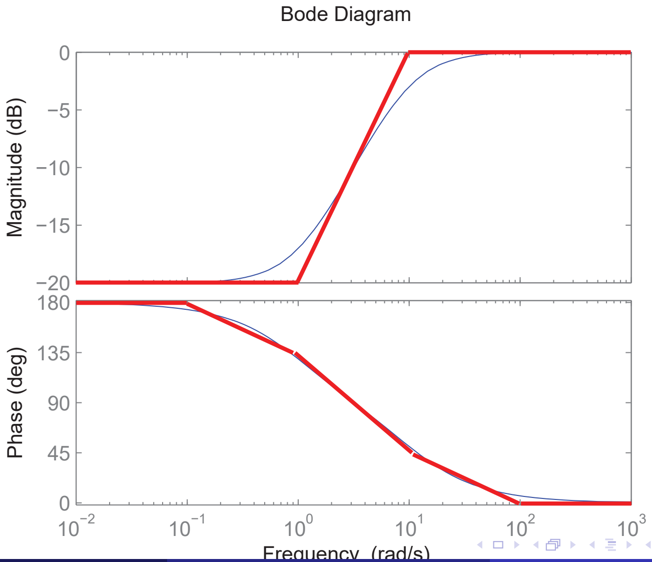
MATLAB Commands to Create Bode Plot

```
» sys = tf([1 -1],[1 10]) % define system using tf
function

sys =
s - 1
----
s + 10

» bode(sys)
```

MATLAB-Generated Bode Plot



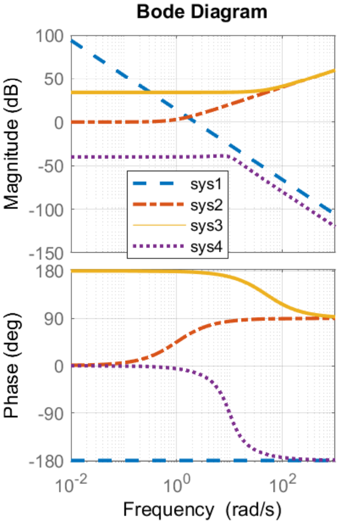
Bode Plot with Asymptotes

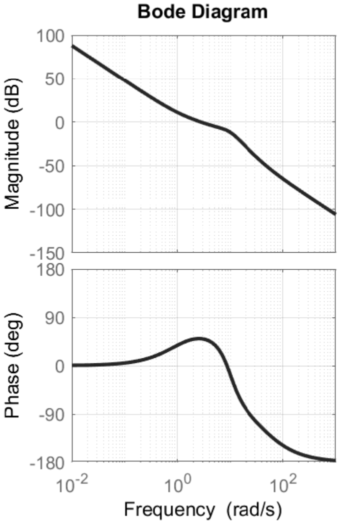


MATLAB Code for Second Example

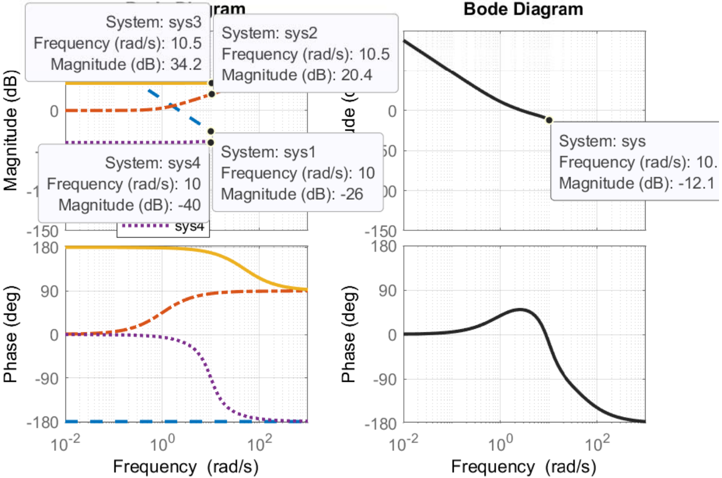
```
» s = tf([1 0],[1]); % define s as the Laplace
variable
» sys1 = 5/s^2;
» sys2 = s+1;
» sys3 = s-50;
» sys4 = 1/(s^2+10*s+100);
» sys=sys1*sys2*sys3*sys4;
» subplot(1,2,1)
» bode(sys1,sys2,sys3,sys4)
» legend('sys1','sys2','sys3','sys4')
» subplot(1,2,2)
» bode(sys)
```

Matlab Results





Matlab Results with Markers at 10 rad/s



$$G(s) = \frac{5(s+1)(s-50)}{s^2(s^2+10s+100)}$$

Step 1: Factor out constant terms and any poles or zeros at $s = 0$ to obtain the transfer function in "Bode form"

$$G(s) = \frac{5(-50)}{100s^2} \frac{(s+1)(\frac{s}{-50}+1)}{\left(\left(\frac{s}{10}\right)^2 + \frac{s}{10} + 1\right)}$$
$$= \frac{-2.5}{s^2} \frac{(s+1)(\frac{s}{-50}+1)}{\left(\left(\frac{s}{10}\right)^2 + \frac{s}{10} + 1\right)}$$

the term $\frac{-2.5}{s^2}$ is called the *low frequency term*

Step 2: List break frequencies and important info

Break Frequency	Item (P/Z? L/RHP? #?)	Magnitude Slope	Phase Slope
1 rad/s	1 LHP zero	20 dB/dec	45°/dec
10 rad/s	2 LHP poles	-40 dB/dec	-90°/dec
50 rad/s	1 RHP zero	20 dB/dec	-45°/dec

Step 3: Calculate gain and phase of low frequency term $(2.5/s^2)$. To calculate the magnitude, we pick a frequency less than or equal to all break frequencies. In this case 1 rad/s is convenient, so we plug in $s = j\omega$ with $\omega = 1$:

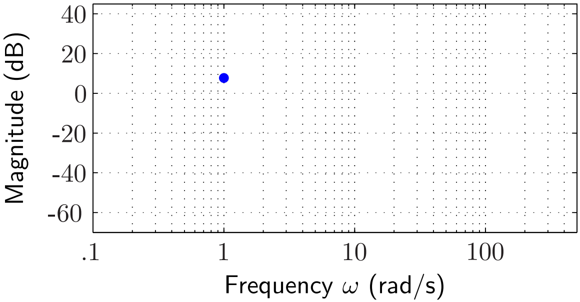
$$\left|-\frac{2.5}{s^2}\right|_{s=j} = \frac{|-2.5|}{|j^2|} = \frac{2.5}{1} = 2.5$$

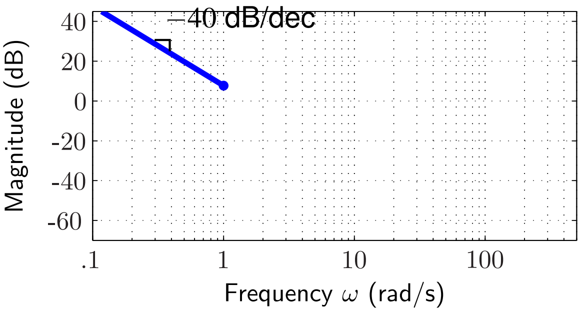
Thus, at 1 rad/s, the magnitude is $20\log_{10}(2.5) = 7.96$. To calculate low frequency phase, you can always just plug in j

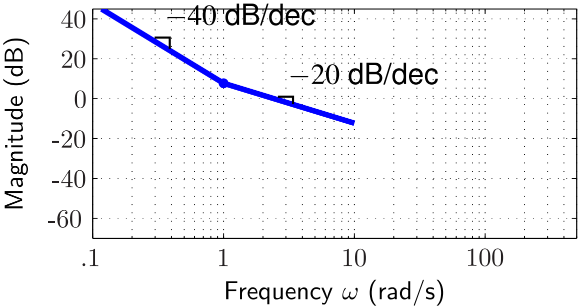
$$\angle -\frac{2.5}{s^2}\bigg._{s=j} = \angle\frac{-2.5}{j^2} = \angle\frac{-2.5}{-1} = \angle 2.5 = 0^\circ$$

In this case, the low frequency phase is 0°.

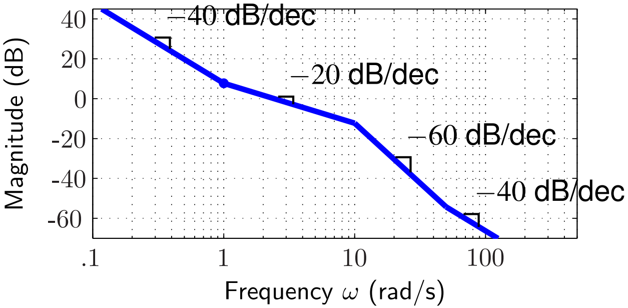
Step 4: Draw magnitude plot, starting from lowest frequencies.



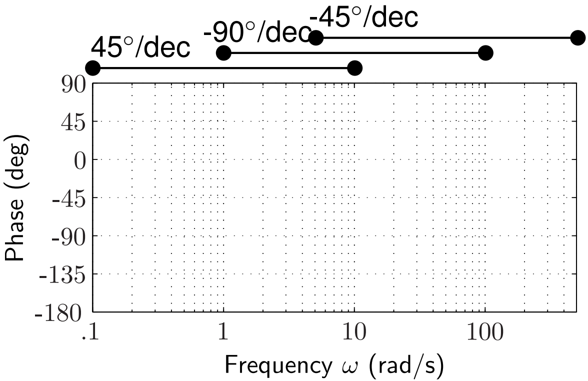

−40 dB/dec
40
20
0
-20
-40
-60
Magnitude (dB)
.1
1
10
100
Frequency ω (rad/s)

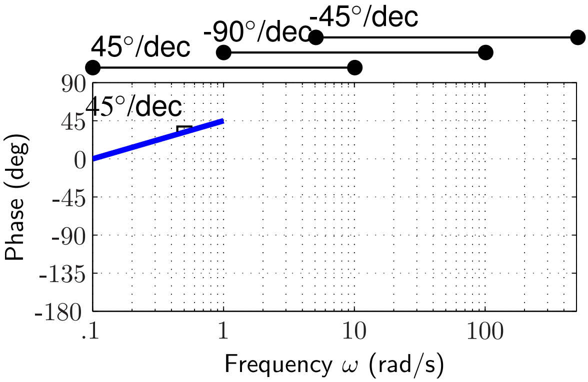


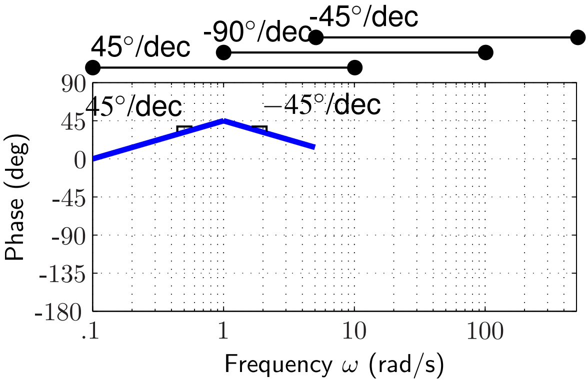
-40 dB/dec
-20 dB/dec
40
20
0
-20
-40
-60
Magnitude (dB)
.1
1
10
100
Frequency ω (rad/s)

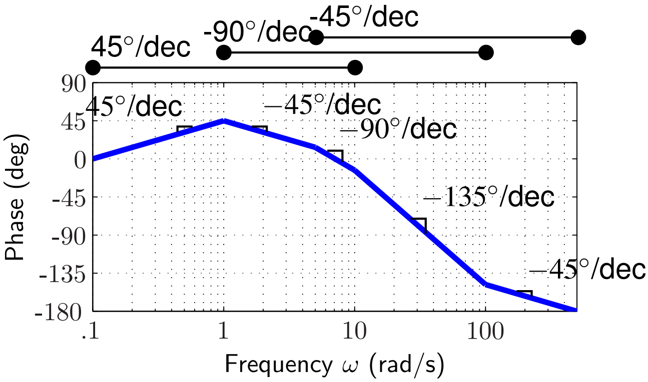

−40 dB/dec
−20 dB/dec
−60 dB/dec
−40 dB/dec
Magnitude (dB)
40
20
0
-20
-40
-60
.1
1
10
100
Frequency ω (rad/s)

Step 5: Draw phase plot




45°/dec
-90°/dec
-45°/dec
45°/dec
90
45
0
-45
-90
-135
-180
.1
1
10
100
Phase (deg)
Frequency ω (rad/s)


-45°/dec
-90°/dec
45°/dec
90
45
0
-45
-90
-135
-180
45°/dec
−45°/dec
Phase (deg)
.1
1
10
100
Frequency ω (rad/s)


45°/dec
-90°/dec
-45°/dec
90
45
0
-45
-90
-135
-180
45°/dec
−45°/dec
−90°/dec
−135°/dec
−45°/dec
.1
1
10
100
Phase (deg)
Frequency ω (rad/s)