

Bode Plots for Higher Order Systems

Build Bode plot for

$$G(s) = \frac{5(s+1)(s-50)}{s^2(s^2+10s+100)}$$

Point by point TechniqueFactor constants, integrators, 1st and 2nd order systems:

$$G(s) = \left(\frac{-5}{2}\right) \left(\frac{1}{s^2}\right) \left(\frac{s+1}{1}\right) \left(\frac{s-50}{-50}\right) \left(\frac{10^2}{s^2+10s+10^2}\right)$$

Complete the following table for G(s). For terms whose magnitude and phase slopes change, list the non-zero slope in the Magnitude and Phase columns. For terms whose slope does not change, list the constant value in the Magnitude and Phase columns. Include units for your answers in the Magnitude and Phase columns.

Term	Break Freq	Type	Magnitude	Phase
-5/2	N/A	Constant	8 dB	+/- 180°
1/s^2	"0" rad/sec	Integrator	-40 dB/dec	+/- 180°
s+1/1	1 rad/sec	1 LHP Z	20 dB/dec	45°/dec
10^2/(s^2+10s+10^2)	10 rad/sec	2 LHP P	-40 dB/dec	90°/dec
s-50/-50	50 rad/sec	1 RHP Z	20 dB/dec	-45°/dec

Determine points on Magnitude portion of Bode Plot

- 1) Find the lowest non-zero break frequency: 1 rad/sec

Determine the magnitude at this frequency:

Constant:

$$20 \log \left(\left| \frac{5}{2} \right| \right) = 8 \text{dB}$$

Integrator:

$$20 \log \left(\left| \frac{1}{(j\omega)^2} \right| \right) |_{\omega=1 \text{ rad/sec}} = 0$$

Determine the slope to left of this frequency:

-40dB/decade

- 2) Find the next larger break frequency: 10 rad/sec

Determine the slope to this frequency: -40dB/decade from 1/s^2

+20dB/decade from s+1/1 total: -20dB/decade

Determine magnitude at this frequency:

#decades: log(10 rad/sec/1 rad sec) = 1 decade

#dB: -20dB/decade * 1 decade = -20dB

Mag: 8dB – 20dB = -12dB

- 3) Find the next larger break frequency: 50 rad/sec

Determine the slope to this frequency: -40dB/decade from 1/s^2

+20dB/decade from s+1/1

-40dB/decade from second order total: -60dB/decade

Determine magnitude at this frequency:

#decades: log(50 rad/sec/10 rad sec) = 0.7 decade

#dB: -60dB/decade * 0.7 decade = -42dB

Mag: -12dB – 42dB = -54dB

- 4) Determine slope to right last break frequency: -40dB/decade from 1/s^2

+20dB/decade from s+1/1

-40dB/decade from second order

+20dB/decade from s-50/-50 total: -40dB/decade

Determine points on Phase portion of Bode Plot

- 1) Find decade below lowest break frequency: 0.1 rad/sec
 Determine the phase at this frequency:
 Constant: $\pm 180^\circ$
 Integrator: $-90^\circ * n = \pm 180^\circ$ total: 0°
- 2) Find the decade below next break frequency: 1 rad/sec
 Determine the slope to this frequency: $+45^\circ/\text{decade}$ from $s+1/1$ total: $+45^\circ/\text{decade}$
 Determine phase at this frequency:
 #decades: $\log(1 \text{ rad/sec} / 0.1 \text{ rad sec}) = 1 \text{ decade}$
 #degrees: $+45^\circ/\text{decade} * 1 \text{ decade} = -45^\circ$
 degrees: $0^\circ + 45^\circ = 45^\circ$
- 3) Find the decade below next break frequency: 5 rad/sec
 Determine the slope to this frequency: $+45^\circ/\text{decade}$ from $s+1/1$
 $-90^\circ/\text{decade}$ from second order total: $-45^\circ/\text{decade}$
 Determine phase at this frequency:
 #decades: $\log(5 \text{ rad/sec} / 1 \text{ rad sec}) = 0.7 \text{ decade}$
 #degrees: $-45^\circ/\text{decade} * 0.7 \text{ decade} = -32^\circ$
 degrees: $45^\circ + -32^\circ = 14^\circ$
- 4) Find the decade below next break frequency: 10 rad/sec
 Determine the slope to this frequency:
 $+45^\circ/\text{decade}$ from $s+1/1$
 $-90^\circ/\text{decade}$ from second order
 $-45^\circ/\text{decade}$ from $s-50/50$ total: $-90^\circ/\text{decade}$
 Determine phase at this frequency:
 #decades: $\log(10 \text{ rad/sec} / 5 \text{ rad sec}) = 0.3 \text{ decade}$
 #degrees: $-90^\circ/\text{decade} * 0.3 \text{ decade} = -27^\circ$
 degrees: $14^\circ + -27^\circ = -13^\circ$
- 5) Find the decade below next break frequency: 100 rad/sec
 Determine the slope to this frequency:
 $-90^\circ/\text{decade}$ from second order
 $-45^\circ/\text{decade}$ from $s-50/50$ total: $-135^\circ/\text{decade}$
 Determine phase at this frequency:
 #decades: $\log(100 \text{ rad/sec} / 10 \text{ rad sec}) = 1 \text{ decade}$
 #degrees: $-135^\circ/\text{decade} * 1 \text{ decade} = -135^\circ$
 degrees: $-13^\circ + -135^\circ = -148^\circ$
- 6) Find the decade below next break frequency: 500 rad/sec
 Determine the slope to this frequency: $-45^\circ/\text{decade}$ from $s-50/50$ total: $-45^\circ/\text{decade}$
 Determine phase at this frequency:
 #decades: $\log(500 \text{ rad/sec} / 100 \text{ rad sec}) = 0.7 \text{ decade}$
 #degrees: $-45^\circ/\text{decade} * 0.7 \text{ decade} = -32^\circ$
 degrees: $-148^\circ + -32^\circ = -180^\circ$

