EENG307: Time Response of Higher Order Systems[1]

Lecture 12

Elenya Grant, Kathryn Johnson, and Hisham Sager[2]

Department of Electrical Engineering
Colorado School of Mines

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[2] Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kupilik, University of Alaska, Anchorage

Step Response Specifications

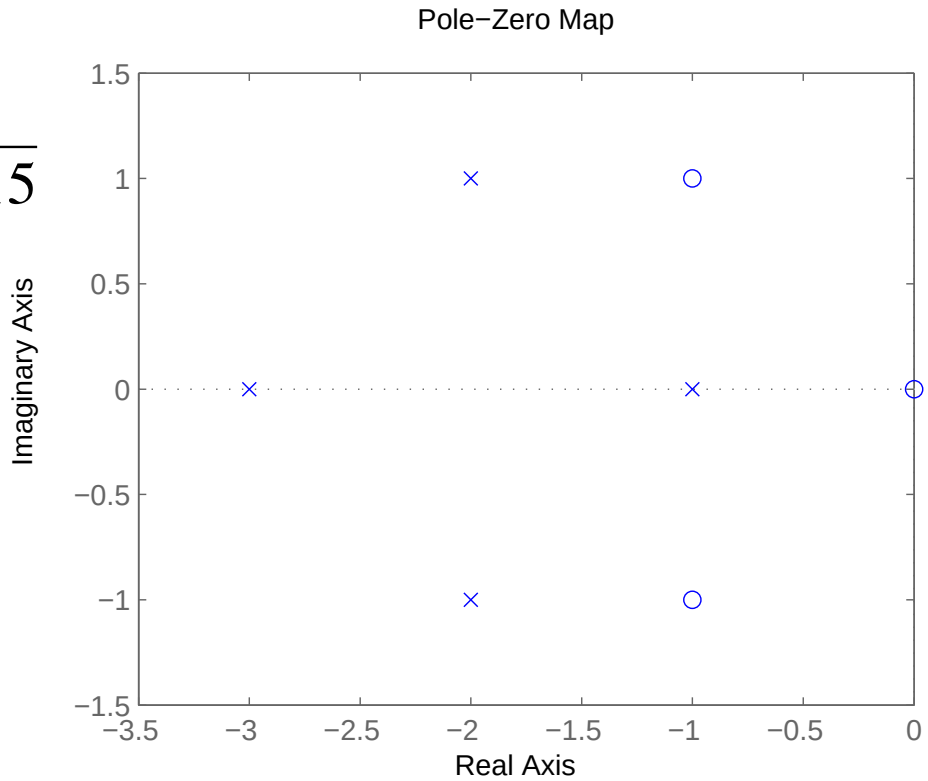
System	System Parameters	Rise Time	Settling Time	% Overshoot
$\frac{K\sigma}{s+\sigma}$	σ - pole magnitude	$t_r = \frac{2.2}{\sigma}$	$t_s = \frac{4.6}{\sigma}$	0
$\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ζ - damping ratio ω_n - natural frequency $\sigma = \zeta\omega_n$ - real part of poles	$t_r = \frac{2.2}{\omega_n}$	$t_s = \frac{4.6}{\zeta\omega_n}$	$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$

Pole-Zero diagram

$$F(s) = \frac{s^3 + 2s^2 + 2s}{s^4 + 8s^3 + 24s^2 + 32s + 15}$$

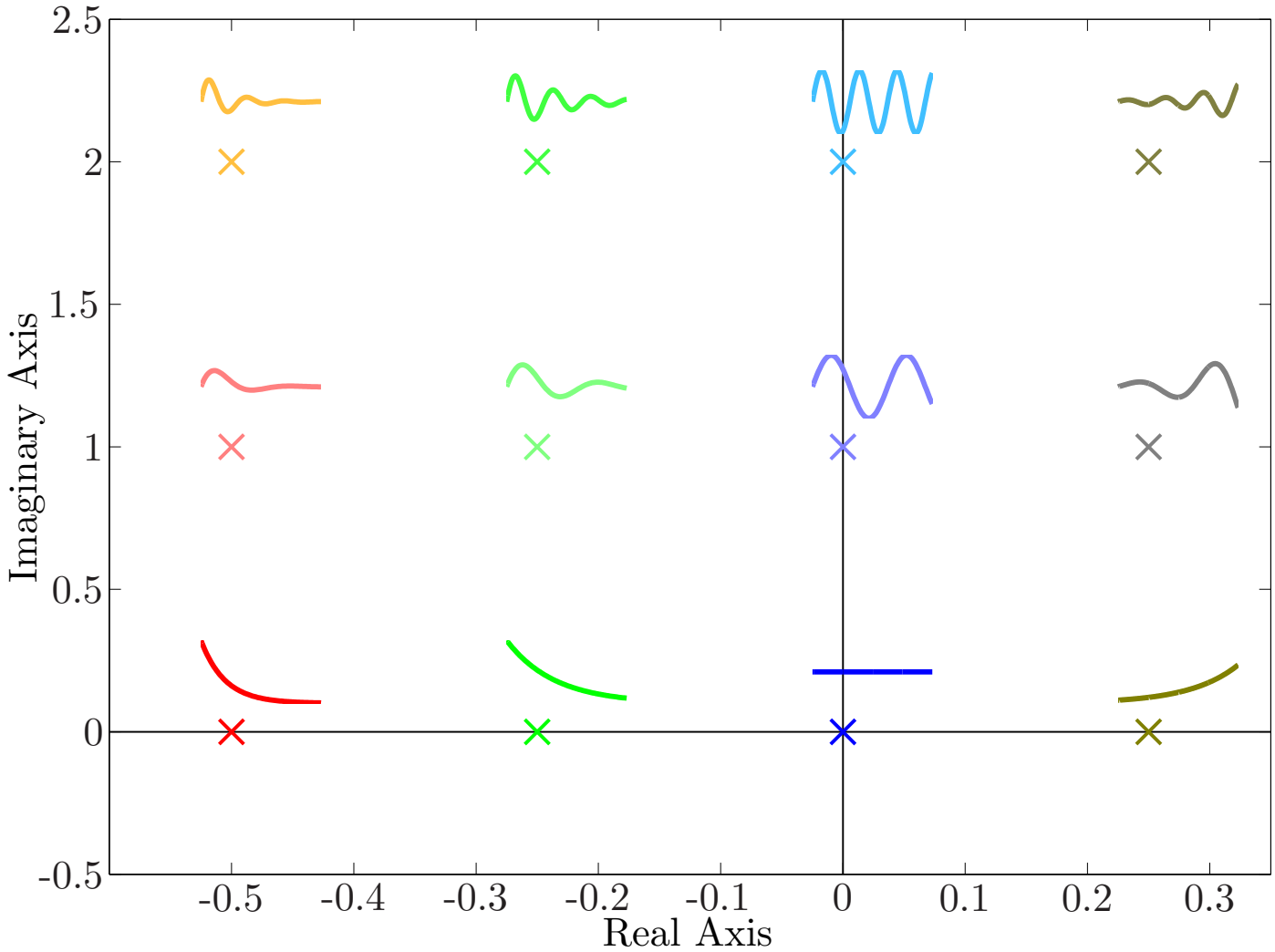
zeros: $-1 + j, -1 - j, 0$

poles: $-3, -1, -2 + j, -2 - j$



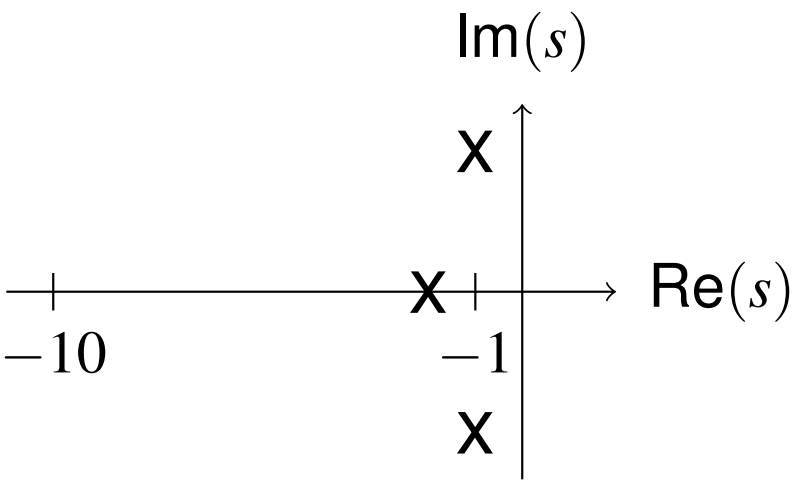
$e^{\sigma t}u(t)$	$\frac{1}{s+\sigma}$
$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

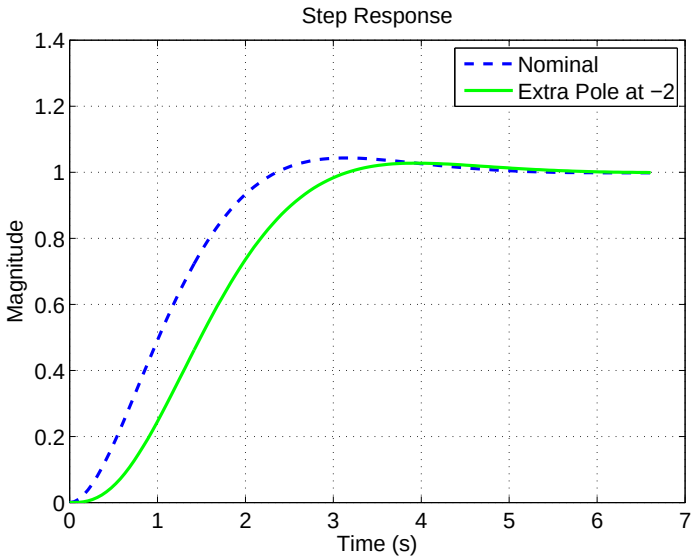
Pole map with responses

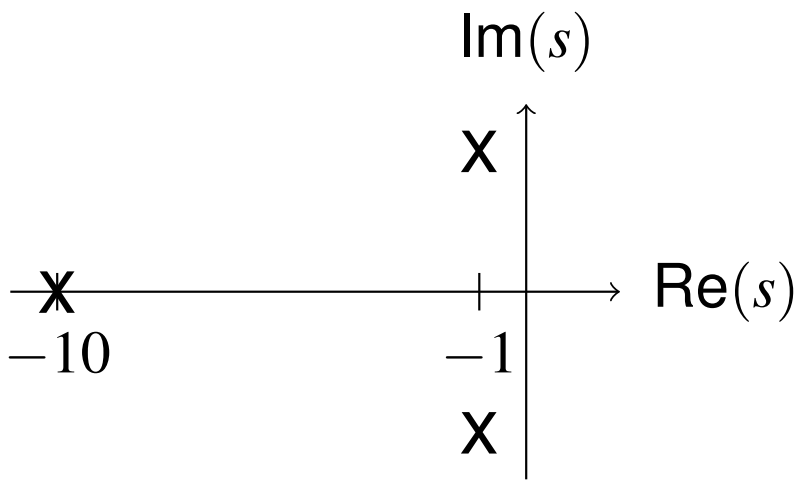


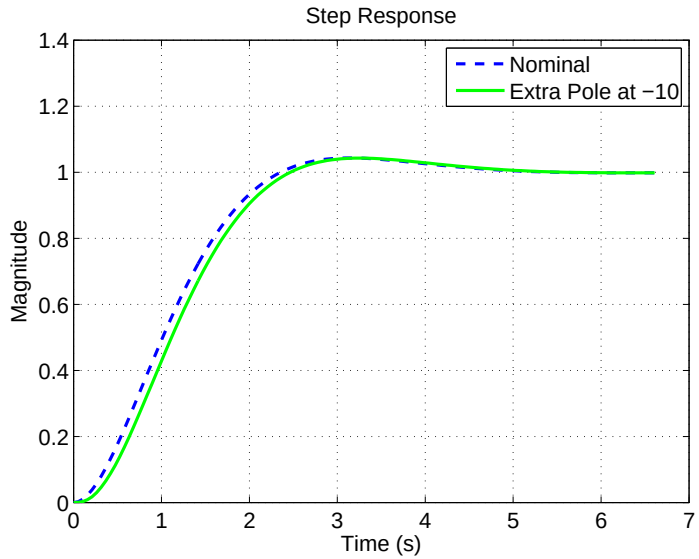
(Conjugate poles not shown)

Effect of extra pole on step response

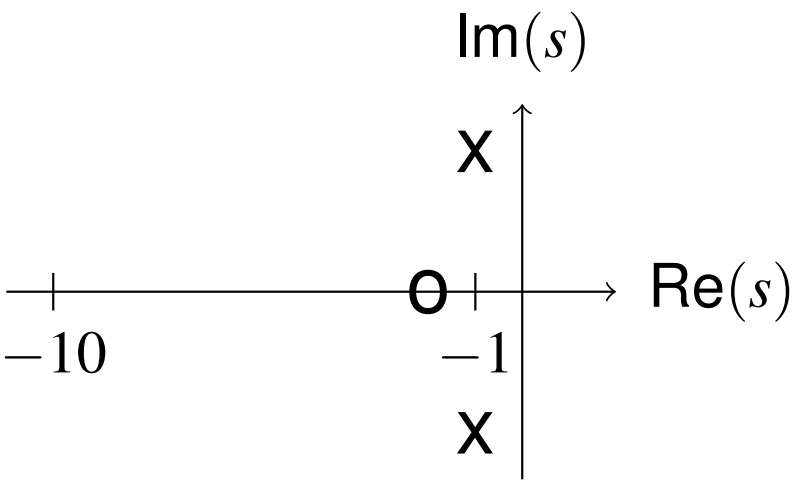


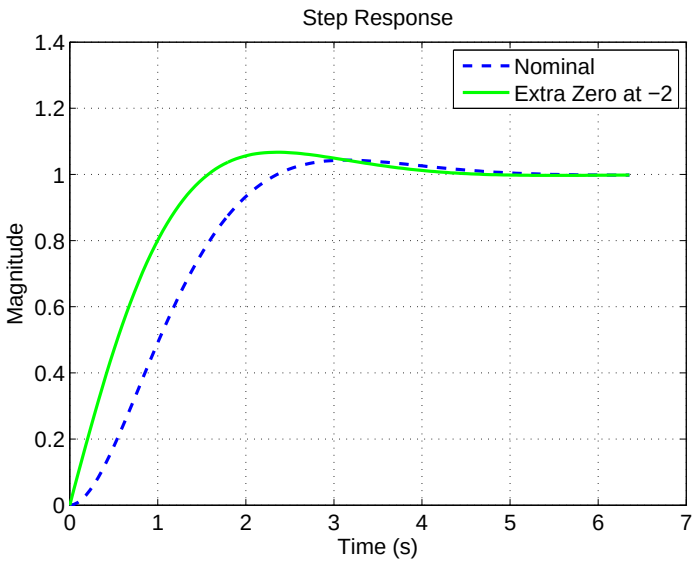


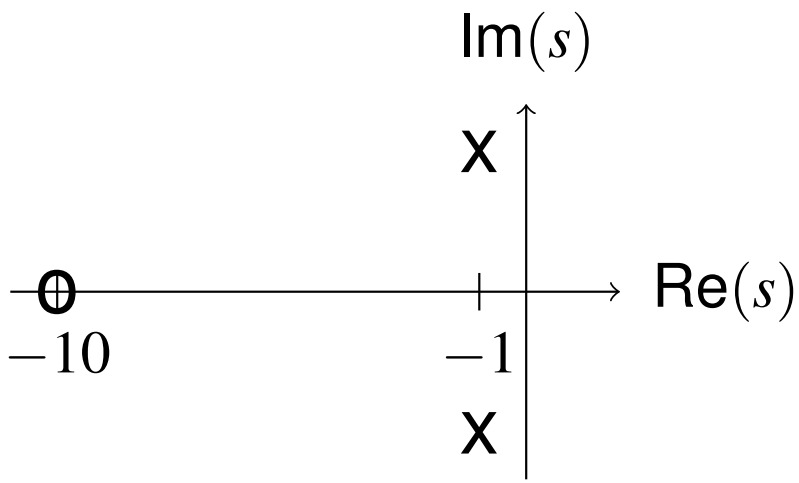


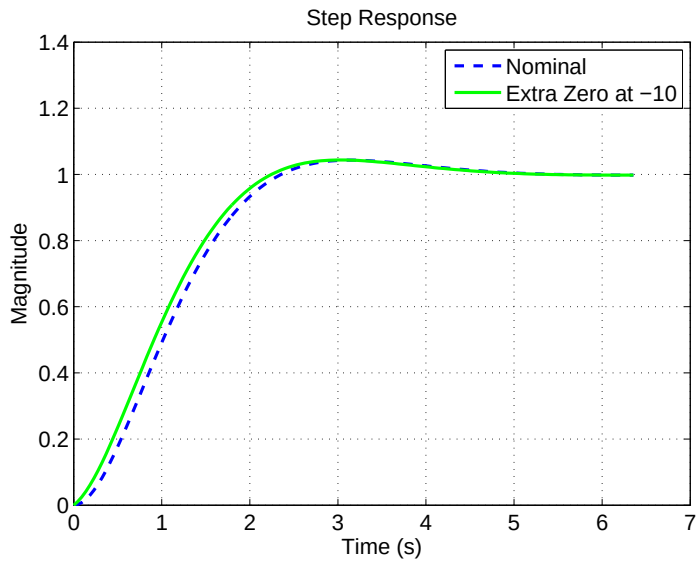


Effect of extra zero on step response



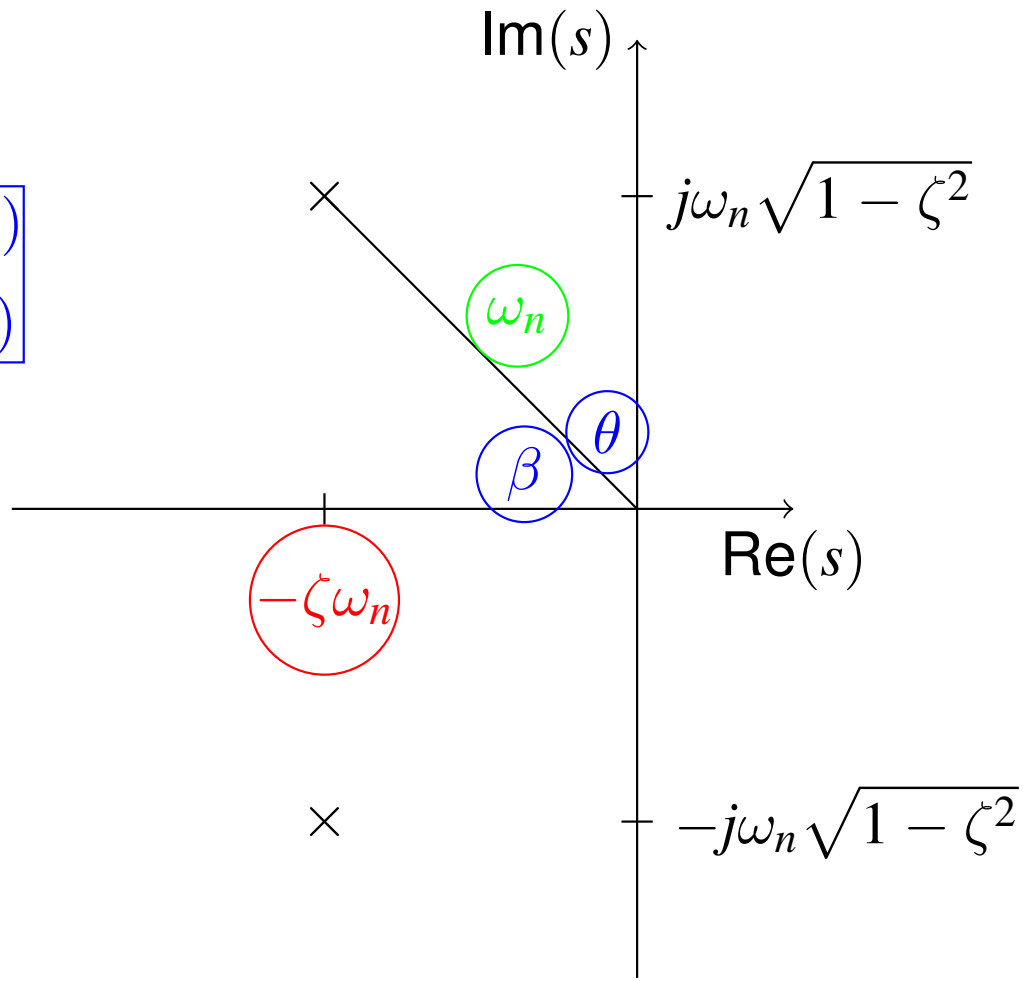




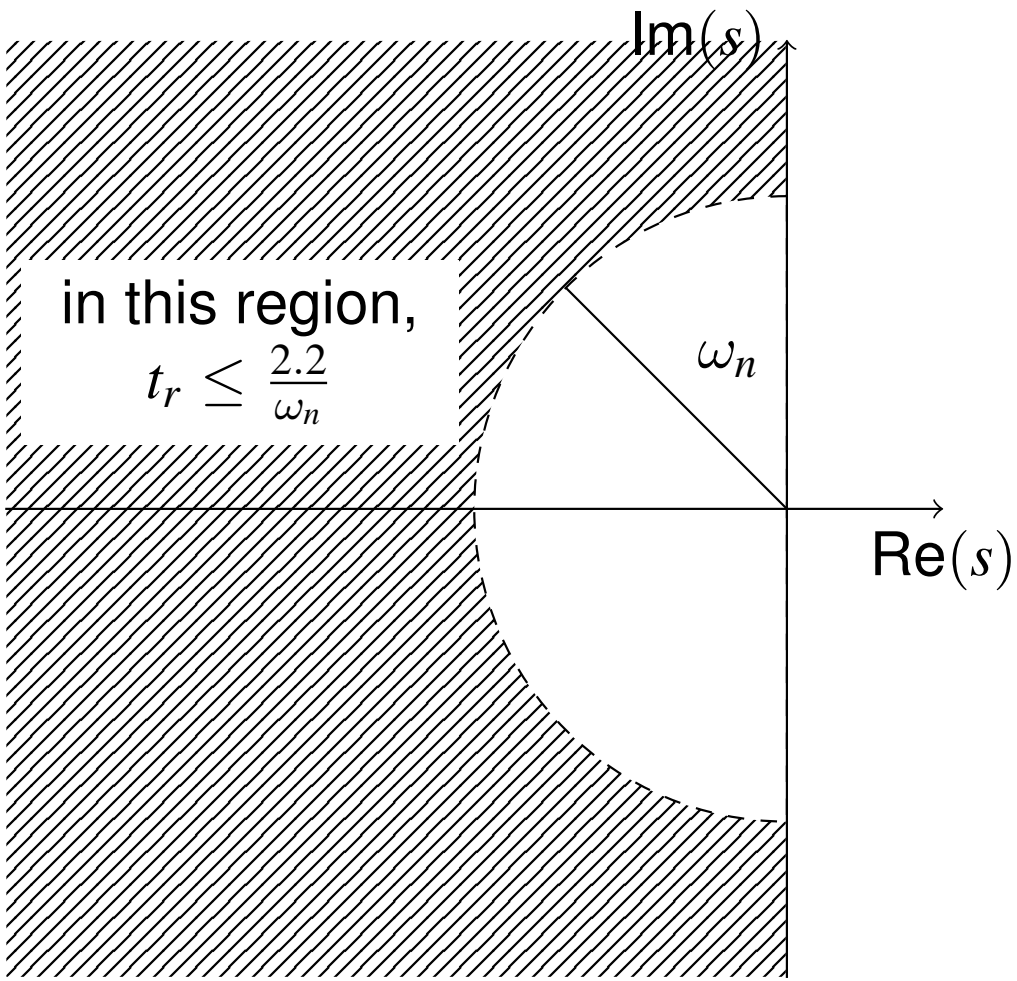


Under-damped poles in the complex plane

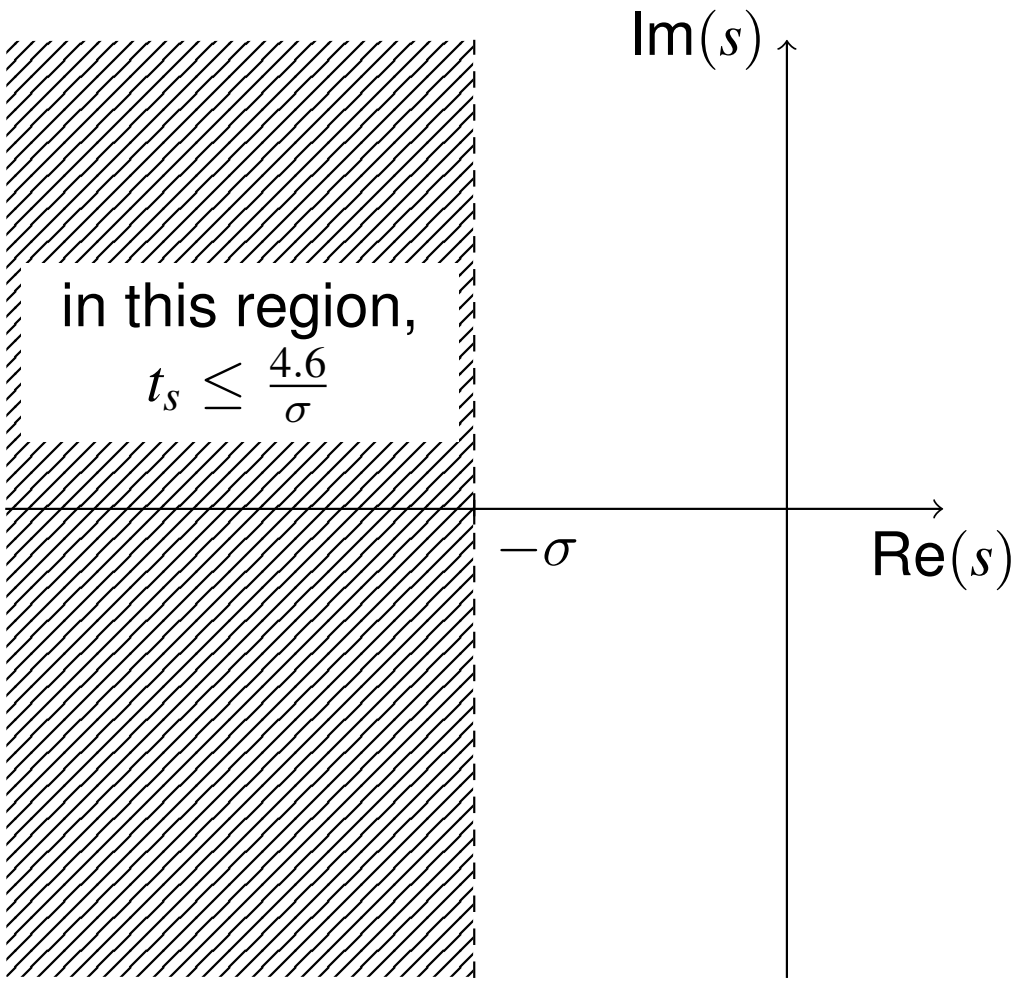
$$\beta = \cos^{-1}(\zeta)$$
$$\theta = \sin^{-1}(\zeta)$$



Rise Time Constraint



Settling Time Constraint



Overshoot Constraint