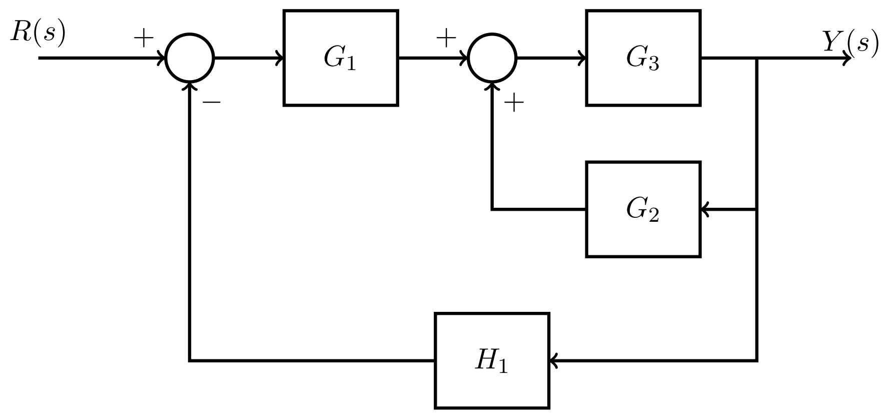
EEENG307: Intro to Feedback Control

Fall 2020

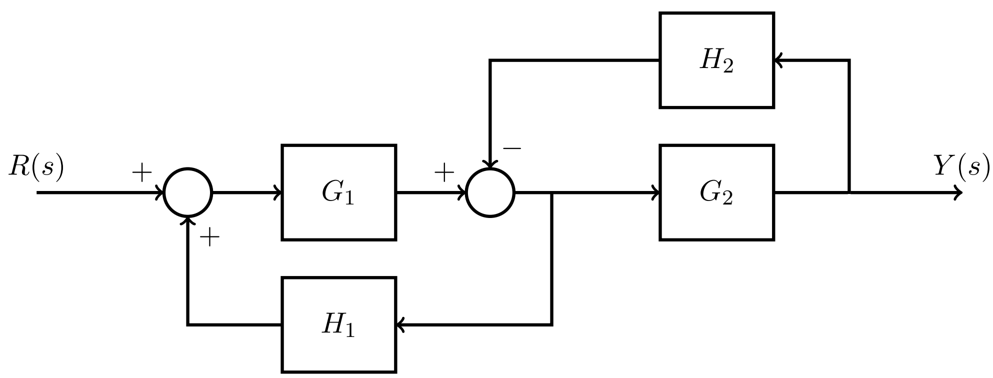
Homework Assignment #4

Due: 11:59pm, Wednesday, Sept 23rd, 2020.

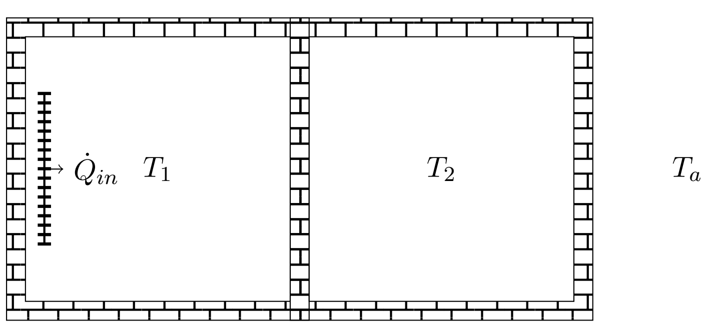
1. Simplify the following block diagram



2. Simplify the block diagram to find the transfer function $\frac{Y(s)}{R(s)}$.

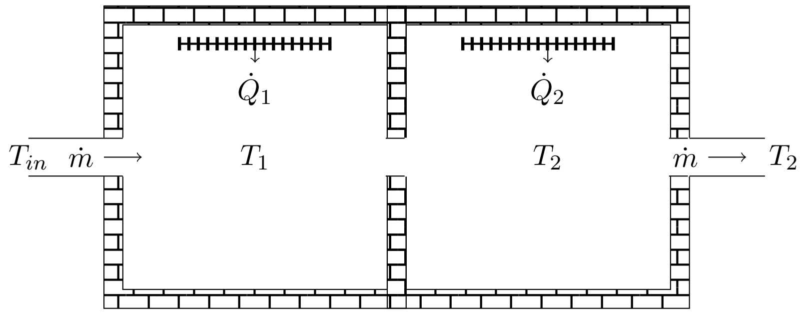


3. A building has two rooms, only one of which is heated. Each room has the same thermal capacitance, C, and the thermal resistance from the room to the outside is also the same, R_1. The thermal resistance between the rooms is R_2. Find the transfer function with input $\dot{Q}_{in}$ and output the temperature of the second room, T_2.



4. An industrial process has a two stage heating system to heat the feedstock for the process. The heating system is diagrammed below. The walls are sufficiently insulated that thermal conductance

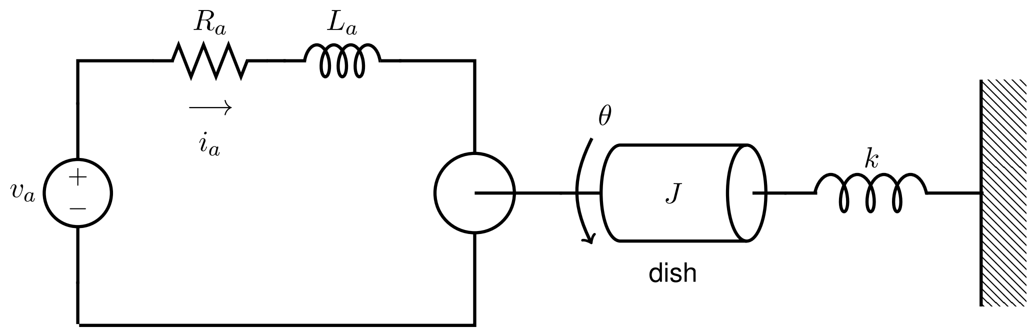
can be neglected. The feedstock with specific capacity c enters with mass flow $\dot{m}$ and temperature T_{in}. Two controllable heaters supply heat flow $\dot{Q}_1$ and $\dot{Q}_2$. The heat capacity of the chambers are C_1 and C_2, left to right.



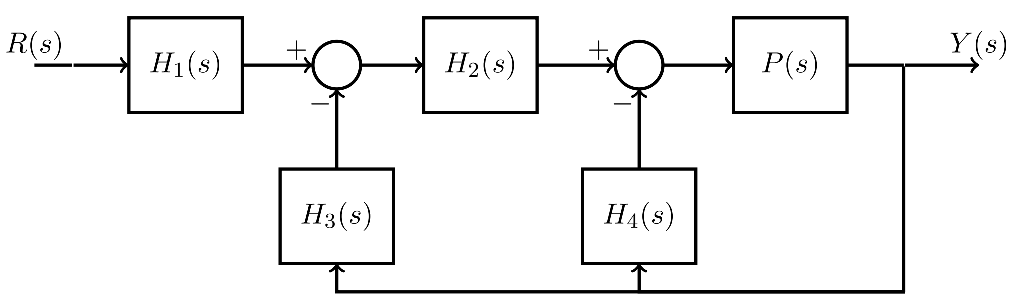
(a) Draw the thermal circuit that represents this system.

(b) Set T_{in} and Q_1 to zero to find the transfer function $\frac{T_2(s)}{\dot{Q}_2(s)}$. (Note, T_{in} and $Q_1 = 0$ will imply $T_1 = 0$ for the correct circuit.)

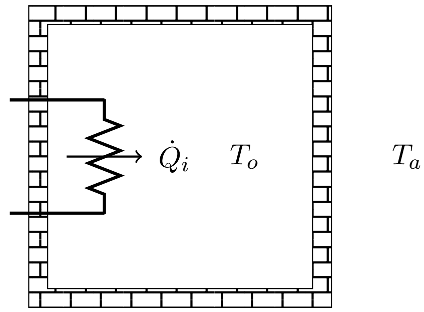
5. A motor is used to rotate a satellite dish that is supported by shocks. This is modeled as a rotational inertia and spring. Find the transfer function with input motor voltage v_a and output rotational position θ. Simplify your answer to a ratio of polynomials.



6. Quiz Question Monday: Simplify the following block diagram to obtain the transfer function $Y(s)/R(s)$.



7. Quiz Question Wednesday: Consider the following oven with a controllable heating element that produces heat flux $\dot{Q}_i$. Suppose the oven has thermal capacity C and conductive heat transfer to the outside characterized by thermal resistance R. The oven temperature is T_o and the external temperature is T_a.



(a) Draw the equivalent impedance network for this system.

(b) Find the transfer function $\frac{T_o(s)}{\dot{Q}_i(s)}$.

(c) If $\dot{Q}_i = 0$ and T_a is constant, then the oven will come to equilibrium with the ambient, so that $T_o = T_a$. The transfer function $G(s) = \frac{T_o(s)}{\dot{Q}_i(s)}$ will tell us how the temperature of the oven will vary away from this equilibrium. That is, in the Laplace domain, we will have

$$T_o(s) = G(s)\dot{Q}_i(s) + \frac{T_a}{s}$$

Suppose the oven is initially at ambient, and then a step input of $\dot{Q}_i(t) = 1$ J/s is applied. Find $T_o(t)$ for $t > 0$. (It will be a function of R and C).

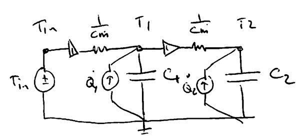
(d) If this step experiment was actually applied, and it took 20 minutes for the oven to reach a temperature of 20 °C above ambient, with a final steady state temperature of 40 °C above ambient, what is R and C?

Solutions:

1. $\frac{Y(s)}{R(s)} = \frac{G_3 G_1}{1 - G_2 G_3 + H_1 G_3 G_1}$

2. $\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 - G_1 H_1}$

3. $\frac{T_2(s)}{\dot{Q}_{in}(s)} = \frac{R_1^2}{(R_1 R_2 C s + R_2 + 2R_1)(R_1 C s + 1)}$

4. (a) 

   (b) $\frac{T_2(s)}{Q_2(s)} = \frac{1}{sC_2 + c\dot{m}}$

5. $\frac{\theta(s)}{V_a(s)} = \frac{K_t}{JL_a s^3 + JR_a s^2 + (KL_a + K_t K_e)s + KR_a}$