EENG307 Unit1: Lecture Summaries

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Fall 2022

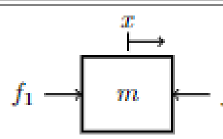
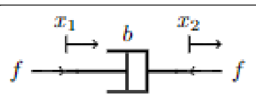
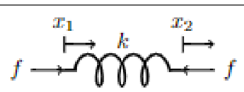
1 Mechanical Translational

For mechanical translational systems, the variables we are modeling are force and position. To model a mechanical translational system, we need to know the relationship *between* force and position. That relationship is described by component laws. The idealized elements (mass, spring and dampers), which have associated component laws, are summarized in Table 1.
What we want: relationship of force to position
Why we want it: to model mechanical translational systems
How do we do it: using component laws of idealized components

1.1 Summary

- Variables (inputs and outputs)
  - Force f (units: [N])
  - Position x (units: [m])
- Idealized Components
  - Mass: models an object (all objects have mass)
  - Damper: models friction or drag forces
  - Spring: models elastic compressibility
- Connection Laws
  - Position variables are equal
  - Forces must be equal and opposite
- Boundary Conditions
  - Specifying variable or trajectory for a force or position

Table 1: L02: Mechanical Translational Component Laws

Element Name	Variable	Schematic	Component Law	Constitutive Relationship	SI Units
Mass	m		$m\ddot{x} = f_1 - f_2$	force and acceleration	[kg] or [N s^2 m^{-1}]
Damper	b		$f = b(\dot{x}_1 - \dot{x}_2)$	force and velocity	[N s m^{-1}]
Spring	k		$f = k(x_1 - x_2)$	force and position	[N m^{-1}]

1.2 Key Reminders

Steps to Make Model

1. Define ground
2. Draw free body diagrams of each component (assign labels to positions and forces)
3. Write component laws
4. Write connection laws
5. Write boundary conditions
6. Determine what input and output you're interested in
7. Eliminate unwanted variables by using equations in Step 5 to simplify equations in Step 3 and Step 4
8. Continue simplifying

Additional context to connection laws: Forces

- Forces are equal and opposite at connection points
- In Figure 1
  - the pink highlighted force f_b is the force *on* the damper *by* the mass
  - the yellow highlighted force f_b is the force *on* the mass *by* the damper
  - these forces are equal in magnitude but opposite in direction (opposite direction indicated by the arrows pointing different directions).
  - Think about it: if you kick a rock, the rock kind of kicks you back. You exerted a force on the rock (force on rock by you) but your toe hurts because of the equal and opposite force (force on you by rock).

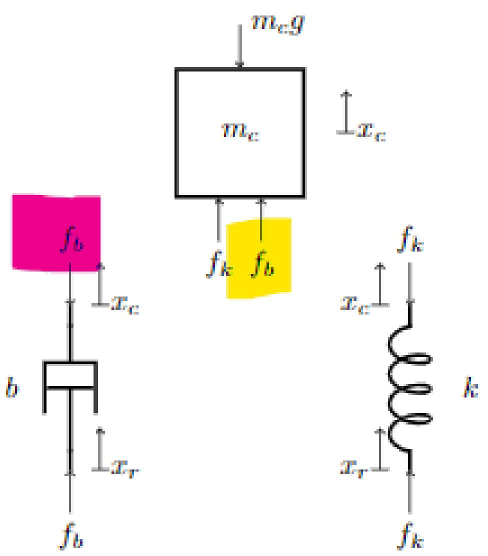


Figure 1: Example 2 from Lecture 2 of Car suspension.

Figure 2 further emphasizes this point: **The forces at a connection point are equal and opposite**. Let's focus on the connection point between the mass and the spring. If your spring is in *compression* the spring force on the mass f_k , denoted with a pink arrow, is pointing *left* (the opposite direction of f_k).

If your spring is in *tension* the spring force on the mass f_k , denoted with a pink arrow, is pointing *right* (the opposite direction of f_k).

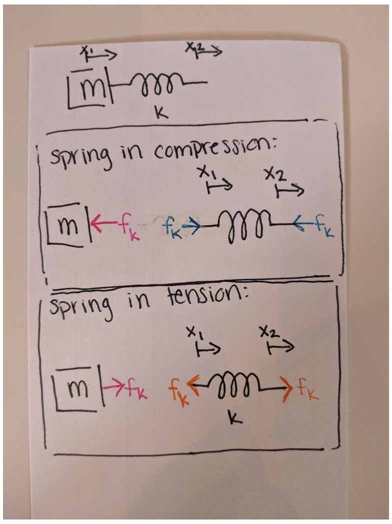


Figure 2: Connection of Direction of Forces

<u>Additional context to connection laws: Positions</u>

Positions are only different across a damper or spring. Think of the position of a mass as the position of the center of mass - a point. Or, think of the mass as having no size.

<u>Additional context to component laws: Forces and Positions</u>

Figure 3 further emphasizes this point: **The positive position value for your spring and damper component laws depends on the direction of the force.** For springs and dampers, we have $f_k = k(x_+ - x_-)$ or $f_b = b(\dot{x}_+ - \dot{x}_-)$ respectively. The force arrows define the direction of positive force, but does not necessarily mean our force will be in that direction.

The left side of Figure 3 shows a spring in compression. Here, f_k is positive in the same direction as x_1, the arrows for f_k and x_1 are both pointing to the right. So, $f_k = k(x_+ - x_-) = k(x_1 - x_2)$.

The right side of Figure 3 shows a spring in tension. Here, f_k is positive in the same direction as x_2, the arrows for f_k and x_2 are both pointing to the right. So, $f_k = k(x_+ - x_-) = k(x_2 - x_1)$.

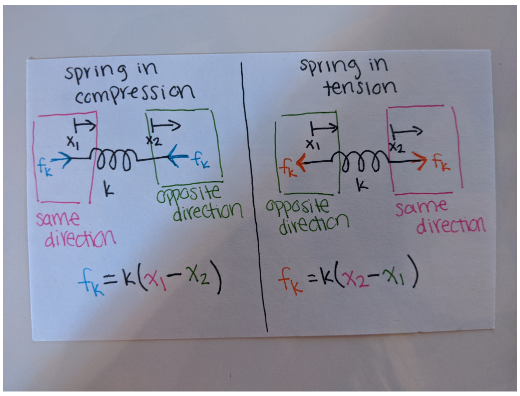


Figure 3: Component Law and Direction of Forces. Pink box means the force and position are in the same direction (arrows pointing in same direction). Green box means the force and position are pointing in the opposite direction (arrows pointing each other).

Let's go back to the the car suspension example from the Lecture 2 article. Looking at the damper in Figure 4, the blue highlighted section shows the force f_b and position x_c are pointing in opposite directions. The green highlighted section shows f_b and position x_r are pointing in the same direction. So, our component law for this is $f_b = b(\dot{x}_r - \dot{x}_c)$.

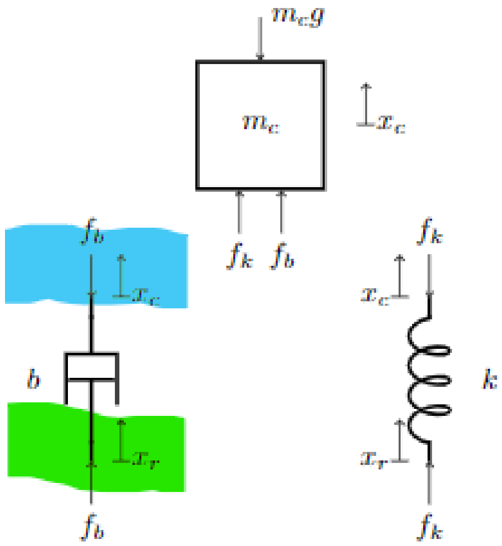


Figure 4: Example 2 from Lecture 2 of Car suspension.