

EENG307 Unit1: Lecture Summaries

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1 Electrical Impedance

1.1 Motivation

We care about how the system output is related to the system input. Previously, we found the relationship between the system output and system input as a series of differential equations. Using Laplace transforms to get the impedance of the system elements (resistor, capacitor, and inductor), we can use algebra to get this relationship instead. This relationship is also known as a transfer function $G(s)$ where $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$.

1.2 Electrical Impedance Summary

Through variable: magnitude is same on each side of the element

Across variable: magnitude is different on each side of the element

$$\text{Impedance(element)} = \frac{\mathcal{L}\{\text{across}\}}{\mathcal{L}\{\text{through}\}} = \frac{\mathcal{L}\{v\}}{\mathcal{L}\{i\}} = \frac{V(s)}{I(s)} \quad (1)$$

We can use Eq. (1) to derive the bottom 2 rows in Figure 1.

Electrical Impedance

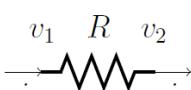
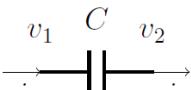
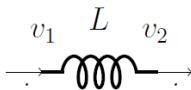
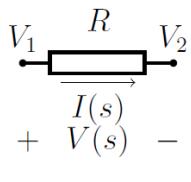
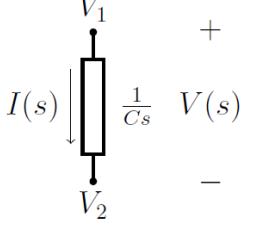
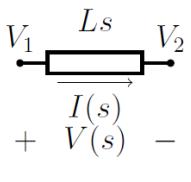
	resistor	capacitor	inductor
Component			
Component law	$v_1 - v_2 = iR$	$i = C \frac{d(v_1 - v_2)}{dt}$	$v_1 - v_2 = L \frac{di}{dt}$
Laplace Transform	$V(s) = I(s)R$	$V(s) = \frac{1}{Cs} I(s)$	$V(s) = LsI(s)$
Impedance Component			

Figure 1: Impedance for Electrical System Components [image credit: Dr.Coulston, Dr. Johnson, Dr. Sager]

1.3 Electrical Systems Review:

- Variables (inputs and outputs)
 - **Across variable:** Voltage v (units: volts [V])
 - **Through Variable:** Current x (units: amps [A])

- Idealized Components
 - Resistor: converts electrical energy to heat
 - Capacitor: stores electrical charge
 - Inductor: store energy in magnetic field
- Connection Laws (when two elements are connected)
 - Voltage is same (**shared across variable**)
 - Current sums to zero (**through variables sum to zero**)
- Boundary Conditions
 - **Set either the through or across variable on one side of the component**
 - Specifying voltage or current source

1.4 Steps to convert electrical system to impedance diagram

1. Convert circuit components to impedance component (reference Figure (1))
 - Switch schematics for elements to rectangles to represent impedance
 - Change variables to their impedance equation derived from “Laplace Transform” Row of Figure (1)
 $\frac{V(s)}{I(s)}$
2. Use algebra and circuit equations to get transfer function $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$ where $Y(s)$ and $R(s)$ may either be voltage $V(s)$ or current $I(s)$ depending on the problem.