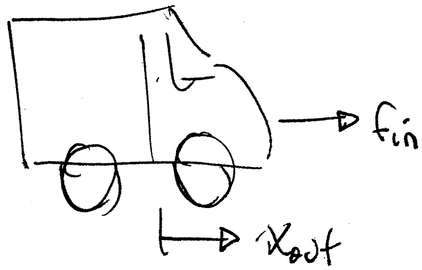
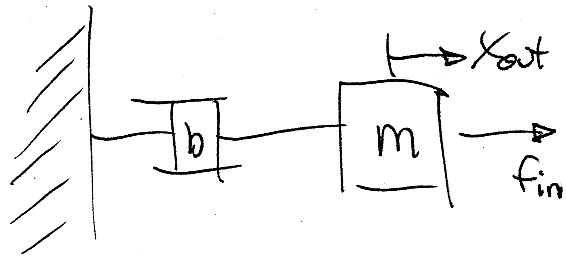
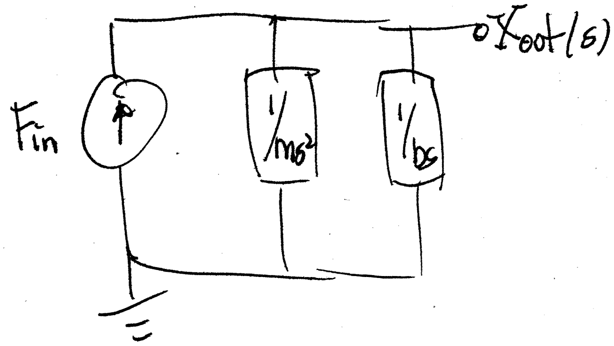
Parking The Honda Element



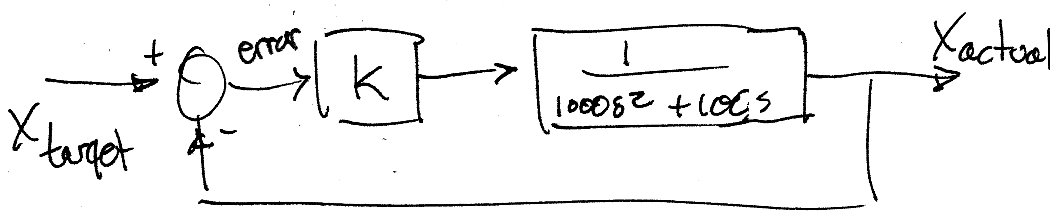




$$\frac{X_{out}(s)}{F_{in}(s)} = \frac{1}{ms^2+bs}$$

let $m = 1000\,kg$ $b = 100$

Proportional Controller



Use Block Diagram to simplify

$$\frac{X_{actual}}{X_{target}} = \frac{\dfrac{K}{1000s^2+100s}}{1+\dfrac{K}{1000s^2+100s}} * \frac{1000s^2+100s}{1000s^2+100s} = \frac{K}{1000s^2+100s+K}$$

Let $K = 10$

Apply $10u(t)$ as X_{target} so $\frac{10}{s}$ as input

$$X_{actual} = \frac{10}{s} \frac{10}{1000s^2 + 100s + 10} = \frac{1}{s(10s^2 + s + 1/10)}$$

Partial Fraction Expansion

$$\frac{1}{s(10s^2 + s + 1/10)} = \frac{A}{s} + \frac{Bs + C}{10s^2 + s + 1/10}$$

A: Multiply by s and eval @ $s=0$ $\quad \frac{1}{1/10} = \boxed{A = 10}$

* eval @ $s = 1$

$$\frac{1}{10 + 1 + 1/10} = 10 + \frac{B + C}{10 + 1 + 1/10} \qquad B + C = 1 - 10 \cdot \frac{111}{10} = -110$$

* eval @ $s = -1$

$$\frac{1}{-1(10 - 1 + 1/10)} = -10 + \frac{C - B}{10 - 1 + 1/10} \qquad -B + C = -1 + 10 \cdot \frac{91}{10} = 90$$

$\therefore \quad 2C = -20 \quad C = -10$

$\therefore \quad B = -110 - C = -100$

$$X_{actual} = \frac{10}{s} + \frac{-100s - 10}{10s^2 + s + 1/10} = \frac{10}{s} - 100 \frac{10s + 1}{100s^2 + 10s + 1}$$

For inverse Laplace complete square in denominator ...

$$X_{actual} = \frac{10}{s} - 100\frac{(10s+1)}{100s^2+10s+1}$$ Complete square

$$-\frac{100\,(10s+1)}{100s^2+10s+1} = -10\,\frac{s+1/10}{s^2+s/10+1/100} =$$

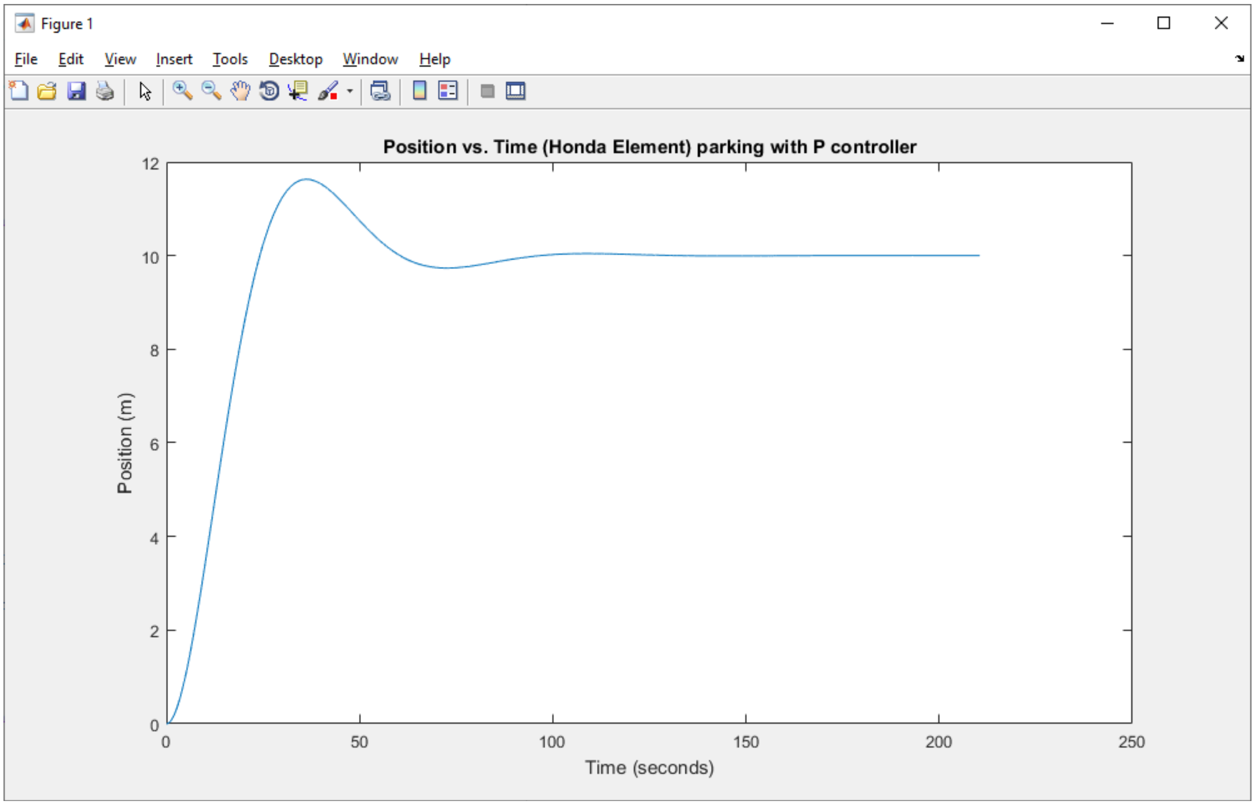
$$-10\,\frac{(s+1/20)+1/20}{(s+1/20)^2+\left(\sqrt{\frac{3}{400}}\right)^2} = -10\,\frac{s+1/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2} - \frac{10}{\sqrt{3}}\,\frac{\sqrt{3}/20}{(s+1/20)^2+\left(\sqrt{3}/20\right)^2}$$

So $$X_{actual} = \frac{10}{s} - 10\,\frac{s+1/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2} - \frac{10}{\sqrt{3}}\,\frac{\sqrt{3}/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2}$$

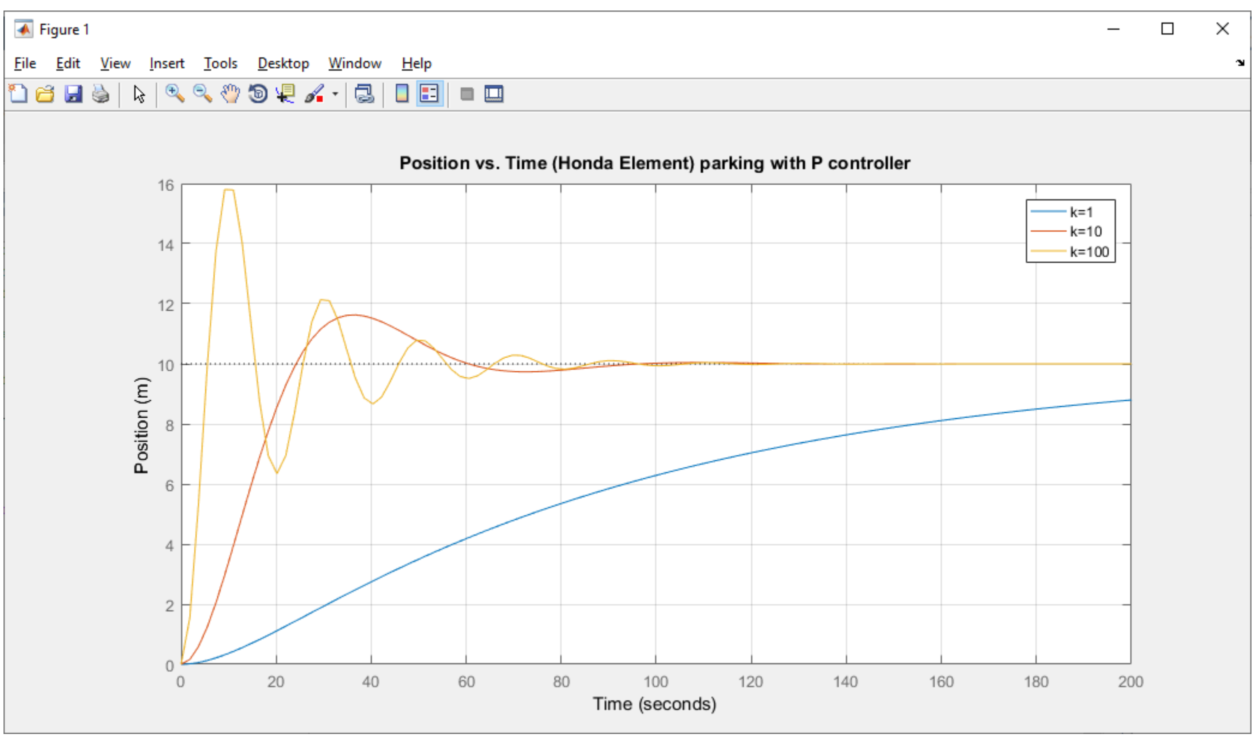
Inverse Laplace

$$X_{actual}(t) = 10u(t) - 10e^{-t/20}\cos\left(\frac{\sqrt{3}}{20}t\right) - \frac{10}{\sqrt{3}}e^{-t/20}\sin\left(\frac{\sqrt{3}}{20}t\right)$$

Analytic solution using plot function in Matlab



Found with Matlab stepplot function for 3 different values of k



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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Chris Coulston      Spring 2023    Colorado School of Mines
% EENG 307  Intro to Feedback Control Systems
% A mathematical of a Honda Element in the frequency domain.
% All measurements are in SI units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The equation for x follows from 3 pages of math and couple of hours
t = 0:pi/100:67*pi;
x = 10 - 10.*exp(-t/20).*cos(0.0866.*t)- 5.77.*exp(-
t/20).*sin(0.0866.*t);
plot(t,x)
xlabel('Time (seconds)');
ylabel('Position (m)');
title('Position vs. Time (Honda Element) parking with P controller');

waitforbuttonpress;


close all;
s = tf('s');

m = 1000;       % units of kilograms
b = 100;        % drag
kp = 10;

X = 1/(m*s^2+ b*s);         % Transfer function of X/F for my Honda
Element
X_k1   = feedback(1*X,1);   % Transfer function for closed loop
X_k10  = feedback(10*X,1);   % Transfer function for closed loop
X_k100 = feedback(100*X,1);   % Transfer function for closed loop

stepplot(10*X_k1,10*X_k10, 10*X_k100, 200);     % Plot the response
for a step input

grid on;
xlabel('Time');
ylabel('Position (m)');
title('Position vs. Time (Honda Element) parking with P controller');
legend('k=1', 'k=10', 'k=100');

waitforbuttonpress;

close all;
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