EENG307 Unit 2: Lecture Summaries

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1 Mechanical Rotational Impedance

1.1 Motivation

We care about how the system output is related to the system input. Previously, we found the relationship between the system output and system input as a series of differential equations. Using Laplace transforms to get the impedance of the system elements (mass, spring, and damper), we can use algebra to get this relationship instead. This relationship is also known as a transfer function $G(s)$ where $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$

1.2 Mechanical Impedance Summary

Through variable: magnitude is same on each side of the element
Across variable: magnitude is different on each side of the element

$$\text{Impedance(element)} = \frac{\mathcal{L}\{\text{across}\}}{\mathcal{L}\{\text{through}\}} = \frac{\mathcal{L}\{\theta\}}{\mathcal{L}\{\tau\}} = \frac{\theta(s)}{\tau(s)} \tag{1}$$

We can use Eq. (1) to derive the bottom 2 rows in Figure 1.

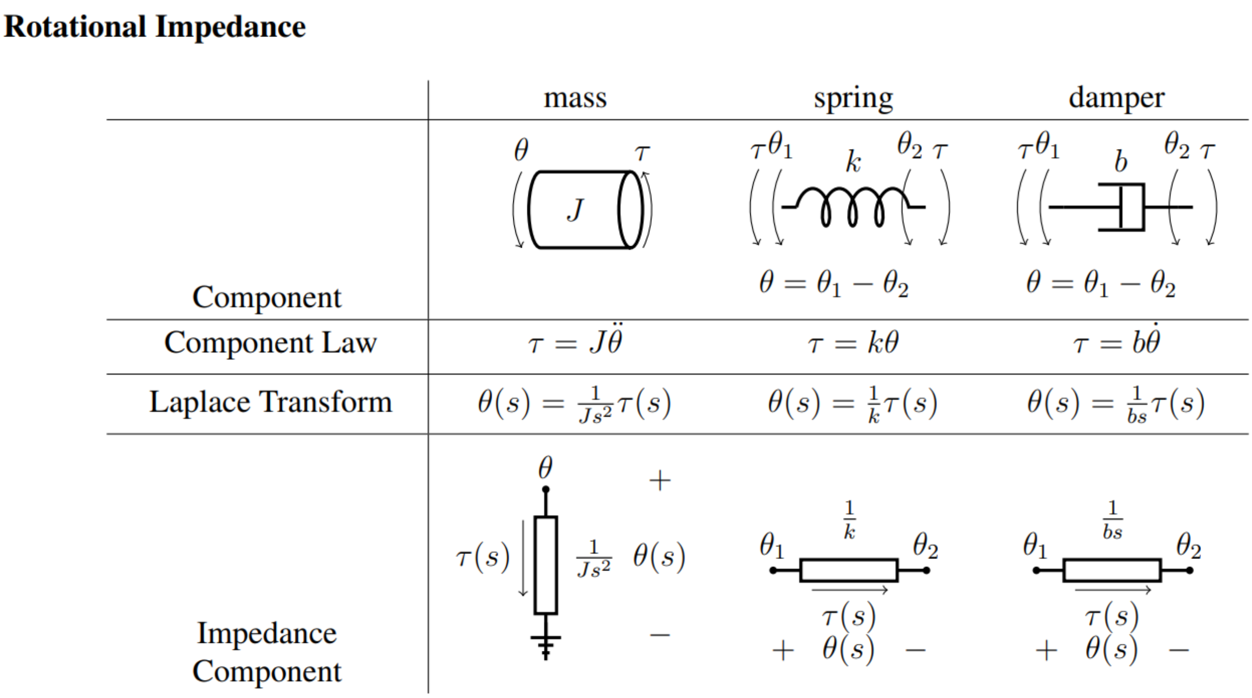

Figure 1: Impedance for Mechanical Rotational System Components [image credit: Dr.Coulston, Dr. Johnson, Dr. Sager]

1.3 Mechanical Rotational Systems Review:

- Variables (inputs and outputs)
  - **Across variable:** Angular Position θ (units: [rad])

- **Through variable:** Torque τ (units: [N-m])

- Idealized Components
  - Mass: models an object (all objects have mass)
  - Damper: models friction or drag forces
  - Spring: models elastic compressibility

- Connection Laws
  - Angular Position variables are equal **(shared across variable)**
  - Torque sums to zero - torque flows *through* connected components **(through variables sum to zero)**

- Boundary Conditions
  - **Set either the through or across variable on one side of the component**
  - Specifying variable or trajectory for a torque or angular position
  - **If your input is torque, then model this with a current input**
  - **If your input is angular position, then model this with a voltage input**

1.4 Steps to convert mechanical rotational system to impedance diagram

1. Identify all node variables
   - nodes are the across variables - in this case, it is the position(s)
   - what used to be $\theta(t)$ is now $\theta(s)$ in impedance diagram (impedance diagram is in Laplace domain). Similarly, what used to be $\tau(t)$ is now $\tau(s)$.

2. Identify one node as ground or add a ground node

3. Connect components between nodes
   - Switch schematics for elements to rectangles to represent impedance
   - Change variables to their impedance equation derived from "Laplace Transform" Row of Figure (1) $\frac{\theta(s)}{\tau(s)}$
   - **Remember:** masses are always connected to ground!!!

4. Apply boundary conditions
   - **If the input is an applied torque, this is modeled with a current input**
   - **If the input is an angular position, this is modeled with a voltage source**

5. Use algebra and circuit equations to get transfer function $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$ where $Y(s)$ and $R(s)$ may either be angular position $\theta(s)$ or torque $\tau(s)$ depending on the problem.