EENG307 Unit 2: Lecture Summaries

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1 Fluid Impedance

1.1 Motivation

We care about how the system output is related to the system input. Previously, we found the relationship between the system output and system input as a series of differential equations. Using Laplace transforms to get the impedance of the system elements (tank and valve), we can use algebra to get this relationship instead. This relationship is also known as a transfer function $G(s)$ where $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$

1.2 Fluid Impedance Summary

Through variable: magnitude is same on each side of the element
Across variable: magnitude is different on each side of the element

$$\text{Impedance(element)} = \frac{\mathcal{L}\{\text{across}\}}{\mathcal{L}\{\text{through}\}} = \frac{\mathcal{L}\{p\}}{\mathcal{L}\{q\}} = \frac{P(s)}{Q(s)} \tag{1}$$

We can use Eq. (1) to derive the bottom 2 rows in Figure 1.

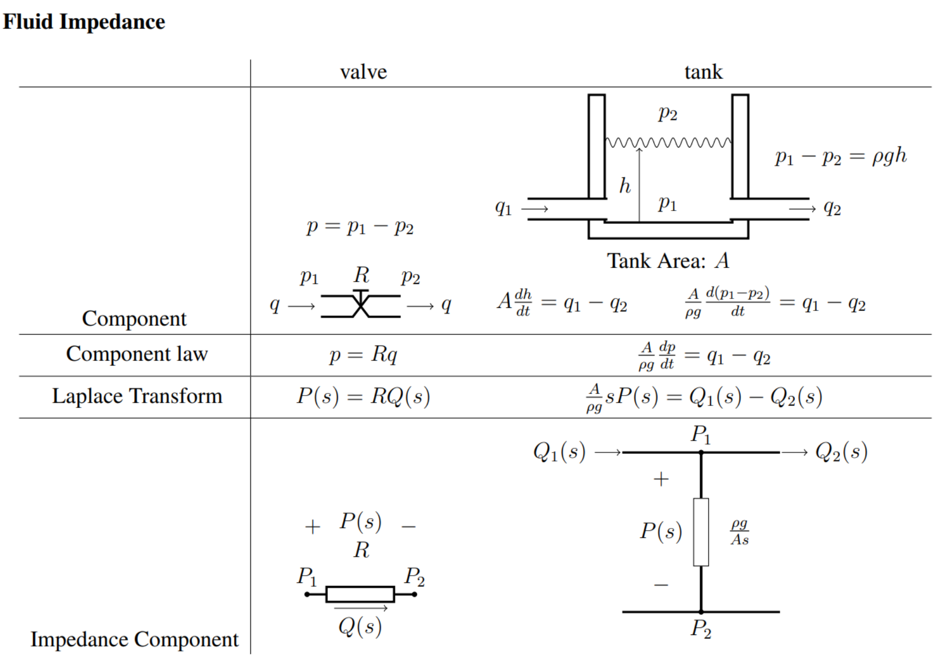

Figure 1: Impedance for Mechanical Translational System Components [image credit: Dr. Coulston, Dr. Johnson, Dr. Sager]

Additional Notes on Figure 1:

- ρ is fluid density ([kg/m^3])
- g is acceleration due to gravity ([m/s^2])

1.3 Fluid Systems Review:

- Variables (inputs and outputs)
  - **Across variable:** Pressure p (units: [N/m^2])
  - **Through variable:** Volumetric Flow q (units: [m^3/s])
- Idealized Components
  - Tank: volume in tank is proportional to volumetric flow in and volumetric flow out (similar to a capacitor)
  - Valve: restricts pressure (similar to a resistor)
- Connection Laws
  - Pressure variables are equal **(shared across variable)**
  - Flow sums to zero - volume flows *through* connected components **(through variables sum to zero)**
- Boundary Conditions
  - **Set either the through or across variable on one side of the component**
  - Specifying variable or trajectory for a pressure or flow
  - **If your input is flow, then model this with a current input**
  - **If your input is pressure, then model this with a voltage input**

1.4 Steps to convert fluid system to impedance diagram

1. Identify all node variables
   - nodes are the across variables - in this case, it is the position(s)
   - what used to be $p(t)$ is now $P(s)$ in impedance diagram (impedance diagram is in Laplace domain). Similarly, what used to be $q(t)$ is now $Q(s)$.
2. Identify one node as ground or add a ground node
3. Connect components between nodes
   - Switch schematics for elements to rectangles to represent impedance
   - Change variables to their impedance equation derived from "Laplace Transform" Row of Figure (1) $\frac{X(s)}{F(s)}$
4. Apply boundary conditions
   - **If the input is an applied flow, this is modeled with a current input**
   - **If the input is a pressure, this is modeled with a voltage source**
5. Use algebra and circuit equations to get transfer function $G(s) = \frac{Y(s)}{R(s)} = \frac{\text{output}}{\text{input}}$ where $Y(s)$ and $R(s)$ may either be pressure $P(s)$ or flow $Q(s)$ depending on the problem.