

# EENG307: Modeling DC Motors<sup>1</sup>

## Lecture 20

Elenya Grant, Kathryn Johnson, and Hisham Sager<sup>2</sup>

Department of Electrical Engineering  
Colorado School of Mines

Fall 2022

---

<sup>1</sup> This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, 171 Second Street, Suite 300, San Francisco, California, 94105, USA.

<sup>2</sup> Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kupilik, University of Alaska, Anchorage

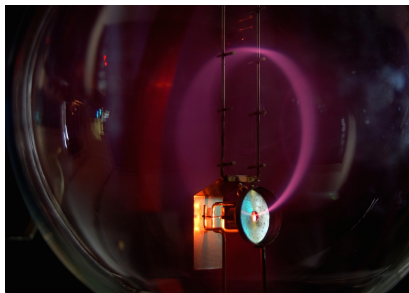
# Electric Motors

- Motors are *transducers* which convert electrical energy to mechanical energy.
- We will look at models for brushed DC motors.

# Key Idea 1: Lorentz Force

- An electron moving in a magnetic field experiences a force perpendicular to its motion and the magnetic field.

$$F = q(v \times B)$$



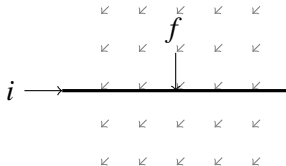
(<http://commons.wikimedia.org/wiki/File:Draaibank.png>)

- Electrons move in a circle when exposed to a constant magnetic field.

# Key Idea 1: Lorentz Force

- Electrons in a wire are constrained to move in a straight line. When a wire is placed in a magnetic field, and a current is set up through the wire, the Lorentz force creates a force perpendicular to the current and the magnetic field.

$$f = Ki$$



- This is the basis for a motor - the force on the wire can drive a mechanical load.

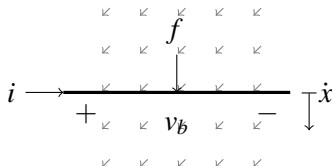
## Key Idea 2: Conservation of Energy

- A force on a wire will cause it to accelerate.
- Work done
  - on the wire:  $\dot{x}f = \dot{x}Ki$ .
  - to create the current:  $iv_b$ .
- Conservation of energy suggests these are equal

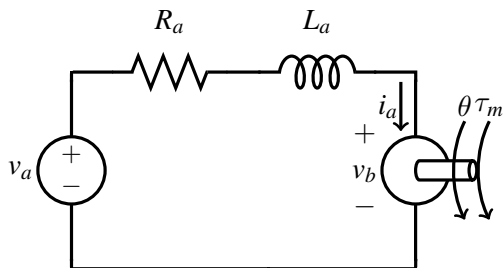
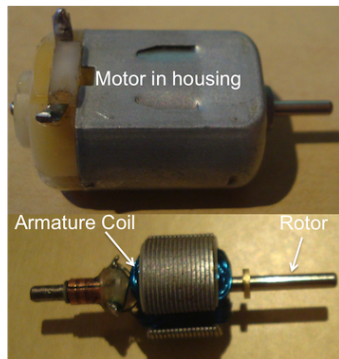
$$iv_b = \dot{x}Ki,$$

$$v_b = \dot{x}K.$$

- voltage (back electro-motive force)  $v_b$  is proportional to the wire's velocity



# DC motor components



Motor Constants:  $K_t, K_e$

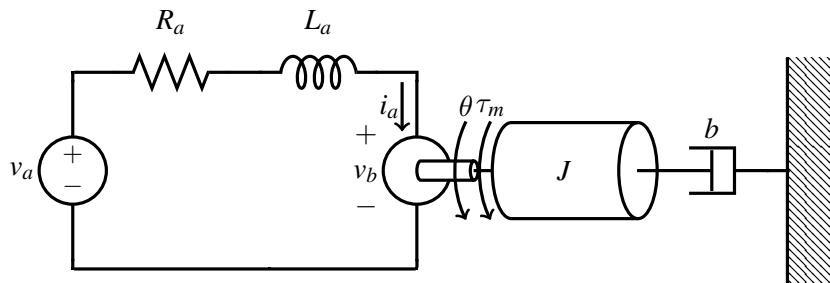
Transducer Relations:

$$v_b = K_e \dot{\theta}$$

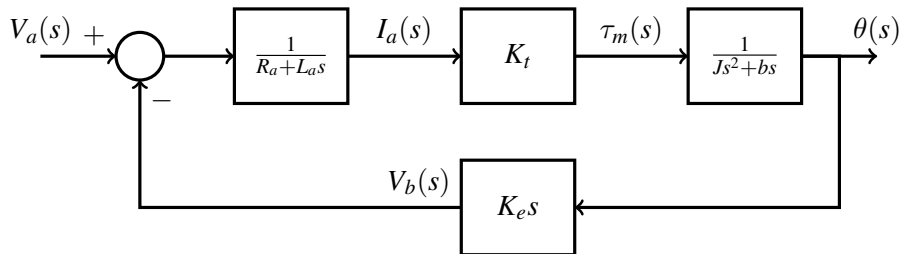
$$\tau_m = K_t i_a$$

[http://en.wikipedia.org/wiki/File:Motor\\_internals.JPG](http://en.wikipedia.org/wiki/File:Motor_internals.JPG)

# DC motor diagram

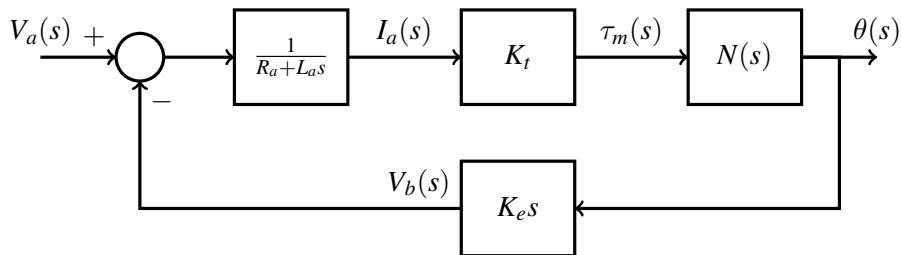


# DC motor block diagram





# DC motor block diagram with arbitrary load



# DC motor with different load

