EEENG307: Nyquist Stability Theorem[1]

Lecture 28

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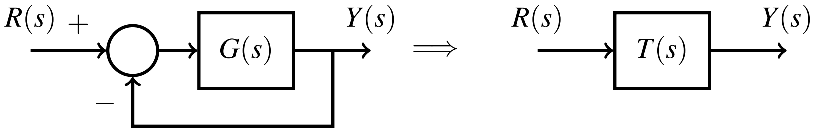
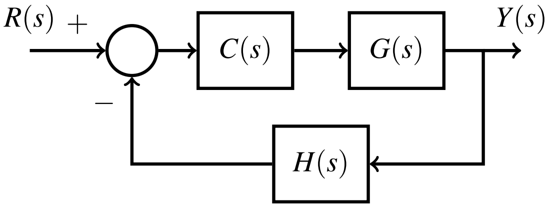
Checking Stability, so far...



$$T(s) = \frac{N(s)}{D(s)} \qquad D(s) = 0 \text{ for } \mathrm{Re}(s) > 0?$$

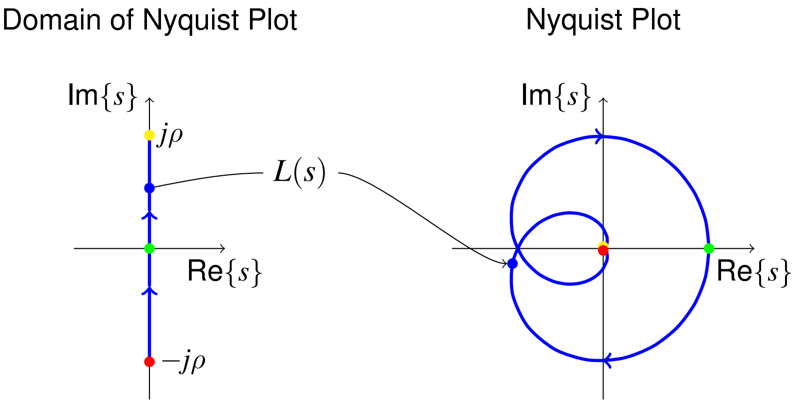
Loop Gain



$$L(s) = H(s)G(s)C(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1 + C(s)G(s)H(s)} = \frac{G(s)C(s)}{1 + L(s)}$$

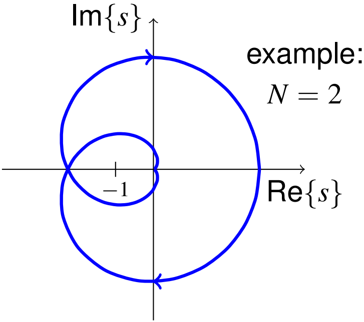
Nyquist Plot Mapping



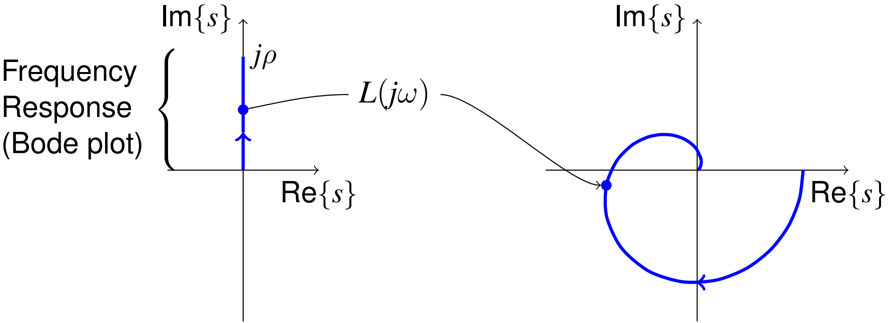
$\rho =$ big enough number so that $L(j\rho)$ approaches a constant

Nyquist Stability Criterion

- Let $P = \#$ of poles of $L(s)$ with *positive* real part (open loop information)
- Let $N = \#$ of clockwise encirclements of the point $-1 + j0$ by the Nyquist plot of $L(s)$
- Let $Z = \#$ of poles of the *closed loop system* $\frac{C(s)G(s)}{1+L(s)}$ with *positive* real part
- result: $Z = N + P$.

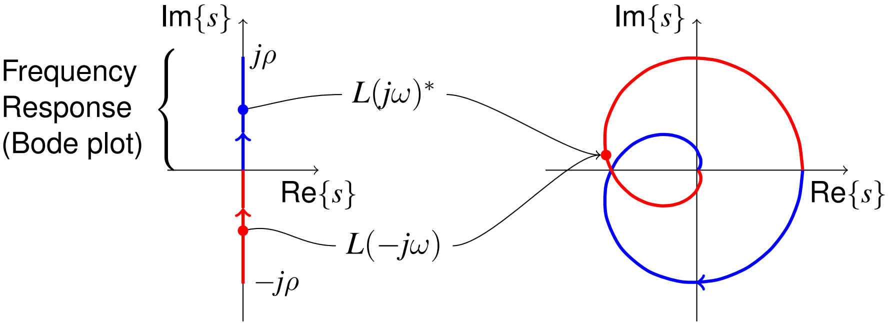


Plotting the Nyquist Plot:
Step 1

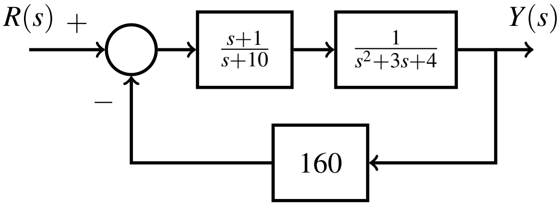


Plotting the Nyquist Plot:

Step 2



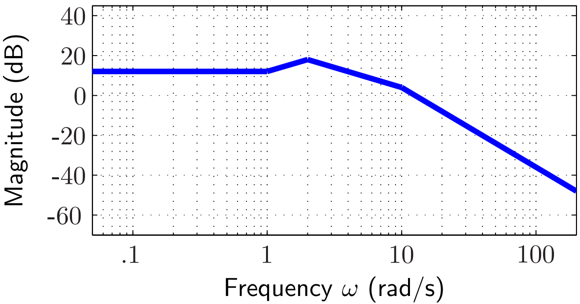
Example feedback system



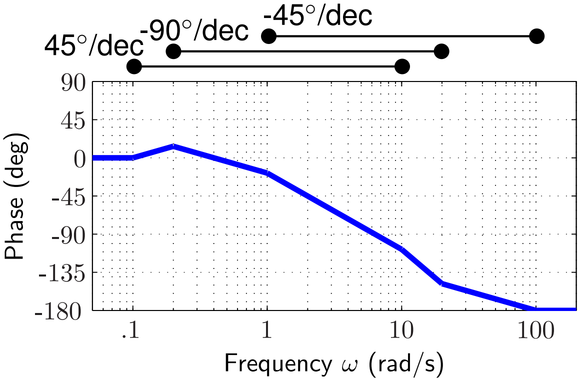
In this case, the loop gain is

$$L(s) = 160 \frac{1}{s^2 + 3s + 4} \frac{s+1}{s+10}$$

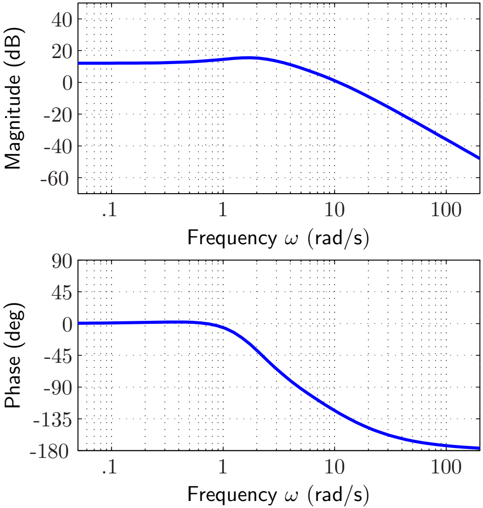
Magnitude Plot



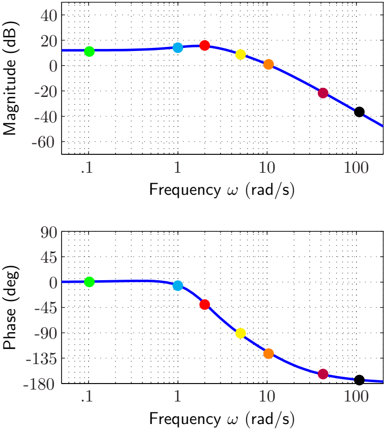
Phase Plot


45°/dec
-90°/dec
-45°/dec
90
45
0
-45
-90
-135
-180
Phase (deg)
.1
1
10
100
Frequency ω (rad/s)

Bode Plot


40
20
0
-20
-40
-60
Magnitude (dB)
.1
1
10
100
Frequency ω (rad/s)
90
45
0
-45
-90
-135
-180
Phase (deg)
.1
1
10
100
Frequency ω (rad/s)

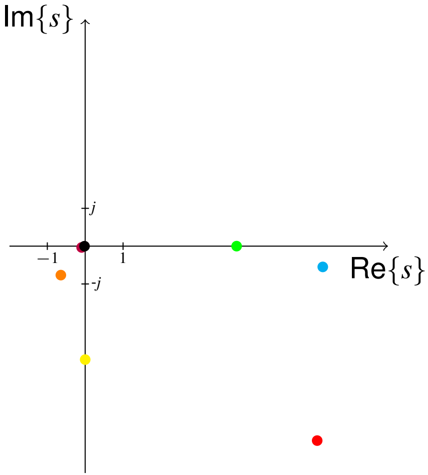
Determine Magnitude and Phase at convenient set of frequencies



Freq (rad/s)	Mag (dB)	Mag	Phase
.1	12	4.0	-0.8°
1	14	5.0	-6°
2	15	5.6	-38°
5	9	2.8	-92°
10	1	1.1	-123°
40	-20	0.10	-163°
100	-36	0.015	-173°

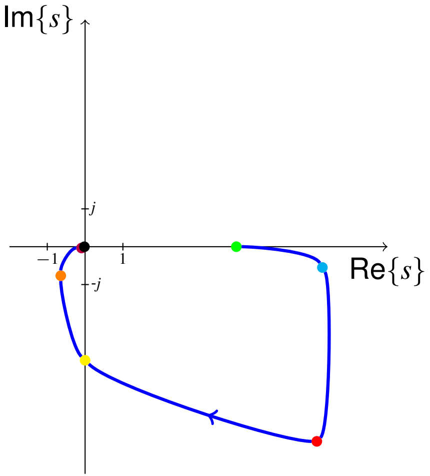
Transfer that information to the complex plane

Mag	Phase
4.0	-0.8°
5.0	-6°
5.6	-38°
2.8	-92°
1.1	-123°
0.10	-163°
0.015	-173°

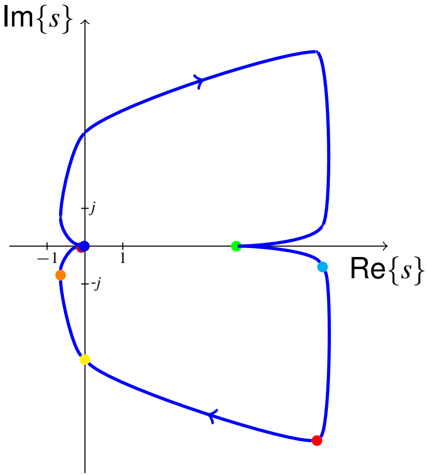


Draw connecting lines, going from low frequency to high frequency

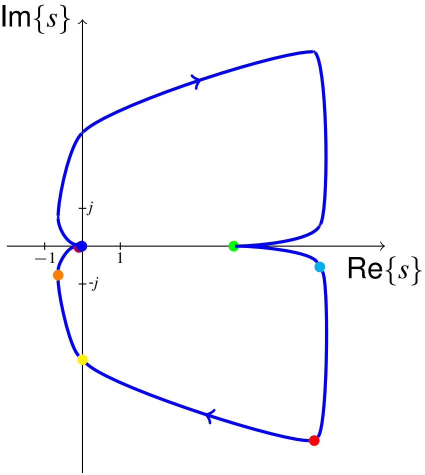
Mag	Phase
4.0	-0.8°
5.0	-6°
5.6	-38°
2.8	-92°
1.1	-123°
0.10	-163°
0.015	-173°



Draw Mirror Image to Complete Nyquist Plot



Determine Number of Clockwise Encirclements



$N = 0$

Nyquist Plot using MATLAB

```
» C = tf([1 1],[1 10]);
» G = tf(1,[1 3 4]);
» H = 160;
» nyquist(H*G*C)
```