Lecture 15: Disturbances and Steady-State Error Handout

The Final Value Theorem:

Theorem 1. *Given Laplace Transform $Y(s)$ such that $sY(s)$ has all poles with negative real part. Then the inverse Laplace transform $y(t)$ satisfies*

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

Exercise 1: Does a signal with Laplace transform with a pole in the RHP, such as

$$Y(s) = \frac{1}{s-1}$$

have a "final value" in the time domain?

Exercise 2: Does a sinusoidal function such as

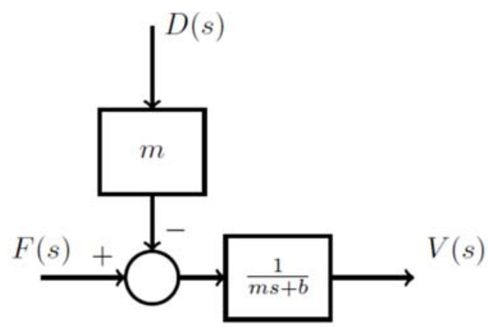
$$Y(s) = \frac{2}{s^2+4} \leftrightarrow y(t) = \sin(2t)u(t)$$

have a "final value" in the time domain? Is your answer consistent with the requirement "*that $sY(s)$ has all poles with negative real part*" in Theorem 1?

Cruise Control Example

Recall the open loop car "plant" block diagram from last time:

$$V(s) = \frac{1}{ms+b}F(s) - \frac{m}{ms+b}D(s)$$



If our system meets the requirements for the final value theorem (FVT), we can use the FVT to find the "final value" (i.e., steady-state value) of the car's velocity in two cases:

1. Suppose the motor supplies a constant force f_m on a flat road. At steady state, what will the car's speed be?

2. Suppose the car is at steady state on a flat road, but then drives on an incline with slope ϕ. By how much will the car slow down?