EENG307: Solving Differential Equations using Laplace Transforms, Part II[1]

Lecture 6

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Laplace Transform Pairs with repeated roots

Repeated roots occur when an exponential or sinusoid is multiplied by t or t^n. First, let's derive the Laplace Transform of t^n. We already have established the Laplace Transform pair

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s},$$

and the integration theorem

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}\mathcal{L}\left\{f(t)\right\}.$$

Laplace Transform Pairs with repeated roots

Note that a ramp is the integral of a step. That is, the ramp function $tu(t)$ can be defined via

$$tu(t) = \int_{0^-}^{t} u(t)dt.$$

Using the integration theorem gives us

$$tu(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^2}.$$

Thus, a ramp has two poles at $s = 0$.

Higher Powers

The Laplace Transform for higher powers of t can be found via further integration. For example,

$$\frac{1}{2}t^2u(t) = \int_{0^-}^{t} tu(t)dt,$$

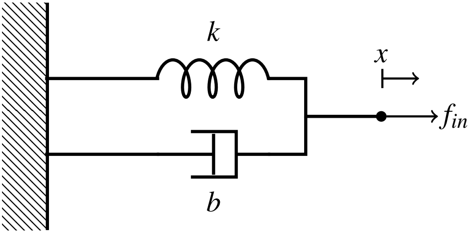
which implies

$$\frac{1}{2}t^2u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^3}.$$

and even higher orders of t can be found similarly. What we see is that powers of t give us a denominator with repeated roots at $s = 0$.

Spring and Damper

A spring with spring constant $k = 4$ N/m and damper with damping coefficient $b = 2$ Ns/m is connected in parallel to a wall. A force of $f_{in} = 1$ N is applied for $t \geq 0$. If the initial displacement of the right side of the spring and damper is $x = 1$ m at $t = 0$, find $x(t)$ for $t \geq 0$



Circuit Problem

An LRC circuit has applied voltage $v_{in} = 1$ for $t \geq 0$. If $v_{out}(t)$ was zero for $t \leq 0$, find $v_{out}(t)$ for $t \geq 0$.