

# EENG307: Time Response of Higher Order Systems<sup>1</sup>

## Lecture 12

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# Step Response Specifications

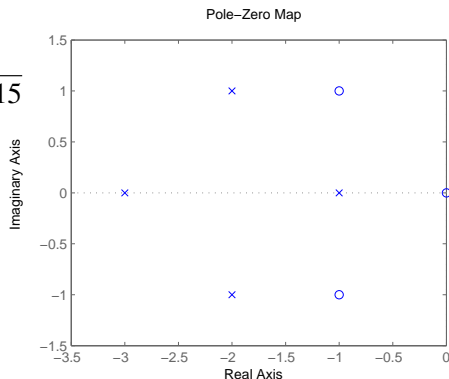
System	System Parameters	Rise Time	Settling Time	% Overshoot
$\frac{K\sigma}{s+\sigma}$	$\sigma$ - pole magnitude	$t_r = \frac{2.2}{\sigma}$	$t_s = \frac{4.6}{\sigma}$	0
$\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\zeta$ - damping ratio $\omega_n$ - natural frequency $\sigma = \zeta\omega_n$ - real part of poles	$t_r = \frac{2.2}{\omega_n}$	$t_s = \frac{4.6}{\zeta\omega_n}$	$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$

# Pole-Zero diagram

$$F(s) = \frac{s^3 + 2s^2 + 2s}{s^4 + 8s^3 + 24s^2 + 32s + 15}$$

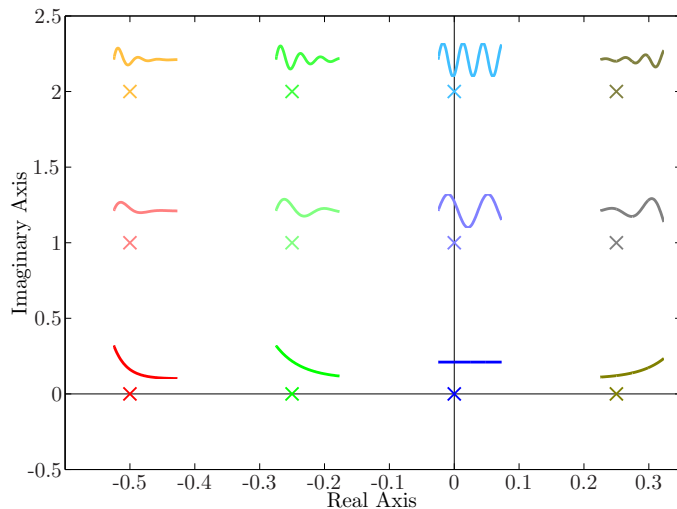
zeros:  $-1 + j, -1 - j, 0$

poles:  $-3, -1, -2 + j, -2 - j$



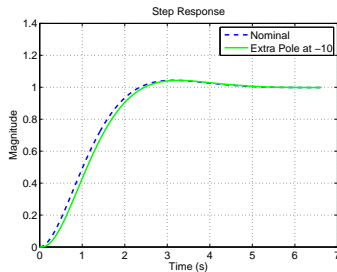
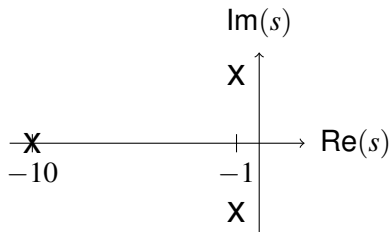
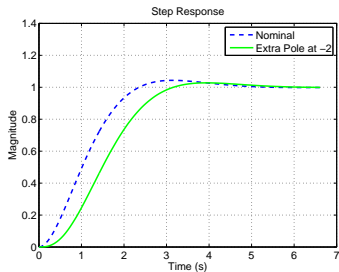
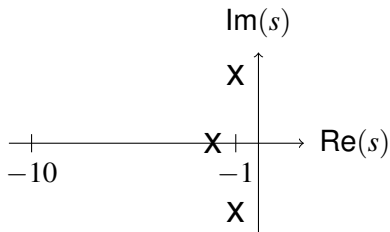
$$\begin{array}{ll} e^{\sigma t} u(t) & \frac{1}{s+\sigma} \\ e^{-at} \sin(\omega t) u(t) & \frac{\omega}{(s+a)^2 + \omega^2} \\ e^{-at} \cos(\omega t) u(t) & \frac{s+a}{(s+a)^2 + \omega^2} \end{array}$$

# Pole map with responses

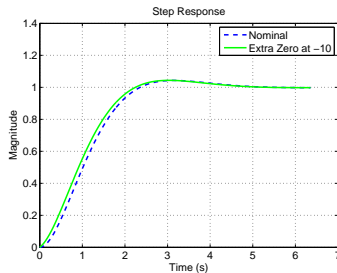
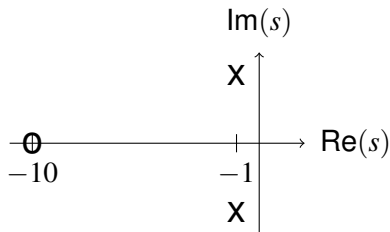
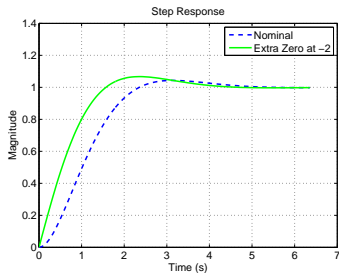
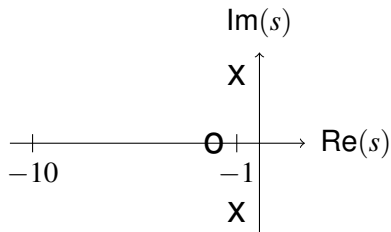


(Conjugate poles not shown)

# Effect of extra pole on step response



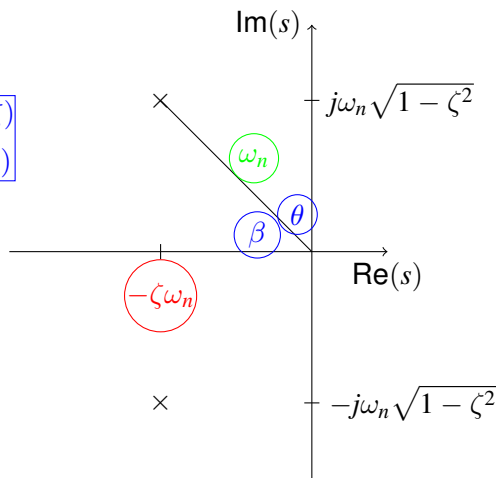
# Effect of extra zero on step response



# Under-damped poles in the complex plane

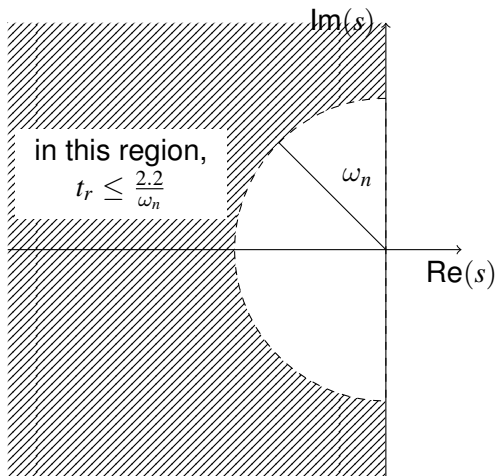
$$\beta = \cos^{-1}(\zeta)$$

$$\theta = \sin^{-1}(\zeta)$$

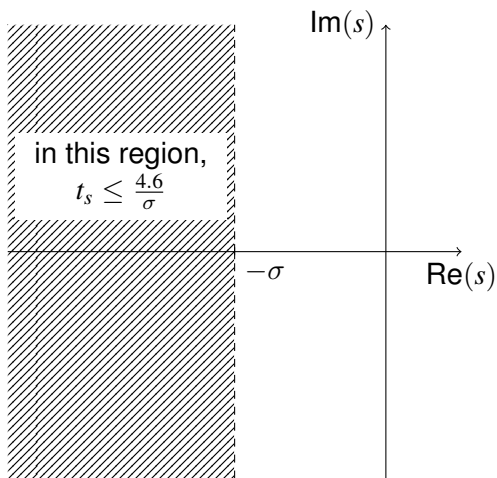




# Rise Time Constraint



# Settling Time Constraint



# Overshoot Constraint

