EEENG307: Stability and Routh Hurwitz Criterion[1]

Lecture 16

Christopher Coulston, Salman Mohagheghi and Jeffrey Schowalter[2]

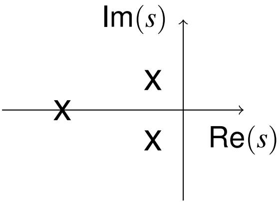
Department of Electrical Engineering
Colorado School of Mines

Fall 2020

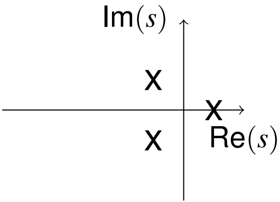
[2] Developed and edited by Tyrone Vincent and Kathryn Johnson, Colorado School of Mines, with contributions from Salman Mohagheghi, Chris Coulston, Kevin Moore, CSM and Matt Kupilik, University of Alaska, Anchorage

BIBO Stability Examples

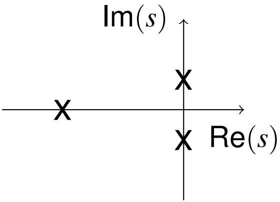
BIBO stable



not BIBO stable



not BIBO stable



Start of Routh Array

$$\begin{array}{rcccccl} s^n: & a_0 & a_2 & a_4 & \cdots & \left.\right\} & \text{polynomial coefficients} \\ s^{n-1}: & a_1 & a_3 & a_5 & \cdots & & \\ s^{n-2}: & b_1 & b_2 & b_3 & \ldots & \left.\right\} & \text{calculated from previous rows} \end{array}$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{-1}{a_1} \begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \quad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

Routh Array Continued

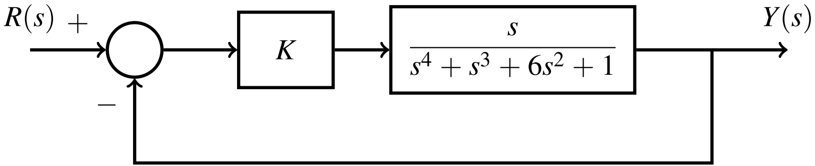
$$
\begin{array}{rcllll}
s^n & : & a_0 & a_2 & a_4 & \cdots \\
s^{n-1} & : & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} & : & b_1 & b_2 & b_3 & \cdots \\
s^{n-3} & : & c_1 & c_2 & c_3 & \cdots \\
 & \vdots & \vdots & & & \\
s^0 & : & z_1 & & &
\end{array}
$$

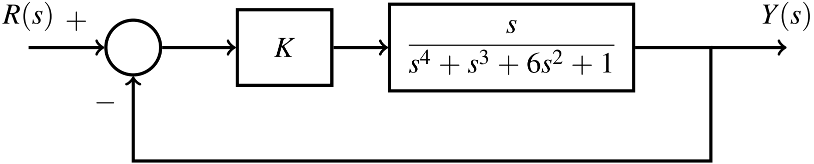
$$
c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \quad \cdots
$$

Routh-Hurwitz Test:

Last Step

$$
\begin{array}{rcccccc}
s^n & : & a_0 & a_2 & a_4 & \cdots \\
s^{n-1} & : & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} & : & b_1 & b_2 & b_3 & \cdots \\
s^{n-3} & : & c_1 & c_2 & c_3 & \cdots \\
 & \vdots & \vdots & & & \\
s^0 & : & z_1 & & &
\end{array}
$$


$R(s)$
+
−
K
$\frac{s}{s^4 + s^3 + 6s^2 + 1}$
$Y(s)$



$$
\begin{aligned}
\frac{Y(s)}{R(s)} &= \frac{K\frac{s}{s^4+s^3+6s^2+1}}{1 + K\frac{s}{s^4+s^3+6s^2+1}} \\
&= \frac{Ks}{s^4 + s^3 + 6s^2 + Ks + 1}
\end{aligned}
$$

$$
\begin{array}{lccc}
s^4: & 1 & 6 & 1 \\
s^3: & 1 & K & \\
s^2: & 6-K & 1 &
\end{array}
$$

$$
\begin{array}{lccc}
s^4: & 1 & 6 & 1 \\
s^3: & 1 & K & \\
s^2: & 6-K & 1 & \\
s^1: & K - \frac{1}{6-K} & &
\end{array}
$$

$$\begin{array}{lccc} s^4: & 1 & 6 & 1 \\ s^3: & 1 & K & \\ s^2: & 6-K & 1 & \\ s^1: & K-\frac{1}{6-K} & & \\ s^0: & 1 & & \end{array}$$

$$-K^2 + 6K - 1 > 0$$