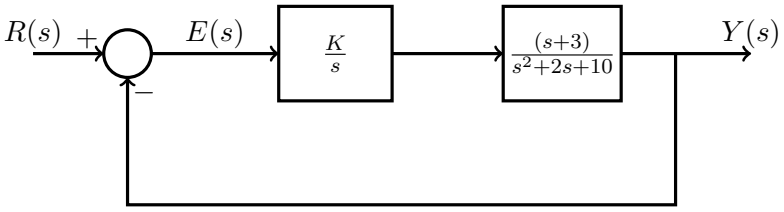
EENG307: Intro to Feedback Control

Fall 2020

Homework Assignment #7

Due: Wednesday, Oct 21st, 11:59pm

1. Consider the following unity gain feedback system

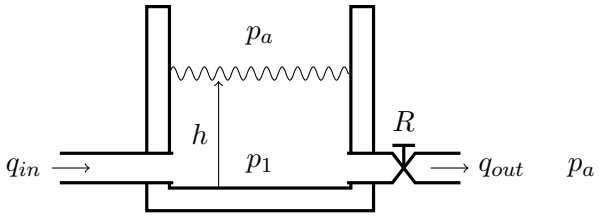


   Find the feasible range for parameter K so that the system is stable and the steady-state error $e(\infty)$ for a unit ramp input $r(t) = tu(t)$ is less than or equal to 0.75.

2. You are to design a Proportional/Integral (PI) control system for an tank with controllable volumetric flow input q_{in}. The level of liquid in the tank must be carefully regulated.

   The desired specifications are:

   - Settling time equal to 10 seconds
   - Overshoot of 15%
   - Zero steady state error for unit step reference



Tank Area: A

   The tank has an area of 100 m^2, the density of the liquid is such that $\rho g = 10000$ kg m^{-2}s^{-2} and the flow resistance at the outlet valve is 400 kg m^{-4}s^{-1}.

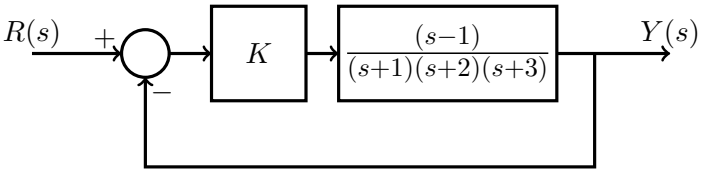
   (a) Find the transfer function for the tank from input q_{in} to output h.

   (b) Draw a block diagram that represents the PI feedback control system. Include the reference signal for the desired height $H_d(s)$

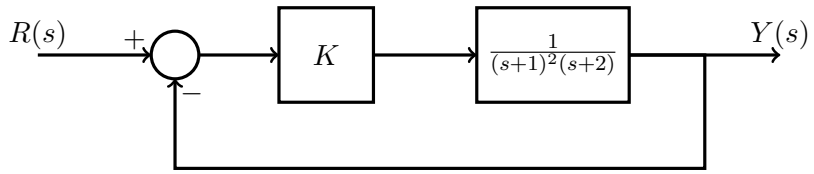
   (c) Determine the acceptable damping ratio and undamped natural frequency of the closed loop poles achieve the desired closed loop specifications

   (d) Find the gains for your controller that achieve the pole locations for part (c)

3. Using the rules covered in lecture, make a sketch of the root locus of closed loop poles for the following feedback system.



4. Use Root-Locus analysis to answer the questions about the following feedback control system (hint on b,c: You can use Routh-Hurwitz, or solve $1 + KG(s) = 0$ with $s = j\omega$)



   (a) Sketch the root locus for this system.

   (b) Find the point on the imaginary axis where the poles cross over from the left-half to the right-half plane.

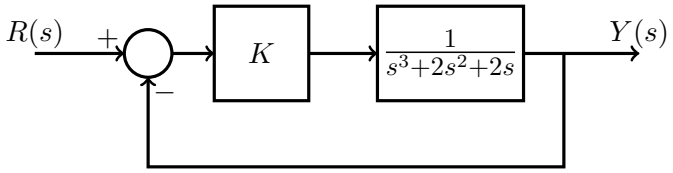
   (c) Find the value K that corresponds to the crossing point found in part (b)

5. Quiz Question Monday: Consider the following unity gain feedback system. By varying K, what is the smallest possible steady state error ($e(\infty) = r(\infty) - y(\infty)$) for

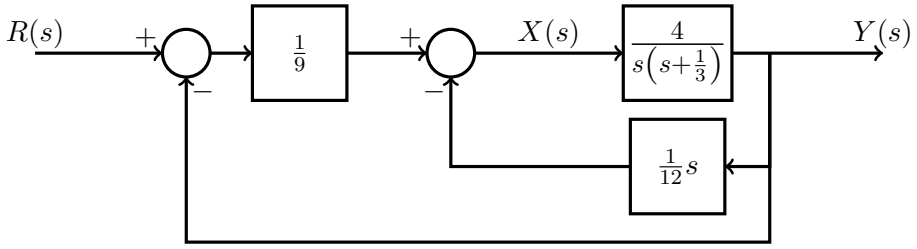
   (a) A unit step reference command $r(t) = u(t)$.

   (b) A unit ramp reference command $r(t) = tu(t)$.

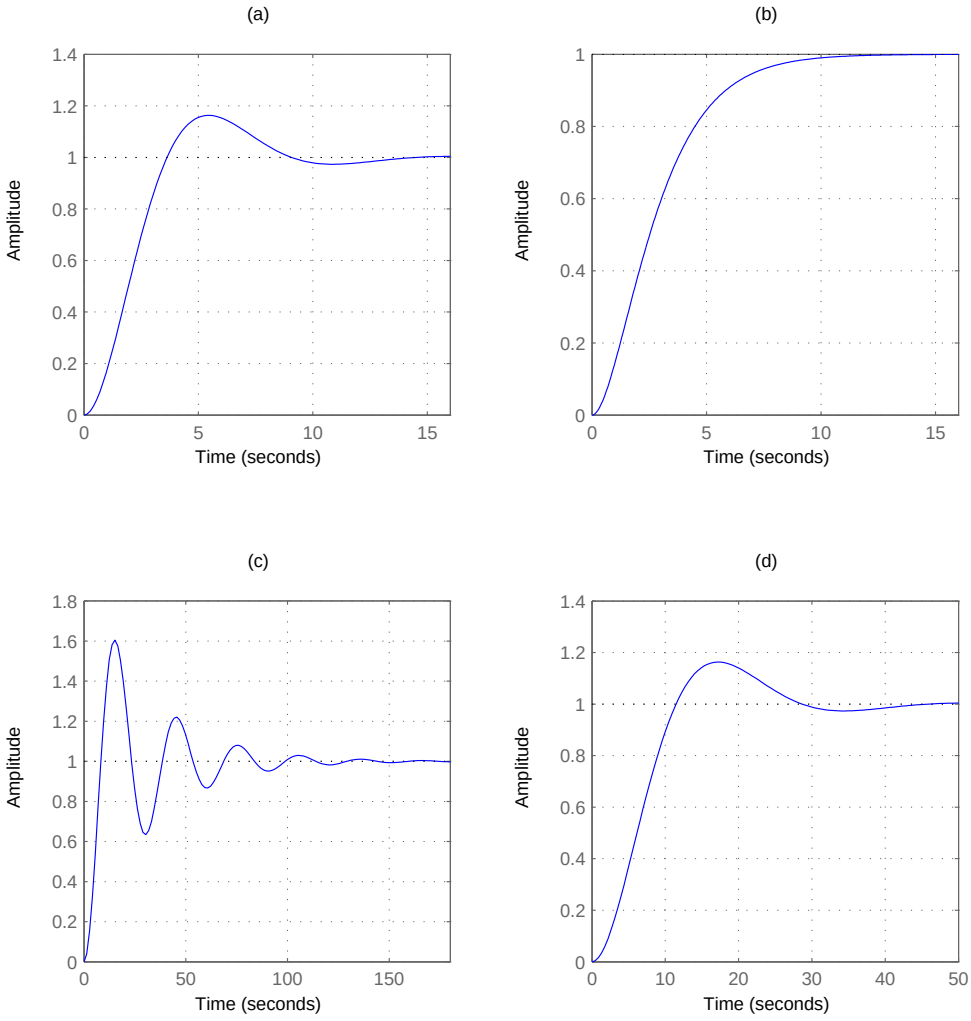
   Remember that the closed loop must be stable to have finite steady state error.



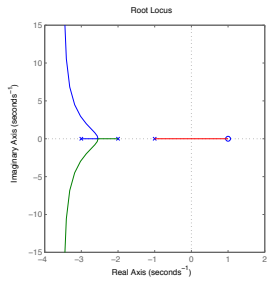
6. Quiz Question Wednesday: The following is a PD control system for a hydraulic actuator.



   A unit step reference command $r(t) = u(t)$ is applied. Which of the following is a plot of the output $y(t)$?


(a)
Amplitude
Time (seconds)
(b)
Amplitude
Time (seconds)
(c)
Amplitude
Time (seconds)
(d)
Amplitude
Time (seconds)

Solutions:

1. $\frac{40}{9} \leq K < 20$
2. (a) $\frac{H(s)}{Q_{in}(s)} = \frac{1/100}{s+1/4}$
   (b) no partial solution
   (c) $\zeta = 0.5$ (approx), $\omega_n = 0.92$
   (d) $K_p = 67$, $K_I = 85$ (using values from (c))
3. 

4. (b) $s = -j\sqrt{5}$
   (c) $K = 18$