EENG307 Unit1: Lecture Summaries

Elenya Grant

Fall 2022

1 L07: Stability

1.1 Mass Spring Damper Example

In Figure 1 we see the input signal $f(t)$ goes to zero as time t approaches infinity and output signal $x(t)$ converges to its initial position (zero). In Figure 2 we see that all components of output signal converge to zero so we expect overall output signal $x(t)$ to also converge to zero.
System Poles: $s = -1, -2, -3$

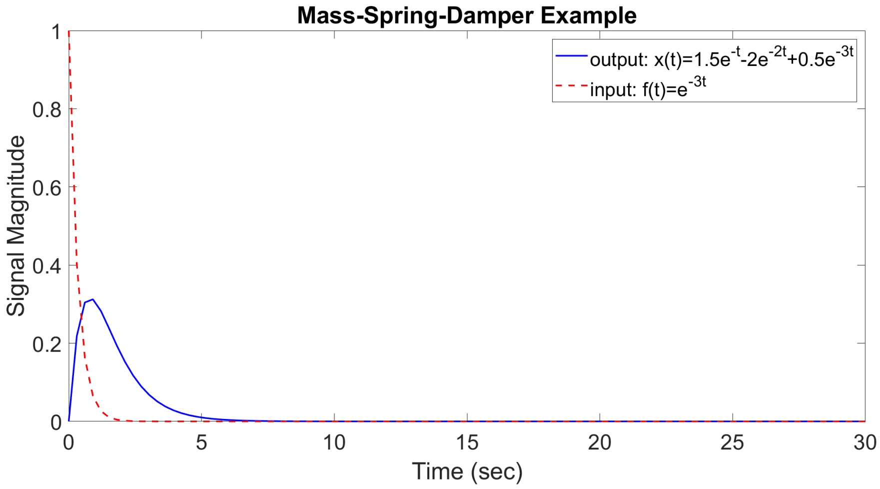

Figure 1: Mass Spring Damper Example - Input (red dotted line) and Output (blue solid line) Signals

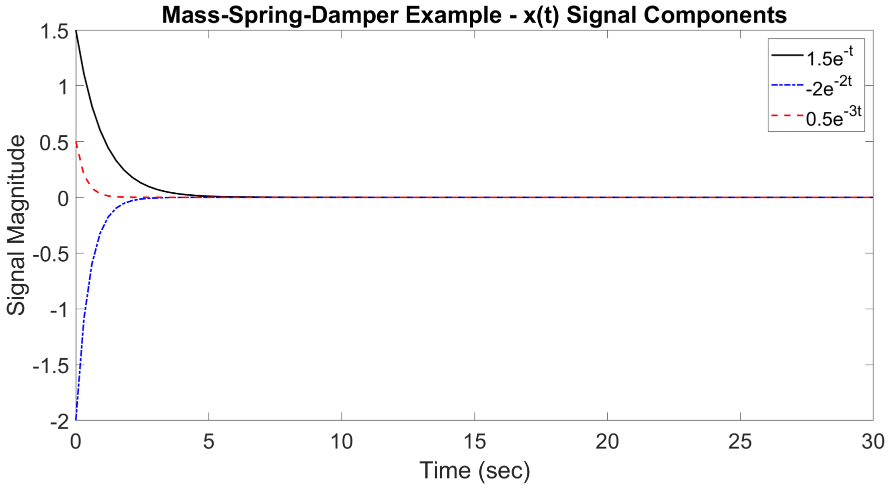

Figure 2: Mass Spring Damper Example - Signal Components of $x(t)$

1.2 Simple Complex Poles Example

In Figure 3 we see that as time t approaches infinity and output signal $f(t)$ converges to its initial position (zero). In Figure 4 we see that all components of output signal converge to zero so we expect overall output signal $f(t)$ to also converge to zero.
System Poles: $s = -1, -1 + j, -1 - j$

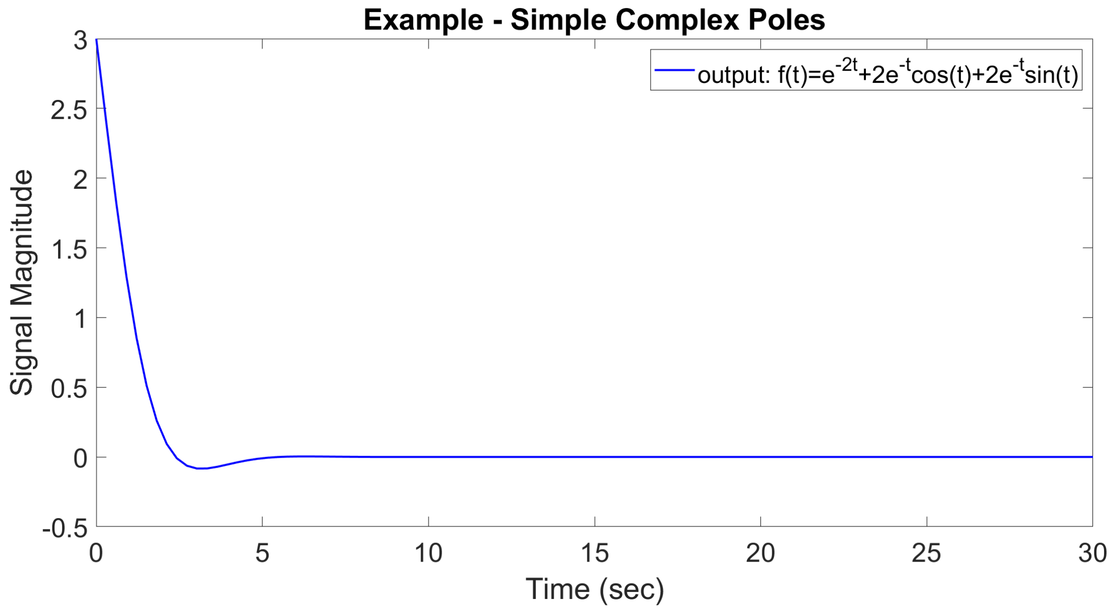

Figure 3: Simple Complex Poles Example - Output Signal

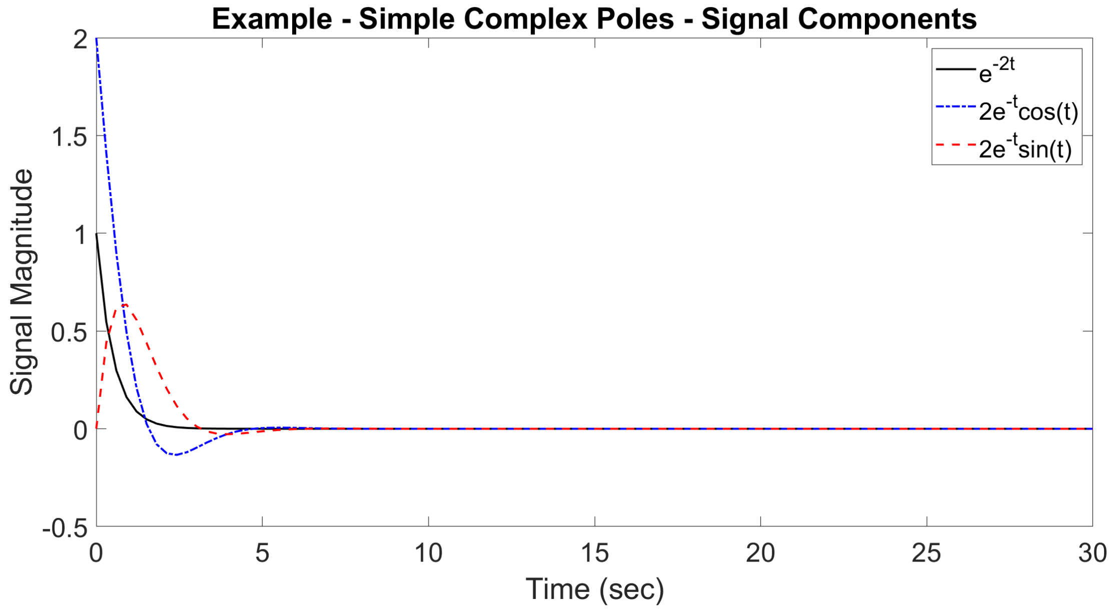

Figure 4: Simple Complex Poles Example - Signal Components of Output Signal

1.3 RLC Circuit Example

In Figure 5 we see the input signal $v_{in}(t)$ goes to one as time t approaches infinity and output signal $v_{out}(t)$ converges to one. In Figure 6 we see that one component of output signal is constantly one and the others converge to zero so we expect overall output signal $v_{out}(t)$ to also converge to one.
System Poles: $s = 0, -2, -2$

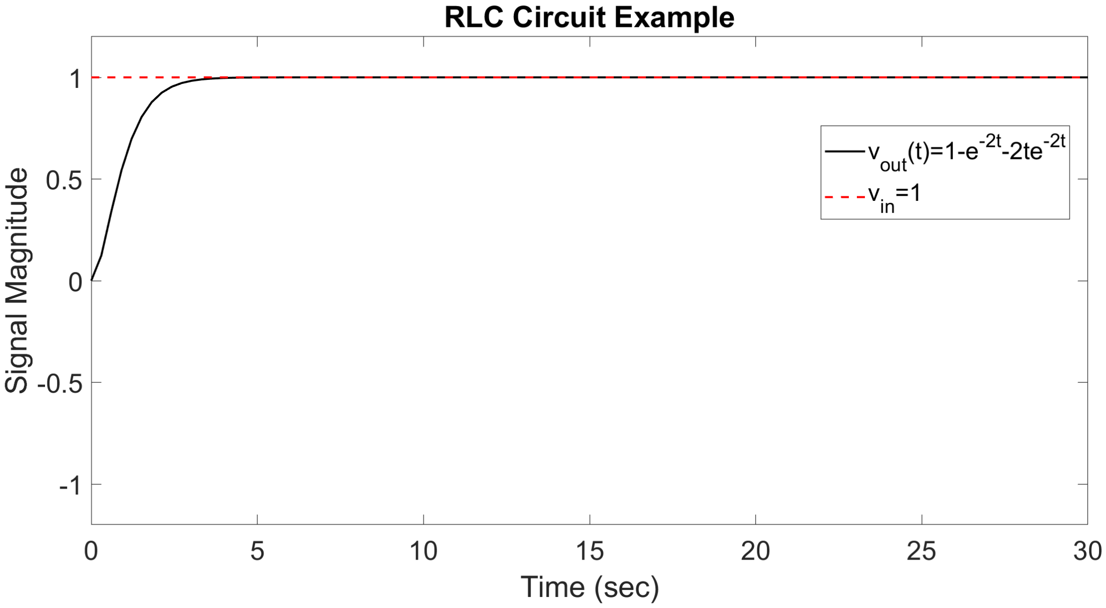

Figure 5: RLC Circuit Example - Input (red dotted line) and Output (black solid line) Signals

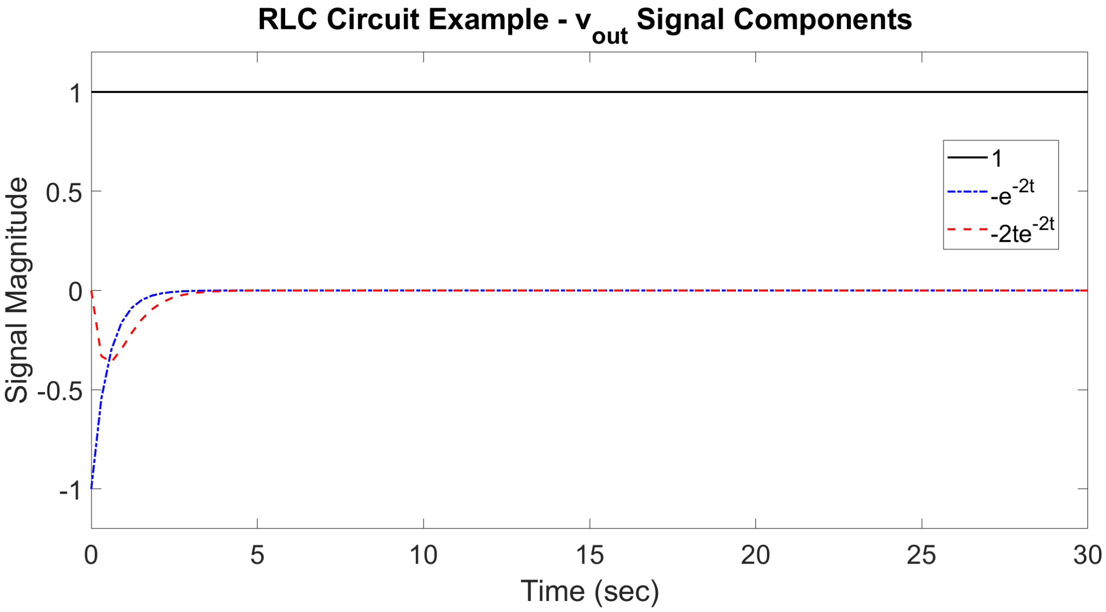

Figure 6: RLC Circuit Example - Signal Components of $v_{out}(t)$

1.4 Spring Damper Example

In Figure 7 we see the input signal $f_{in}(t)$ goes to one as time t approaches infinity and output signal $x(t)$ converges to $\frac{1}{4}$. In Figure 8 we see that one component of output signal is constantly $\frac{1}{4}$ and the other converges to zero so we expect overall output signal $x(t)$ to also converge to $\frac{1}{4}$.
System Poles: $s = 0, -2$

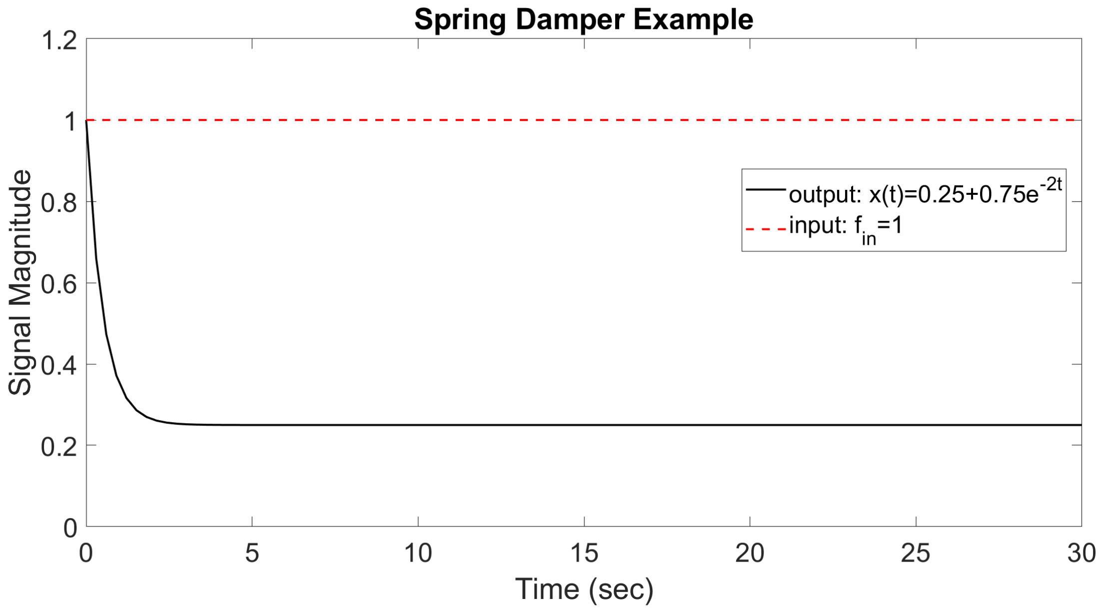

Figure 7: Spring Damper Example - Input (red dotted line) and Output (black solid line) Signals

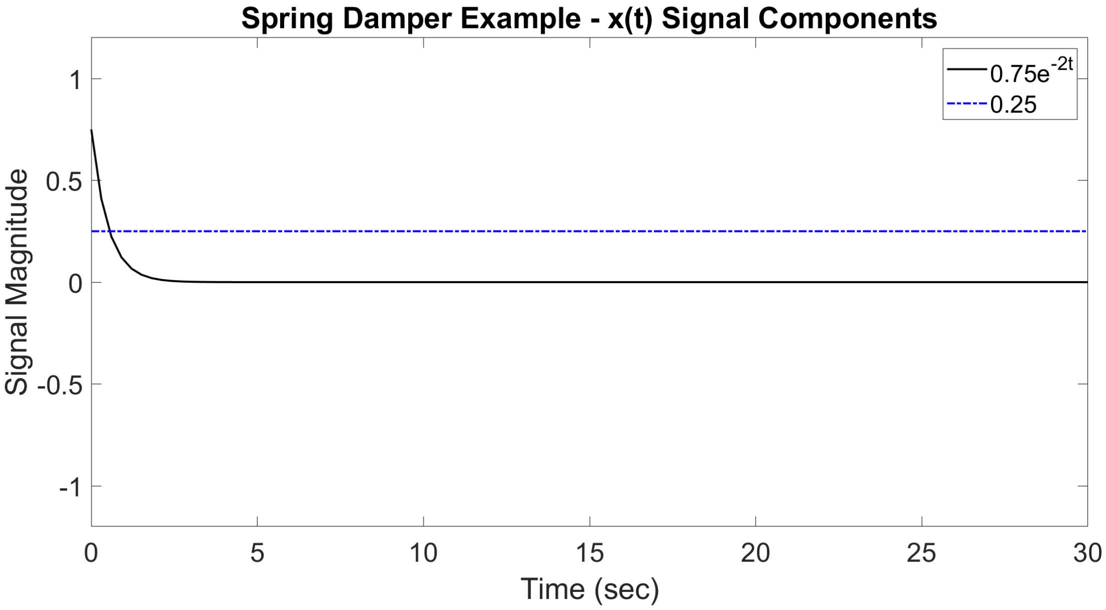

Figure 8: Spring Damper Example - Signal Components of $x(t)$

1.5 Thought Experiment

Notice that the coefficients for t in the exponentials are always negative in the examples, so our outputs converge to some value. What would you expect to happen to our output signal if one of these was positive? Suppose our system had repeated poles at $s = 0.2$ and the Laplace transform of our output signal was $\frac{1}{(s-0.2)^2+1}$ which is $e^{0.2t} sin(t)$ in the time domain (plotted in Figure 9).

In this thought experiment, as time approaches infinity, our system does not diverge, and will also approach infinity eventually. Does this look stable to you? What if this was a plot of an object's position? Does it seem like a "stable" system if it diverges? What was different about this system and the examples? If you thought that the difference is the pole has a positive real part - you're correct! We will discuss that concept in more detail in later lecture.

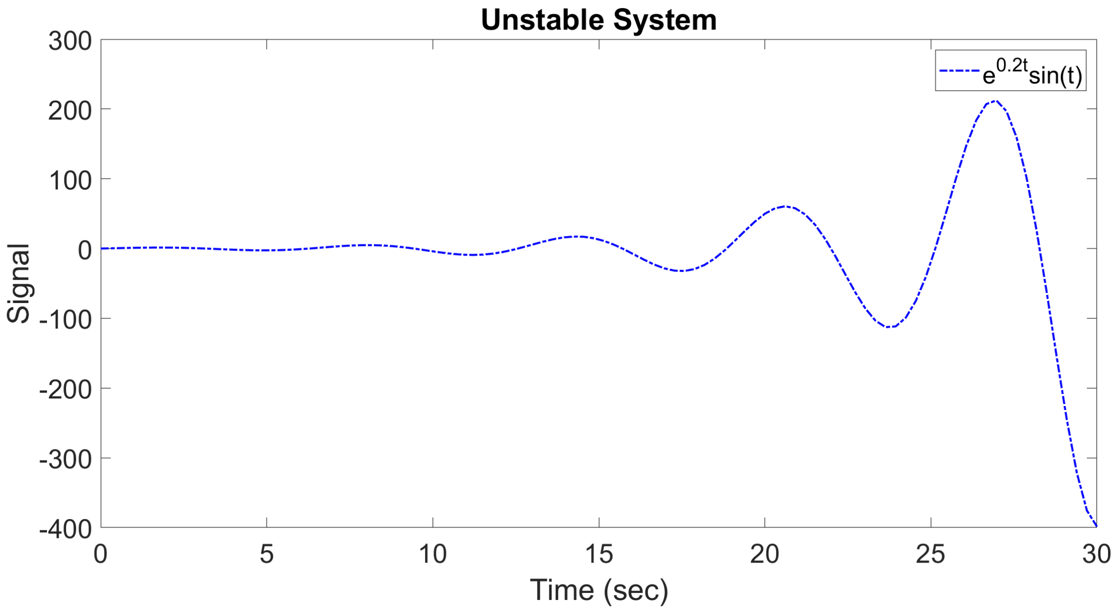

Figure 9: Random Unstable System Example