EENG307: Fluid Systems and System Analogies[1]

Lecture 10

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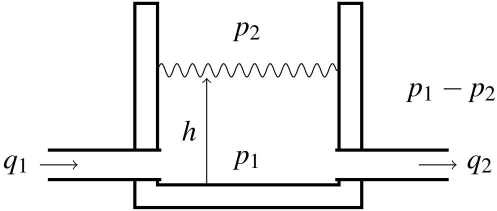
Tank

The change in the volume of fluid inside the tank is equal to the difference between the input and output volumetric flow.



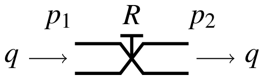
$$p_1 - p_2 = \rho g h$$

Tank Area: A

$$A\frac{dh}{dt} = q_1 - q_2 \qquad \frac{A}{\rho g}\frac{d(p_1 - p_2)}{dt} = q_1 - q_2$$

Linear Valve

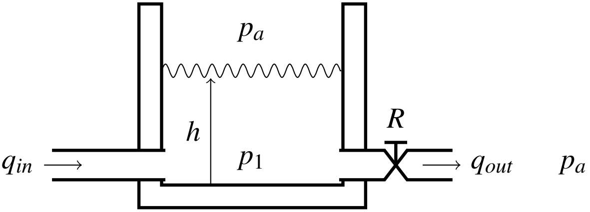
A valve causes a restriction that causes the pressure on one end of the value to be higher than the other end. When this pressure drop is proportional to the flow, the valve is linear with valve constant R. (Most valves are nonlinear, however.) By unit analysis, the units of valve resistance are [N s m^{-5}] or equivalently [kg $m^{-4}s^{-1}$].



$$p_1 - p_2 = Rq$$

Tank and Valve Example

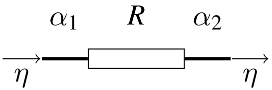
Assume the valve is linear with valve constant R and that the density of the fluid is ρ. The valve empties to atmospheric pressure, p_a, which is the same as the pressure at the top of the tank.



Tank Area: A

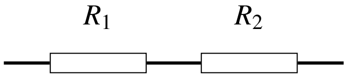
Generic Lumped Element

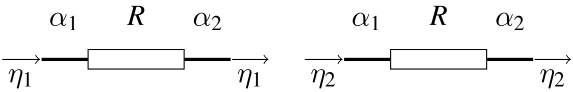
across variable: α
through variable: η



component law: $\alpha_1 - \alpha_2 = R\eta$
or $\frac{d(\alpha_1 - \alpha_2)}{dt} = R\eta$
or $\alpha_1 - \alpha_2 = R\frac{d\eta}{dt}$
or $\cdots$

Generic Connection Rules

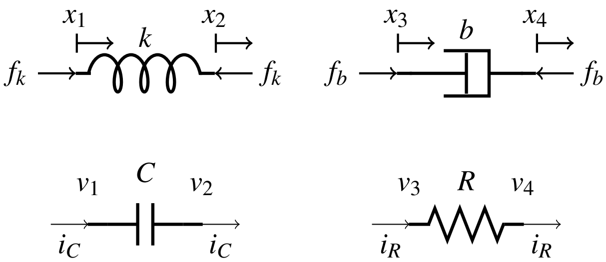




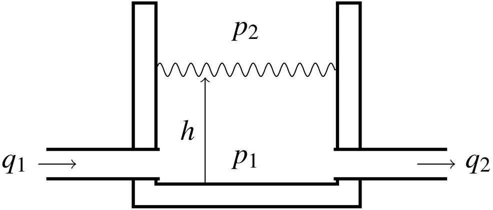
Across and through variables

Domain	Across Variable	Through Variable
Electrical	Voltage	Current
Translational Mechanical	Position	Force
Fluid	Pressure	Flow
Rotational Mechanical	Angular Position	Torque
Thermal	Temperature	Heat Flow

Force flow vs Current flow

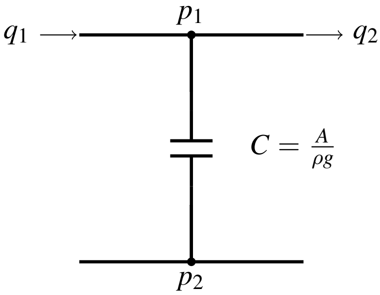


Electrical Analogy for Fluid Elements:
Tank



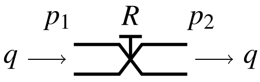
Tank Area: A

$$\frac{A}{\rho g}\frac{d(p_1 - p_2)}{dt} = q_1 - q_2$$

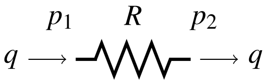


$$C\frac{d(p_1 - p_2)}{dt} = q_1 - q_2$$

Electrical Analogy for Fluid Elements:
Valve

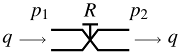
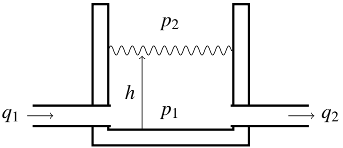
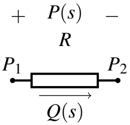
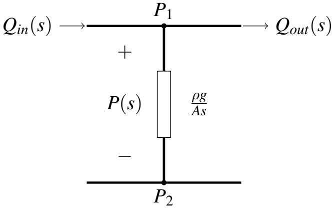


$$p_1 - p_2 = Rq$$



$$p_1 - p_2 = Rq$$

Fluid Impedances

<table>
<tr><th></th><th>valve</th><th>tank</th></tr>
<tr><td>Component</td><td>$p = p_1 - p_2$
</td><td>
Tank Area: A</td></tr>
<tr><td>Component law</td><td>$p = Rq$</td><td>$\frac{A}{\rho g}\frac{dp}{dt} = q_{in} - q_{out}$</td></tr>
<tr><td>Laplace Transform</td><td>$P(s) = RQ(s)$</td><td>$\frac{A}{\rho g} sP(s) = Q_{in}(s) - Q_{out}(s)$</td></tr>
<tr><td>Impedance Component</td><td></td><td></td></tr>
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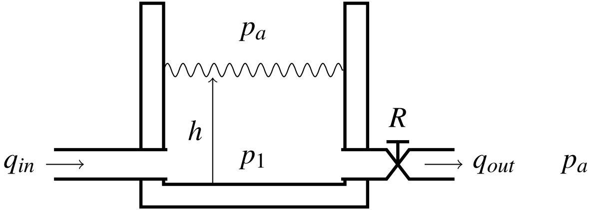
Tank and Valve Problem

Example: Find the equivalent impedance model of the tank and valve system below with input flow q_{in} and output flow q_{out}.



Tank Area: A

Tank system nodes

Tank system impedance model