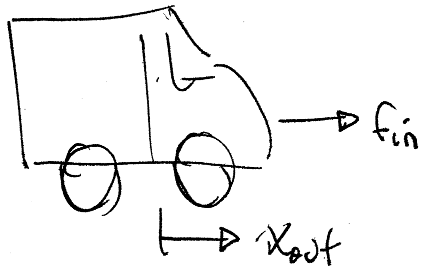
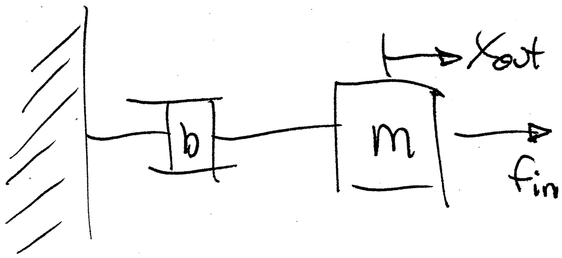
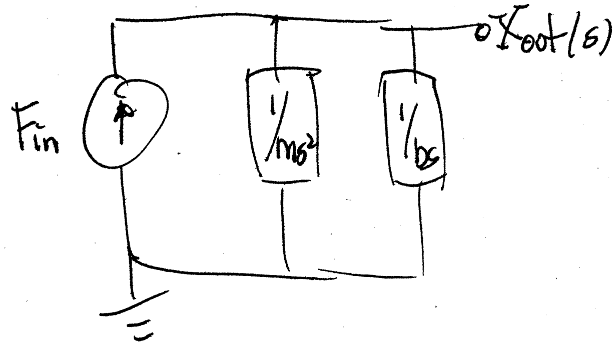
Parking The Honda Element



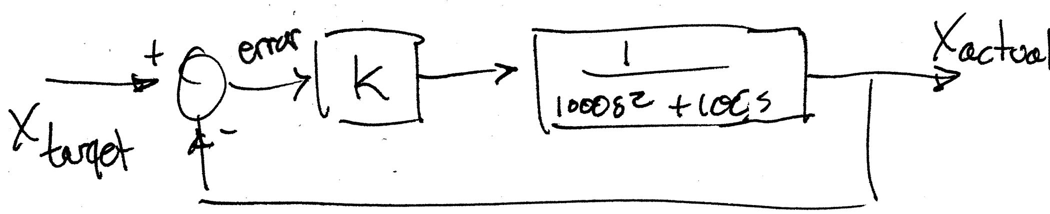




$$\frac{X_{out}(s)}{F_{in}(s)} = \frac{1}{ms^2+bs}$$

let $m = 1000\,kg$ $b = 100$

Proportional Controller



Use Block Diagram to simplify

$$\frac{X_{actual}}{X_{target}} = \frac{\frac{K}{1000s^2+100s}}{1+\frac{K}{1000s^2+100s}} * \frac{1000s^2+100s}{1000s^2+100s} = \frac{K}{1000s^2+100s+K}$$

Let $K = 10$

Apply $10u(t)$ as X_{target} so $\frac{10}{s}$ as input

$$X_{actual} = \frac{10}{s}\,\frac{10}{1000s^2+100s+10} = \frac{1}{s(10s^2+s+1/10)}$$

Partial Fraction Expansion

$$\frac{1}{s(10s^2+s+1/10)} = \frac{A}{s} + \frac{Bs+C}{10s^2+s+1/10}$$

A: Multiply by s and eval @ $s=0$ $\quad \frac{1}{1/10} = \boxed{A = 10}$

* eval @ $s=1$

$$\frac{1}{10+1+1/10} = 10 + \frac{B+C}{10+1+1/10} \qquad B+C = 1 - 10\cdot\frac{111}{10} = -110$$

* eval @ $s=-1$

$$\frac{1}{-1(10-1+1/10)} = -10 + \frac{C-B}{10-1+1/10} \qquad -B+C = -1+10\cdot\frac{91}{10} = 90$$

$\therefore \quad 2C = -20 \quad C = -10$

$\therefore \quad B = -110 - C = -100$

$$X_{actual} = \frac{10}{s} + \frac{-100s-10}{10s^2+s+1/10} = \frac{10}{s} - 100\,\frac{10s+1}{100s^2+10s+1}$$

For inverse Laplace complete square in denominator...

$$X_{actual} = \frac{10}{s} - 100\frac{(10s+1)}{100s^2+10s+1}$$ Complete square

$$-\frac{100\,(10s+1)}{100s^2+10s+1} = -10\,\frac{s+1/10}{s^2+s/10+1/100} =$$

$$-10\,\frac{(s+1/20)+1/20}{(s+1/20)^2+\left(\sqrt{\frac{3}{400}}\right)^2} = -10\,\frac{s+1/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2} - \frac{10}{\sqrt{3}}\,\frac{\sqrt{3}/20}{(s+1/20)^2+\left(\sqrt{3}/20\right)^2}$$

So $$X_{actual} = \frac{10}{s} - 10\,\frac{s+1/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2} - \frac{10}{\sqrt{3}}\,\frac{\sqrt{3}/20}{(s+1/20)^2+\left(\frac{\sqrt{3}}{20}\right)^2}$$

Inverse Laplace

$$X_{actual}(t) = 10u(t) - 10e^{-t/20}\cos\left(\frac{\sqrt{3}}{20}t\right) - \frac{10}{\sqrt{3}}e^{-t/20}\sin\left(\frac{\sqrt{3}}{20}t\right)$$