CPE vs RCs

April 27, 2023

1 CPE element approximation

The behavior of a constant phase element can be approximated by N RC-circuits.

$$Z_{RC} = \sum_{k=1}^{N} \frac{1}{\frac{1}{R_k} + j\omega C_k} \approx \frac{1}{A(j\omega)^{\phi}}$$

To build a real CPE Element, either an adjustable inductance or an adjustable capacitance is necessary [1].

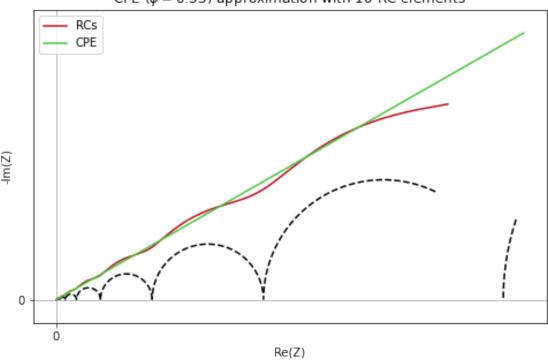
[1] S. Holm, T. Holm, und Ø. G. Martinsen, "Simple circuit equivalents for the constant phase element", PLOS ONE, Bd. 16, Nr. 3, S. e0248786, März 2021, doi: 10.1371/journal.pone.0248786.

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[39]: import sympy as sp
      import numpy as np
      import matplotlib.pyplot as plt
      N = 10 # Number of RC circuits
      R = np.logspace(-4, -1, N) # R's
      C = np.logspace(1, 7, N) # C's
      f_low = -6 # Lower frequency in 1eHz
      f_high = 6 # Upper frequency in 1eHz
      f_n = 100000  # Sample points
      f = np.logspace(f_low, f_high, f_n) # Frequency array in Hz
      w = np.multiply(f, 2*np.pi) # Pulsatance in 1/s
      phi cpe = 0.33
      omega = sp.symbols('omega')
      parts = []
      for i in range(N): # Create parts of impedance equations
          parts.append(1/(1/R[i] + 1j*omega*C[i]))
      Z_RC = sum(parts) # Sum all parts up to one expression
      Z_RC = sp.lambdify(omega, Z_RC, "numpy") # Evaluate expression numericaly with_
       \hookrightarrow numpy
      Z_RC = Z_RC(w)
      Z_RCs = []
      RO = 0
      for i in range(N):
          Z_RCs.append(RO + 1/(1/R[i] + 1j*w*C[i]))
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RO += R[i]

Z_CPE = 1/(500*(1j*w)**phi_cpe)
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[40]: <matplotlib.legend.Legend at 0x26f6ef22e90>



CPE (ϕ = 0.33) approximation with 10 RC elements