

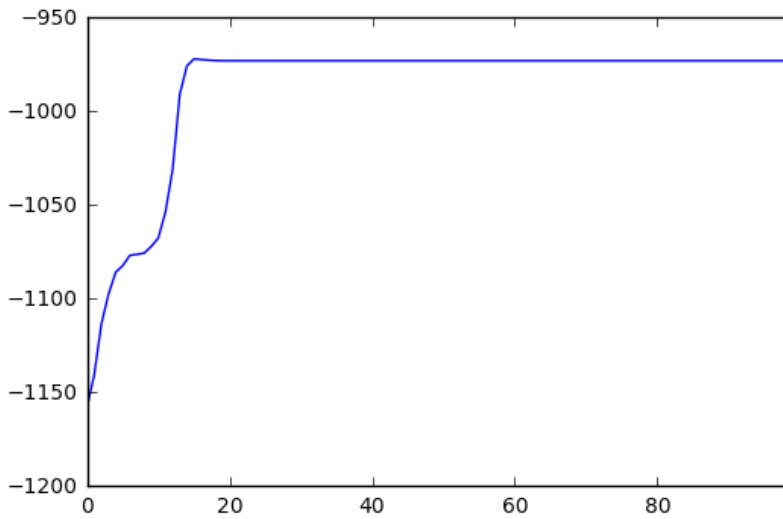
Problem 1

a)

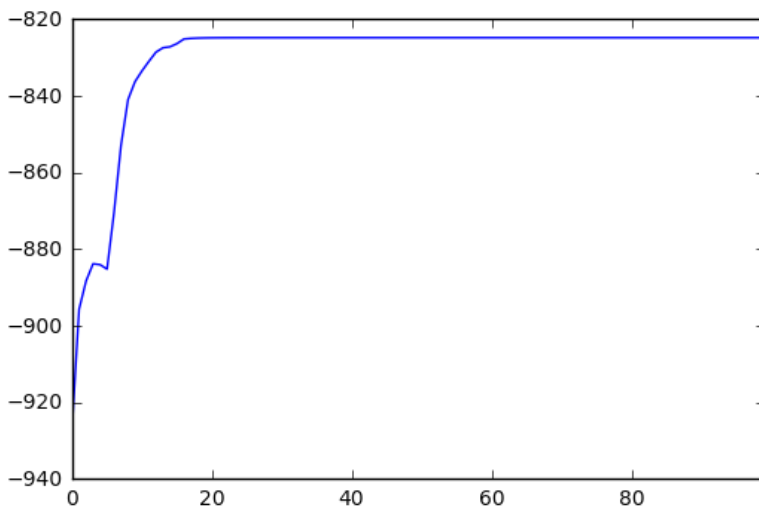
Class EM_GMM in GMM.py

b)

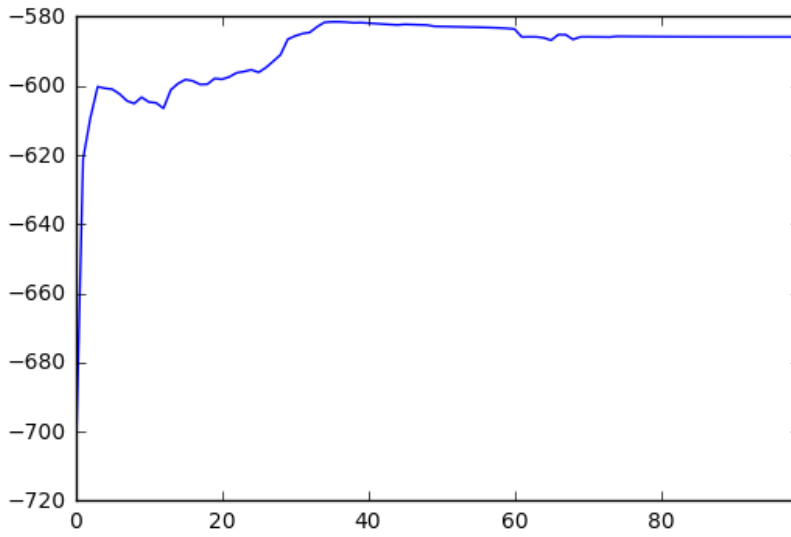
$K = 2$



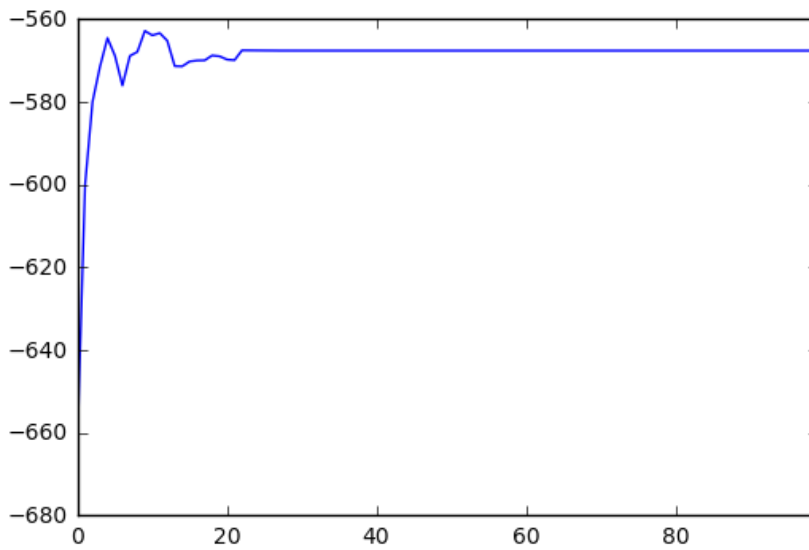
$K = 4$



$K = 8$



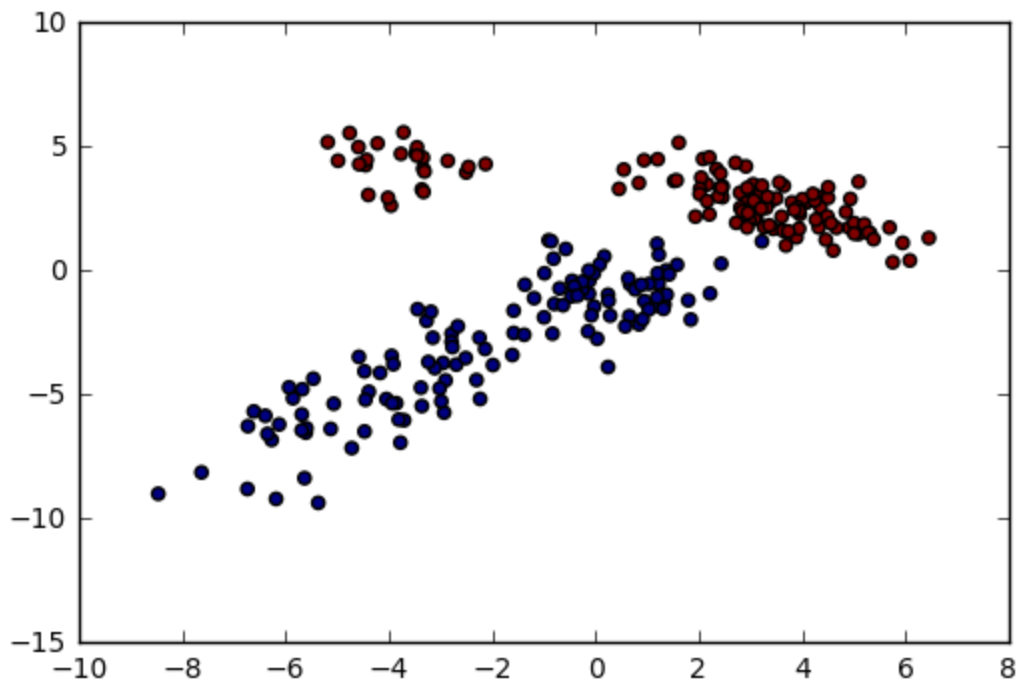
$K = 10$



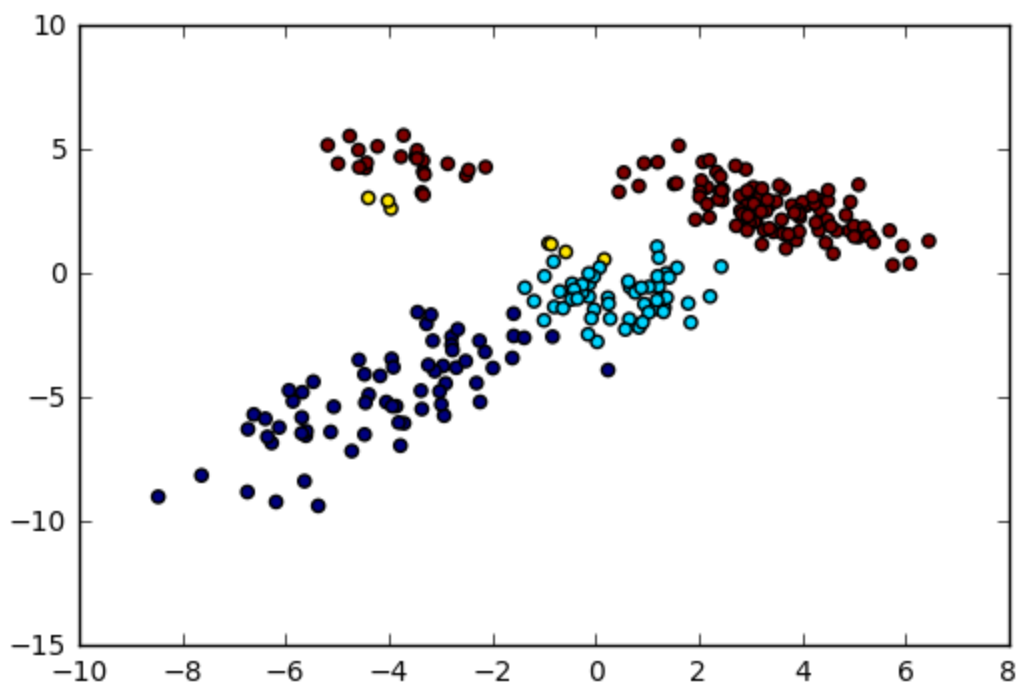
The objective function is converging relatively quickly.

c)

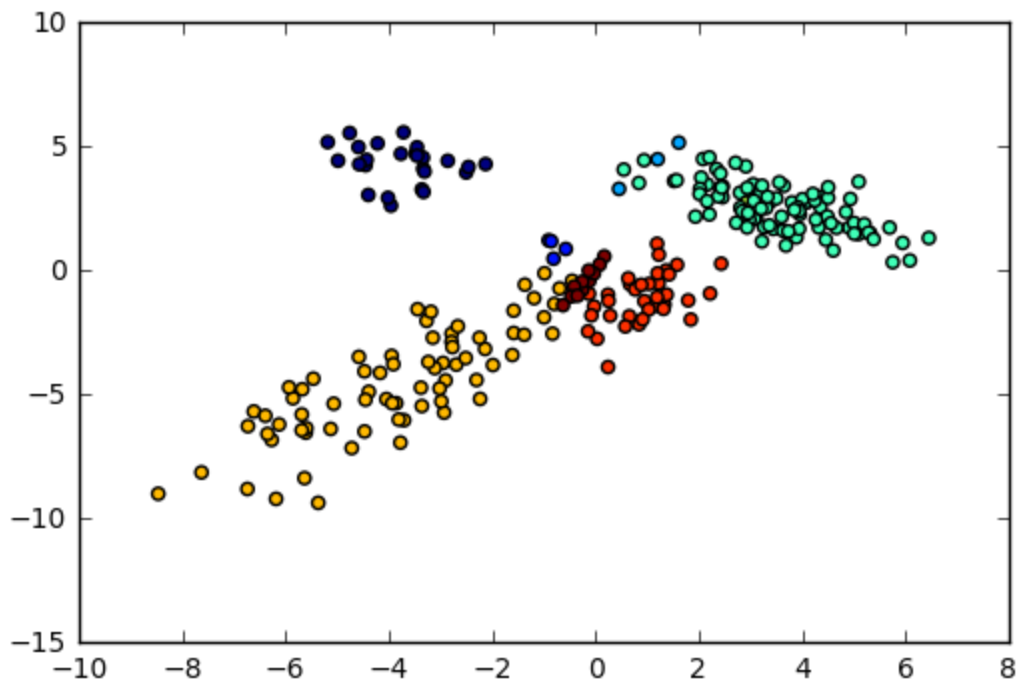
K=2



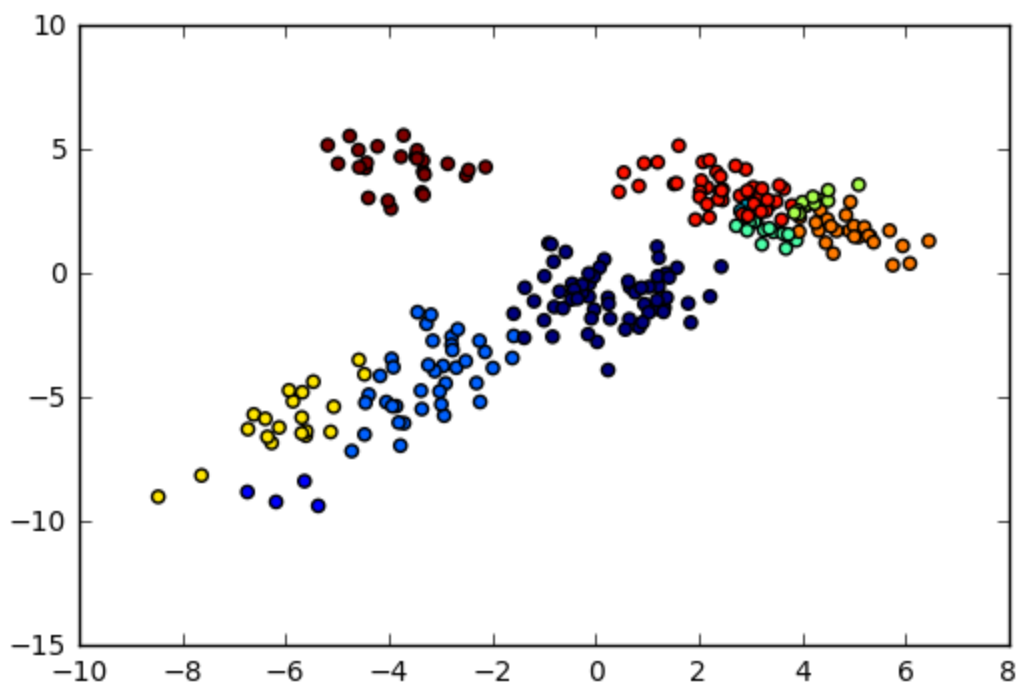
K=4



$K = 8$



$K = 10$



Since we are forcing the number of clusters the data is being forced into these clusters regardless.

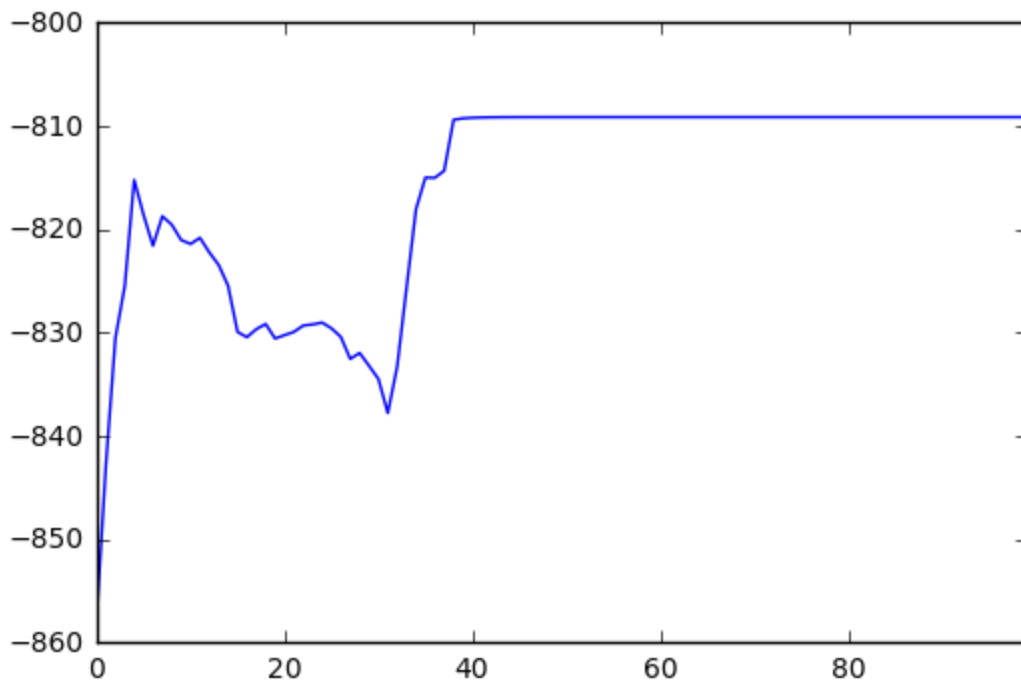
Problem 2

a)

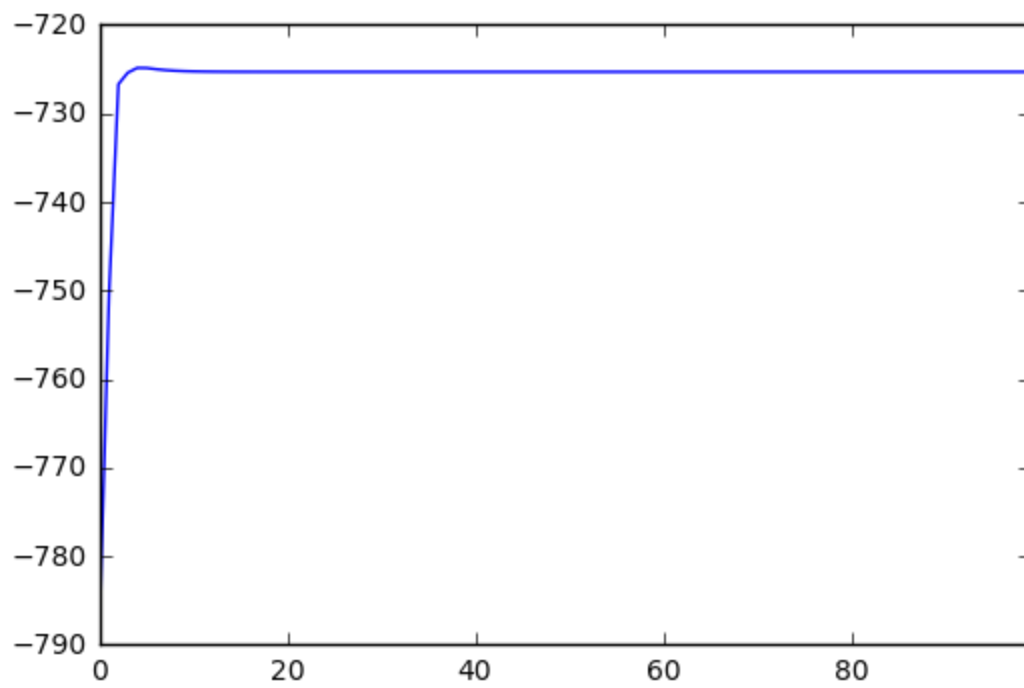
Class VI_GMM in GMM.py

b)

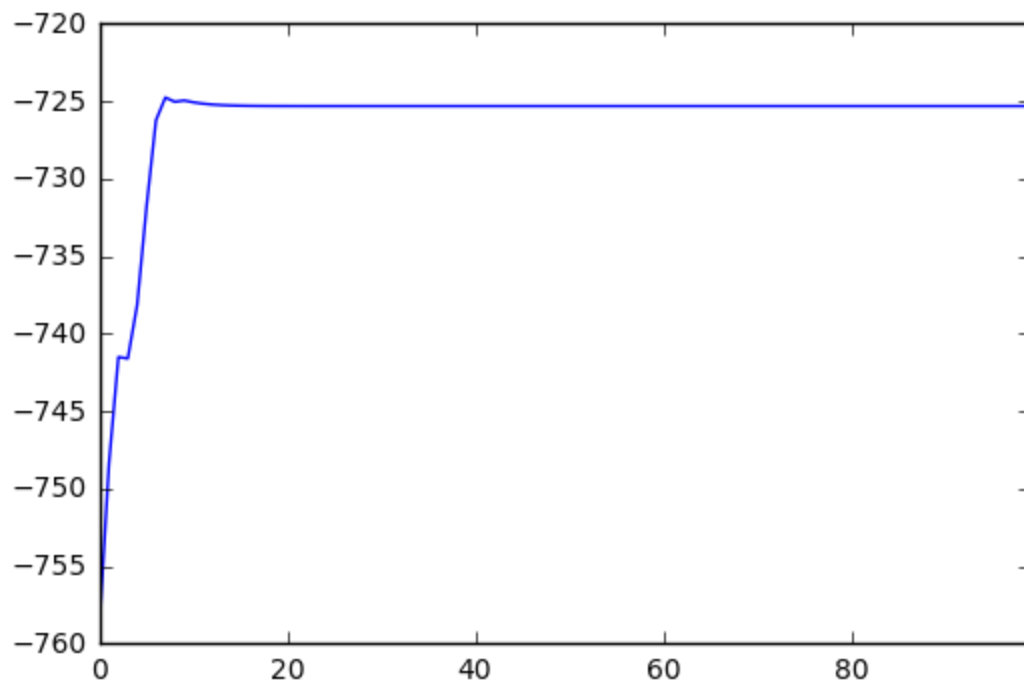
K = 2



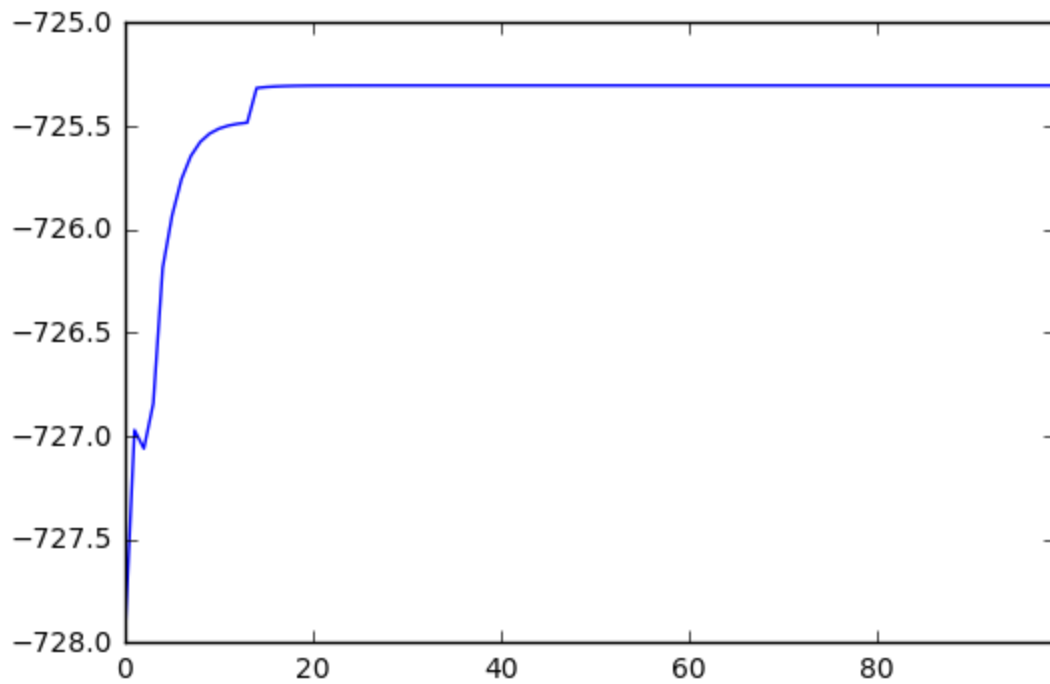
K = 4



$K = 10$



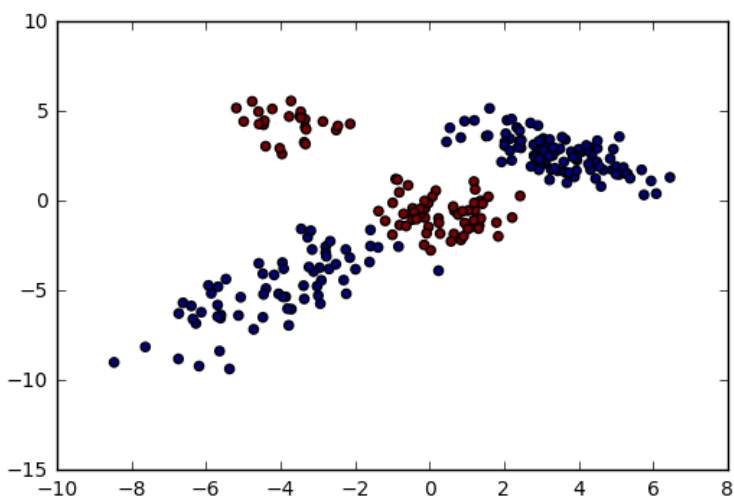
$K = 25$



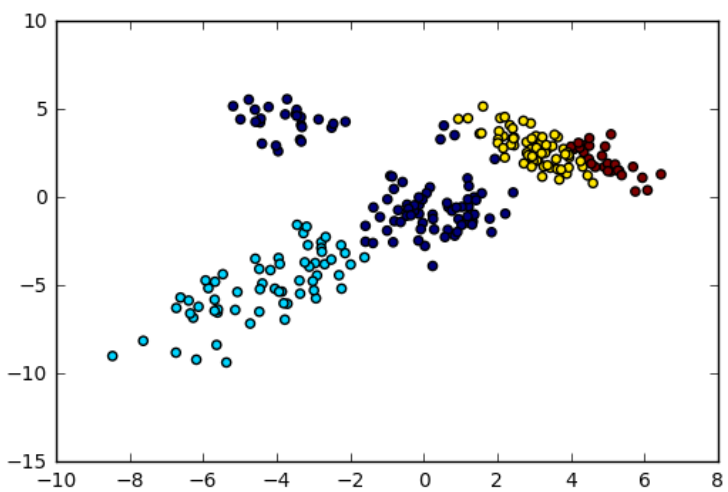
Something seem to be off with the objective function either the derivation or the code. I wasn't able to get it better than that but the data seem to be converging with no issues

c)

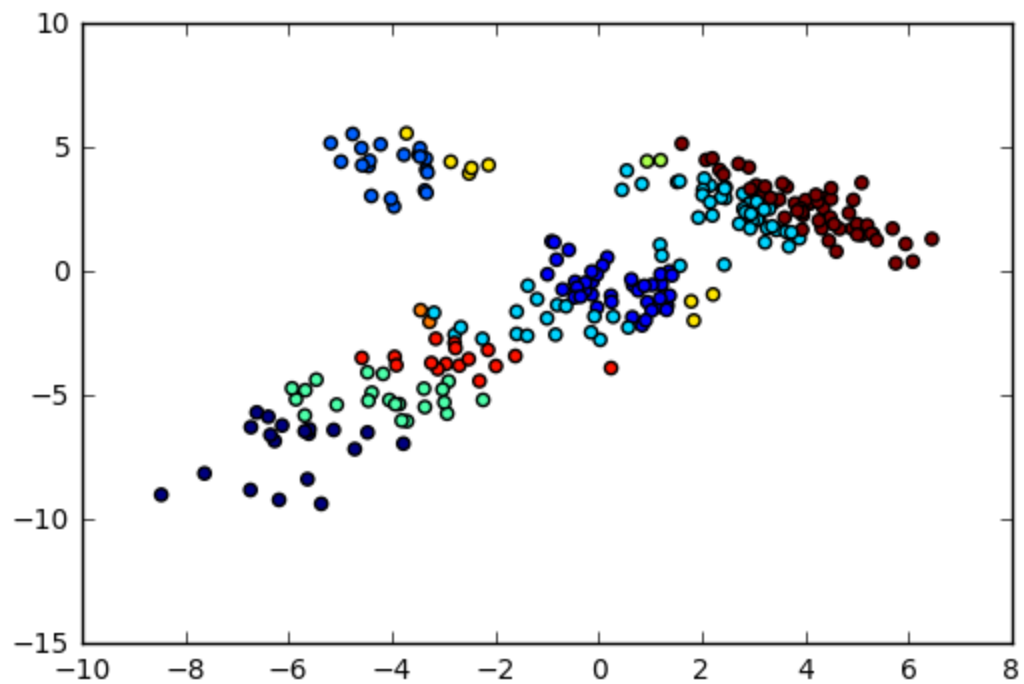
$K = 2$



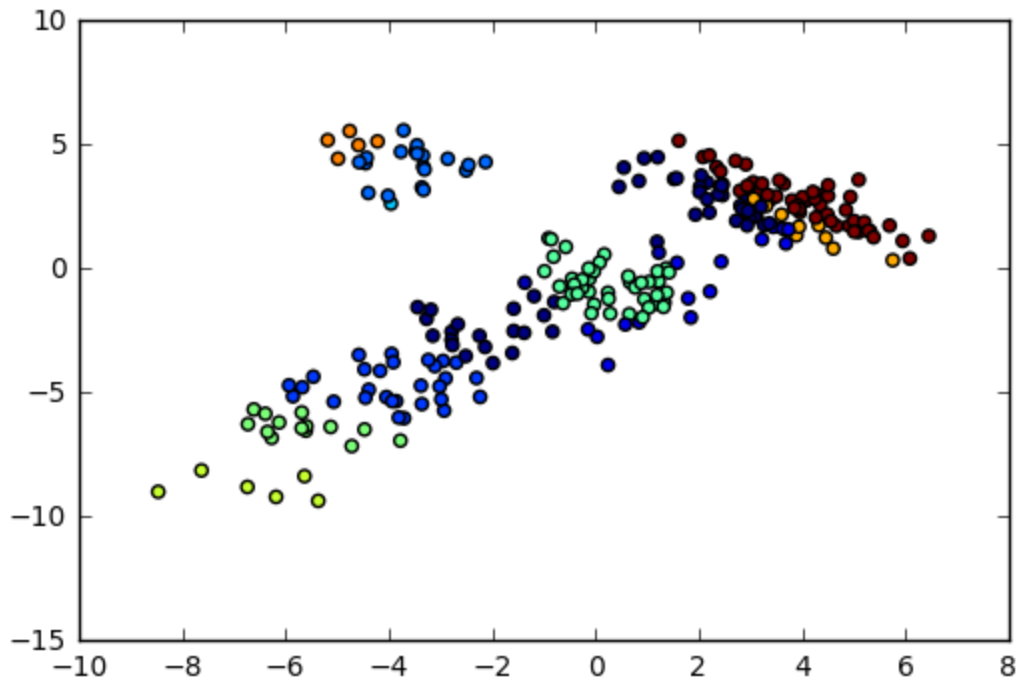
$K = 4$



K=10



K=25



The clusters start overlapping when we increase K a lot
 Problem 3

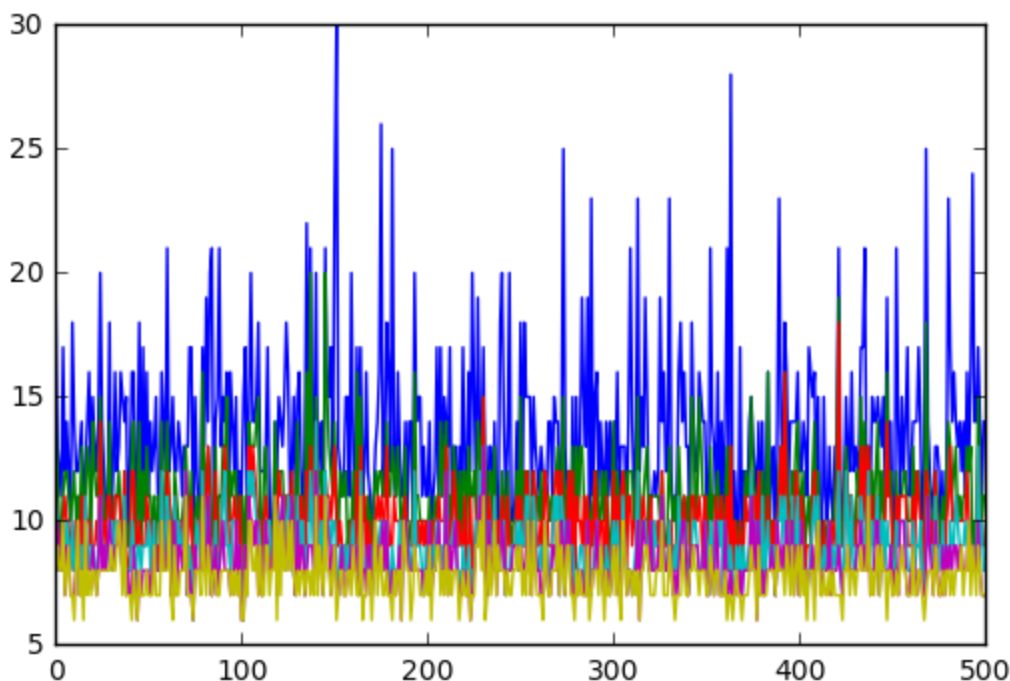
1)

Class GS_GMM() in GMM.pu

2)

I wasn't sure how to plot this one so I am also going to copy the matrix. Each color below is an iteration.

The matrix is a list of lists each list has the number of points in the top 6 clusters



[[19, 14, 11, 11, 8, 8], [13, 12, 10, 10, 10, 9], [12, 10, 9, 8, 8, 8], [11, 9, 9, 8, 8, 8], [17, 11, 10, 10, 10, 10], [12, 12, 11, 8, 7, 7], [14, 12, 10, 10, 9, 8], [12, 11, 10, 9, 9, 9], [10, 10, 8, 8, 7, 7], [18, 11, 10, 10, 7, 7], [14, 12, 8, 8, 7, 6], [12, 10, 10, 8, 8, 8], [12, 11, 10, 10, 10, 10], [13, 10, 8, 7, 7, 7], [14, 13, 11, 9, 8, 8], [10, 9, 8, 8, 7, 6], [12, 11, 11, 11, 10, 10], [14, 13, 10, 9, 8, 7], [16, 12, 10, 8, 8, 8], [12, 9, 9, 9, 8, 7], [15, 14, 9, 9, 9, 8], [13, 12, 9, 8, 8, 8], [12, 11, 10, 8, 7, 7], [14, 11, 9, 9, 8, 8], [20, 15, 14, 11, 11, 8], [14, 12, 10, 9, 8, 8], [12, 10, 10, 9, 9, 8], [10, 10, 8, 8, 8, 8], [11, 11, 11, 9, 9, 8], [18, 14, 11, 10, 10, 8], [10, 10, 10, 10, 9, 9], [12, 9, 9, 8, 8, 8], [16, 11, 11, 10, 9, 8], [12, 11, 11, 10, 9, 9], [13, 12, 11, 10, 10, 10], [16, 10, 10, 9, 9, 9], [15, 9, 9, 8, 8, 7], [14, 12, 12, 11, 10, 10], [15, 10, 8, 7, 7, 7], [9, 9, 8, 8, 8, 7], [12, 10, 9, 8, 7, 7], [16, 13, 12, 10, 7, 6], [16, 14, 12, 9, 9, 8], [10, 9, 9, 7, 7, 7], [12, 10, 9, 9, 6, 6], [18, 14, 10, 9, 8, 8], [10, 9, 9, 8, 7, 7], [17, 12, 9, 9, 8, 7], [11, 10, 9, 9, 8, 7], [16, 13, 11, 9, 8, 8], [11, 11, 10, 10, 8, 7], [14, 11, 10, 8, 7, 7], [12, 9, 9, 9, 8, 7], [13, 11, 10, 10, 9, 9], [15, 8, 8, 8, 8, 7], [11, 11, 9, 8, 8, 7], [10, 10, 8, 8, 8, 7], [16, 14, 12, 12, 10, 10], [14, 10, 9, 9, 8, 8], [12, 10, 9, 8, 8, 8], [21, 14, 12, 11, 9, 8], [12, 11, 11, 10, 10, 10], [13, 10, 10, 7, 7, 7], [15, 8, 8, 8, 7, 6], [12, 12, 11, 11, 10, 9], [12, 11, 8, 8, 7, 7], [14, 10, 10, 10, 9, 7], [13, 10, 9, 9, 9, 8], [11, 10, 10, 9, 8, 8], [13, 11, 10, 10, 9, 8], [13, 13, 12, 12, 12, 11], [13, 11, 9, 7, 7, 7], [17, 10, 10, 9, 8, 7], [17, 12, 11, 8, 7, 7], [9, 8, 7, 6, 6, 6], [15, 10, 9, 9, 9, 8], [12, 11, 10, 10, 8, 8], [10, 10, 10, 9, 8, 8], [11, 9, 8, 8, 8, 7], [17, 16, 12, 12, 11, 11], [13, 10, 9, 8, 8, 8], [19, 12, 11, 10, 8, 7], [14, 13, 13, 11, 11, 9], [20, 12, 10, 9, 9, 8], [21, 10, 7, 7, 7, 7], [10, 9, 8, 8, 8, 8], [14, 13, 12, 10, 9, 8], [15, 9, 8, 8, 7, 7], [21, 11, 8, 8, 8, 7], [13, 11, 10, 9, 9, 8], [15, 13, 11, 11, 10, 8], [14, 13, 13, 12, 8, 8], [16, 15, 11, 9, 8, 8], [10, 10, 10, 9, 8, 8], [16, 12, 8, 8, 7, 7], [14, 11, 10, 10, 8, 8], [12, 12, 11, 10, 8, 7], [15, 12, 10, 10, 10, 8], [10, 9, 8, 8, 7, 7], [12, 11, 11, 9, 9, 8], [11, 11, 8, 7, 6, 6], [15, 8, 7, 6, 6, 6], [17, 11, 9, 9, 9, 8], [10, 9, 8, 7, 7, 7], [15, 14, 13, 12, 11, 10], [20, 13, 11, 11, 10, 10], [14, 13, 13, 12, 10, 8], [14, 13, 12, 11, 9, 8], [15, 14, 11, 10, 10, 9], [18, 15, 10, 10, 7, 7], [12, 10, 10, 9, 9, 9], [12, 9, 7, 7, 7, 7], [12, 12, 10, 8, 8, 7], [13, 13, 11, 10, 10, 9], [17, 10, 10, 10, 9, 9], [11, 10, 10, 10, 8, 7], [9, 9, 8, 8, 8, 8], [10, 10, 9, 9, 9, 8], [14, 14, 11, 11, 11, 11], [12, 11, 10, 10, 7, 6], [15, 12, 12, 10, 10, 10], [14, 12, 11, 10, 10, 7], [13, 12, 11, 11,

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