Total Points: *150*

1. (a) $(x_1+x_2+x_4)(x_1+x_2+x_5)(x_1+x_2+x_6)(x_1+x_3+x_4)(x_1+x_3+x_5)(x_1+x_3+x_6)$
   (b) $n!$

2. (a) $F=(b').(a+c').(b+d+e')$
   - Set b to 0. (Unit Clause) $F=(a+c').(d+e')$
   - Assume $a=1$. $F=(d+e')$
   - Assume $d=1$. $F=1$

   Satisfiable with one possible assignment as $(a,b,c,d,e)=(1,0,X,1,X)$

   (b) $F=(a+b').(a'+b).(b+c').(b'+c).(a+c').(a'+c)$
   - Assume $a=1$. $F=(b).(b+c').(b'+c).(c)$
   - Set b to 1 (Unit Clause). $F=c$
   - Set c to 1 (Unit Clause). $F=1$

   Satisfiable with one possible assignment as $(a,b,c)=(1,1,1)$

3. (a) NAND:
   - $y\leftrightarrow(ab)'$
   - $(y+ab)(y'+(ab)')$
   - $(y+a)(y+b)(y'+a'+b')$

   NOR:
   - $y\leftrightarrow(a+b)'$
   - $(y+a+b)(y'+(a+b)')$
   - $(y+a+b)(y'+a'b')$
   - $(y+a+b)(y'+a')(y'+b')$

   NOT:
   - $y\leftrightarrow a'$
   - $(y+a)(y'+a')$

(b)

$((z'+h)(z'+i)(z+h'+i'))((e+h)(e'+h'))((i'+f)(i'+g)(i+f'+g'))((b+f)(b'+f'))((e+a+b)(e+a')(e+b'))((g+c+d)(g+c')(g+d'))$

Since we require an assignment such that $z = 1$, we add another unit clause - z. Therefore, $F = z(z'+h)(z'+i)(z+h'+i')(e+h)(e'+h')(i'+f)(i'+g)(i+f'+g')(b+f)(b'+f')(e+a+b)(e+a')(e+b')(g+c+d)(g+c')(g+d')$

Using the SAT algorithm, this expression gives one possible assignment as
$(a,b,c,d,e,f,g,h,i,z) = (0,0,1,1,0,1,1,1,1,1)$

4. (a)

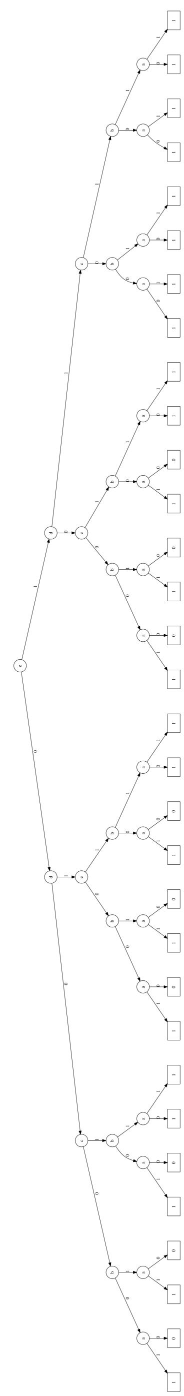

Figure 1: BDD for 4(a)

(b)

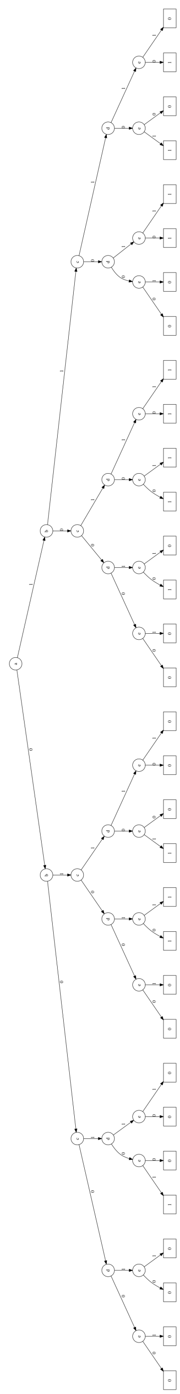

Figure 2: BDD for 4(b)

(c)

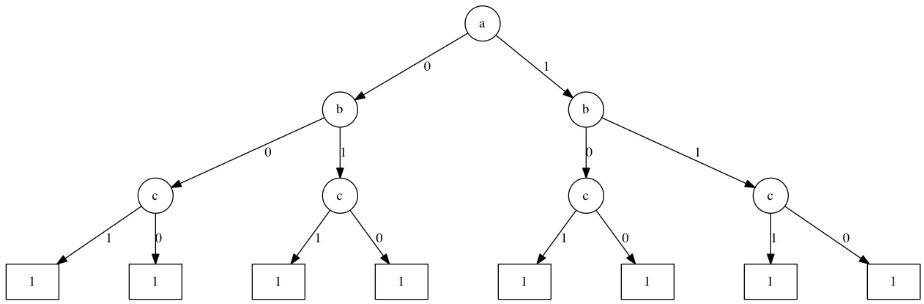

Figure 3: BDD for 4(c)

5. (a)

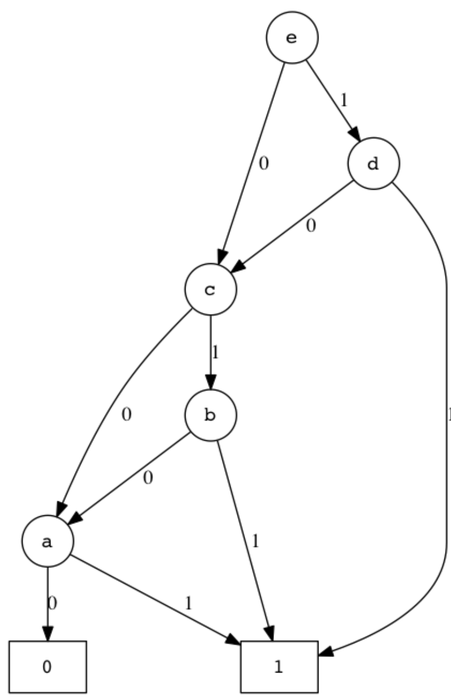

Figure 4: BDD for 5(a)

(b)

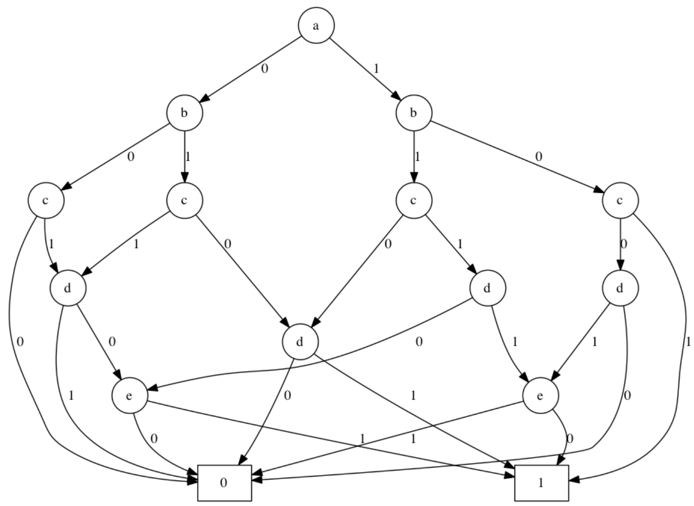

Figure 5: BDD for 5(b)

(c)

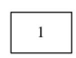

Figure 6: BDD for 5(c)

6. No, the BDD in (5-a) is the smallest graph for this expression with respect to the size of the graph.

7. (a) $F = (a).(a' + b + c).(c' + d + e + f)$
   - Neither Horn nor Definite Horn formulas
   - Can be made into a definite horn by substituting $newB = b, newD = d, newE = e$

   (b) $F = (a + b).(a' + b' + c).(a + b' + c' + d).(a + b' + c + d' + e)$
   - Neither Horn nor Definite Horn formulas
   - Can be made into a definite horn by substituting $newA = a, newC = c, newD = d, newE = e$

   (c) $F = (a + b).(a + b' + d').(a' + b + d' + e)$
   - Neither Horn nor Definite Horn formulas
   - Cannot be converted to a definite horn formula.

   (d) $F = (a').(b' + c' + d).(c + e' + f').(b' + e + f' + g')$
   - Horn Formula
   - Can be made into a definite horn by substituting $newA = a$

8. Definite horn formulas are always satisfiable. An assignment of $True$ to all the variables will satisfy the formula.

   Consider two main observations on general horn formulas:

   1. A unit clause decides what assignment the variable in the clause should have to possibly satisfy the complete formula.
   2. A horn formula without unit clauses will always be satisfiable, since an assignment of $False$ to all the variables will satisfy the formula.

   Let ϕ be the set of horn clauses on Boolean variables from the set V.

   Step 1: Let U be the set of unit clauses in ϕ.

   Step 2: If (U is empty) return "Satisfiable"

   Step 3: Pick a unit clause from U and propagate that literal in ϕ. (Unit propagation)

   Step 4: If the above step forces a clause in ϕ to be False, return "Unsatisfiable"

   Step 5: Goto Step 2.

   The above algorithm will be polynomial time.