Off-the-grid, in the clouds, and beyond

Jean-Baptiste Courbot
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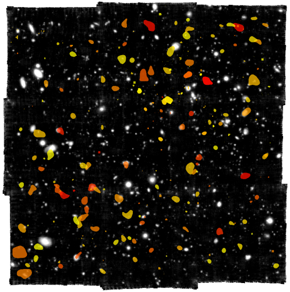
Plan

Before Mokaplan (2014-17):

- Bayesian image segmentation
- Collab. astronomers



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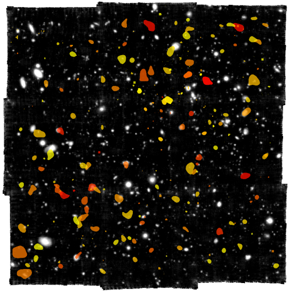

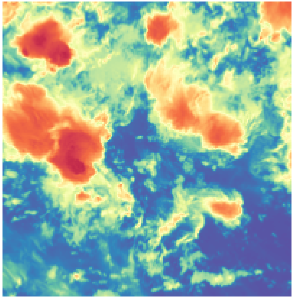

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- Bayesian image segmentation
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Mokaplan (2017-18):

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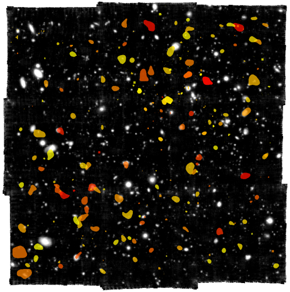
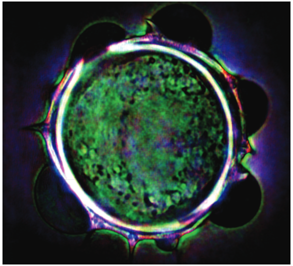
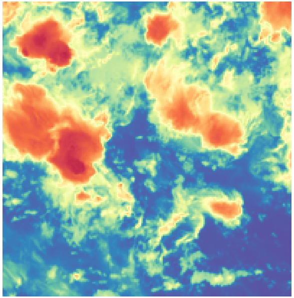
- Collab. meteorologists

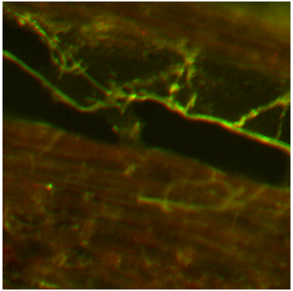
After Mokaplan (2018 onwards):

- Bayesian, sparse, and time-frequency
- Collab. microscopists, biologists









Plan

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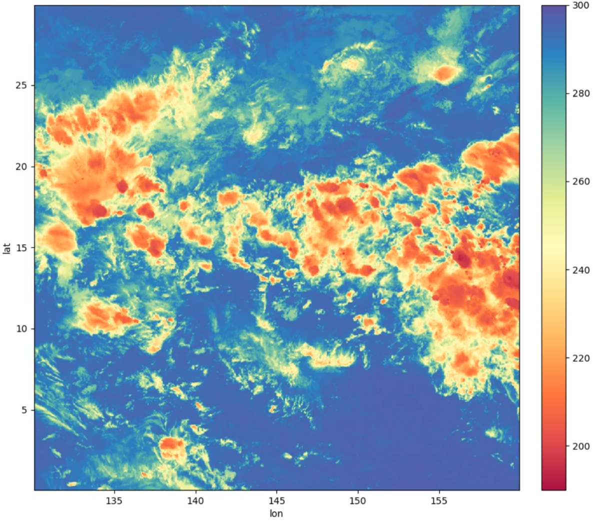
The problems with clouds



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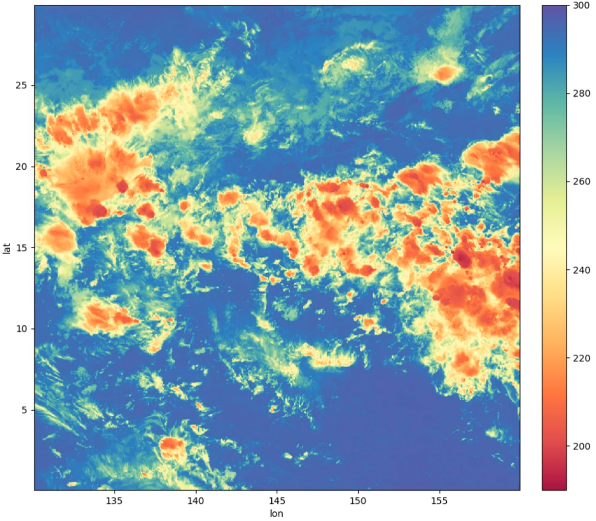

Context:

- Cloud convection is partially known
- Data sources are huge

$\rightarrow$ Track and analyze cloud systems

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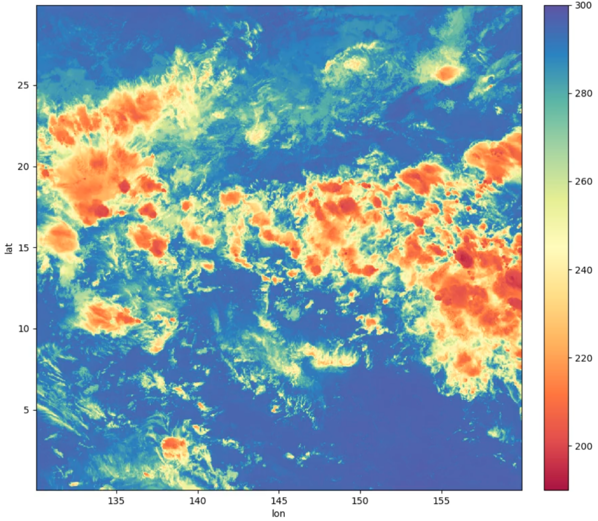

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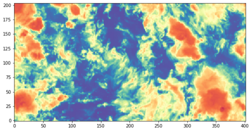

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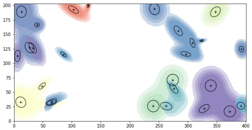

The problems with clouds

Several problems arise:

- Given one cloud image, how to estimate the corresponding Gaussian mixture ?
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Cloud observation model

Given one cloud image, how to estimate the corresponding Gaussian mixture ?

Observation model:

- $\mathbf{y}$ an image
- A mixture is depicted as a measure $\boldsymbol{\mu}$ – a set of Dirac masses with weights $\boldsymbol{w}$
- Each mass indicates the parameter $\boldsymbol{\theta}$ of the corresponding Gaussian

Then, we can solve:

$$\min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} C(\mathbf{y}, \lambda, \boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}) \stackrel{\text{def.}}{=} \min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} \frac{1}{2} \|\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}\|_2^2 + \lambda |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}|. \qquad (\mathcal{P}_\lambda)$$

The Sliding Frank-Wolfe algorithm

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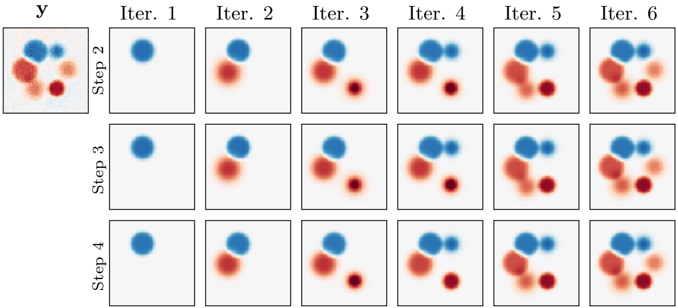
SFW main steps:

1. add one spike to the solution with $\eta_\lambda(\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}) \stackrel{\text{def.}}{=} \frac{1}{\lambda}\boldsymbol{\Phi}^T(\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}})$.
2. adjust all the weights $\boldsymbol{w}$
3. local descent: adjust both $\boldsymbol{\theta}$ and $\boldsymbol{w}$
4. repeat until $\|\eta_\lambda\|_\infty \leq 1$



Denoyelle, Q., Duval, V., Peyré, G., & Soubies, E. (2019). The sliding Frank–Wolfe algorithm and its application to super-resolution microscopy. Inverse Problems.

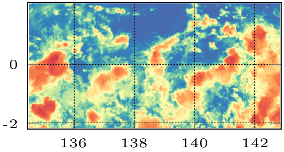
Sketch of a solution

Initialization with a coarse-to-fine SFW.

Then, repeat (1)–(4) for each time t.

(1) Propagation from the previous step

Observation



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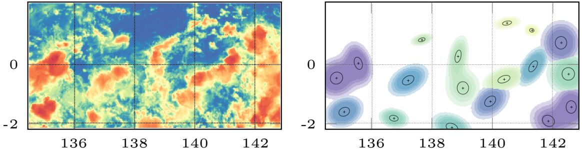

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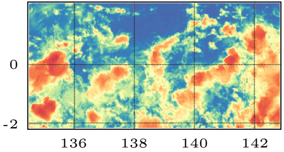

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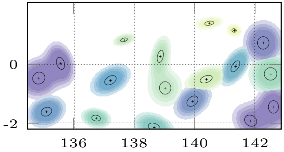
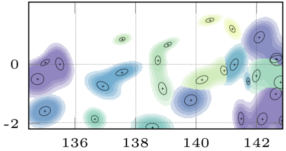


$\rightarrow$ (1) Progagation



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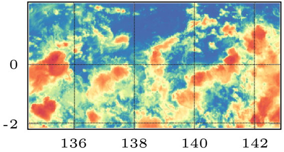

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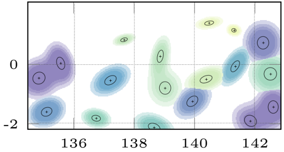

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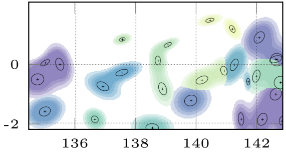
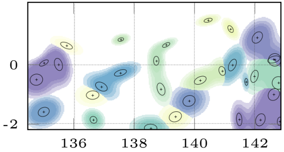


(2) Creation



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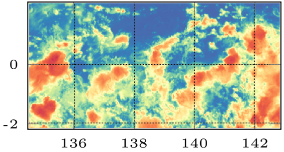

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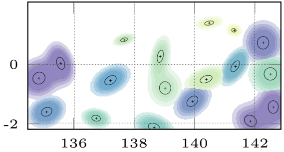

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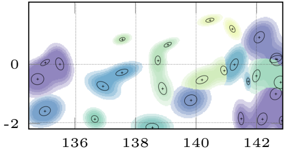

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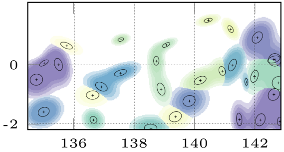
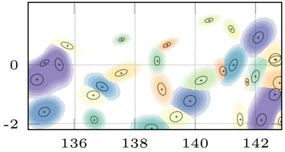


(3) Splitting



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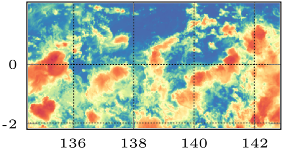

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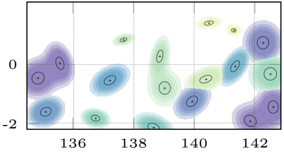

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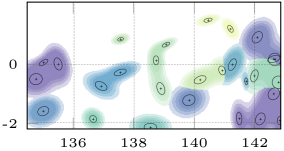

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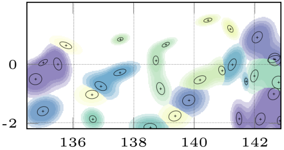

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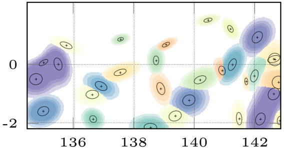
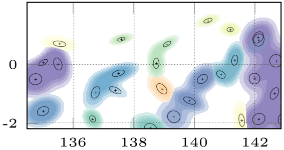
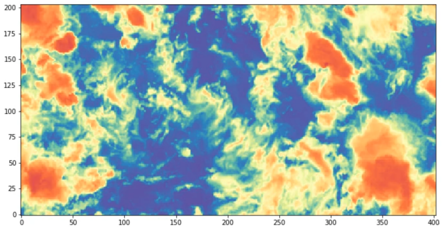
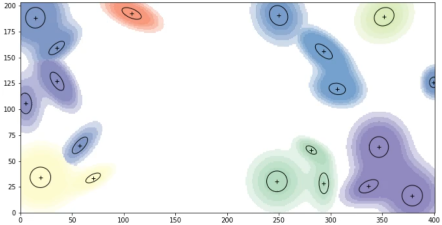


(4) Merging



Some results





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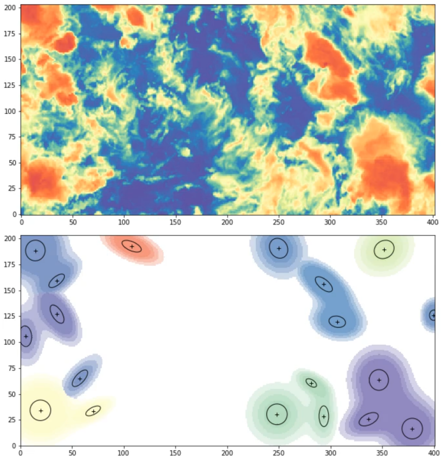


Open issues:

- Validation
- Scale
- Remaining parameters

Courbot, J. B., Duval, V., & Legras, B. (2020). Sparse analysis for mesoscale convective systems tracking. Signal Processing: Image Communication.
Wang, X., Iwabuchi, H., & Courbot, J. B. (2022). Analysis of Diurnal Evolution of Cloud Properties and Convection Tracking over the South China Coastal Area. Remote Sensing.

Plan

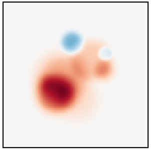
An other observation model

$$\mathbf{y} = \mathbf{H} * \sum_{n=1}^{N} \mathcal{G}(\boldsymbol{\theta}_n, w_n) + \boldsymbol{\epsilon}$$

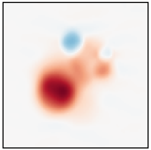
We want to estimate:

- the parameters $\boldsymbol{\theta}_n$ & related weights w_n
- the number of atoms N

$\mathbf{x}$



$\mathbf{Hx} = \boldsymbol{\Phi}\mu$



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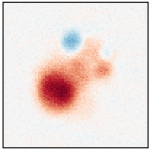

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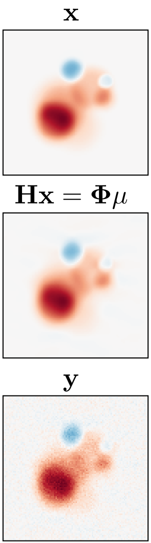
Here, $\mathcal{G}$ is a generalized 3D Gaussian:

$$\mathcal{G}(\boldsymbol{\theta}_n, w_n; \boldsymbol{s}) = w_n \exp\left(-\frac{1}{2\sigma_n^{d_n}} \|\boldsymbol{m}_n - \boldsymbol{s}\|_2^{d_n}\right)$$

$\rightarrow \mathcal{G}$ can be changed, as long as it is well formed (derivation, etc).



An other observation model

Other formulation:

$$\mathbf{y} = \boldsymbol{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} + \boldsymbol{\epsilon}$$

- $\boldsymbol{\Phi} : \mathcal{M}(\mathcal{D}) \mapsto \mathbb{R}^S$ = imaging operator encoding $\mathbf{H}$, $\mathcal{G}$
- $\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} = \sum_{n=1}^{N} w_n \delta_{\boldsymbol{\theta}_n}$ = Dirac masses locating the N parameters

Then, the ℓ_1 minimization is:

$$\min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} \frac{1}{2} \|\boldsymbol{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} - \mathbf{y}\|_2^2 + \lambda |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}| \qquad (\mathcal{P}_\lambda)$$

$\rightarrow |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}| = \sum_{n=1}^{N} |w_n|$

Issues

$$\min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} \frac{1}{2} \|\boldsymbol{\Phi} \boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} - \mathbf{y}\|_2^2 + \lambda |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}| \tag{$\mathcal{P}_\lambda$}$$

In practice...

- Can we go faster than the vanilla SFW ?

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Problem(s)

BLASSO solvers (SFW, BSFW) solve:

$$\min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} \frac{1}{2} \|\boldsymbol{\Phi} \boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} - y\|_2^2 + \lambda |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}| \tag{$\mathcal{P}_\lambda$}$$

$$\left.\begin{array}{l} \blacktriangleright \ \text{high } \lambda \to \text{sparse solution} \\ \blacktriangleright \ \text{low } \lambda \to \text{dense solution} \end{array}\right\} \to \text{no explicit link !}$$

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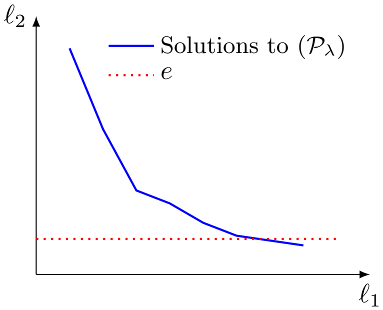
Going back to the constrained problem:

$$\min_{\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}} \in \mathcal{M}(\mathcal{D})} |\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}| \text{ subject to } \|\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}\|_2 \leq e \tag{$\mathcal{P}_1$}$$

$\to$ how can we benefit from solving $(\mathcal{P}_\lambda)$ to solve $(\mathcal{P}_1)$?

Homotopy: principles



Principle: explore the solutions to $(\mathcal{P}_\lambda)$ until a solution to $(\mathcal{P}_1)$ is found.

Property: the path is piecewise linear, with 1 segment $=$ 1 number of atoms.

Homotopy-embedding BLASSO solvers

Produces two sequences:

- $\lambda_0, \lambda_1, \ldots, \lambda_T$
- $\mu_{[0]}, \mu_{[1]}, \ldots, \mu_{[T]}$

So that:

- $\forall t, \mu_{[t]}$ solves $(\mathcal{P}_{\lambda_t})$
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Main steps:

- $\lambda_0 = \|\mathbf{\Phi y}\|_\infty$ as a starting point
- for each t:
  - starting from $\mu_{[t-1]}$, solve $(\mathcal{P}_{\lambda_t})$ to obtain $\mu_{[t]}$
  - if $\|\mathbf{y} - \mathbf{\Phi}\boldsymbol{\mu}_{\boldsymbol{w},\boldsymbol{\theta}}\|_2 \leq e$: $\mu_{[t]}$ is a solution.
  - else: choose λ_{t+1} so that $\mu_{[t+1]}$ contains more spike

Property: stops in a finite number of iterations

Plan

On synthetic data

2 test cases:

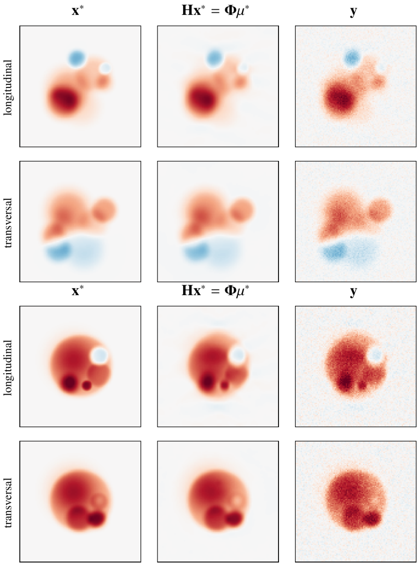
- Gaussian mix
- Gaussian mix within a "pseudo-cell"

What we get:

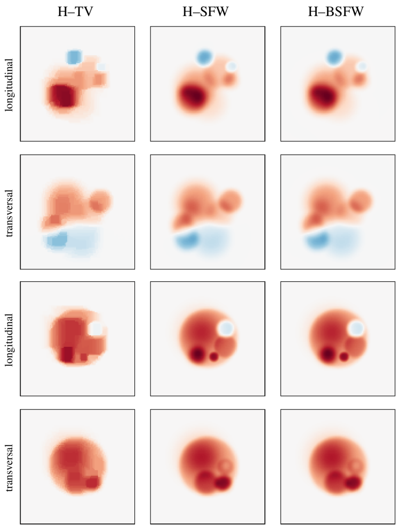
- Distance w.r.t. original
- Computation time

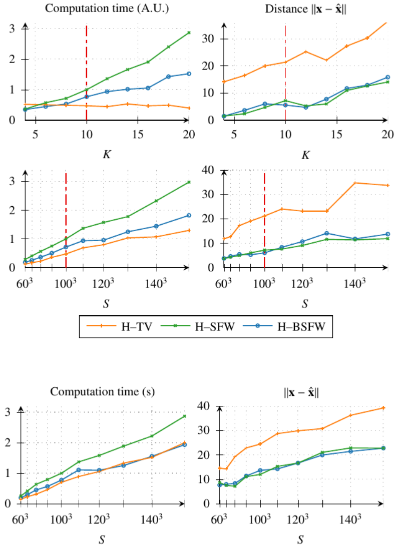
Parameters:

- Atom number K
- Voxel number S

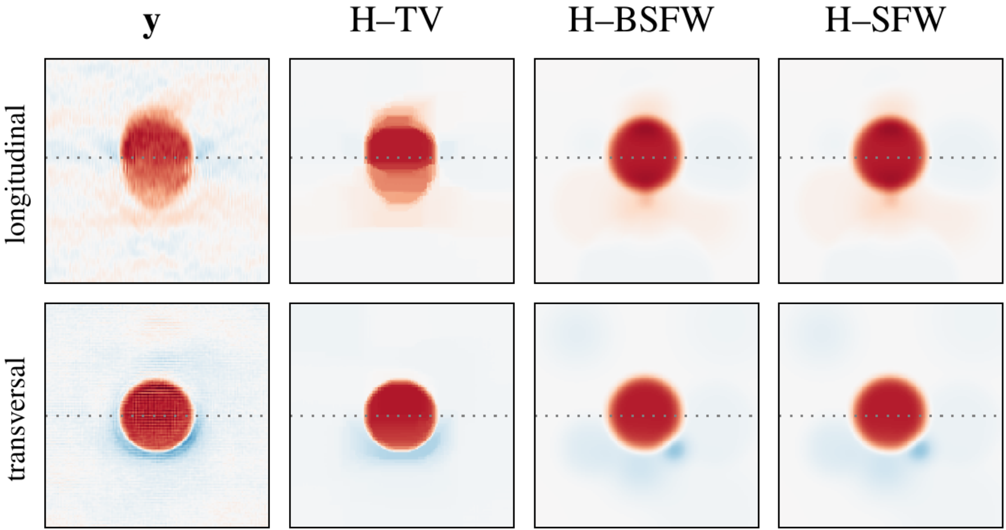


On synthetic data





In tomographic diffractive microscopy image: polymer bead



Concluding remarks

- Gridless sparsity & parametric estimation
- Numerical issues: local optimizations are expensive $\rightarrow$ BSFW
- Practical issue: choosing λ is not obvious $\rightarrow$ homotopy

Further works:

- further numerical speedup with a wise data subsampling
- beyond Dirac masses ?

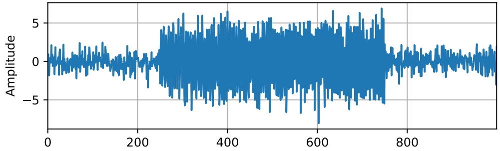
Courbot, J. B., & Colicchio, B. (2021). A fast homotopy algorithm for gridless sparse recovery. Inverse Problems.

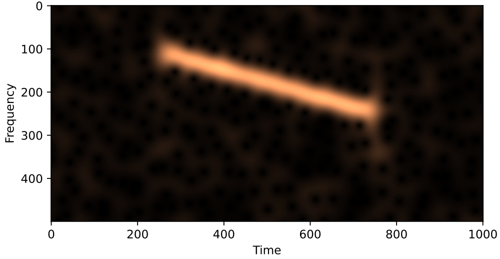
Plan

Context

Let's look at spectrograms – here, modulus of the short-term Fourier transform:

- a local frequency analysis





Bardenet, R., Flamant, J., & Chainais, P. (2020). On the zeros of the spectrogram of white noise. Applied and Computational Harmonic Analysis, 48(2), 682-705.

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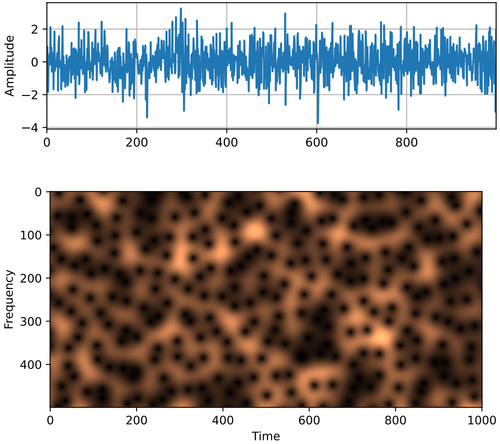
- the spectrogram of white Gaussian noise



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Let's look at spectrograms – here, modulus of the short-term Fourier transform:

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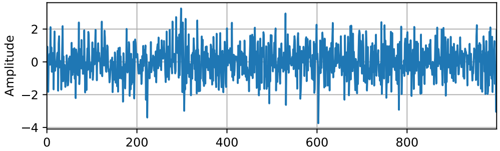
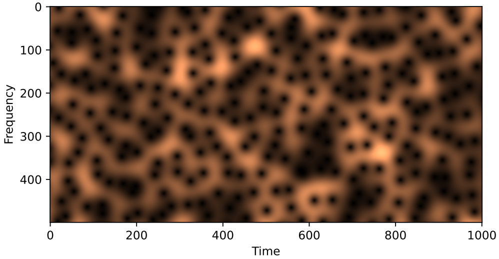
- the spectrogram of white Gaussian noise
- its zeroes have known properties





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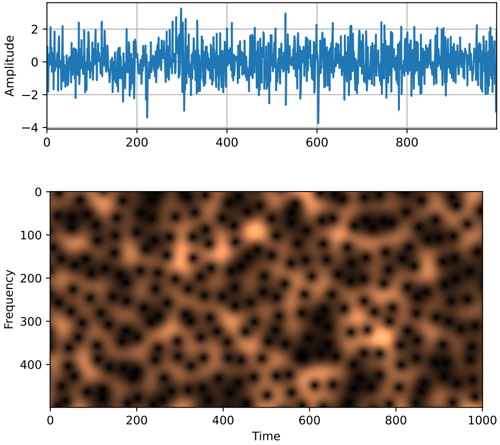


What if tried to locate the zeros beyond the initial sampling ?

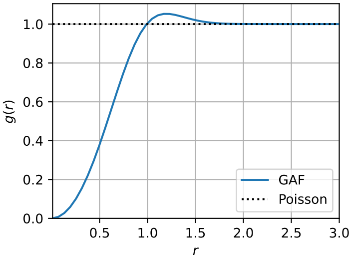
Bardenet, R., Flamant, J., & Chainais, P. (2020). On the zeros of the spectrogram of white noise. Applied and Computational Harmonic Analysis, 48(2), 682-705.

Some properties of the zeros of the STFT

Properties:

- stationarity
- isotropy

The pair correlation function $g(r)$,



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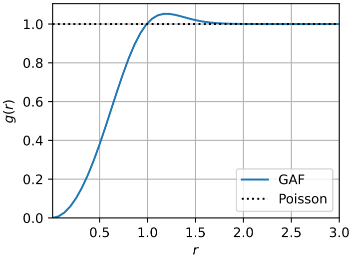

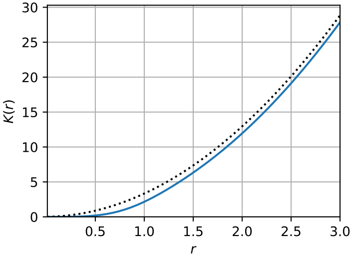

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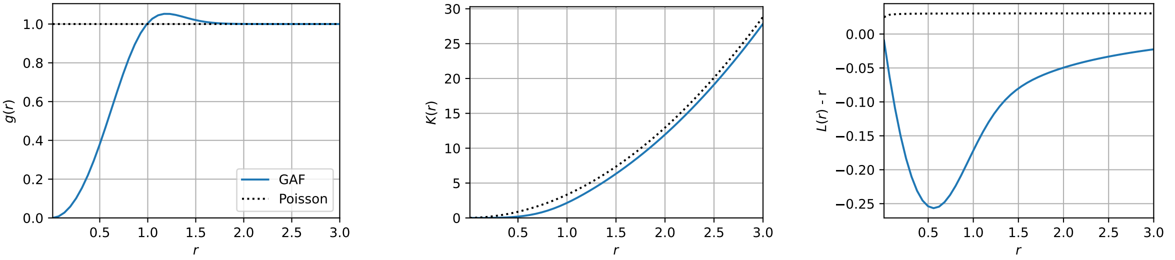
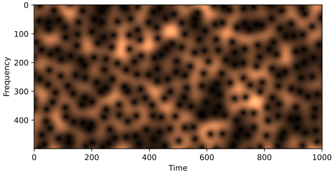
The pair correlation function $g(r)$, Ripley's K-function $K(r)$, its variance-stabilized version $L(r)$.



Observation model

As previously:

$$\operatorname*{arg\,min}_{\boldsymbol{\mu}\in\mathcal{M}(\mathcal{D})} C_\lambda(\boldsymbol{\mu}) = \operatorname*{arg\,min}_{\boldsymbol{\mu}\in\mathcal{M}(\mathcal{D})} \|\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\mu}\|_2^2 + \lambda|\boldsymbol{\mu}|_{\mathrm{TV}}. \qquad (\mathcal{P}_\lambda)$$



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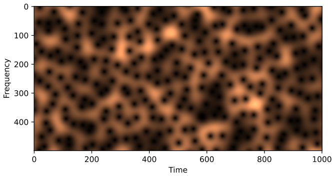

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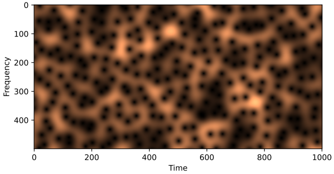

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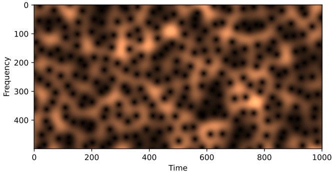

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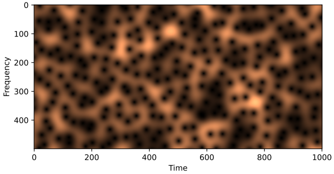

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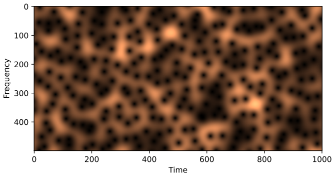

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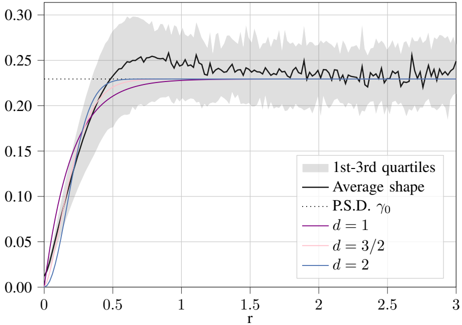

Homotopy in the TF domain

There is no "noise": an other meaningful criterion is needed for homotopy.

Proposal: a ℓ_g criterion measuring the cost of a point set $\mathbf{z}$ seen through the theoretical g_0.

$$\ell_g(\boldsymbol{\mu}) = \frac{1}{\mathrm{Card}(\mathcal{P}(\boldsymbol{\mu}))} \sum_{(i,j)\in\mathcal{P}(\boldsymbol{\mu})} -\log(g_0\left(|\mathbf{z}_i - \mathbf{z}_j|\right)).$$

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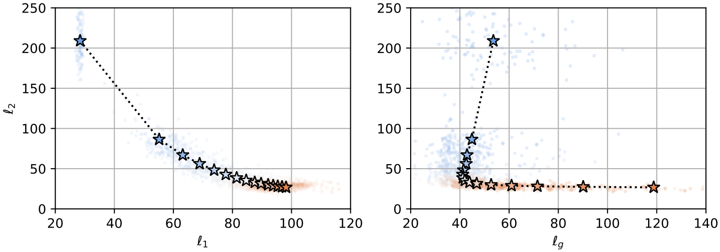
Behavior when probing λ (color):



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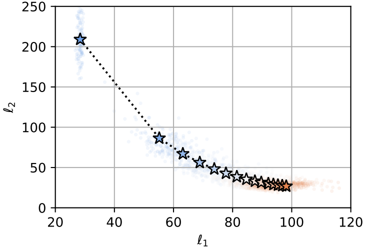
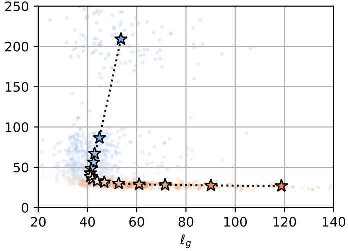
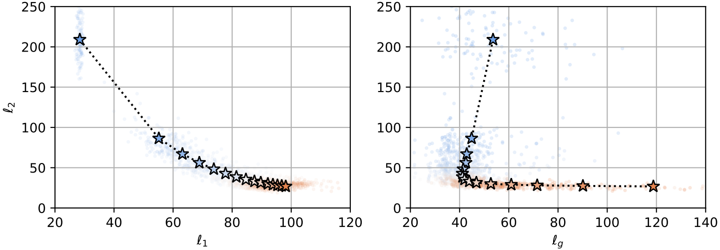




→ Let's minimize ℓ_g !

Homotopy in the TF domain



Then, the problem becomes:

$$\underset{\boldsymbol{\mu}_\lambda \in \mathcal{M}(\mathcal{D})}{\arg\min}\ \ell_g(\boldsymbol{\mu}_\lambda) \text{ subject to: } \boldsymbol{\mu}_\lambda \text{ is a solution to } (\mathcal{P}_\lambda)$$

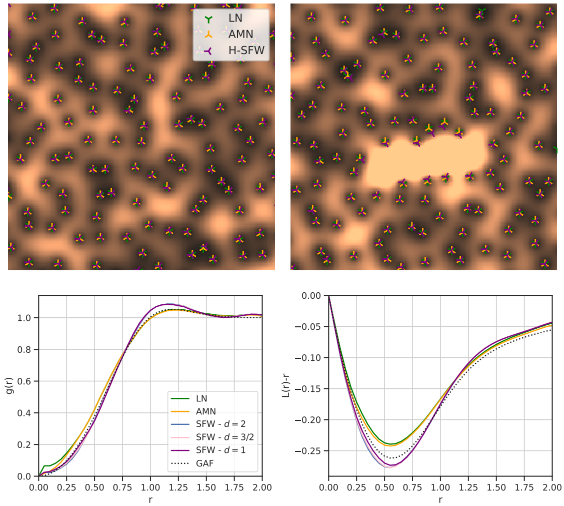
Homotopy: explore the solutions to $(\mathcal{P}_\lambda)$ until a minimum of $\ell_g(\boldsymbol{\mu}_\lambda)$ is found.

Some results

Improvements in...

- Localization of zeros
- Estimation of g, L
- Detection test

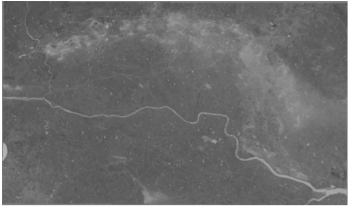
with respect to the discrete counterparts.



Courbot, J. B., Moukadem, A., Colicchio, B., & Dieterlen, A. (2023). Sparse off-the-grid computation of the zeros of STFT. IEEE Signal Processing Letters.

Plan

Some perspectives



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  - Curve analysis
  - Connected curves / networks

Some perspectives

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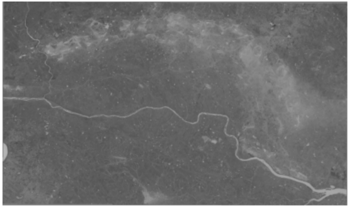
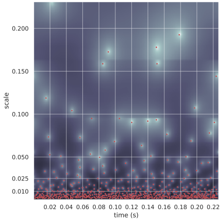
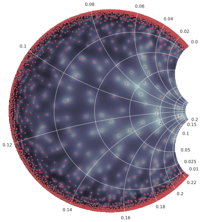
- More time-frequency:
  - Beyond the STFT
  - Synchrosqueezing







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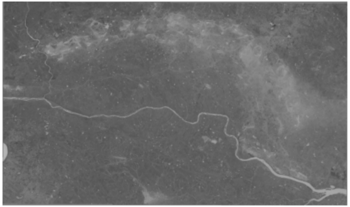

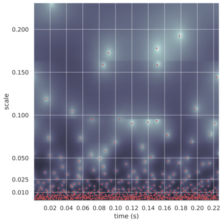

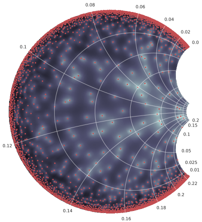

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