

# HNCO

## Influence of the learning rate on the performances of PBIL

October 17, 2016

### Abstract

PBIL is applied many times to the same collection of fitness functions, each time with a different learning rate taken from a finite set of values. All learning rates are ranked according to their median fitness over 20 independent runs, first for each fitness function, then across the entire collection of fitness functions. The mean and standard deviation of fitness are also plotted as a function of the learning rate.

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# 1 Default parameters

```
# algorithm = 100
# bm_mc_reset_strategy = 1
# bm_num_gs_cycles = 1
# bm_num_gs_steps = 100
# bm_sampling = 1
# budget = 10000
# bv_size = 100
# ea_lambda = 100
# ea_mu = 10
# exhaustive_neighborhood = 0
# fun_num_traps = 10
# fun_threshold = 10
# function = 0
# ga_crossover_probability = 0.5
# ga_tournament_size = 10
# hea_binary_dynamics = 0
# hea_delay = 10000
# hea_num_par_updates = 1
# hea_num_seq_updates = 100
# hea_rate_strategy = 0
# hea_reset_period = 0
# hea_sampling_method = 0
# hea_time_constant = 1000
# hea_weight = 1
# learning_rate = 0.001
# map = 0
# map_input_size = 100
# map_path = nopath
# neighborhood = 0
# noise_stddev = 1
# num_iterations = 0
# path = nopath
# patience = 50
# plugin_function_name = nofunction
# population_size = 10
# radius = 2
# sa_initial_acceptance_probability = 0.6
# sa_num_transitions = 50
# sa_num_trials = 100
# sa_rate = 1.2
# scaled_mutation_probability = 1
# seed = 0
# selection_size = 1
# print_default_parameters
# last_parameter
# exec_name = hnco
# version = 1.13
# Generated from hnco.json
```

# 2 Plan

```
{
  "exec": "hnco",
  "opt": "--no-header -s 100 --map 1 --map-random -i 0 -b 200000 --print-performances",
  "num_runs": 20,
  "results": "results",
  "graphics": "graphics",
  "report": "report",
  "parameter": {
    "id": "learning-rate",
    "values": [ 1e-4, 2e-4, 5e-4, 1e-3, 2e-3, 5e-3, 1e-2, 2e-2, 5e-2, 1e-1, 2e-1, 5e-1, 1 ],
  }
}
```

```

    "logscale": true,
    "boxwidth": "($1*0.3)"
  },
  "functions": [
    {
      "id": "one-max",
      "opt": "-F 0 --stop-on-maximum",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "lin",
      "opt": "-F 1 -p instances/lin.100",
      "col": ">{\nnprouddigits{2}}N{2}{2}"
    },
    {
      "id": "leading-ones",
      "opt": "-F 10 --stop-on-maximum",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "ridge",
      "opt": "-F 11 --stop-on-maximum",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "jmp-5",
      "opt": "-F 30 --stop-on-maximum -t 5",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "jmp-10",
      "opt": "-F 30 --stop-on-maximum -t 10",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "djmp-5",
      "opt": "-F 31 --stop-on-maximum -t 5",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "djmp-10",
      "opt": "-F 31 --stop-on-maximum -t 10",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "fp-5",
      "opt": "-F 40 --stop-on-maximum -t 5",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "fp-10",
      "opt": "-F 40 --stop-on-maximum -t 10",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "quad",
      "opt": "-F 50 -p instances/quad.100 --cache",
      "col": ">{\nnprouddigits{2}}N{3}{2}"
    },
    {
      "id": "nk",
      "opt": "-F 60 -p instances/nk.100.4",
      "col": ">{\nnprouddigits{2}}N{1}{2}"
    }
  ]
}

```

```

    },
    {
      "id": "max-sat",
      "opt": "-F 70 -p instances/ms.100.3.1000 --cache",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "labs",
      "opt": "-F 80",
      "col": ">{\nnprouddigits{2}}N{1}{2}"
    },
    {
      "id": "ep",
      "opt": "-F 90 -p instances/ep.100",
      "reverse": true,
      "logscale": true,
      "col": ">{\nnprouddigits{2}}N{1}{2}"
    },
    {
      "id": "cancel",
      "opt": "-F 100 -s 99",
      "reverse": true,
      "col": ">{\nnprouddigits{2}}N{1}{2}"
    },
    {
      "id": "trap",
      "opt": "-F 110 --stop-on-maximum --fun-num-traps 10",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "hiff",
      "opt": "-F 120 --stop-on-maximum -s 128",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    },
    {
      "id": "plateau",
      "opt": "-F 130 --stop-on-maximum",
      "col": ">{\nnprouddigits{0}}N{3}{0}"
    }
  ],
  "algorithms": [
    {
      "id": "pbil",
      "opt": "-A 500 -x 10 -y 1"
    }
  ]
}

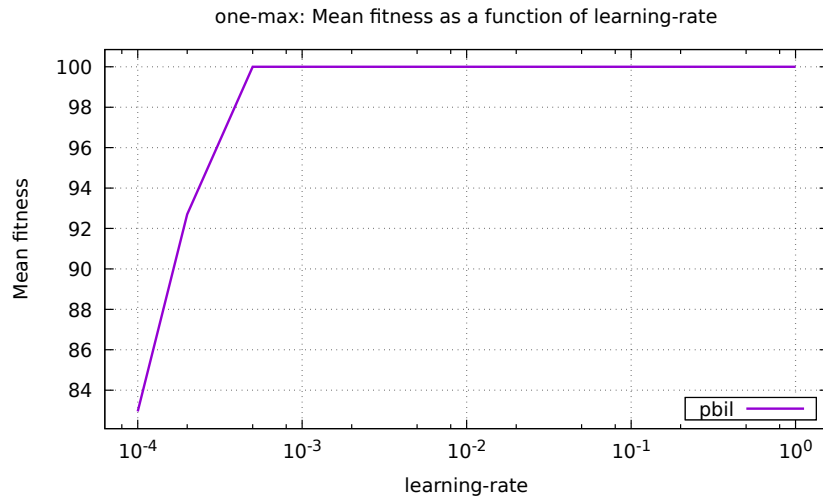
```

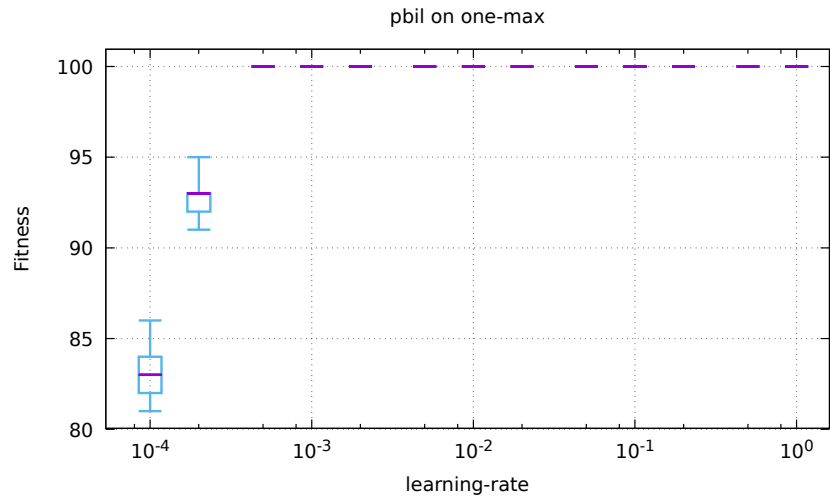
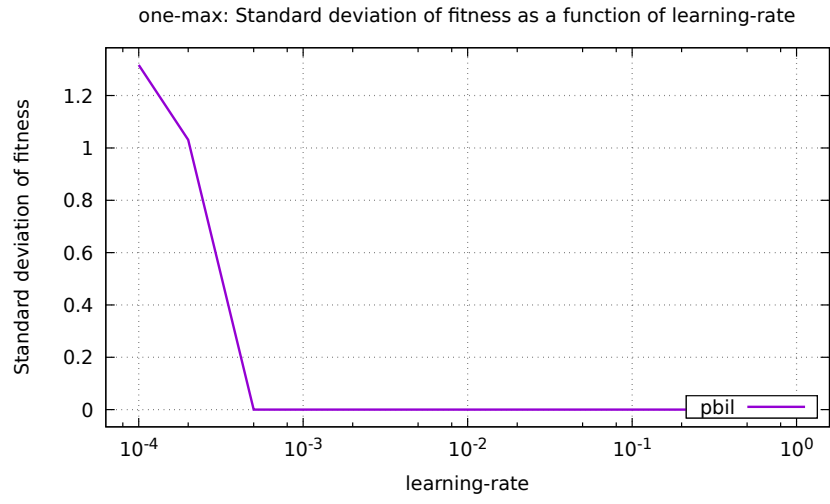
### 3 Rankings

| algorithm   | rank distribution |   |   |   |   |   |   |   |   |    |    |    |    |
|-------------|-------------------|---|---|---|---|---|---|---|---|----|----|----|----|
|             | 1                 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| pbil-1      | 12                | 0 | 0 | 0 | 2 | 1 | 0 | 2 | 0 | 2  | 0  | 0  | 0  |
| pbil-0.002  | 8                 | 4 | 2 | 0 | 0 | 0 | 0 | 2 | 3 | 0  | 0  | 0  | 0  |
| pbil-0.005  | 8                 | 3 | 3 | 2 | 0 | 1 | 1 | 1 | 0 | 0  | 0  | 0  | 0  |
| pbil-0.01   | 8                 | 1 | 3 | 2 | 2 | 1 | 0 | 1 | 1 | 0  | 0  | 0  | 0  |
| pbil-0.02   | 8                 | 1 | 0 | 2 | 4 | 3 | 1 | 0 | 0 | 0  | 0  | 0  | 0  |
| pbil-0.05   | 8                 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 1 | 2  | 0  | 0  | 1  |
| pbil-0.001  | 7                 | 2 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 5  | 0  | 0  | 1  |
| pbil-0.0005 | 5                 | 1 | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 1  | 8  | 1  | 0  |
| pbil-0.1    | 5                 | 1 | 0 | 1 | 0 | 2 | 3 | 3 | 2 | 0  | 1  | 1  | 0  |
| pbil-0.2    | 4                 | 2 | 0 | 2 | 0 | 2 | 1 | 3 | 2 | 1  | 1  | 1  | 0  |
| pbil-0.5    | 4                 | 2 | 0 | 1 | 1 | 3 | 2 | 2 | 1 | 2  | 0  | 1  | 0  |
| pbil-0.0001 | 0                 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2  | 0  | 1  | 15 |
| pbil-0.0002 | 0                 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 0  | 1  | 13 | 1  |

### 4 Function one-max

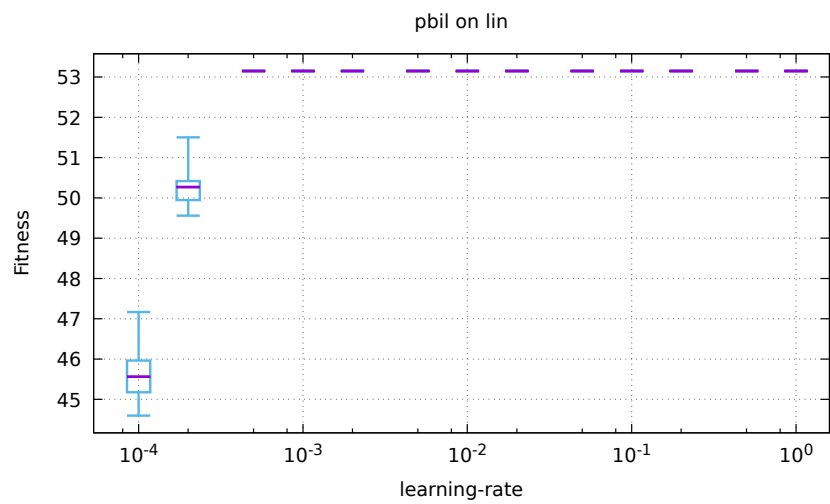
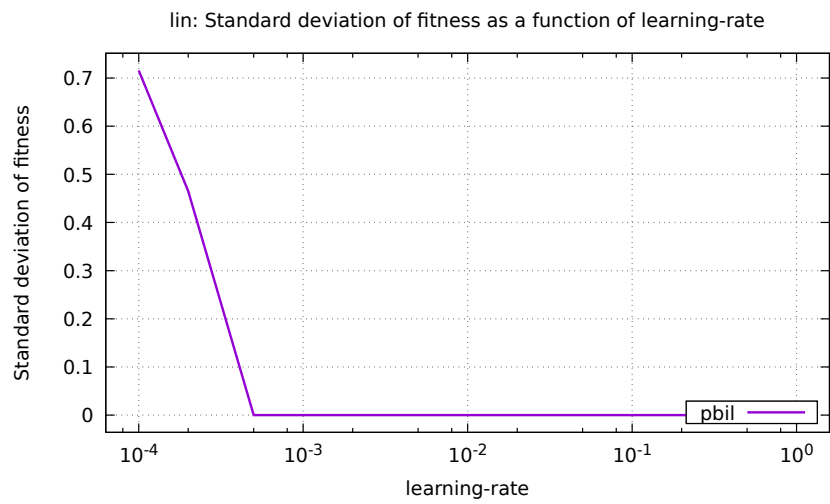
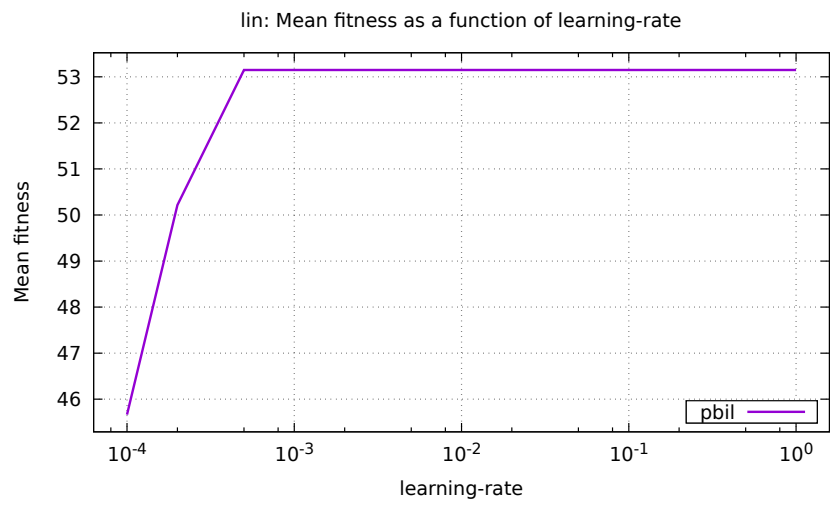
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 81  | 82    | 83   | 84    | 86  | 13 |
| pbil-0.0002 | 91  | 92    | 93   | 93    | 95  | 12 |
| pbil-0.0005 | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.001  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.002  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.005  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.01   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.02   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.05   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.1    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.2    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.5    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-1      | 100 | 100   | 100  | 100   | 100 | 1  |





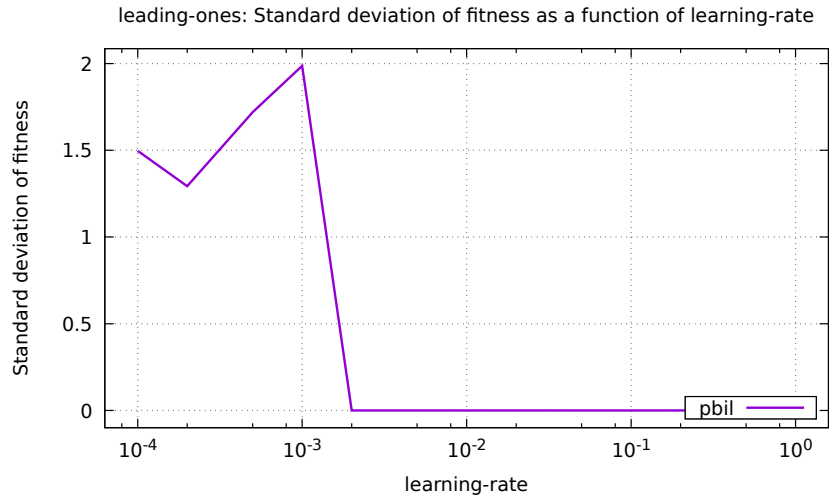
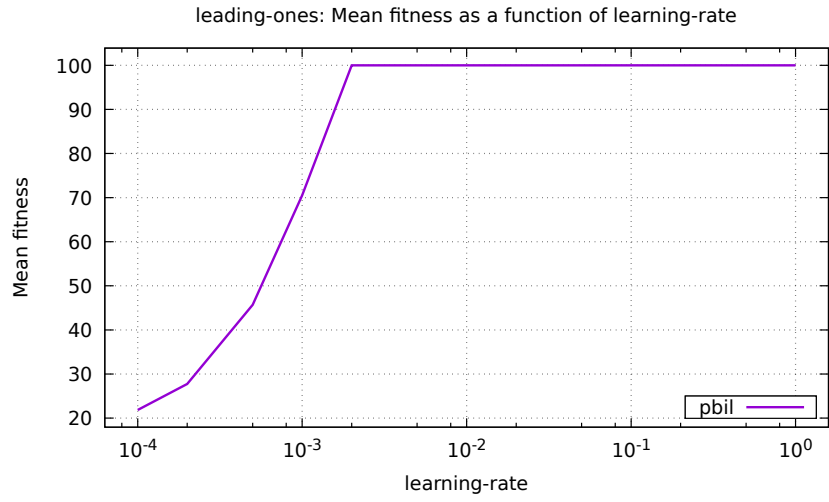
## 5 Function lin

| algorithm   | min   | $Q_1$ | med.  | $Q_3$ | max   | rk |
|-------------|-------|-------|-------|-------|-------|----|
| pbil-0.0001 | 44.60 | 45.18 | 45.56 | 45.96 | 47.17 | 13 |
| pbil-0.0002 | 49.56 | 49.95 | 50.27 | 50.42 | 51.50 | 12 |
| pbil-0.0005 | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.001  | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.002  | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.005  | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.01   | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.02   | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.05   | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.1    | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.2    | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-0.5    | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |
| pbil-1      | 53.15 | 53.15 | 53.15 | 53.15 | 53.15 | 1  |

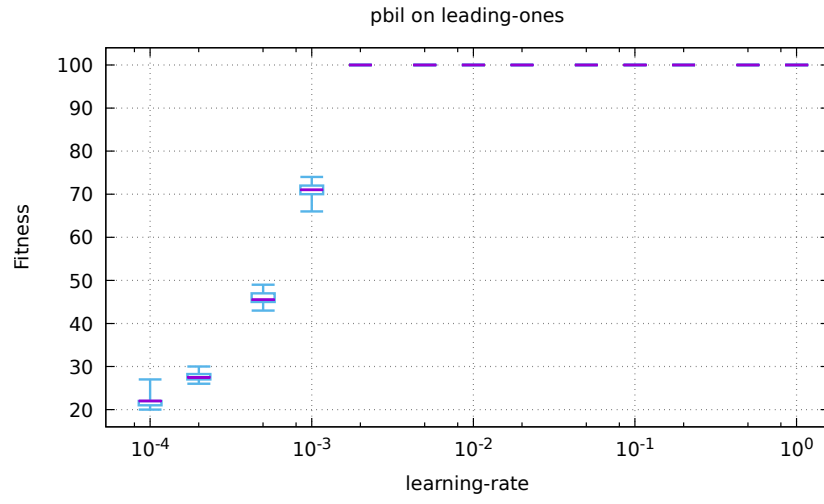


## 6 Function leading-ones

| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 20  | 21    | 22   | 22    | 27  | 13 |
| pbil-0.0002 | 26  | 27    | 28   | 28    | 30  | 12 |
| pbil-0.0005 | 43  | 45    | 46   | 47    | 49  | 11 |
| pbil-0.001  | 66  | 70    | 71   | 72    | 74  | 10 |
| pbil-0.002  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.005  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.01   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.02   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.05   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.1    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.2    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.5    | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-1      | 100 | 100   | 100  | 100   | 100 | 1  |

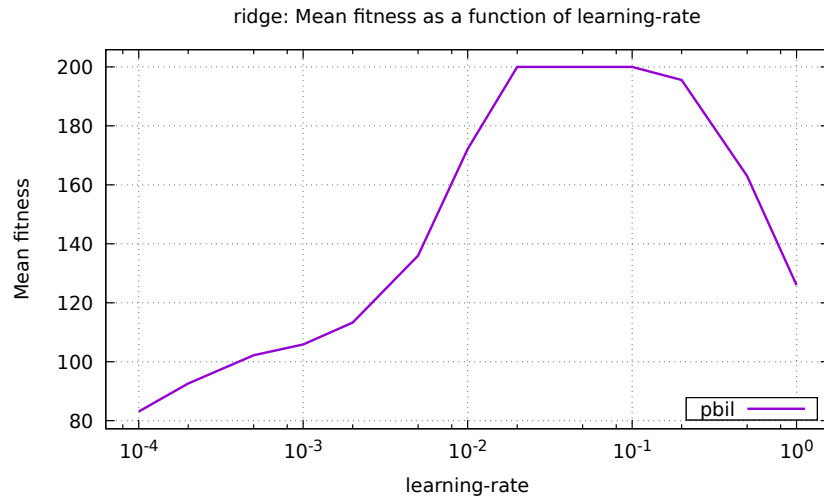


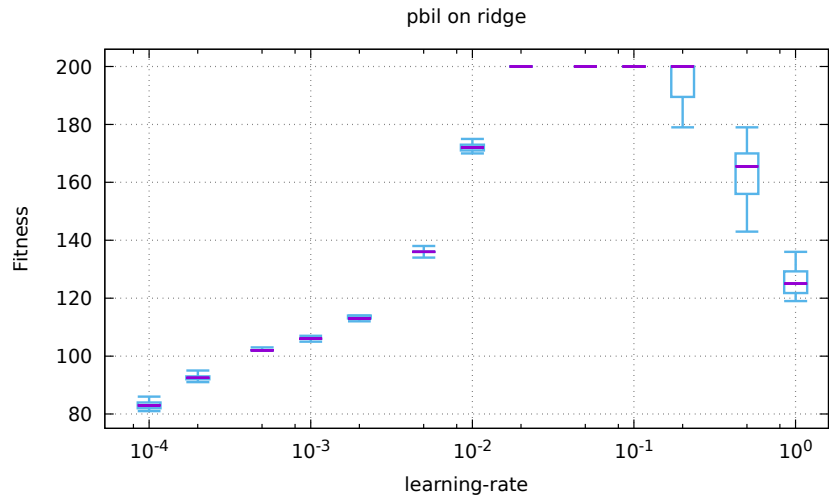
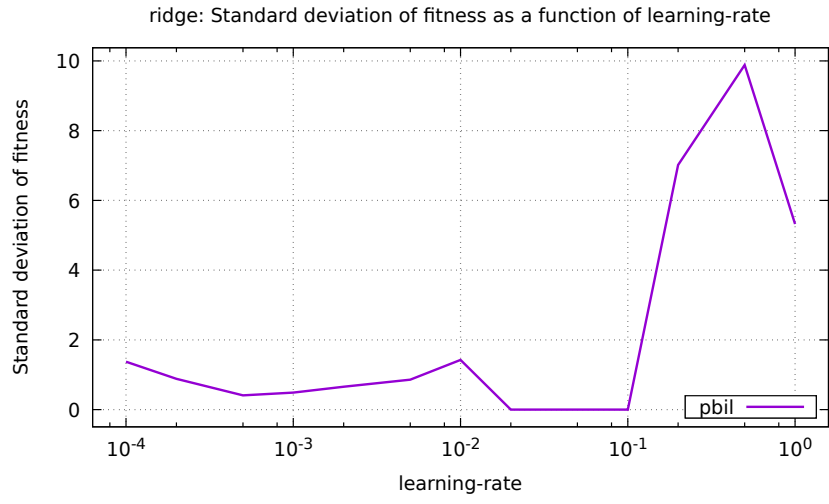




## 7 Function ridge

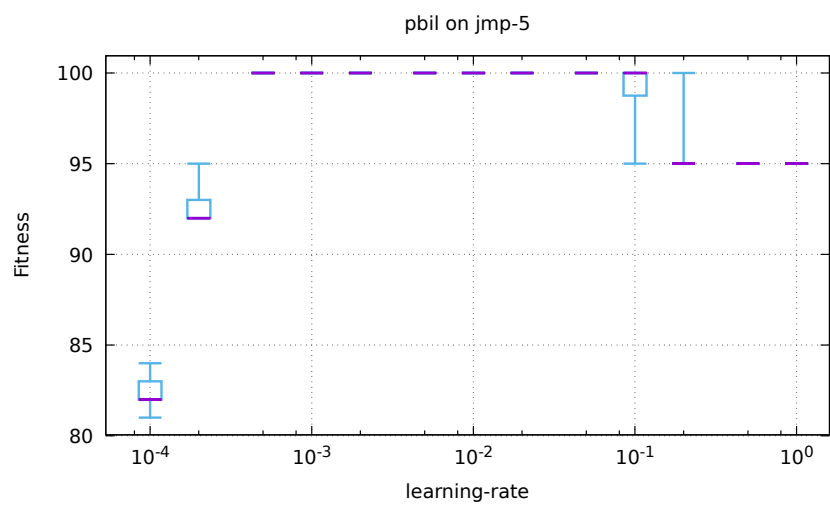
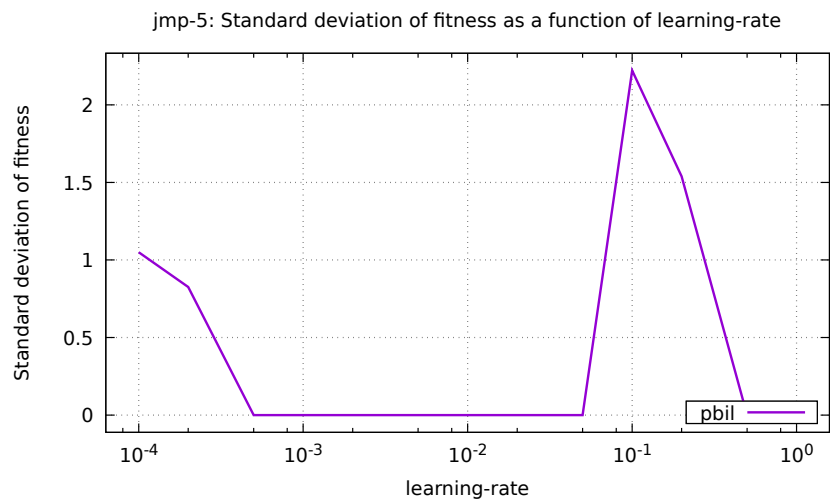
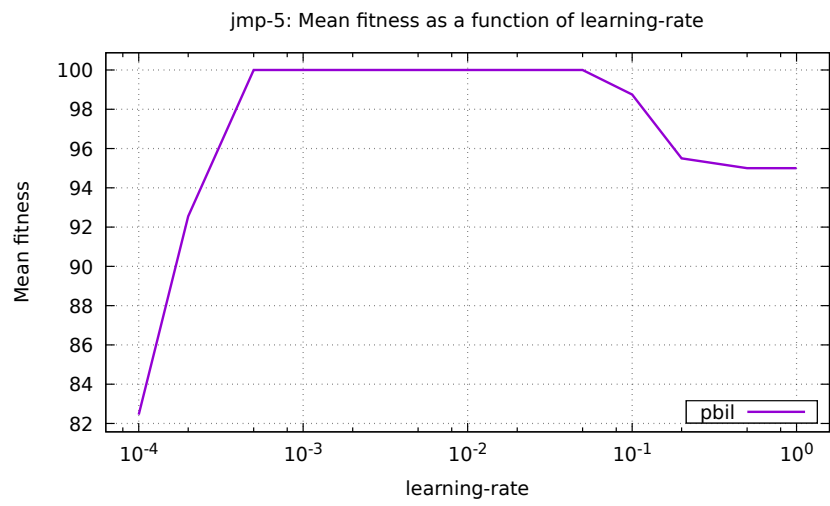
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 81  | 82    | 83   | 84    | 86  | 13 |
| pbil-0.0002 | 91  | 92    | 93   | 93    | 95  | 12 |
| pbil-0.0005 | 102 | 102   | 102  | 102   | 103 | 11 |
| pbil-0.001  | 105 | 106   | 106  | 106   | 107 | 10 |
| pbil-0.002  | 112 | 113   | 113  | 114   | 114 | 9  |
| pbil-0.005  | 134 | 136   | 136  | 136   | 138 | 7  |
| pbil-0.01   | 170 | 171   | 172  | 173   | 175 | 5  |
| pbil-0.02   | 200 | 200   | 200  | 200   | 200 | 1  |
| pbil-0.05   | 200 | 200   | 200  | 200   | 200 | 1  |
| pbil-0.1    | 200 | 200   | 200  | 200   | 200 | 1  |
| pbil-0.2    | 179 | 190   | 200  | 200   | 200 | 4  |
| pbil-0.5    | 143 | 156   | 166  | 170   | 179 | 6  |
| pbil-1      | 119 | 122   | 125  | 129   | 136 | 8  |





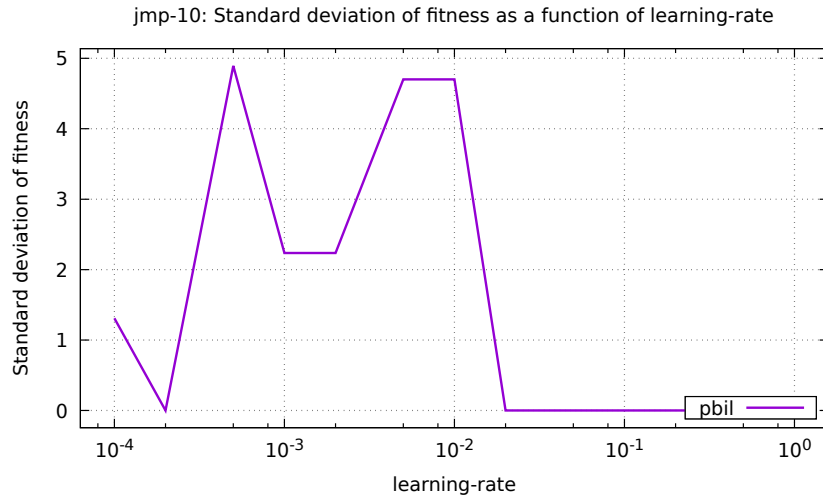
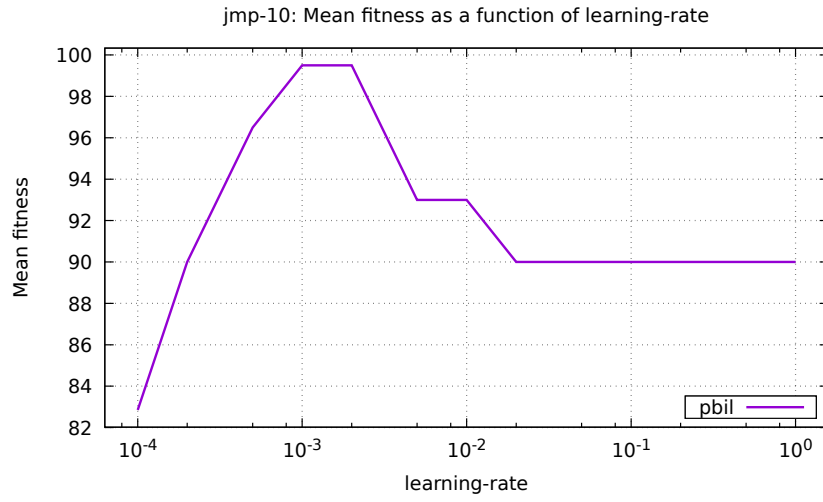
## 8 Function jmp-5

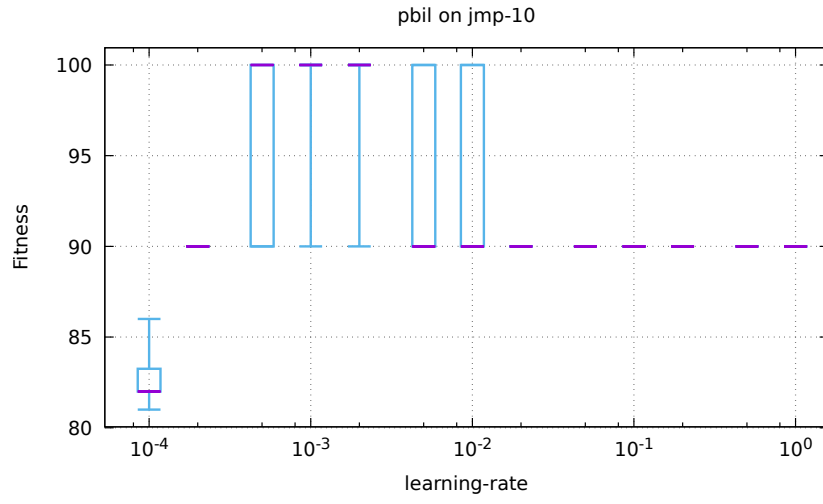
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 81  | 82    | 82   | 83    | 84  | 13 |
| pbil-0.0002 | 92  | 92    | 92   | 93    | 95  | 12 |
| pbil-0.0005 | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.001  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.002  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.005  | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.01   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.02   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.05   | 100 | 100   | 100  | 100   | 100 | 1  |
| pbil-0.1    | 95  | 99    | 100  | 100   | 100 | 8  |
| pbil-0.2    | 95  | 95    | 95   | 95    | 100 | 9  |
| pbil-0.5    | 95  | 95    | 95   | 95    | 95  | 10 |
| pbil-1      | 95  | 95    | 95   | 95    | 95  | 10 |



## 9 Function jmp-10

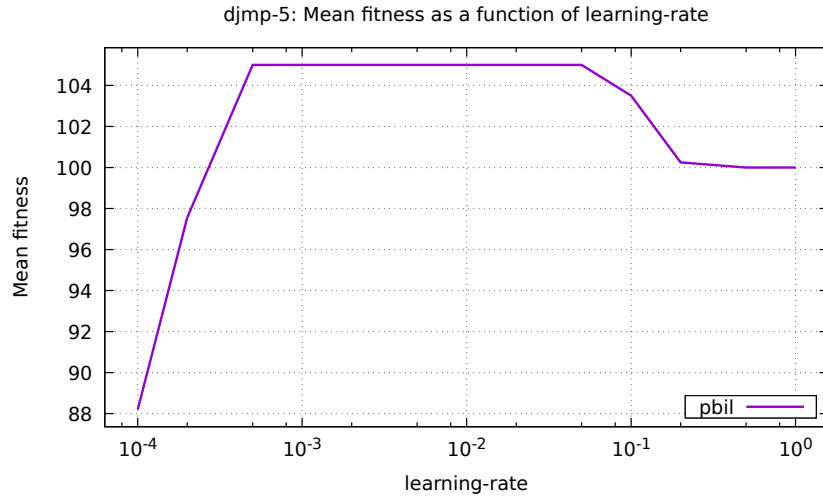
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 81  | 82    | 82   | 83    | 86  | 13 |
| pbil-0.0002 | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-0.0005 | 90  | 90    | 100  | 100   | 100 | 3  |
| pbil-0.001  | 90  | 100   | 100  | 100   | 100 | 1  |
| pbil-0.002  | 90  | 100   | 100  | 100   | 100 | 1  |
| pbil-0.005  | 90  | 90    | 90   | 100   | 100 | 4  |
| pbil-0.01   | 90  | 90    | 90   | 100   | 100 | 4  |
| pbil-0.02   | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-0.05   | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-0.1    | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-0.2    | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-0.5    | 90  | 90    | 90   | 90    | 90  | 6  |
| pbil-1      | 90  | 90    | 90   | 90    | 90  | 6  |

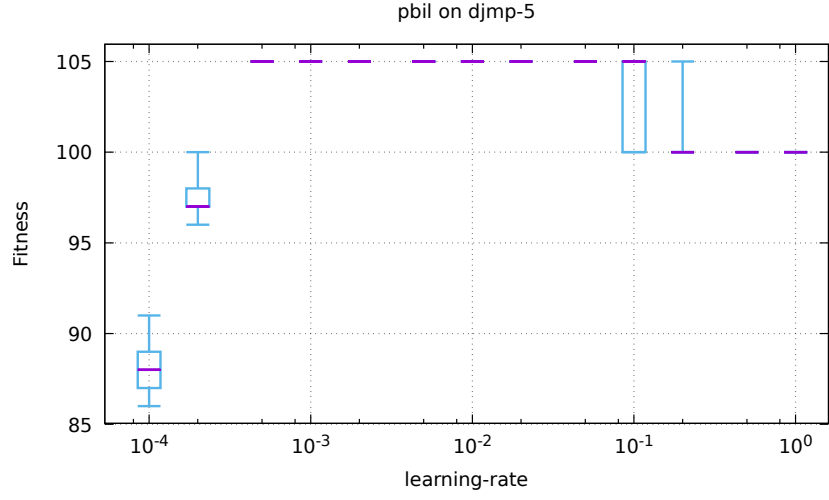
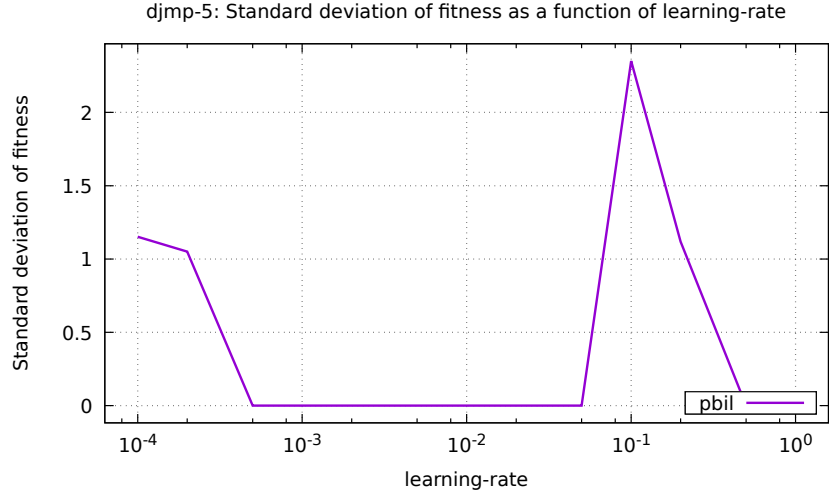




## 10 Function djmp-5

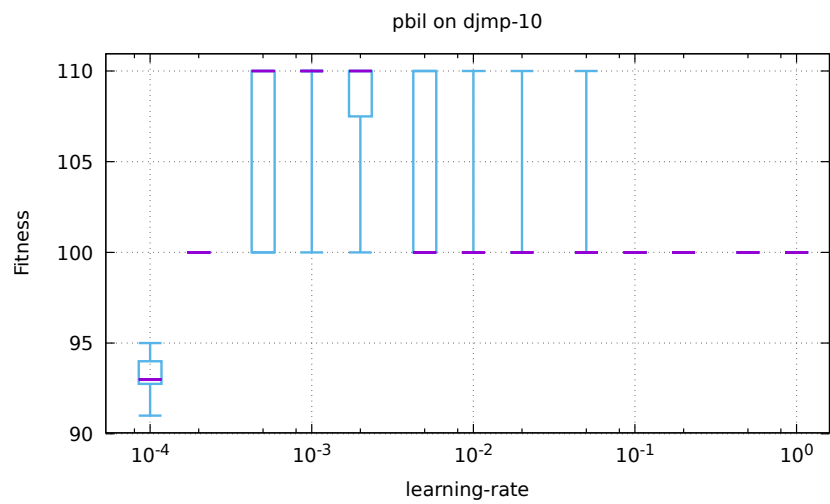
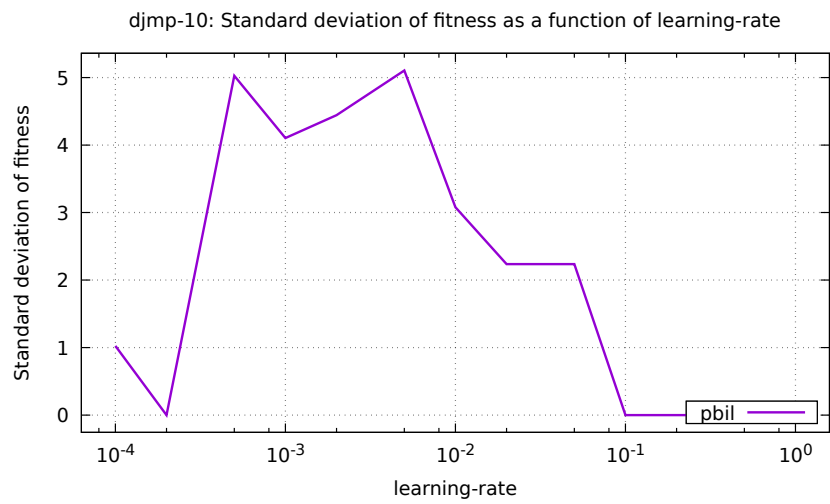
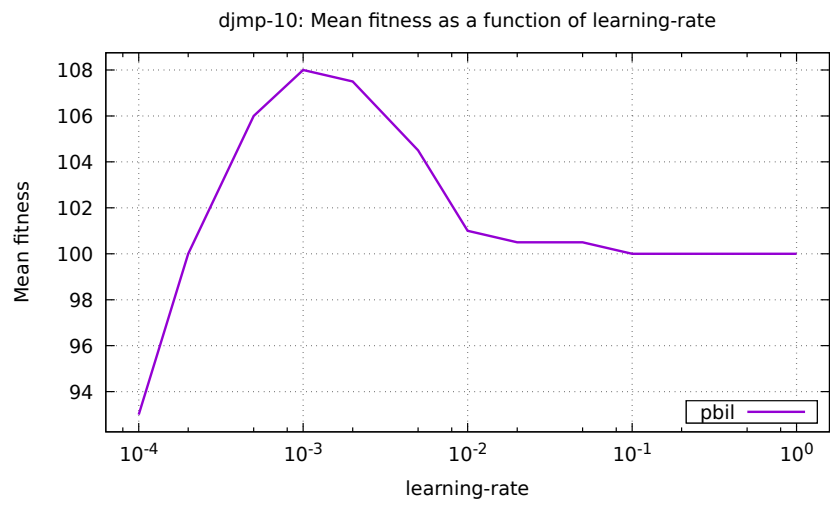
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 86  | 87    | 88   | 89    | 91  | 13 |
| pbil-0.0002 | 96  | 97    | 97   | 98    | 100 | 12 |
| pbil-0.0005 | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.001  | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.002  | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.005  | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.01   | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.02   | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.05   | 105 | 105   | 105  | 105   | 105 | 1  |
| pbil-0.1    | 100 | 100   | 105  | 105   | 105 | 8  |
| pbil-0.2    | 100 | 100   | 100  | 100   | 105 | 9  |
| pbil-0.5    | 100 | 100   | 100  | 100   | 100 | 10 |
| pbil-1      | 100 | 100   | 100  | 100   | 100 | 10 |





## 11 Function djmp-10

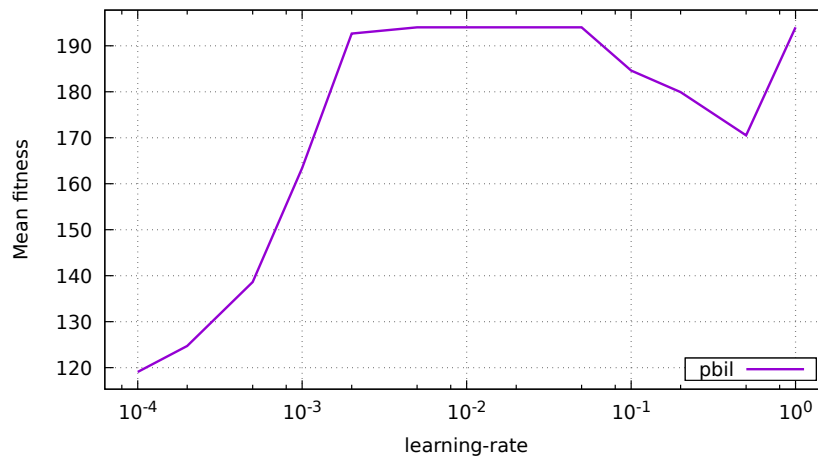
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 91  | 93    | 93   | 94    | 95  | 13 |
| pbil-0.0002 | 100 | 100   | 100  | 100   | 100 | 8  |
| pbil-0.0005 | 100 | 100   | 110  | 110   | 110 | 3  |
| pbil-0.001  | 100 | 110   | 110  | 110   | 110 | 1  |
| pbil-0.002  | 100 | 108   | 110  | 110   | 110 | 2  |
| pbil-0.005  | 100 | 100   | 100  | 110   | 110 | 4  |
| pbil-0.01   | 100 | 100   | 100  | 100   | 110 | 5  |
| pbil-0.02   | 100 | 100   | 100  | 100   | 110 | 5  |
| pbil-0.05   | 100 | 100   | 100  | 100   | 110 | 5  |
| pbil-0.1    | 100 | 100   | 100  | 100   | 100 | 8  |
| pbil-0.2    | 100 | 100   | 100  | 100   | 100 | 8  |
| pbil-0.5    | 100 | 100   | 100  | 100   | 100 | 8  |
| pbil-1      | 100 | 100   | 100  | 100   | 100 | 8  |



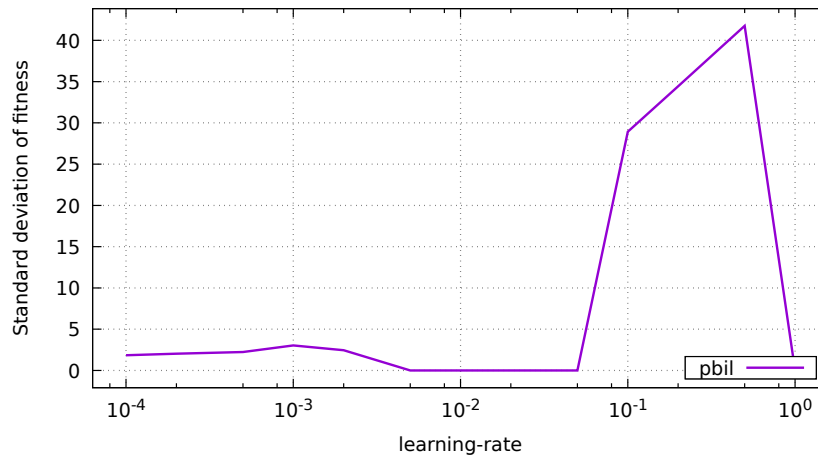
## 12 Function fp-5

| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 116 | 118   | 119  | 120   | 124 | 13 |
| pbil-0.0002 | 122 | 123   | 124  | 126   | 130 | 12 |
| pbil-0.0005 | 134 | 137   | 139  | 140   | 142 | 11 |
| pbil-0.001  | 156 | 162   | 164  | 166   | 169 | 10 |
| pbil-0.002  | 185 | 193   | 194  | 194   | 194 | 8  |
| pbil-0.005  | 194 | 194   | 194  | 194   | 194 | 1  |
| pbil-0.01   | 194 | 194   | 194  | 194   | 194 | 1  |
| pbil-0.02   | 194 | 194   | 194  | 194   | 194 | 1  |
| pbil-0.05   | 194 | 194   | 194  | 194   | 194 | 1  |
| pbil-0.1    | 100 | 194   | 194  | 194   | 194 | 6  |
| pbil-0.2    | 100 | 194   | 194  | 194   | 194 | 6  |
| pbil-0.5    | 100 | 171   | 194  | 194   | 194 | 9  |
| pbil-1      | 194 | 194   | 194  | 194   | 194 | 1  |

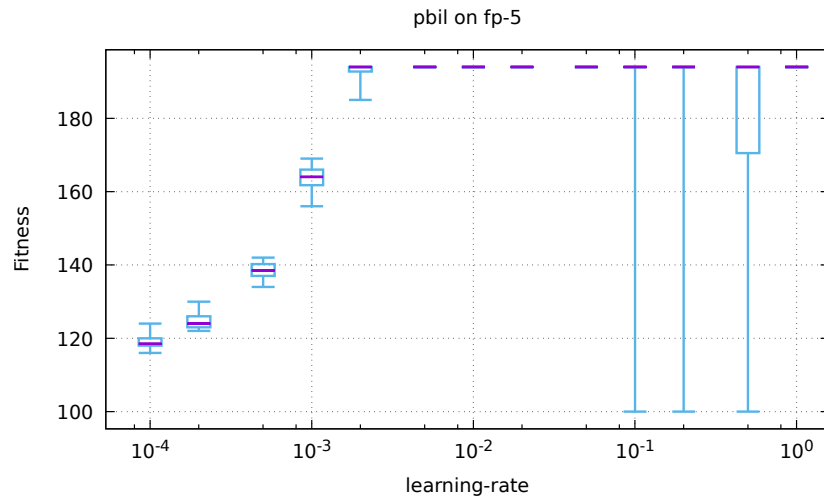
fp-5: Mean fitness as a function of learning-rate



fp-5: Standard deviation of fitness as a function of learning-rate

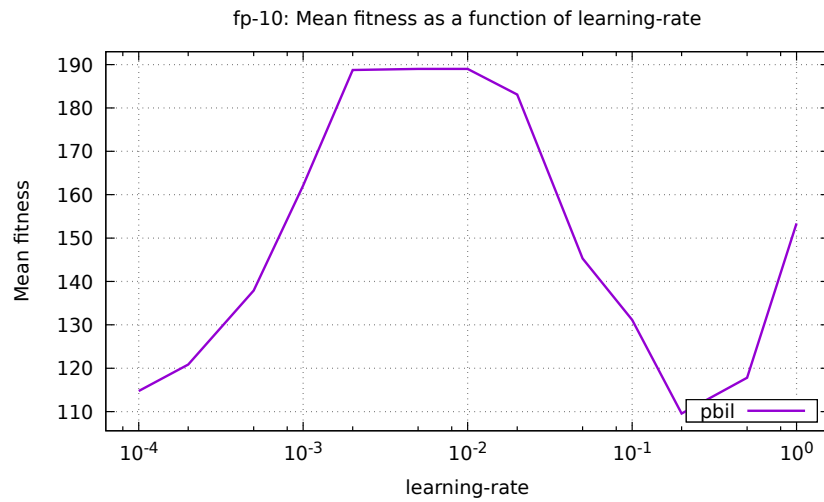


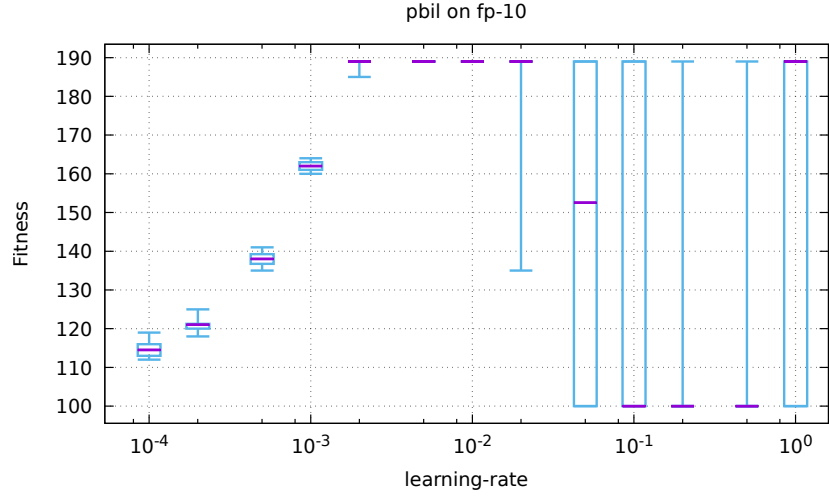
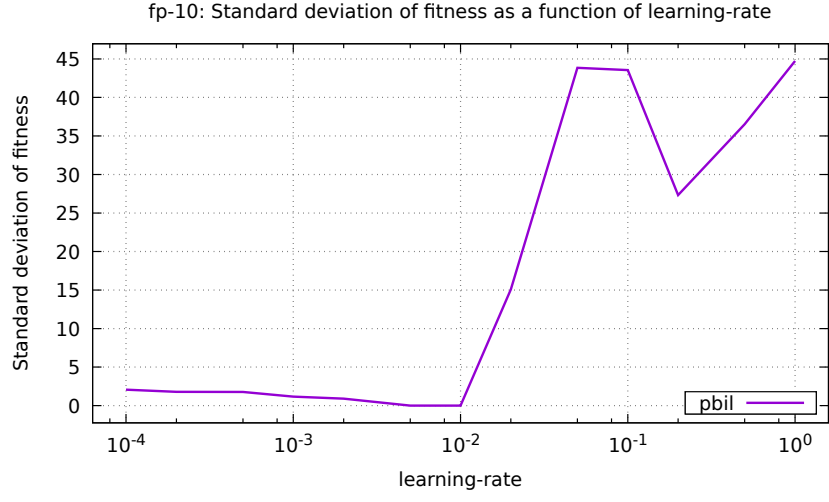




### 13 Function fp-10

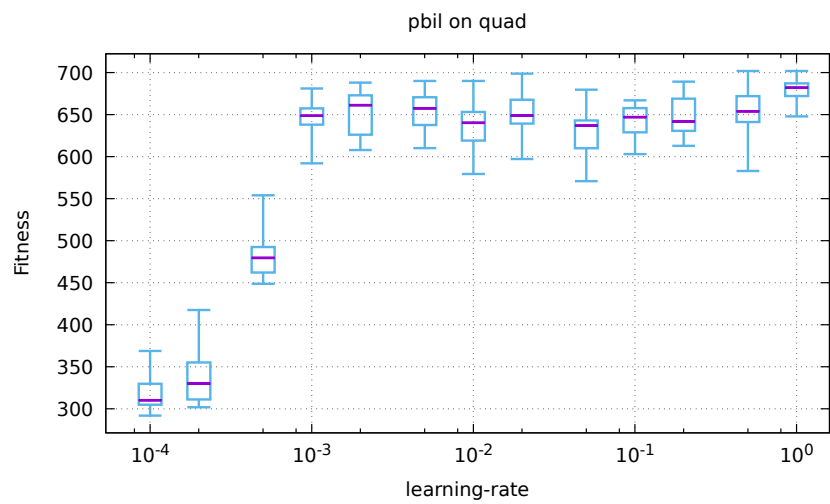
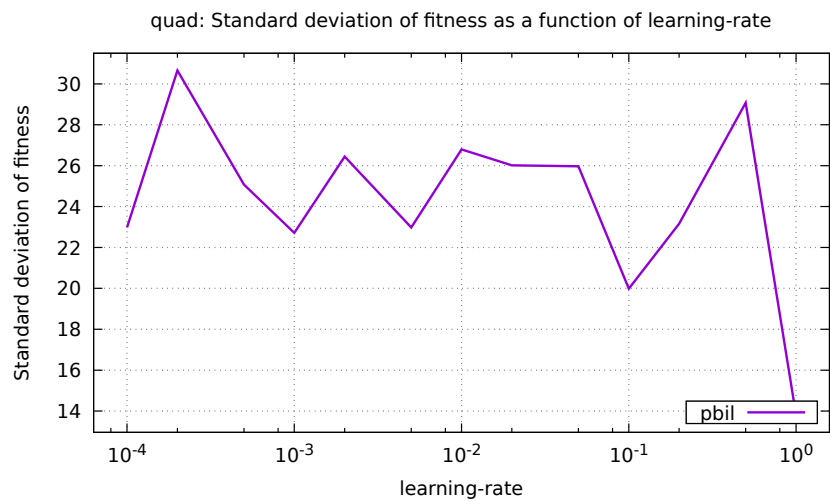
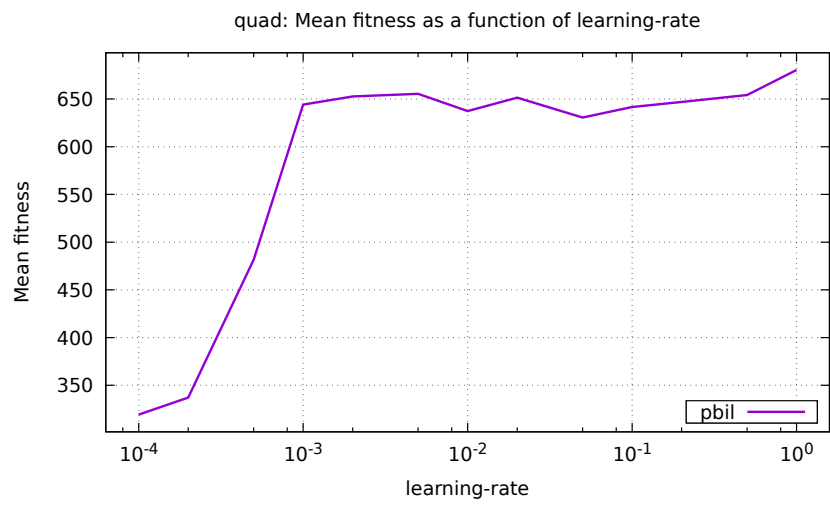
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 112 | 113   | 115  | 116   | 119 | 10 |
| pbil-0.0002 | 118 | 120   | 121  | 121   | 125 | 9  |
| pbil-0.0005 | 135 | 137   | 138  | 139   | 141 | 8  |
| pbil-0.001  | 160 | 161   | 162  | 163   | 164 | 6  |
| pbil-0.002  | 185 | 189   | 189  | 189   | 189 | 3  |
| pbil-0.005  | 189 | 189   | 189  | 189   | 189 | 1  |
| pbil-0.01   | 189 | 189   | 189  | 189   | 189 | 1  |
| pbil-0.02   | 135 | 189   | 189  | 189   | 189 | 4  |
| pbil-0.05   | 100 | 100   | 153  | 189   | 189 | 7  |
| pbil-0.1    | 100 | 100   | 100  | 189   | 189 | 11 |
| pbil-0.2    | 100 | 100   | 100  | 100   | 189 | 12 |
| pbil-0.5    | 100 | 100   | 100  | 100   | 189 | 12 |
| pbil-1      | 100 | 100   | 189  | 189   | 189 | 5  |





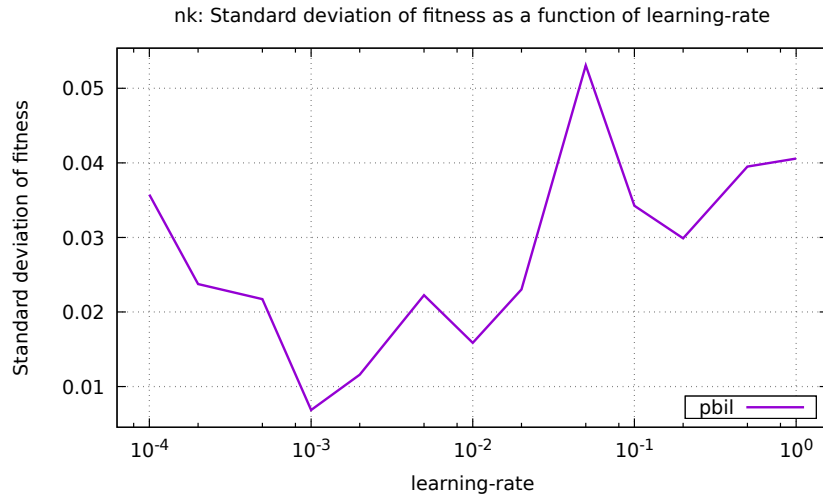
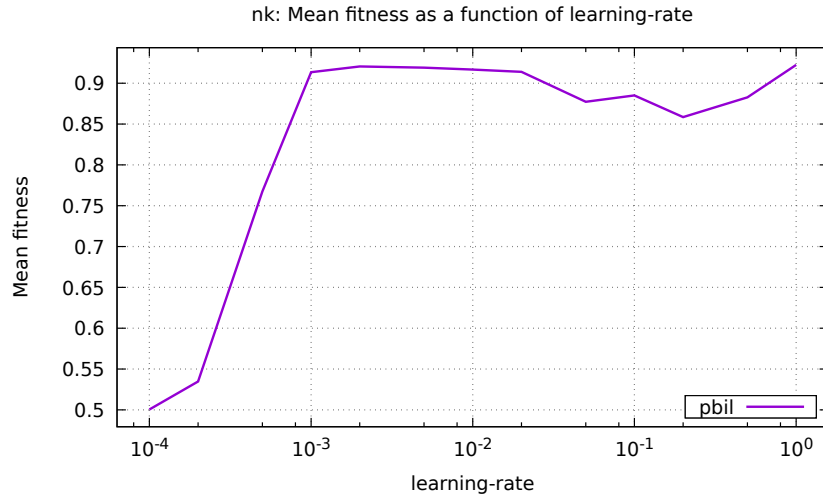
## 14 Function quad

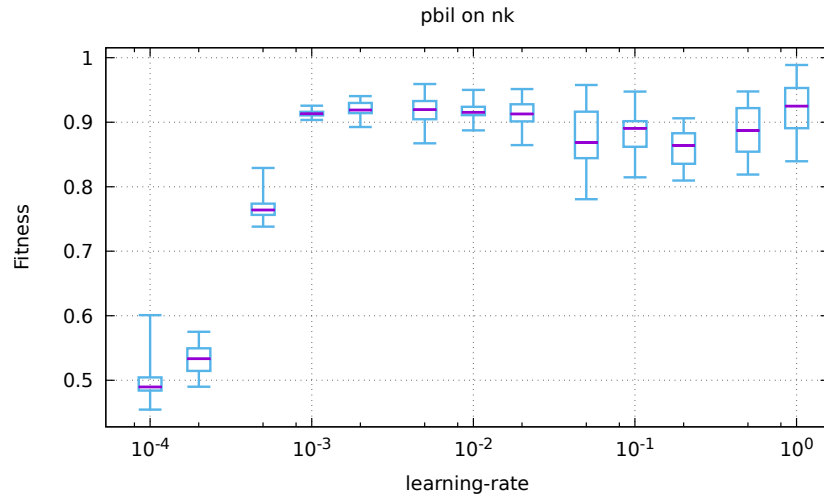
| algorithm   | min    | $Q_1$  | med.   | $Q_3$  | max    | rk |
|-------------|--------|--------|--------|--------|--------|----|
| pbil-0.0001 | 291.88 | 304.90 | 310.28 | 329.77 | 368.73 | 13 |
| pbil-0.0002 | 301.85 | 311.09 | 329.86 | 355.17 | 417.47 | 12 |
| pbil-0.0005 | 448.69 | 462.13 | 479.51 | 492.37 | 554.00 | 11 |
| pbil-0.001  | 592.01 | 638.04 | 648.51 | 657.42 | 681.01 | 6  |
| pbil-0.002  | 607.82 | 626.01 | 661.15 | 672.85 | 687.95 | 2  |
| pbil-0.005  | 610.02 | 637.59 | 657.46 | 670.67 | 689.91 | 3  |
| pbil-0.01   | 579.33 | 619.02 | 640.10 | 653.15 | 689.91 | 9  |
| pbil-0.02   | 597.07 | 639.26 | 648.63 | 667.65 | 698.66 | 5  |
| pbil-0.05   | 570.71 | 609.95 | 636.72 | 642.91 | 679.64 | 10 |
| pbil-0.1    | 602.94 | 628.83 | 646.90 | 657.67 | 666.83 | 7  |
| pbil-0.2    | 612.73 | 630.55 | 641.61 | 668.93 | 689.12 | 8  |
| pbil-0.5    | 582.86 | 641.16 | 653.99 | 671.84 | 701.81 | 4  |
| pbil-1      | 647.75 | 672.07 | 681.88 | 687.14 | 701.81 | 1  |



## 15 Function nk

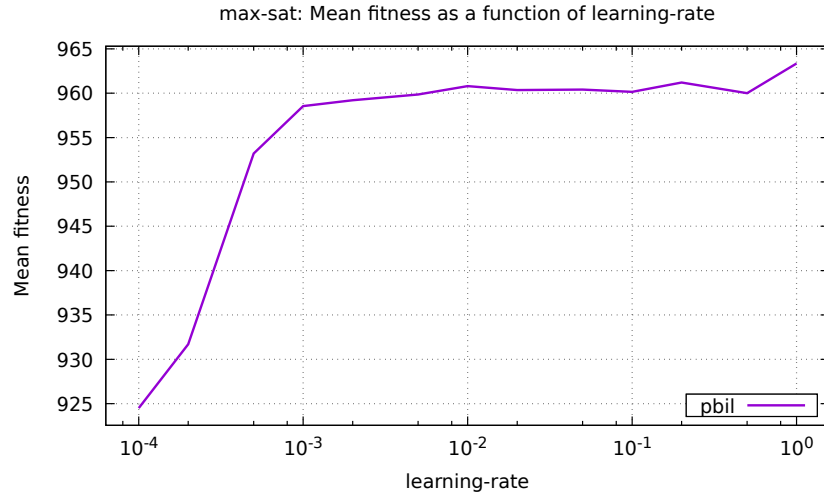
| algorithm   | min  | $Q_1$ | med. | $Q_3$ | max  | rk |
|-------------|------|-------|------|-------|------|----|
| pbil-0.0001 | 0.45 | 0.48  | 0.49 | 0.50  | 0.60 | 13 |
| pbil-0.0002 | 0.49 | 0.51  | 0.53 | 0.55  | 0.58 | 12 |
| pbil-0.0005 | 0.74 | 0.76  | 0.76 | 0.77  | 0.83 | 11 |
| pbil-0.001  | 0.90 | 0.91  | 0.91 | 0.92  | 0.93 | 5  |
| pbil-0.002  | 0.89 | 0.91  | 0.92 | 0.93  | 0.94 | 3  |
| pbil-0.005  | 0.87 | 0.90  | 0.92 | 0.93  | 0.96 | 2  |
| pbil-0.01   | 0.89 | 0.91  | 0.92 | 0.92  | 0.95 | 4  |
| pbil-0.02   | 0.86 | 0.90  | 0.91 | 0.93  | 0.95 | 6  |
| pbil-0.05   | 0.78 | 0.84  | 0.87 | 0.92  | 0.96 | 9  |
| pbil-0.1    | 0.81 | 0.86  | 0.89 | 0.90  | 0.95 | 7  |
| pbil-0.2    | 0.81 | 0.84  | 0.86 | 0.88  | 0.91 | 10 |
| pbil-0.5    | 0.82 | 0.85  | 0.89 | 0.92  | 0.95 | 8  |
| pbil-1      | 0.84 | 0.89  | 0.92 | 0.95  | 0.99 | 1  |

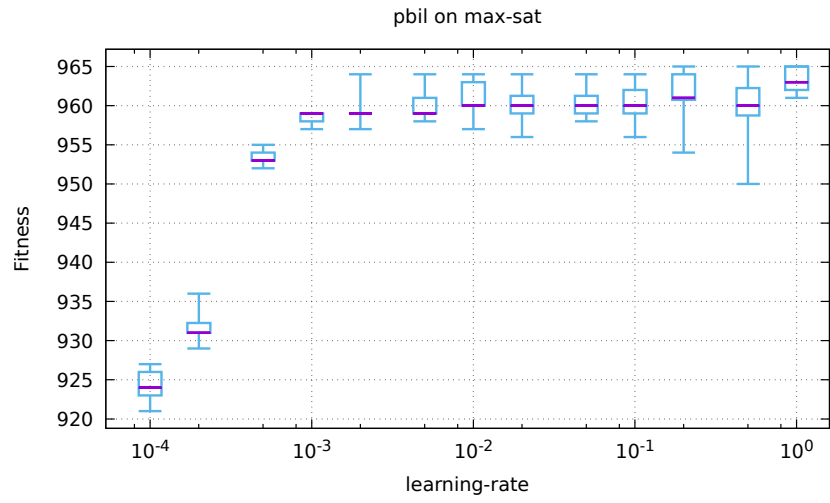
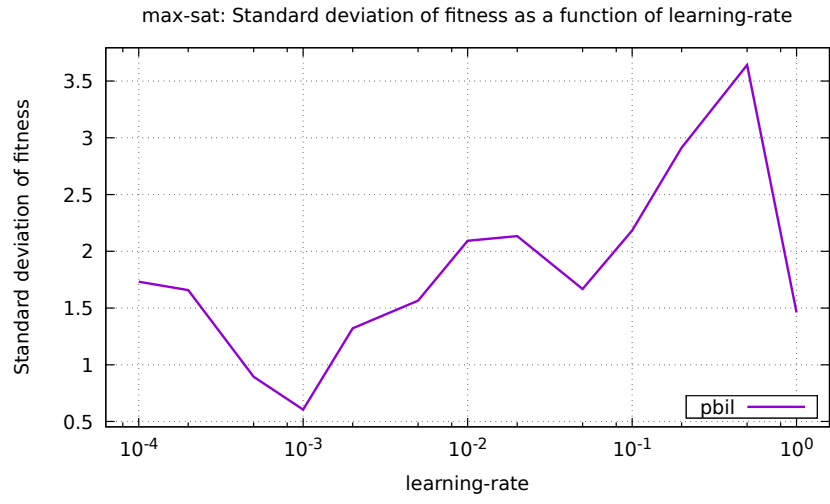




## 16 Function max-sat

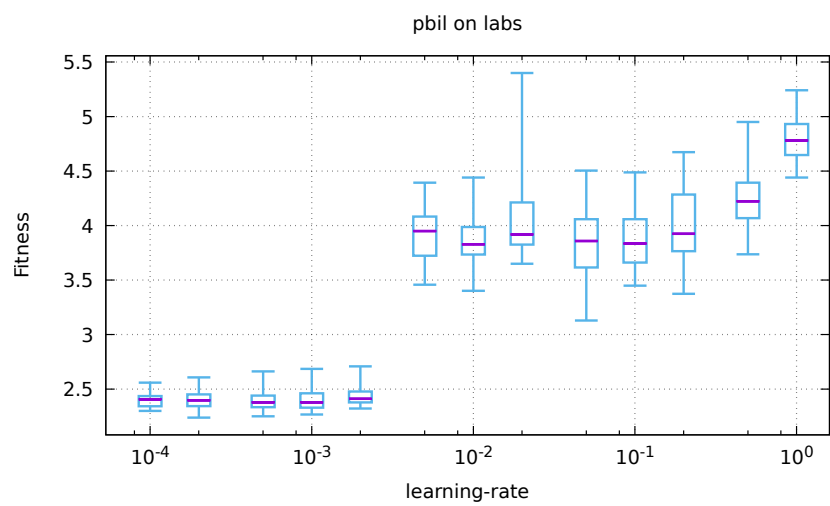
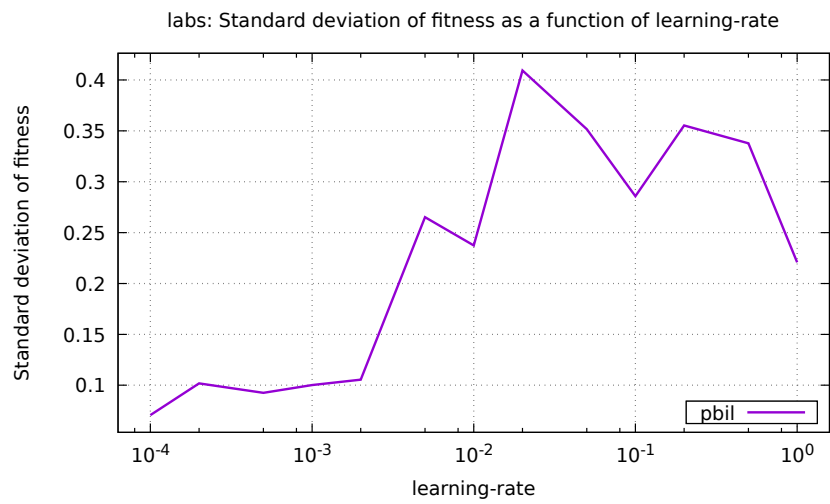
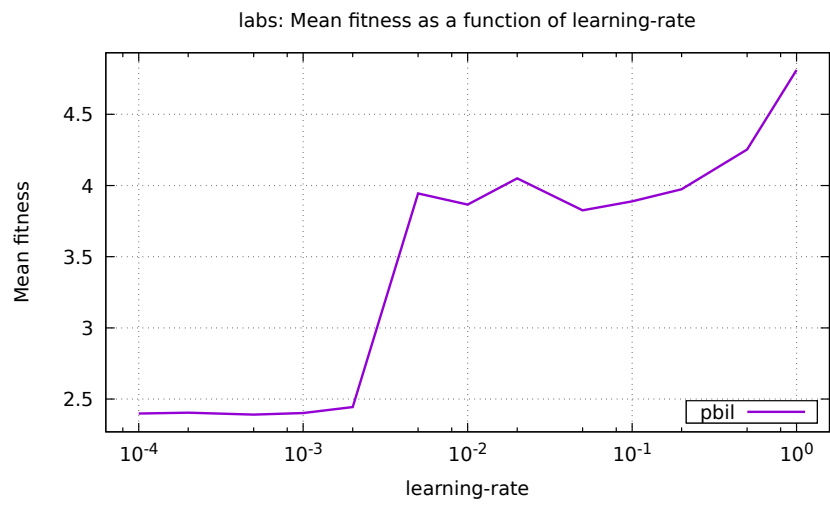
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 921 | 923   | 924  | 926   | 927 | 13 |
| pbil-0.0002 | 929 | 931   | 931  | 932   | 936 | 12 |
| pbil-0.0005 | 952 | 953   | 953  | 954   | 955 | 11 |
| pbil-0.001  | 957 | 958   | 959  | 959   | 959 | 10 |
| pbil-0.002  | 957 | 959   | 959  | 959   | 964 | 9  |
| pbil-0.005  | 958 | 959   | 959  | 961   | 964 | 8  |
| pbil-0.01   | 957 | 960   | 960  | 963   | 964 | 3  |
| pbil-0.02   | 956 | 959   | 960  | 961   | 964 | 6  |
| pbil-0.05   | 958 | 959   | 960  | 961   | 964 | 5  |
| pbil-0.1    | 956 | 959   | 960  | 962   | 964 | 4  |
| pbil-0.2    | 954 | 961   | 961  | 964   | 965 | 2  |
| pbil-0.5    | 950 | 959   | 960  | 962   | 965 | 7  |
| pbil-1      | 961 | 962   | 963  | 965   | 965 | 1  |





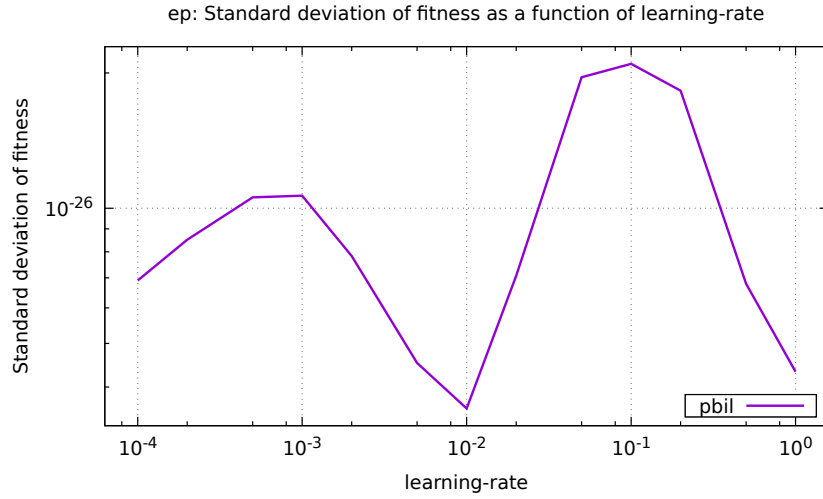
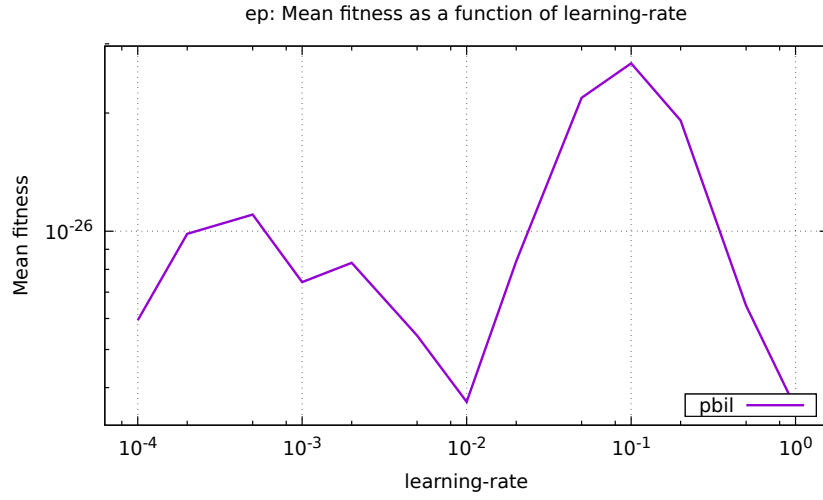
## 17 Function labs

| algorithm   | min  | $Q_1$ | med. | $Q_3$ | max  | rk |
|-------------|------|-------|------|-------|------|----|
| pbil-0.0001 | 2.30 | 2.34  | 2.41 | 2.44  | 2.56 | 10 |
| pbil-0.0002 | 2.24 | 2.34  | 2.40 | 2.45  | 2.61 | 11 |
| pbil-0.0005 | 2.25 | 2.33  | 2.38 | 2.44  | 2.66 | 12 |
| pbil-0.001  | 2.27 | 2.33  | 2.38 | 2.46  | 2.69 | 13 |
| pbil-0.002  | 2.32 | 2.38  | 2.41 | 2.48  | 2.71 | 9  |
| pbil-0.005  | 3.46 | 3.72  | 3.95 | 4.08  | 4.39 | 3  |
| pbil-0.01   | 3.40 | 3.73  | 3.83 | 3.99  | 4.44 | 8  |
| pbil-0.02   | 3.65 | 3.83  | 3.92 | 4.21  | 5.40 | 5  |
| pbil-0.05   | 3.13 | 3.62  | 3.86 | 4.06  | 4.50 | 6  |
| pbil-0.1    | 3.45 | 3.66  | 3.83 | 4.06  | 4.49 | 7  |
| pbil-0.2    | 3.37 | 3.77  | 3.92 | 4.28  | 4.67 | 4  |
| pbil-0.5    | 3.74 | 4.07  | 4.22 | 4.39  | 4.95 | 2  |
| pbil-1      | 4.44 | 4.65  | 4.78 | 4.93  | 5.24 | 1  |

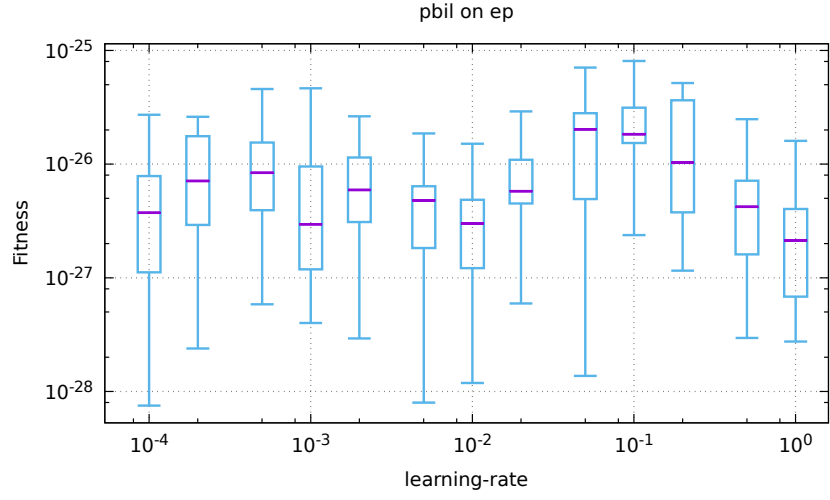


## 18 Function ep

| algorithm   | min                    | $Q_1$                  | med.                   | $Q_3$                  | max                    | rk |
|-------------|------------------------|------------------------|------------------------|------------------------|------------------------|----|
| pbil-0.0001 | $7.50 \times 10^{-29}$ | $1.12 \times 10^{-27}$ | $3.73 \times 10^{-27}$ | $7.86 \times 10^{-27}$ | $2.72 \times 10^{-26}$ | 4  |
| pbil-0.0002 | $2.39 \times 10^{-28}$ | $2.92 \times 10^{-27}$ | $7.12 \times 10^{-27}$ | $1.76 \times 10^{-26}$ | $2.61 \times 10^{-26}$ | 9  |
| pbil-0.0005 | $5.84 \times 10^{-28}$ | $3.93 \times 10^{-27}$ | $8.38 \times 10^{-27}$ | $1.55 \times 10^{-26}$ | $4.58 \times 10^{-26}$ | 10 |
| pbil-0.001  | $4.00 \times 10^{-28}$ | $1.19 \times 10^{-27}$ | $2.96 \times 10^{-27}$ | $9.54 \times 10^{-27}$ | $4.65 \times 10^{-26}$ | 2  |
| pbil-0.002  | $2.93 \times 10^{-28}$ | $3.09 \times 10^{-27}$ | $5.93 \times 10^{-27}$ | $1.14 \times 10^{-26}$ | $2.64 \times 10^{-26}$ | 8  |
| pbil-0.005  | $7.98 \times 10^{-29}$ | $1.83 \times 10^{-27}$ | $4.80 \times 10^{-27}$ | $6.40 \times 10^{-27}$ | $1.86 \times 10^{-26}$ | 6  |
| pbil-0.01   | $1.19 \times 10^{-28}$ | $1.21 \times 10^{-27}$ | $3.01 \times 10^{-27}$ | $4.85 \times 10^{-27}$ | $1.51 \times 10^{-26}$ | 3  |
| pbil-0.02   | $5.94 \times 10^{-28}$ | $4.51 \times 10^{-27}$ | $5.78 \times 10^{-27}$ | $1.09 \times 10^{-26}$ | $2.91 \times 10^{-26}$ | 7  |
| pbil-0.05   | $1.37 \times 10^{-28}$ | $4.93 \times 10^{-27}$ | $2.03 \times 10^{-26}$ | $2.81 \times 10^{-26}$ | $7.07 \times 10^{-26}$ | 13 |
| pbil-0.1    | $2.38 \times 10^{-27}$ | $1.53 \times 10^{-26}$ | $1.83 \times 10^{-26}$ | $3.14 \times 10^{-26}$ | $8.09 \times 10^{-26}$ | 12 |
| pbil-0.2    | $1.16 \times 10^{-27}$ | $3.77 \times 10^{-27}$ | $1.03 \times 10^{-26}$ | $3.65 \times 10^{-26}$ | $5.18 \times 10^{-26}$ | 11 |
| pbil-0.5    | $2.95 \times 10^{-28}$ | $1.61 \times 10^{-27}$ | $4.21 \times 10^{-27}$ | $7.15 \times 10^{-27}$ | $2.49 \times 10^{-26}$ | 5  |
| pbil-1      | $2.75 \times 10^{-28}$ | $6.83 \times 10^{-28}$ | $2.13 \times 10^{-27}$ | $4.03 \times 10^{-27}$ | $1.60 \times 10^{-26}$ | 1  |

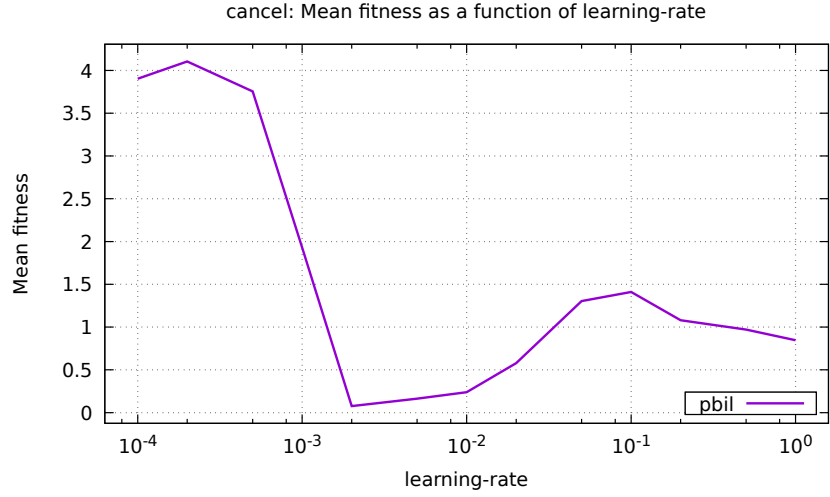


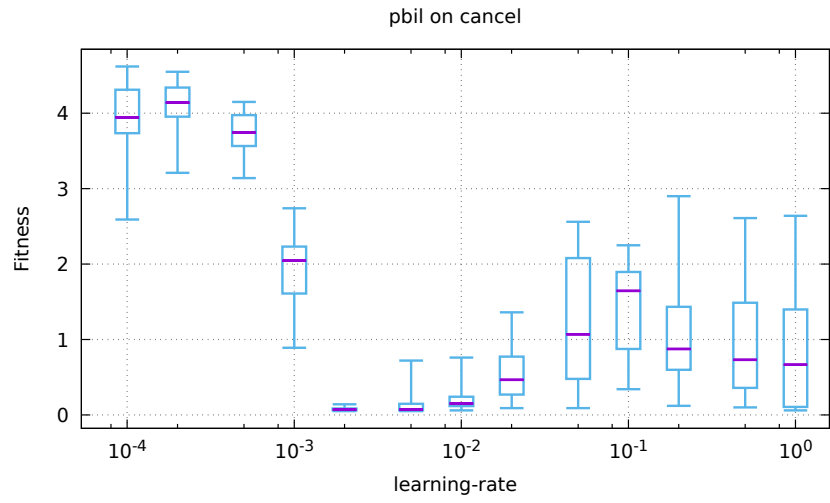
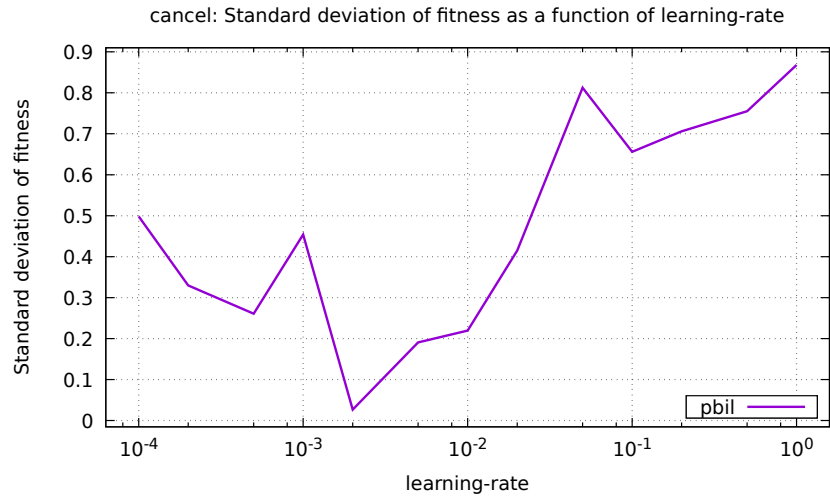




## 19 Function cancel

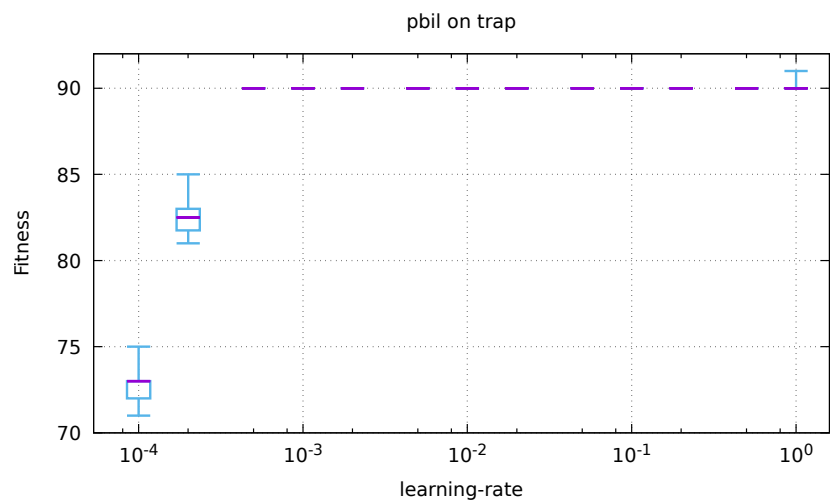
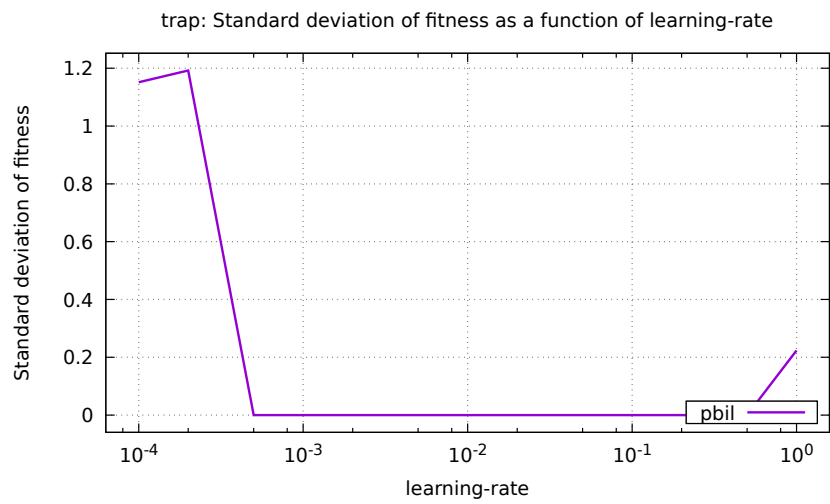
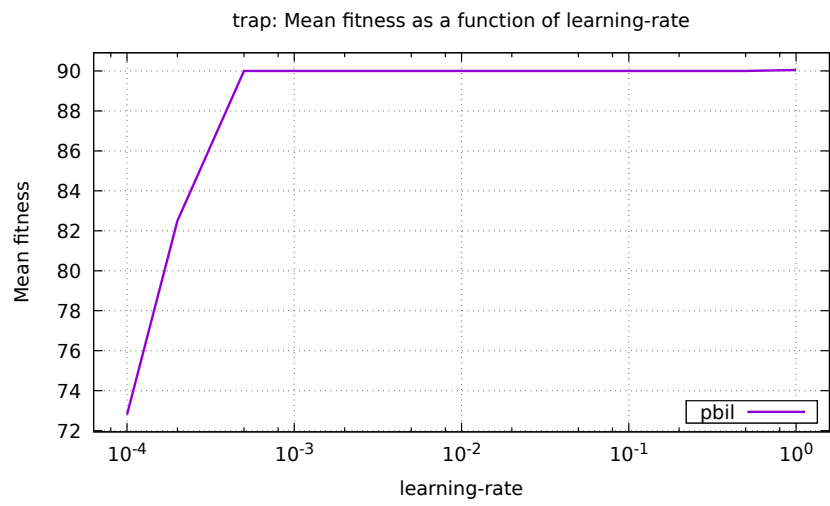
| algorithm   | min  | $Q_1$ | med. | $Q_3$ | max  | rk |
|-------------|------|-------|------|-------|------|----|
| pbil-0.0001 | 2.59 | 3.74  | 3.95 | 4.31  | 4.62 | 12 |
| pbil-0.0002 | 3.21 | 3.96  | 4.14 | 4.34  | 4.55 | 13 |
| pbil-0.0005 | 3.14 | 3.57  | 3.75 | 3.98  | 4.15 | 11 |
| pbil-0.001  | 0.89 | 1.61  | 2.05 | 2.23  | 2.74 | 10 |
| pbil-0.002  | 0.05 | 0.06  | 0.07 | 0.09  | 0.14 | 1  |
| pbil-0.005  | 0.05 | 0.06  | 0.07 | 0.15  | 0.72 | 2  |
| pbil-0.01   | 0.06 | 0.12  | 0.15 | 0.24  | 0.76 | 3  |
| pbil-0.02   | 0.09 | 0.27  | 0.47 | 0.77  | 1.36 | 4  |
| pbil-0.05   | 0.09 | 0.48  | 1.07 | 2.08  | 2.56 | 8  |
| pbil-0.1    | 0.34 | 0.88  | 1.65 | 1.90  | 2.25 | 9  |
| pbil-0.2    | 0.12 | 0.60  | 0.88 | 1.43  | 2.90 | 7  |
| pbil-0.5    | 0.10 | 0.36  | 0.73 | 1.49  | 2.61 | 6  |
| pbil-1      | 0.06 | 0.11  | 0.67 | 1.40  | 2.64 | 5  |





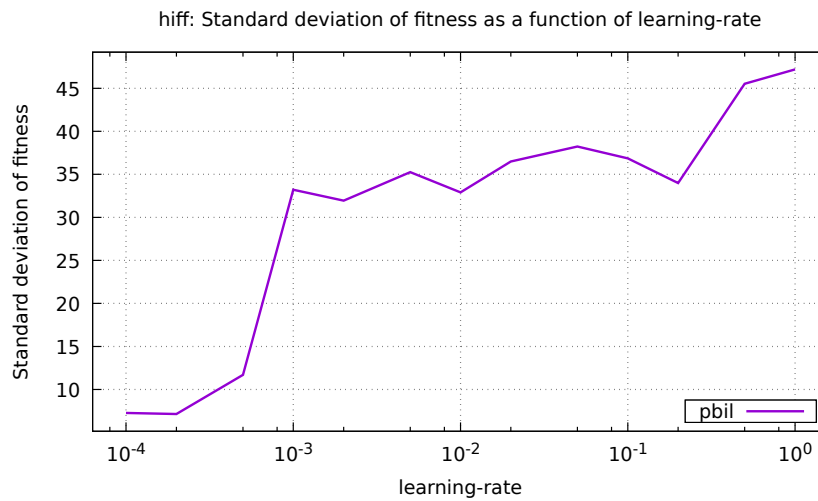
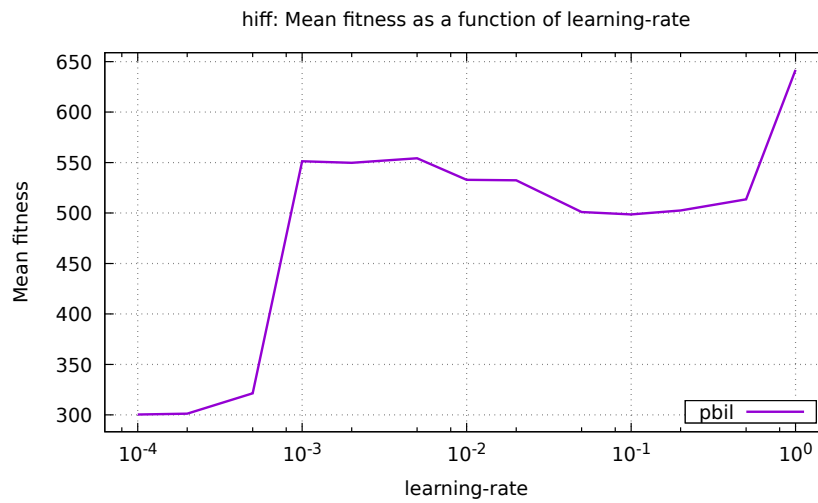
## 20 Function trap

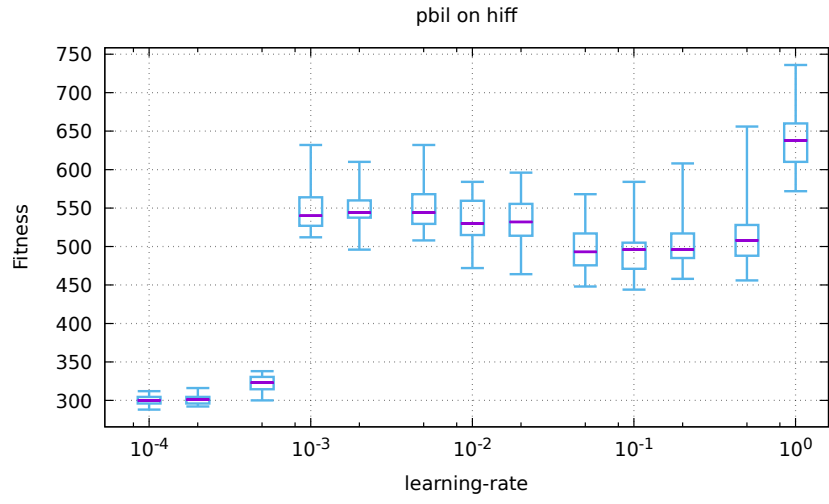
| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 71  | 72    | 73   | 73    | 75  | 13 |
| pbil-0.0002 | 81  | 82    | 83   | 83    | 85  | 12 |
| pbil-0.0005 | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.001  | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.002  | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.005  | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.01   | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.02   | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.05   | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.1    | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.2    | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-0.5    | 90  | 90    | 90   | 90    | 90  | 2  |
| pbil-1      | 90  | 90    | 90   | 90    | 91  | 1  |



## 21 Function hiff

| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 288 | 296   | 300  | 305   | 312 | 13 |
| pbil-0.0002 | 292 | 296   | 301  | 305   | 316 | 12 |
| pbil-0.0005 | 300 | 315   | 323  | 331   | 338 | 11 |
| pbil-0.001  | 512 | 527   | 540  | 564   | 632 | 4  |
| pbil-0.002  | 496 | 538   | 544  | 560   | 610 | 2  |
| pbil-0.005  | 508 | 530   | 544  | 568   | 632 | 3  |
| pbil-0.01   | 472 | 515   | 530  | 560   | 584 | 6  |
| pbil-0.02   | 464 | 514   | 532  | 556   | 596 | 5  |
| pbil-0.05   | 448 | 476   | 493  | 517   | 568 | 10 |
| pbil-0.1    | 444 | 471   | 496  | 505   | 584 | 9  |
| pbil-0.2    | 458 | 485   | 496  | 517   | 608 | 8  |
| pbil-0.5    | 456 | 488   | 508  | 528   | 656 | 7  |
| pbil-1      | 572 | 610   | 638  | 660   | 736 | 1  |





## 22 Function plateau

| algorithm   | min | $Q_1$ | med. | $Q_3$ | max | rk |
|-------------|-----|-------|------|-------|-----|----|
| pbil-0.0001 | 81  | 82    | 83   | 84    | 86  | 13 |
| pbil-0.0002 | 92  | 92    | 92   | 93    | 94  | 12 |
| pbil-0.0005 | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.001  | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.002  | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.005  | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.01   | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.02   | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.05   | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.1    | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.2    | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-0.5    | 101 | 101   | 101  | 101   | 101 | 1  |
| pbil-1      | 101 | 101   | 101  | 101   | 101 | 1  |

