

# HNCO

## Influence of the learning rate on the performance of PBIL

August 7, 2020

### Abstract

PBIL is applied many times to the same collection of fitness functions (bit vector size  $n = 100$ ), each time with a different learning rate taken from a finite set of values. All learning rates are ranked according to their median fitness over 20 independent runs, first for each fitness function, then across the entire collection of fitness functions. The mean and standard deviation of fitness are also plotted as a function of the learning rate.

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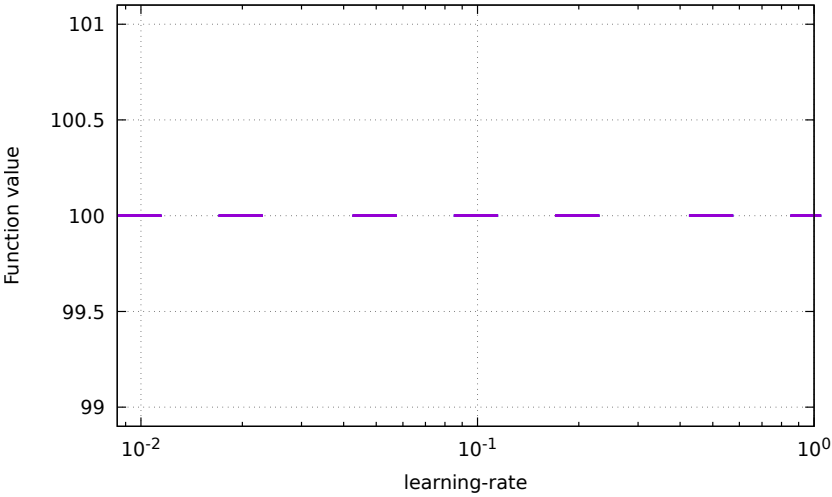
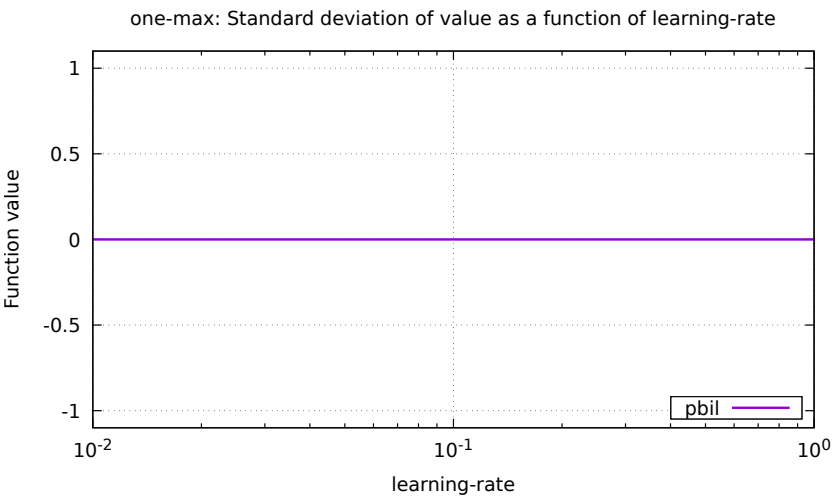
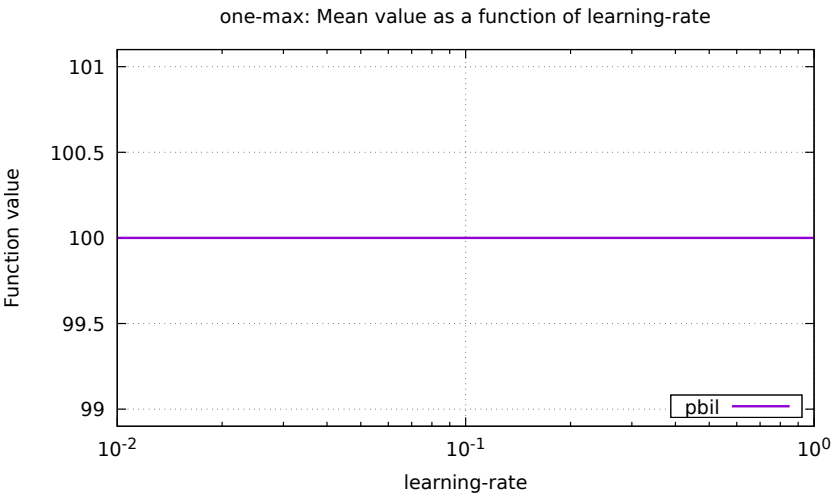
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## 1 Rankings

| algorithm | learning-rate | rank distribution |   |   |   |   |   |   |
|-----------|---------------|-------------------|---|---|---|---|---|---|
|           |               | 1                 | 2 | 3 | 4 | 5 | 6 | 7 |
| pbil      | 1             | 6                 | 1 | 0 | 0 | 1 | 1 | 0 |
| pbil      | 0.01          | 5                 | 3 | 0 | 0 | 0 | 1 | 0 |
| pbil      | 0.02          | 3                 | 1 | 2 | 0 | 1 | 1 | 1 |
| pbil      | 0.05          | 3                 | 0 | 0 | 2 | 2 | 2 | 0 |
| pbil      | 0.5           | 2                 | 1 | 3 | 0 | 1 | 2 | 0 |
| pbil      | 0.2           | 2                 | 0 | 1 | 1 | 1 | 0 | 4 |
| pbil      | 0.1           | 2                 | 0 | 0 | 4 | 1 | 1 | 1 |

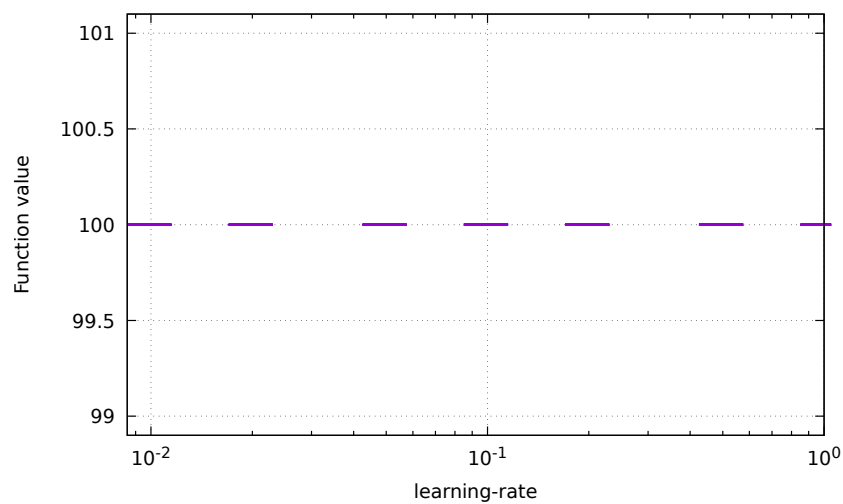
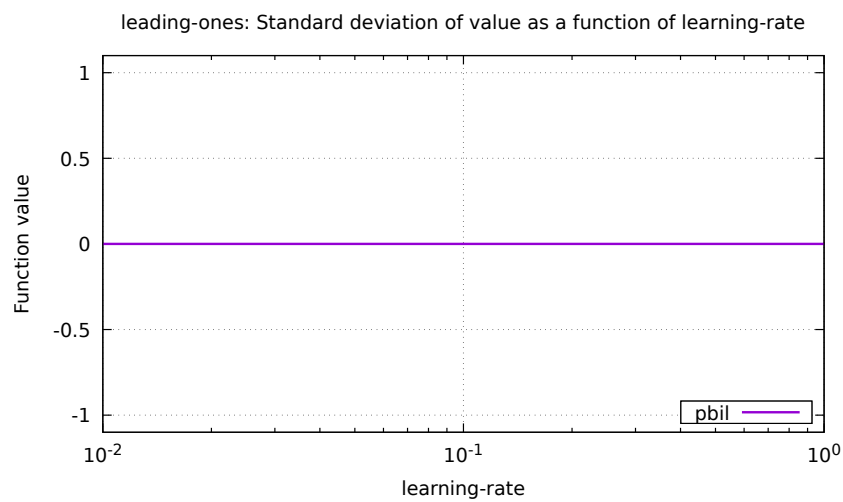
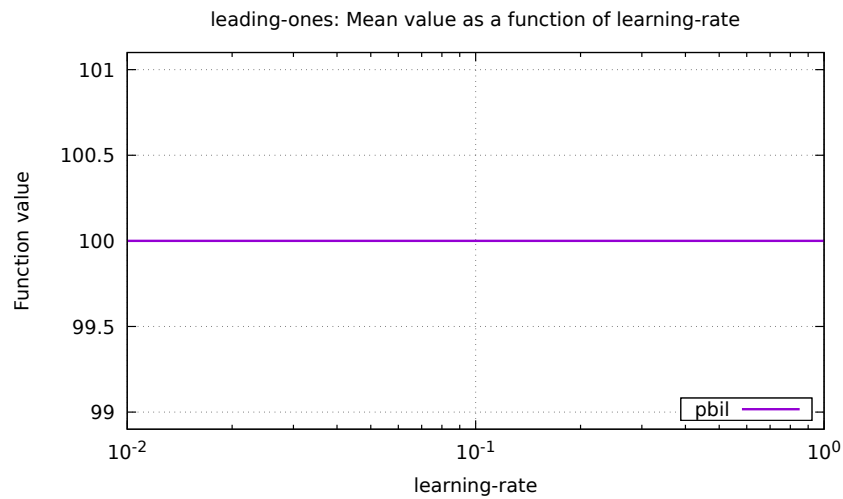
2    Function one-max

| algorithm | learning-rate | function value |       |      |       |     |    |
|-----------|---------------|----------------|-------|------|-------|-----|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max | rk |
| pbil      | 0.01          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.02          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.05          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.1           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.2           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.5           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 1             | 100            | 100   | 100  | 100   | 100 | 1  |



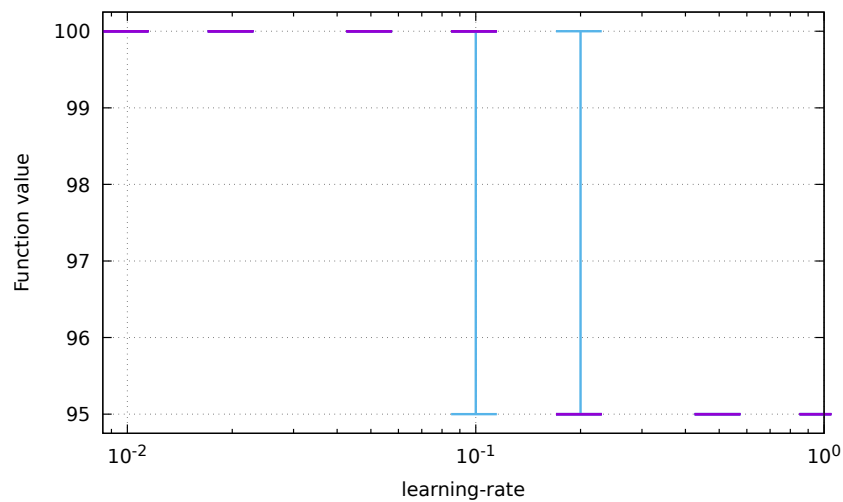
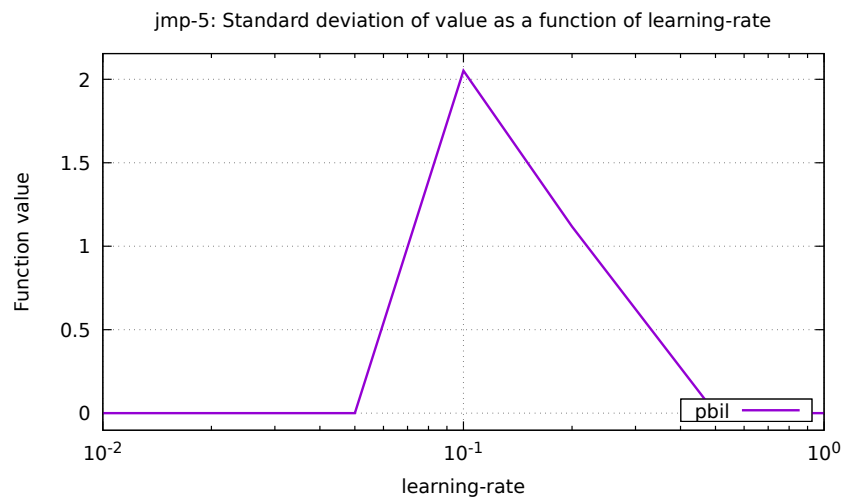
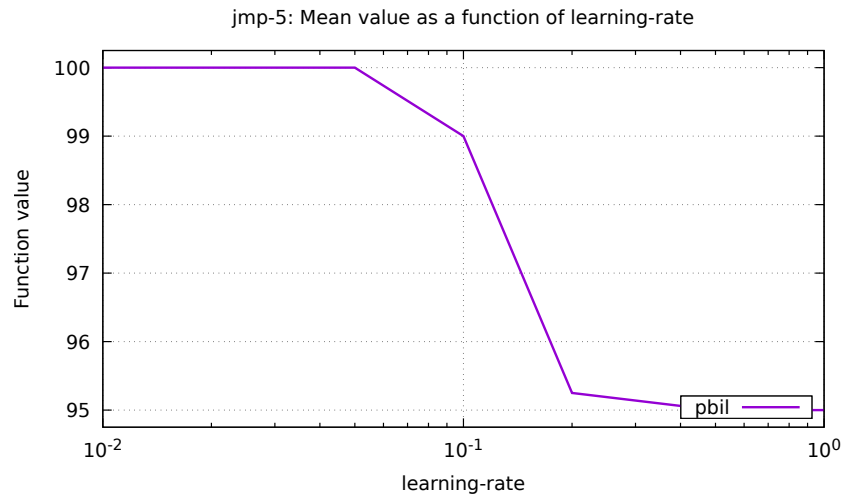
### 3 Function leading-ones

| algorithm | learning-rate | function value |       |      |       |     |    |
|-----------|---------------|----------------|-------|------|-------|-----|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max | rk |
| pbil      | 0.01          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.02          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.05          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.1           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.2           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.5           | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 1             | 100            | 100   | 100  | 100   | 100 | 1  |



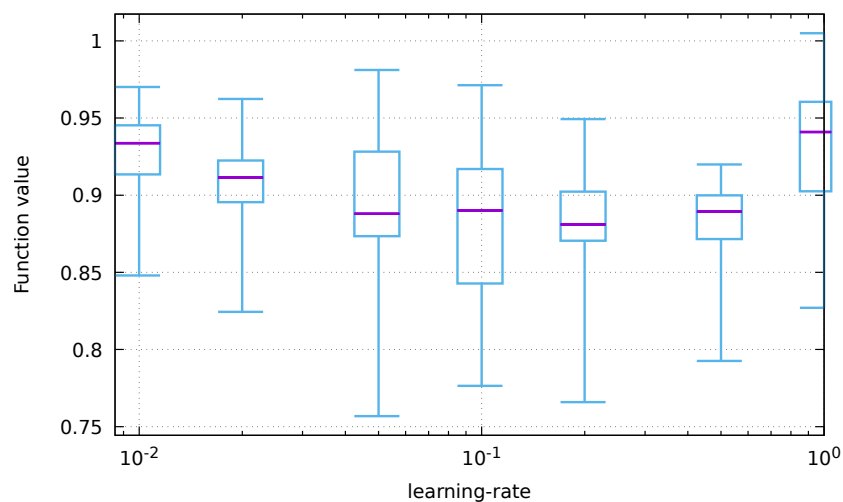
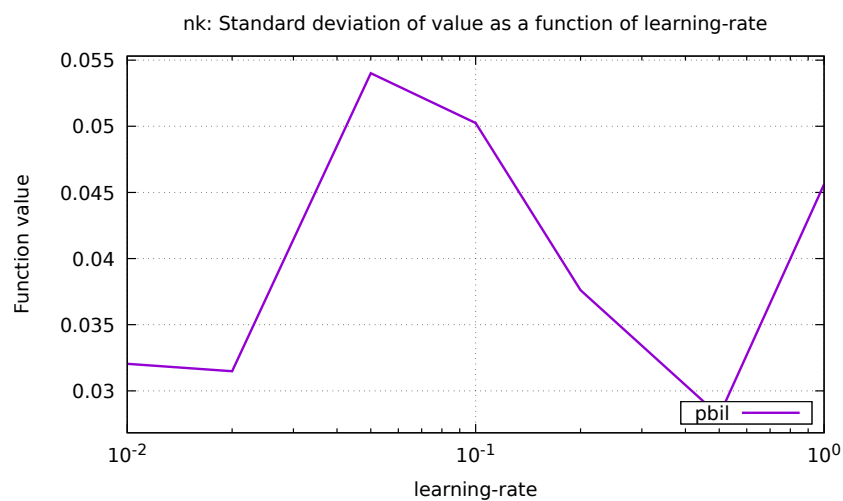
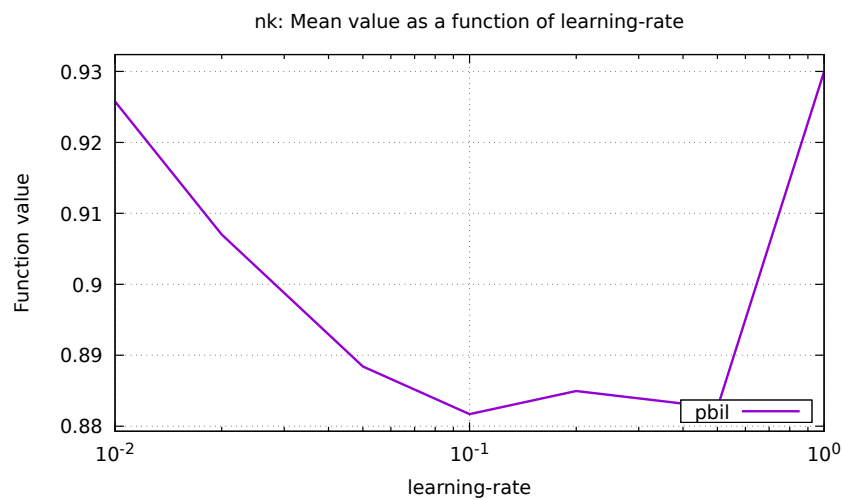
## 4 Function jmp-5

| algorithm | learning-rate | function value |       |      |       |     |    |
|-----------|---------------|----------------|-------|------|-------|-----|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max | rk |
| pbil      | 0.01          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.02          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.05          | 100            | 100   | 100  | 100   | 100 | 1  |
| pbil      | 0.1           | 95             | 100   | 100  | 100   | 100 | 4  |
| pbil      | 0.2           | 95             | 95    | 95   | 95    | 100 | 5  |
| pbil      | 0.5           | 95             | 95    | 95   | 95    | 95  | 6  |
| pbil      | 1             | 95             | 95    | 95   | 95    | 95  | 6  |



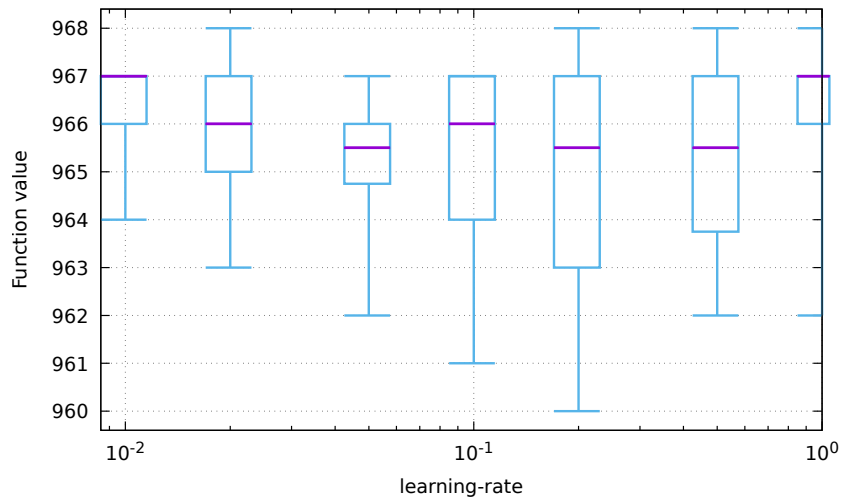
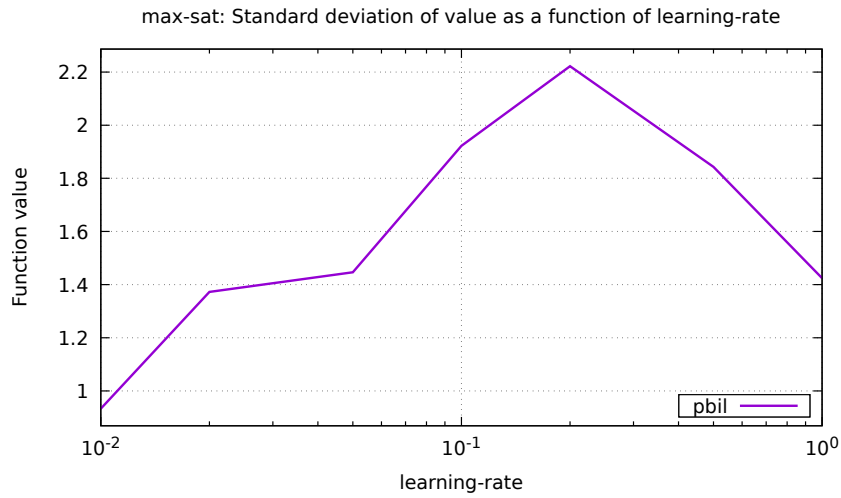
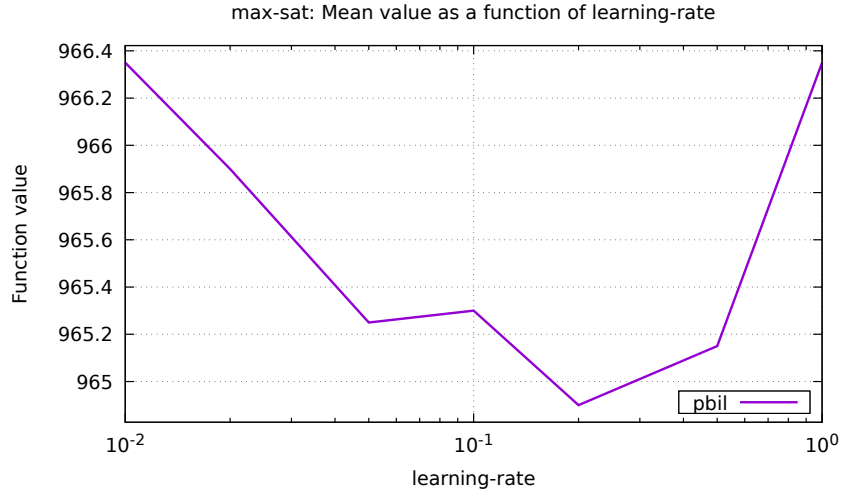
## 5 Function nk

| algorithm | learning-rate | function value |       |      |       |      |    |
|-----------|---------------|----------------|-------|------|-------|------|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max  | rk |
| pbil      | 0.01          | 0.85           | 0.91  | 0.93 | 0.95  | 0.97 | 2  |
| pbil      | 0.02          | 0.82           | 0.90  | 0.91 | 0.92  | 0.96 | 3  |
| pbil      | 0.05          | 0.76           | 0.87  | 0.89 | 0.93  | 0.98 | 6  |
| pbil      | 0.1           | 0.78           | 0.84  | 0.89 | 0.92  | 0.97 | 4  |
| pbil      | 0.2           | 0.77           | 0.87  | 0.88 | 0.90  | 0.95 | 7  |
| pbil      | 0.5           | 0.79           | 0.87  | 0.89 | 0.90  | 0.92 | 5  |
| pbil      | 1             | 0.83           | 0.90  | 0.94 | 0.96  | 1.00 | 1  |



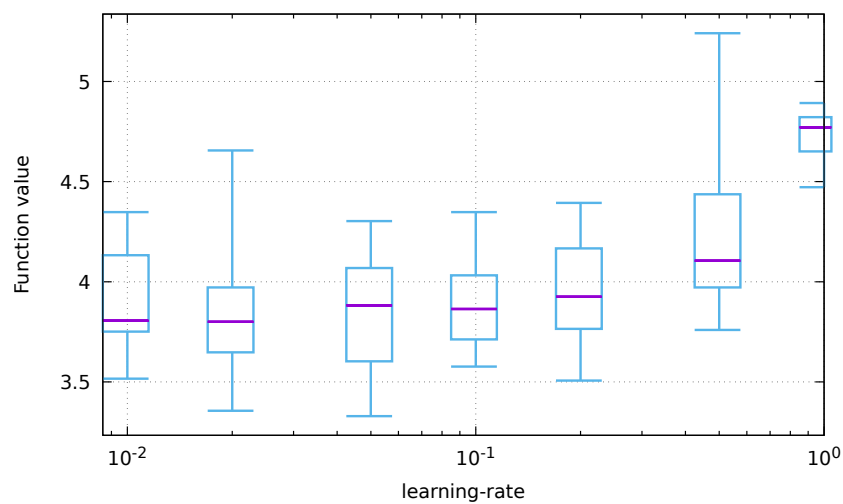
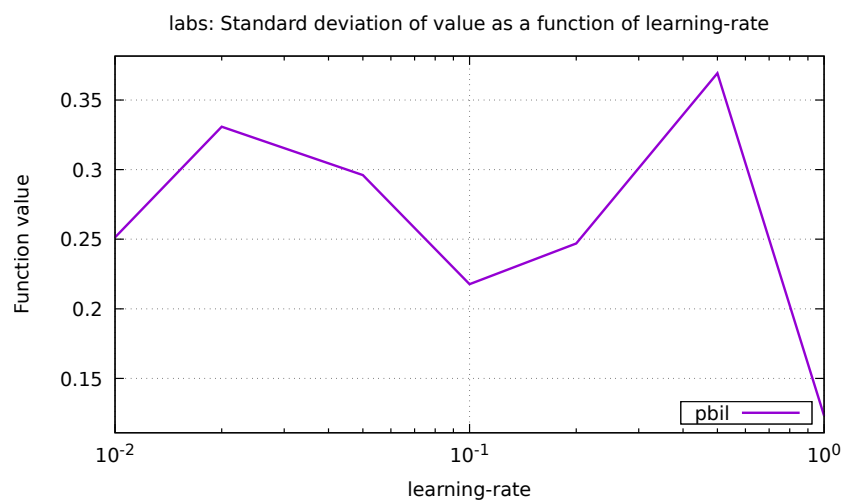
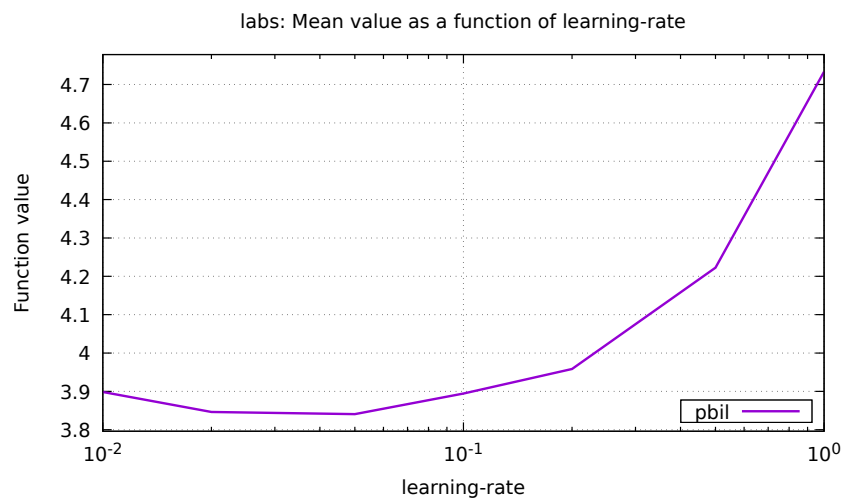
## 6 Function max-sat

| algorithm | learning-rate | function value |       |      |       |     |    |
|-----------|---------------|----------------|-------|------|-------|-----|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max | rk |
| pbil      | 0.01          | 964            | 966   | 967  | 967   | 967 | 1  |
| pbil      | 0.02          | 963            | 965   | 966  | 967   | 968 | 3  |
| pbil      | 0.05          | 962            | 965   | 966  | 966   | 967 | 5  |
| pbil      | 0.1           | 961            | 964   | 966  | 967   | 967 | 4  |
| pbil      | 0.2           | 960            | 963   | 966  | 967   | 968 | 7  |
| pbil      | 0.5           | 962            | 964   | 966  | 967   | 968 | 6  |
| pbil      | 1             | 962            | 966   | 967  | 967   | 968 | 2  |



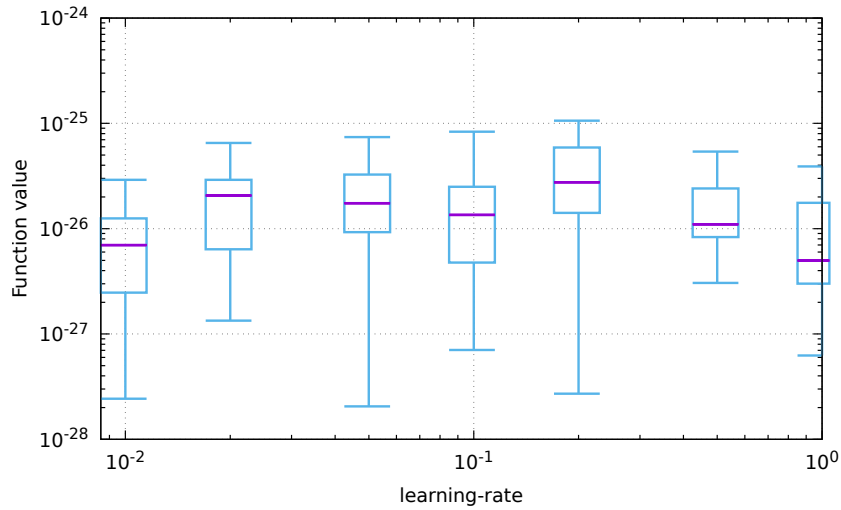
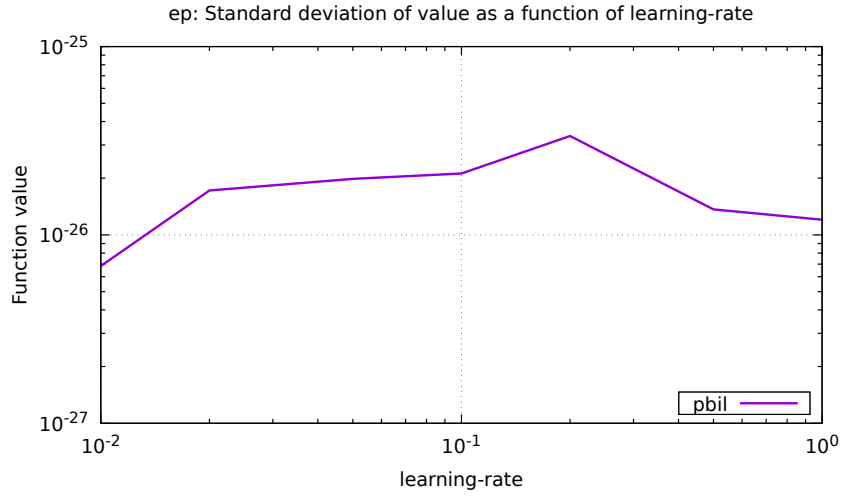
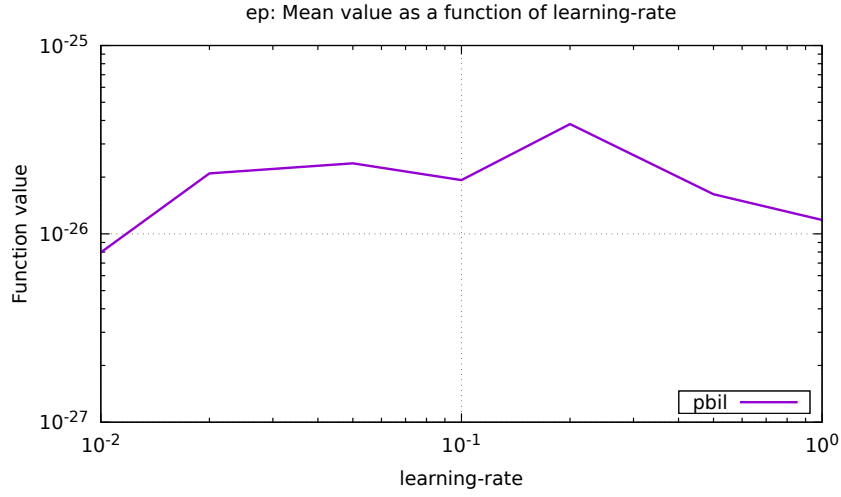
## 7 Function labs

| algorithm | learning-rate | function value |       |      |       |      |    |
|-----------|---------------|----------------|-------|------|-------|------|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max  | rk |
| pbil      | 0.01          | 3.52           | 3.75  | 3.81 | 4.13  | 4.35 | 6  |
| pbil      | 0.02          | 3.36           | 3.65  | 3.80 | 3.97  | 4.66 | 7  |
| pbil      | 0.05          | 3.33           | 3.60  | 3.88 | 4.07  | 4.30 | 4  |
| pbil      | 0.1           | 3.58           | 3.71  | 3.86 | 4.03  | 4.35 | 5  |
| pbil      | 0.2           | 3.51           | 3.77  | 3.92 | 4.17  | 4.39 | 3  |
| pbil      | 0.5           | 3.76           | 3.97  | 4.11 | 4.44  | 5.24 | 2  |
| pbil      | 1             | 4.47           | 4.65  | 4.77 | 4.82  | 4.89 | 1  |



## 8 Function ep

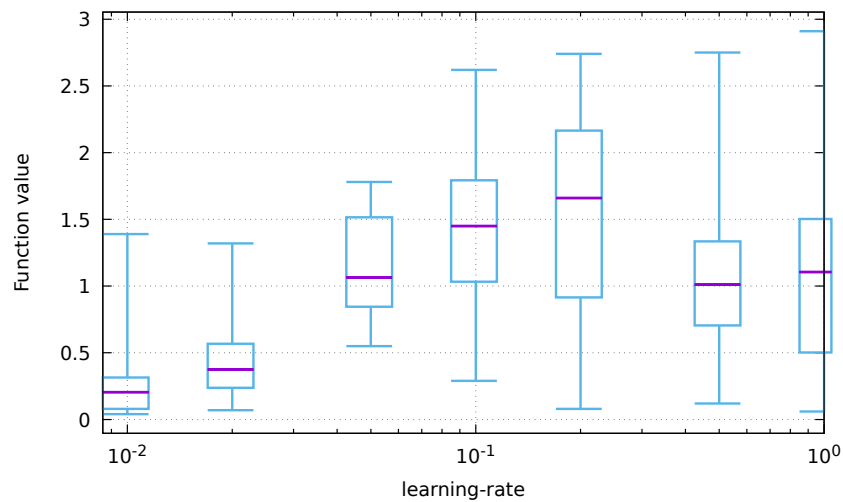
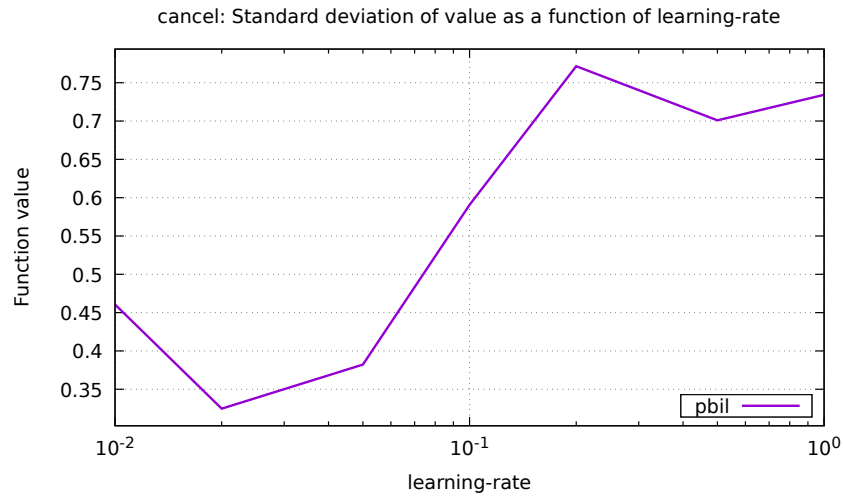
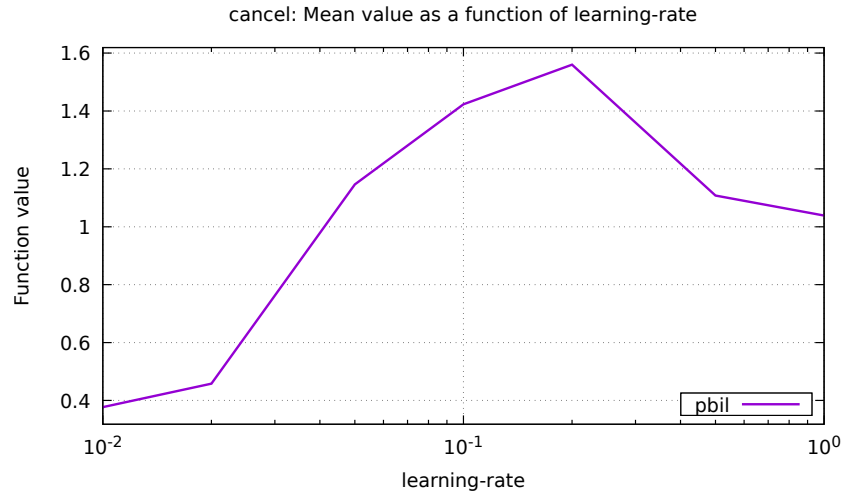
| algorithm | learning-rate | function value        |                       |                       |                       |                       |    |
|-----------|---------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|----|
|           |               | min                   | $Q_1$                 | med.                  | $Q_3$                 | max                   | rk |
| pbil      | 0.01          | $2.4 \times 10^{-28}$ | $2.5 \times 10^{-27}$ | $7.0 \times 10^{-27}$ | $1.3 \times 10^{-26}$ | $2.9 \times 10^{-26}$ | 2  |
| pbil      | 0.02          | $1.3 \times 10^{-27}$ | $6.4 \times 10^{-27}$ | $2.1 \times 10^{-26}$ | $2.9 \times 10^{-26}$ | $6.5 \times 10^{-26}$ | 6  |
| pbil      | 0.05          | $2.1 \times 10^{-28}$ | $9.3 \times 10^{-27}$ | $1.7 \times 10^{-26}$ | $3.3 \times 10^{-26}$ | $7.4 \times 10^{-26}$ | 5  |
| pbil      | 0.1           | $7.0 \times 10^{-28}$ | $4.8 \times 10^{-27}$ | $1.4 \times 10^{-26}$ | $2.5 \times 10^{-26}$ | $8.3 \times 10^{-26}$ | 4  |
| pbil      | 0.2           | $2.7 \times 10^{-28}$ | $1.4 \times 10^{-26}$ | $2.8 \times 10^{-26}$ | $5.9 \times 10^{-26}$ | $1.1 \times 10^{-25}$ | 7  |
| pbil      | 0.5           | $3.1 \times 10^{-27}$ | $8.3 \times 10^{-27}$ | $1.1 \times 10^{-26}$ | $2.4 \times 10^{-26}$ | $5.4 \times 10^{-26}$ | 3  |
| pbil      | 1             | $6.2 \times 10^{-28}$ | $3.0 \times 10^{-27}$ | $5.0 \times 10^{-27}$ | $1.8 \times 10^{-26}$ | $3.9 \times 10^{-26}$ | 1  |





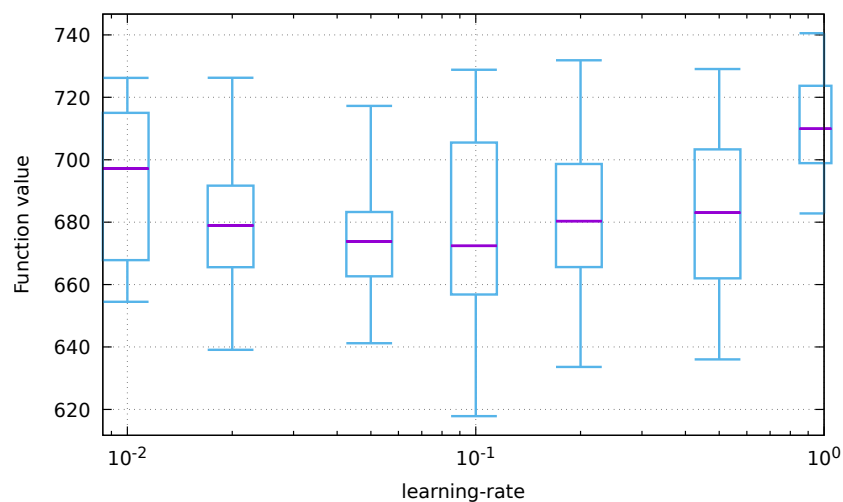
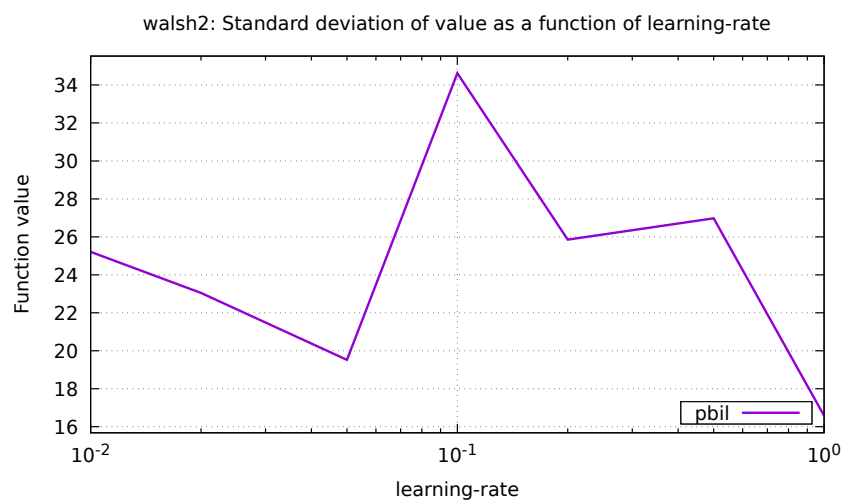
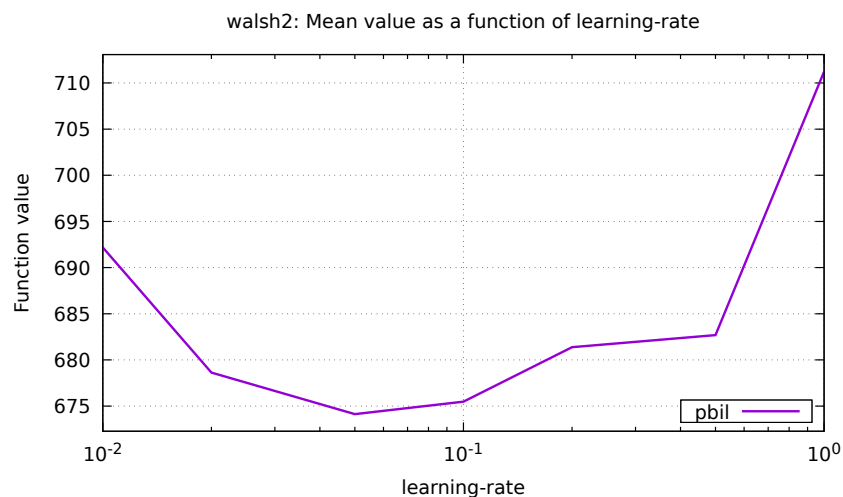
## 9 Function cancel

| algorithm | learning-rate | function value |       |      |       |      |    |
|-----------|---------------|----------------|-------|------|-------|------|----|
|           |               | min            | $Q_1$ | med. | $Q_3$ | max  | rk |
| pbil      | 0.01          | 0.04           | 0.08  | 0.21 | 0.32  | 1.39 | 1  |
| pbil      | 0.02          | 0.07           | 0.24  | 0.38 | 0.57  | 1.32 | 2  |
| pbil      | 0.05          | 0.55           | 0.85  | 1.07 | 1.52  | 1.78 | 4  |
| pbil      | 0.1           | 0.29           | 1.03  | 1.45 | 1.79  | 2.62 | 6  |
| pbil      | 0.2           | 0.08           | 0.92  | 1.66 | 2.17  | 2.74 | 7  |
| pbil      | 0.5           | 0.12           | 0.71  | 1.01 | 1.34  | 2.75 | 3  |
| pbil      | 1             | 0.06           | 0.50  | 1.11 | 1.50  | 2.91 | 5  |



## 10 Function walsh2

| algorithm | learning-rate | function value |        |        |        |        |    |
|-----------|---------------|----------------|--------|--------|--------|--------|----|
|           |               | min            | $Q_1$  | med.   | $Q_3$  | max    | rk |
| pbil      | 0.01          | 654.47         | 667.82 | 697.14 | 715.02 | 726.23 | 2  |
| pbil      | 0.02          | 639.08         | 665.57 | 678.92 | 691.71 | 726.29 | 5  |
| pbil      | 0.05          | 641.21         | 662.65 | 673.83 | 683.24 | 717.26 | 6  |
| pbil      | 0.1           | 617.83         | 656.83 | 672.39 | 705.50 | 728.83 | 7  |
| pbil      | 0.2           | 633.62         | 665.62 | 680.33 | 698.63 | 731.88 | 4  |
| pbil      | 0.5           | 636.03         | 662.00 | 683.02 | 703.33 | 729.08 | 3  |
| pbil      | 1             | 682.81         | 698.91 | 709.93 | 723.70 | 740.55 | 1  |



## A Plan

```
{
  "exec": "hnco",
  "opt": "--print-results --map 1 --map-random -s 100",
  "budget": 200000,
  "num_runs": 20,
  "parallel": true,
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    "values": [ 1e-2, 2e-2, 5e-2, 1e-1, 2e-1, 5e-1, 1 ]
  },
  "graphics": {
    "logscale": true,
    "candlesticks": {
      "boxwidth": "$1 * 0.3"
    }
  },
  "functions": [
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      "opt": "-F 0 --stop-on-maximum",
      "rounding": {
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        "time": { "before": 1, "after": 2 } }
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    {
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      "opt": "-F 10 --stop-on-maximum",
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        "time": { "before": 1, "after": 2 } }
    },
    {
      "id": "jmp-5",
      "opt": "-F 30 --stop-on-maximum -t 5",
      "rounding": {
        "value": { "before": 3, "after": 0 },
        "time": { "before": 1, "after": 2 } }
    },
    {
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        "time": { "before": 1, "after": 2 } }
    },
    {
      "id": "max-sat",
      "opt": "-F 70 -p instances/ms.100.3.1000",
      "rounding": {
        "value": { "before": 3, "after": 0 },
        "time": { "before": 1, "after": 2 } }
    },
    {
      "id": "labs",
      "opt": "-F 81",
      "rounding": {
        "value": { "before": 1, "after": 2 },
        "time": { "before": 1, "after": 2 } }
    },
    {
      "id": "ep",
```

```

        "opt": "-F 90 -p instances/ep.100",
        "reverse": true,
        "logscale": true,
        "rounding": {
            "value": { "before": 1, "after": 1 },
            "time": { "before": 1, "after": 2 } }
    },
    {
        "id": "cancel",
        "opt": "-F 100 -s 99",
        "reverse": true,
        "rounding": {
            "value": { "before": 1, "after": 2 },
            "time": { "before": 1, "after": 2 } }
    },
    {
        "id": "walsh2",
        "opt": "-F 162 -p instances/walsh2.100",
        "rounding": {
            "value": { "before": 3, "after": 2 },
            "time": { "before": 1, "after": 2 } }
    }
],
"algorithms": [
    {
        "id": "pbil",
        "opt": "-A 500 -x 10 -y 1"
    }
]
}

```

## B Default parameters

```

# algorithm = 100
# bm_mc_reset_strategy = 1
# bm_num_gs_cycles = 1
# bm_num_gs_steps = 100
# bm_sampling = 1
# budget = 10000
# bv_size = 100
# description_path = description.txt
# ea_lambda = 100
# ea_mu = 10
# expression = x
# fn_name = noname
# fn_num_traps = 10
# fn_prefix_length = 2
# fn_threshold = 10
# fp_expression = (1-x)^2+100*(y-x^2)^2
# fp_lower_bound = -2
# fp_num_bits = 8
# fp_upper_bound = 2
# function = 0
# ga_crossover_bias = 0.5
# ga_crossover_probability = 0.5
# ga_tournament_size = 10
# hea_bit_herding = 0
# hea_num_seq_updates = 100
# hea_reset_period = 0
# hea_sampling_method = 0
# hea_weight = 1
# learning_rate = 0.001

```

```
# map = 0
# map_input_size = 100
# map_path = map.txt
# map_ts_length = 10
# map_ts_sampling_mode = 0
# mutation_rate = 1
# neighborhood = 0
# neighborhood_iterator = 0
# noise_stddev = 1
# num_iterations = 0
# num_threads = 1
# path = function.txt
# pn_mutation_rate = 1
# pn_neighborhood = 0
# pn_radius = 2
# population_size = 10
# pv_log_num_components = 5
# radius = 2
# results_path = results.json
# rls_patience = 50
# sa_beta_ratio = 1.2
# sa_initial_acceptance_probability = 0.6
# sa_num_transitions = 50
# sa_num_trials = 100
# seed = 0
# selection_size = 1
# solution_path = solution.txt
# target = 100
# print_defaults
# last_parameter
# exec_name = hnco
# version = 0.15
# Generated from hnco.json
```