HNCO
Visualization of Walsh transforms of various functions defined on bit vectors

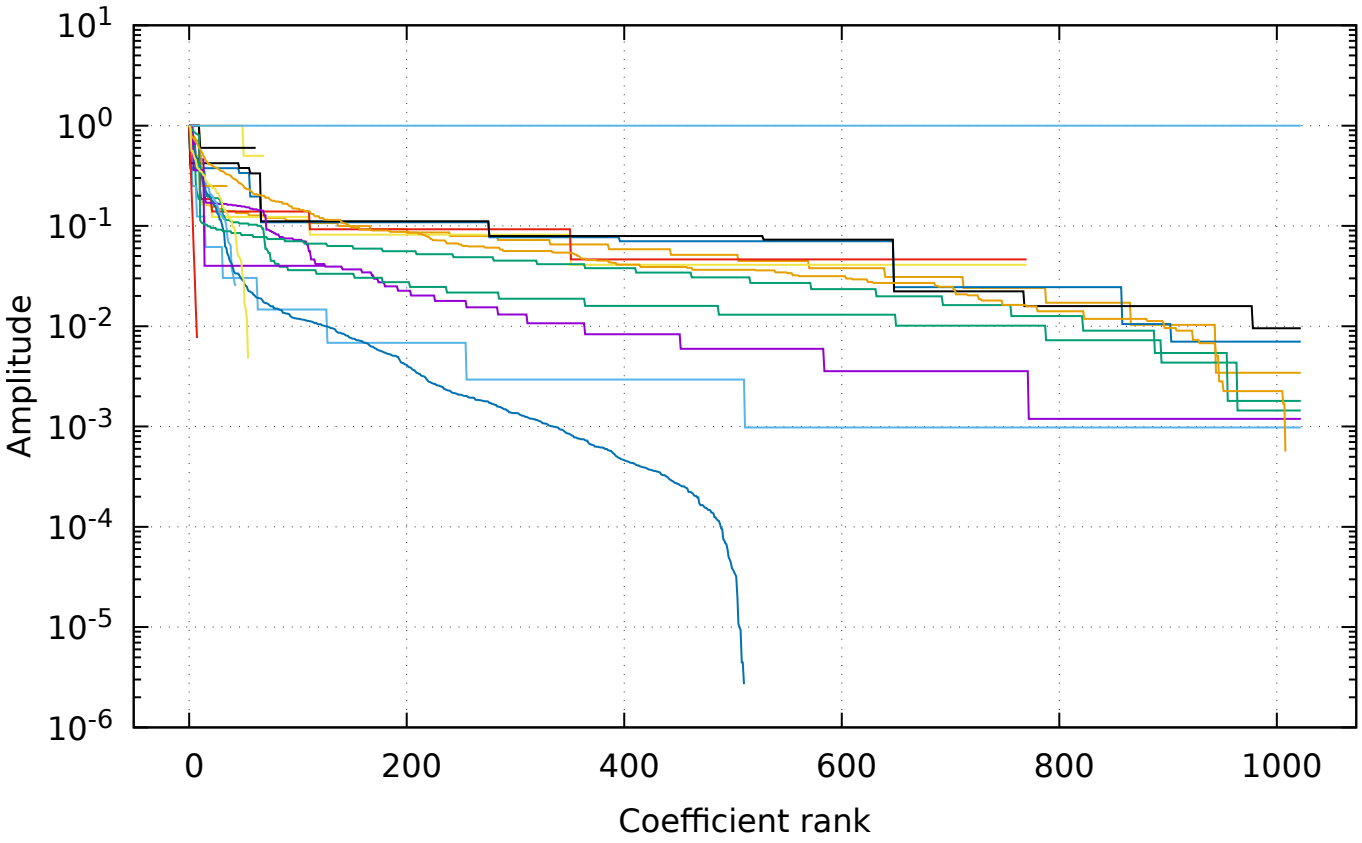
August 15, 2021

Abstract

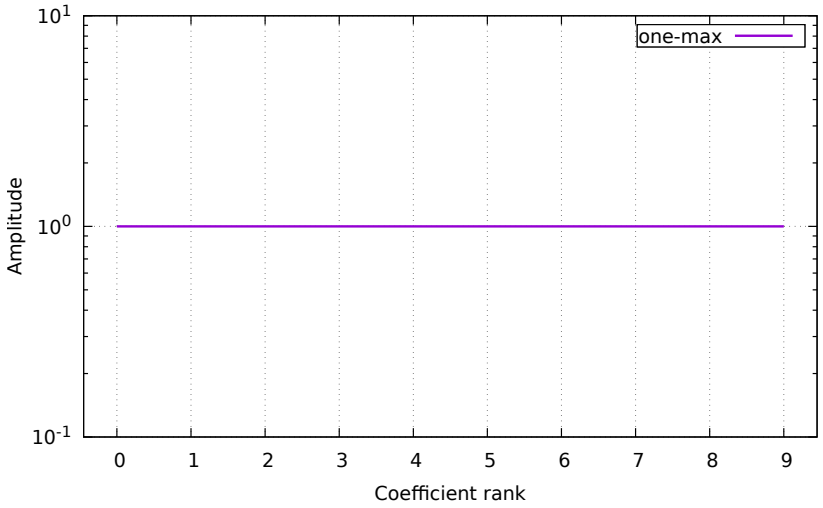
This document proposes a visualization of Walsh (or Fourier) transforms of various functions defined on bit vectors (hypercube) of size $n = 10$. For each function, two graphics are displayed. In the first one, coefficients of the Walsh transform are sorted in decreasing order of amplitude and normalized relatively to the largest amplitude. The second graphics displays the energy (sum of squares of coefficients) as a function of the feature Hamming weight. This can be thought of as a power spectrum. The coefficient of the feature zero has been filtered out. Coefficients c such that $0 < |c/c_{\max}| < 10^{-10}$ have also been filtered out as they mostly result from accumulated errors in floating point arithmetic.

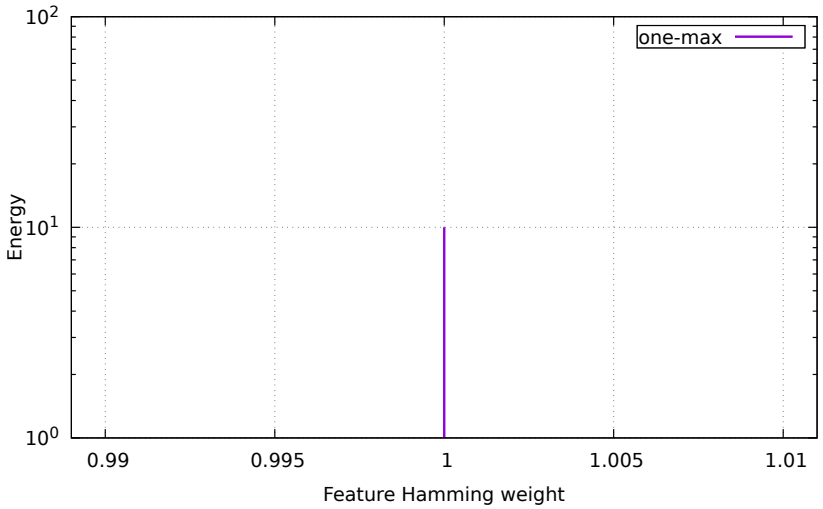
Contents

1 All functions

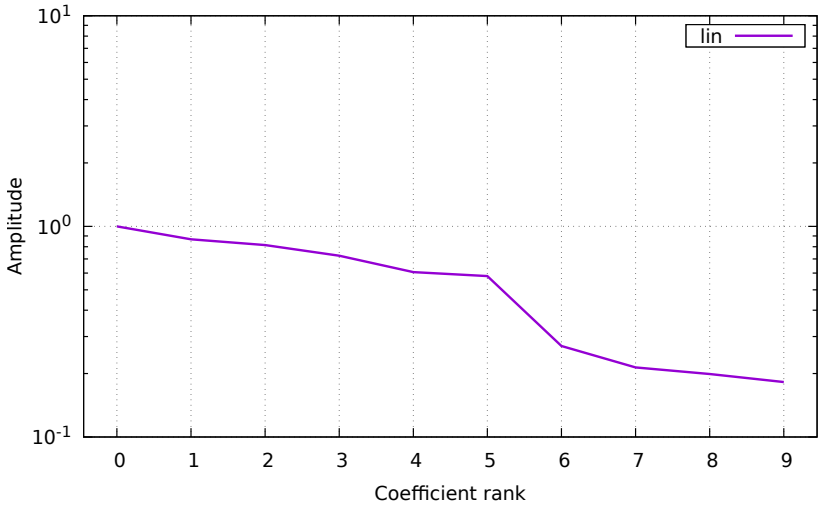


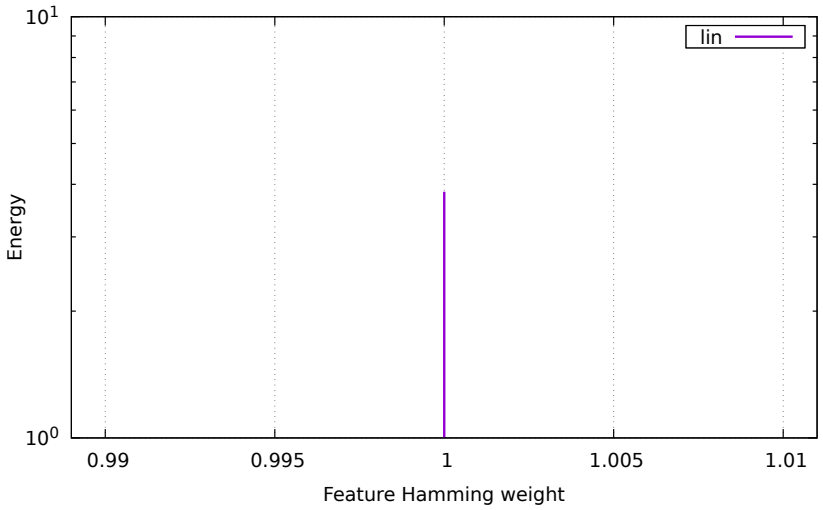
2 one-max


one-max
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
0
1
2
3
4
5
6
7
8
9

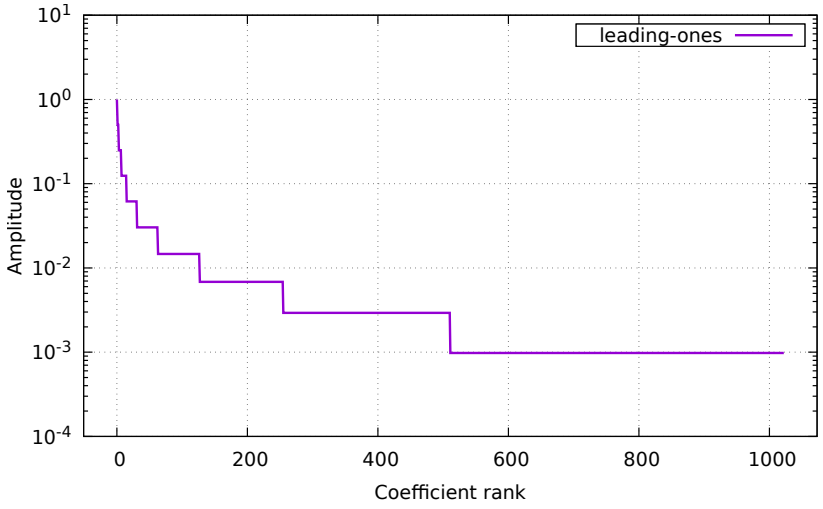

one-max
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
0.99
0.995
1
1.005
1.01

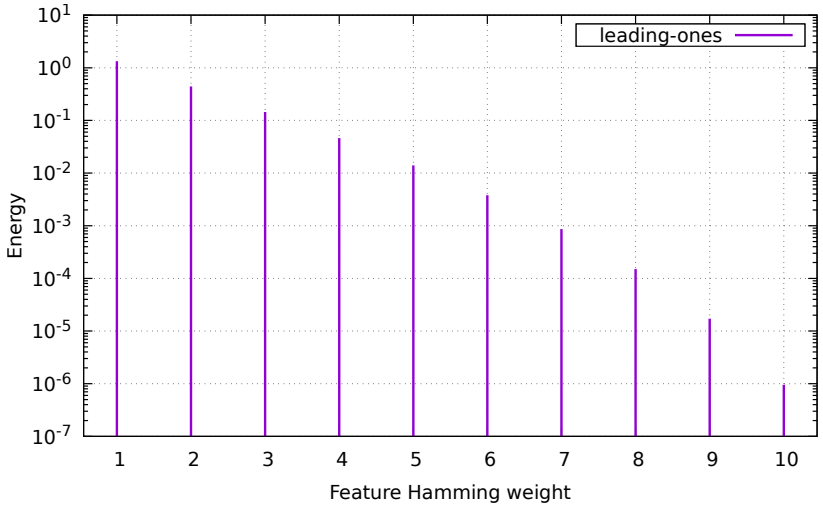
3 lin


lin
Amplitude
Coefficient rank
10^1
10^0
10^{-1}
0
1
2
3
4
5
6
7
8
9

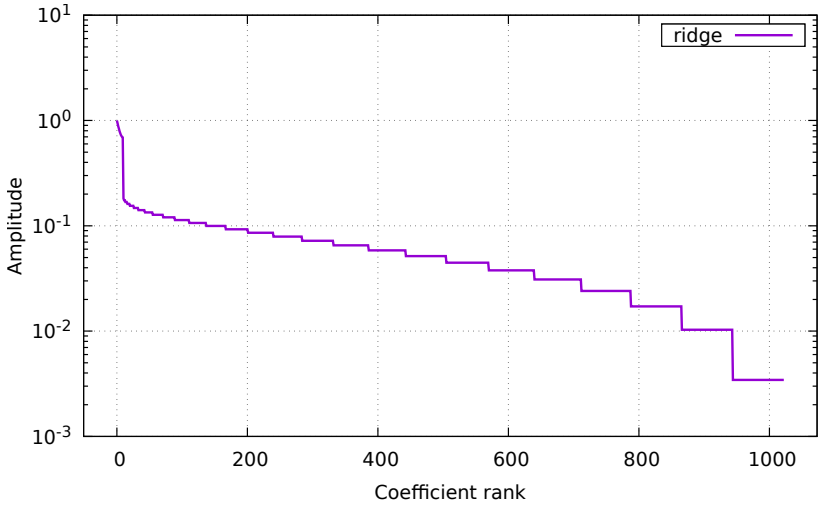

lin
Energy
Feature Hamming weight
10^1
10^0
0.99
0.995
1
1.005
1.01

4 leading-ones





5 ridge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Amplitude
Coefficient rank
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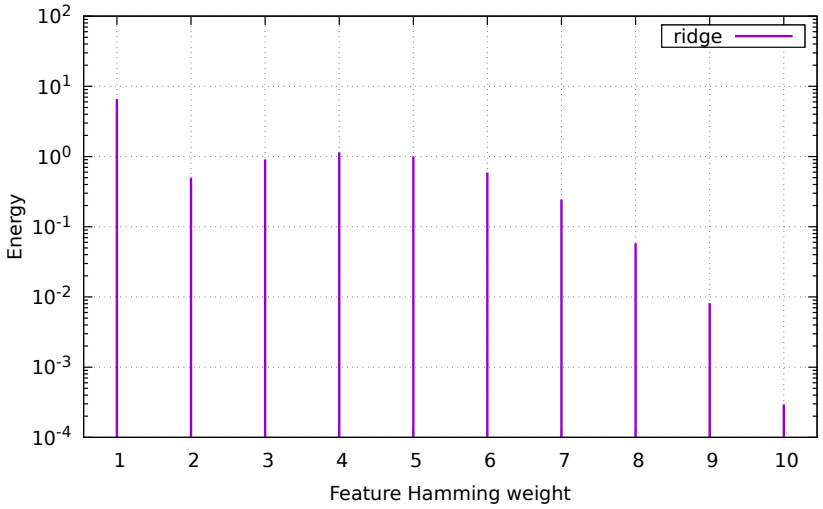
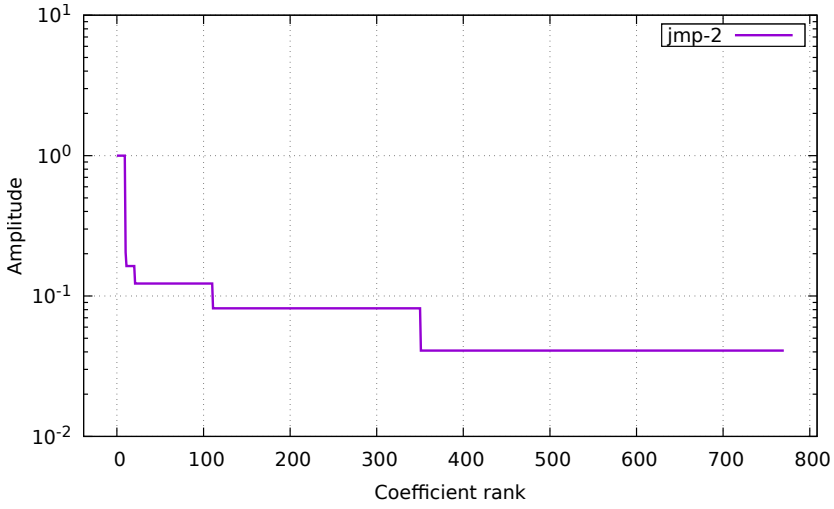
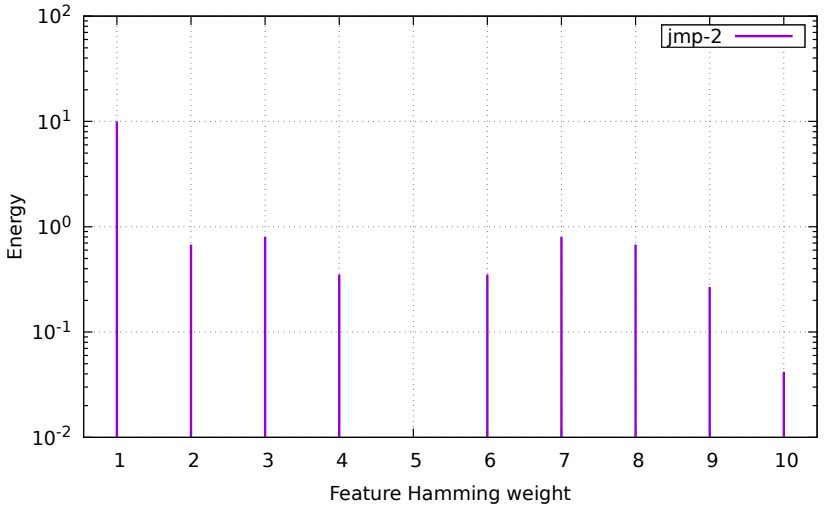
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
200
400
600
800
1000

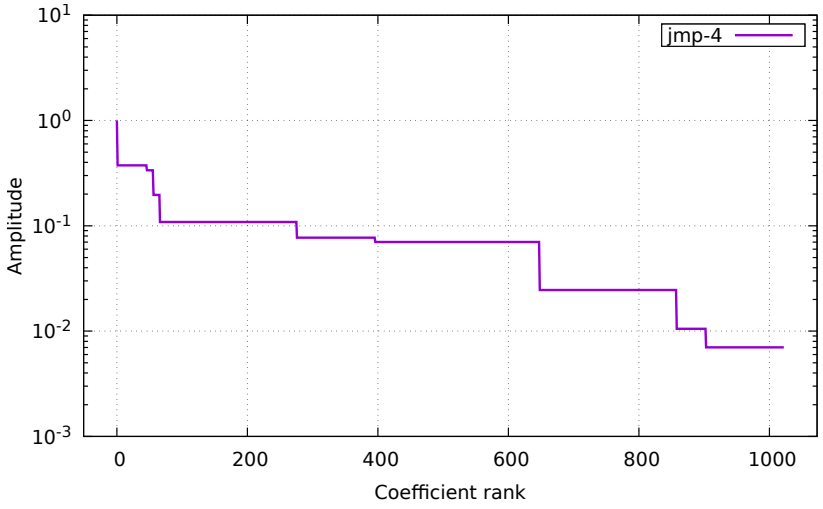

ridge
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1
2
3
4
5
6
7
8
9
10

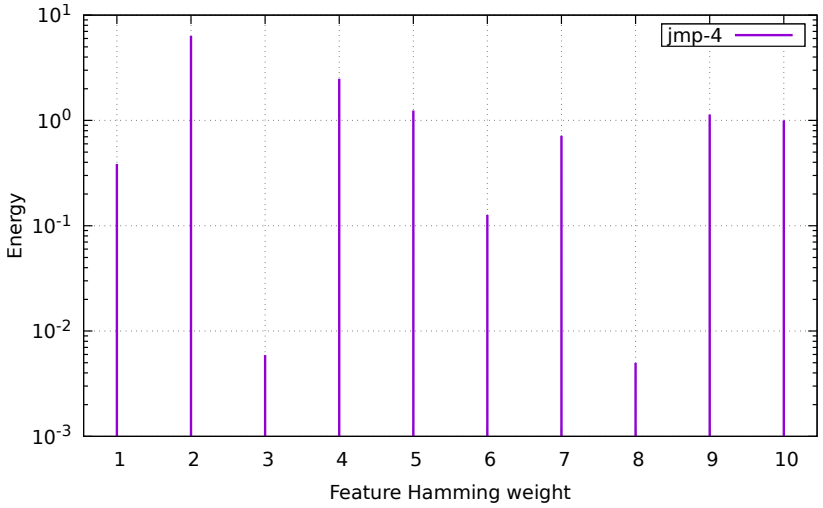
6 jmp-2


jmp-2
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
0
100
200
300
400
500
600
700
800

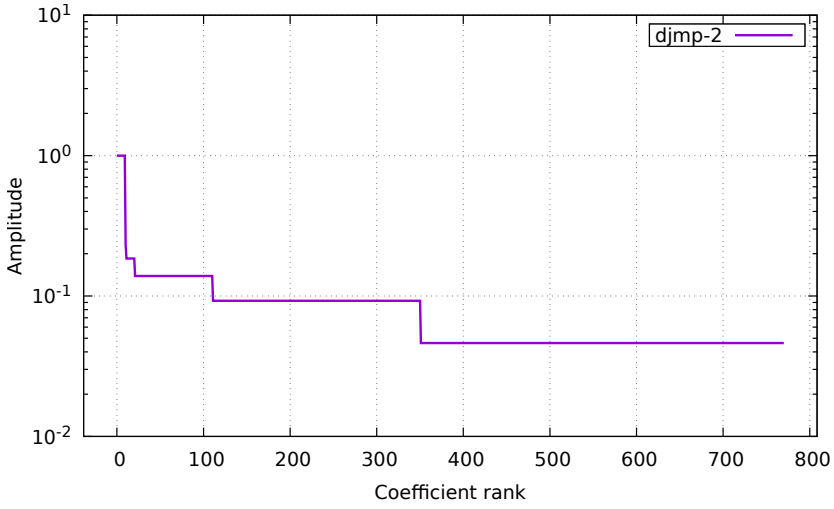

jmp-2
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
1
2
3
4
5
6
7
8
9
10

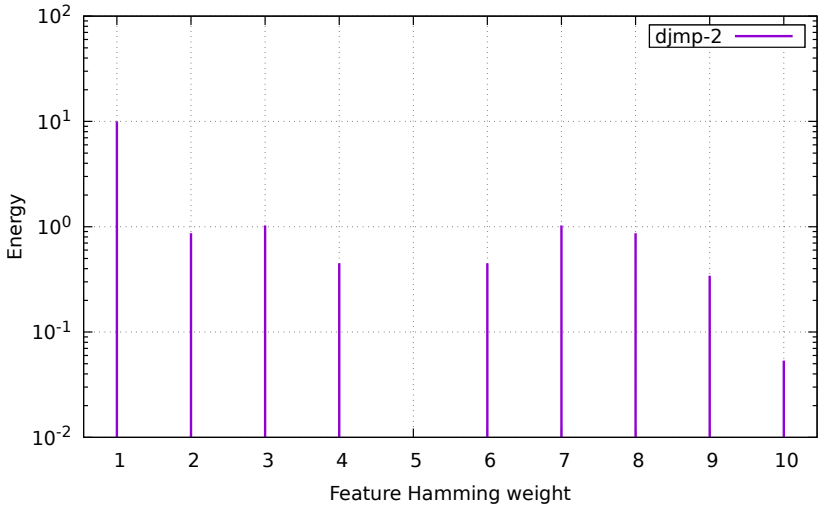
7 jmp-4


jmp-4
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
200
400
600
800
1000

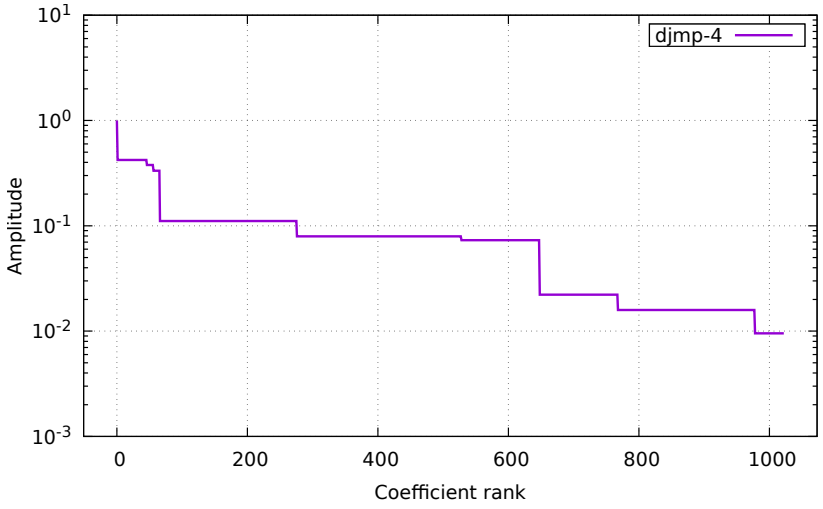

jmp-4
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
1
2
3
4
5
6
7
8
9
10

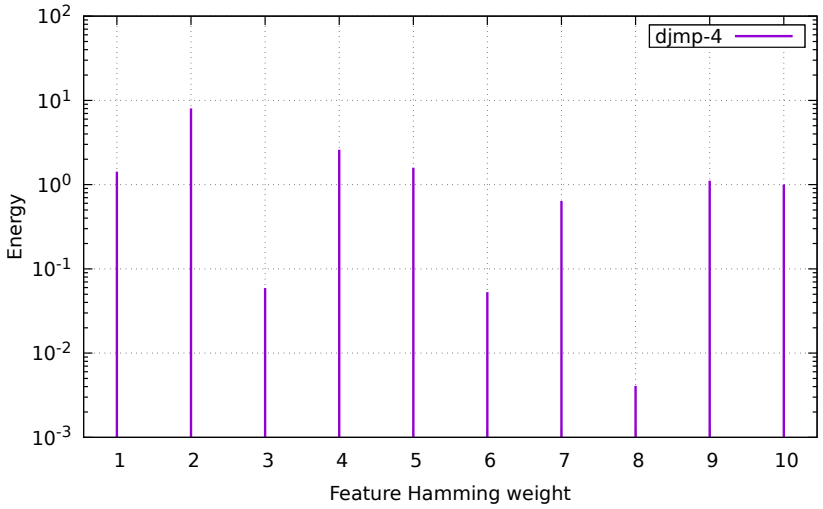
8 djmp-2


djmp-2
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
0
100
200
300
400
500
600
700
800

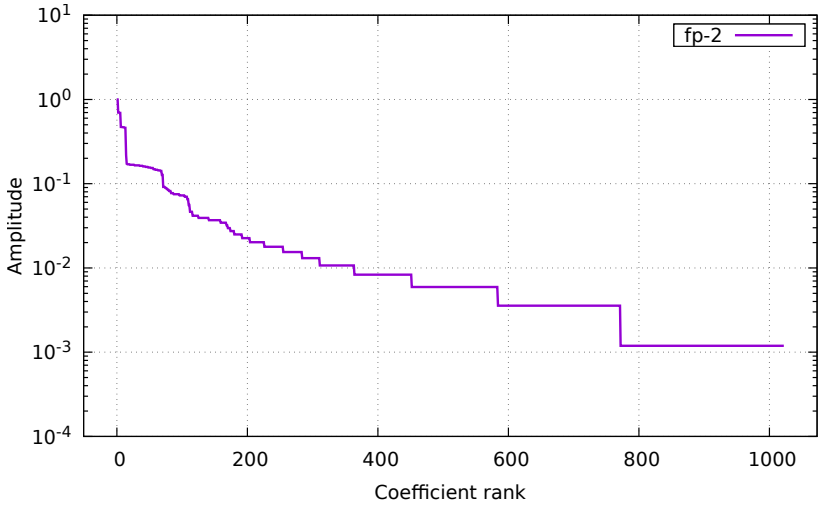

djmp-2
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
1
2
3
4
5
6
7
8
9
10

9 djmp-4


djmp-4
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
200
400
600
800
1000


djmp-4
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
1
2
3
4
5
6
7
8
9
10

10 fp-2


fp-2
Amplitude
Coefficient rank
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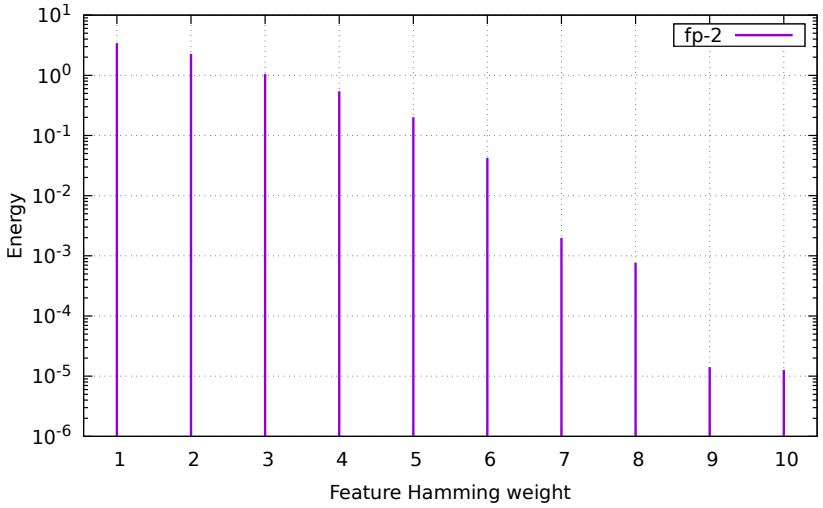
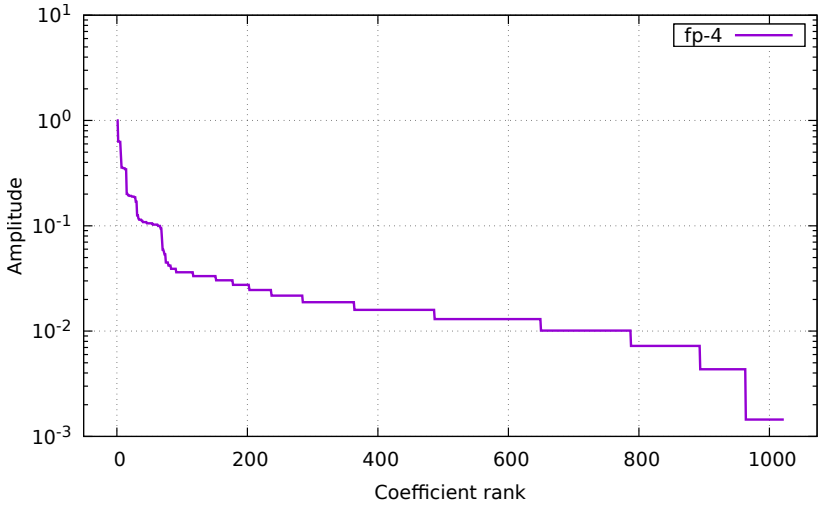
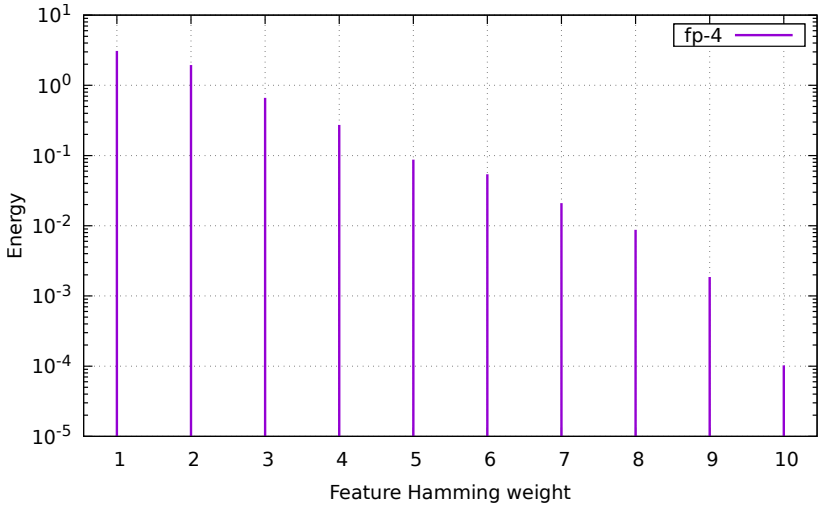
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
200
400
600
800
1000

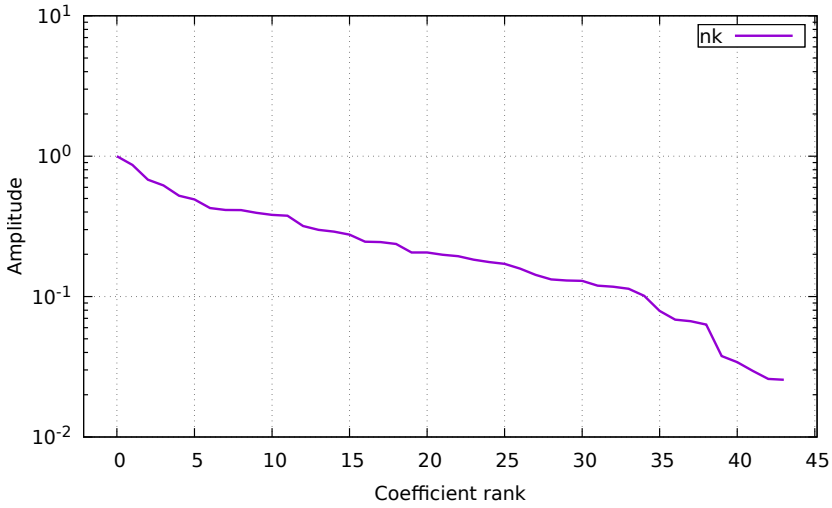

fp-2
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1
2
3
4
5
6
7
8
9
10

11 fp-4


fp-4
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
200
400
600
800
1000


fp-4
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
2
3
4
5
6
7
8
9
10

12 nk


nk
Amplitude
Coefficient rank
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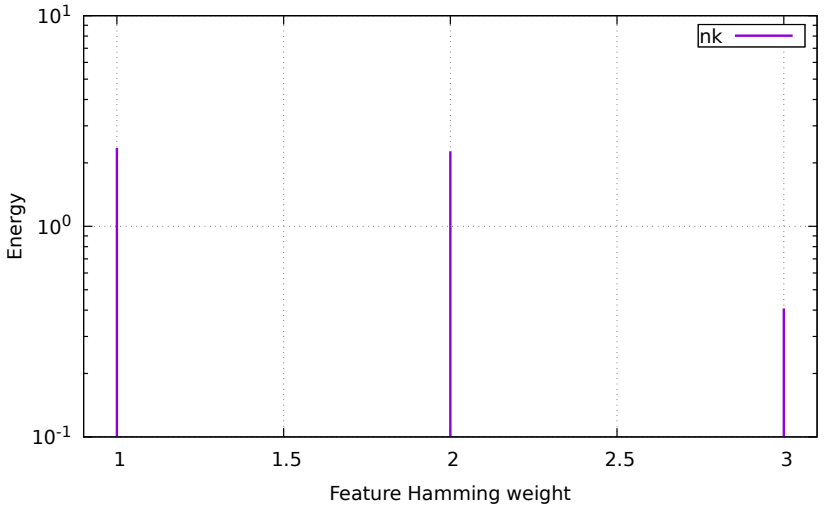
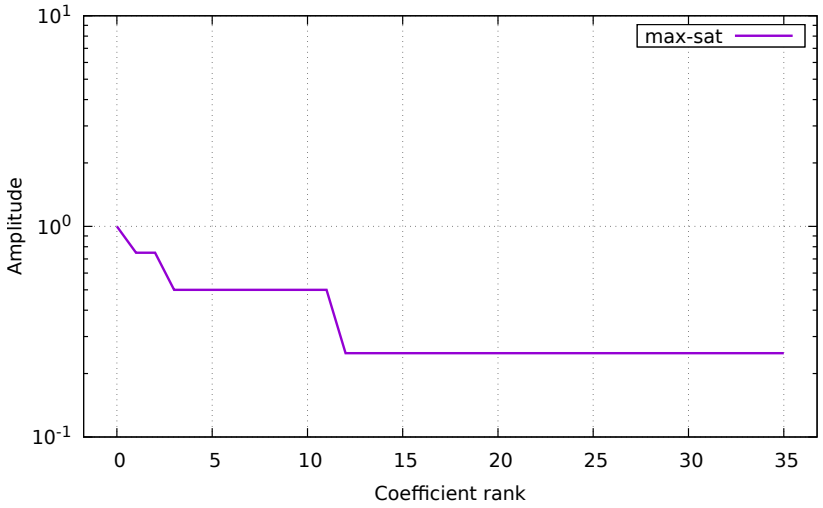
10^{0}
10^{-1}
10^{-2}
0
5
10
15
20
25
30
35
40
45


nk
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
1
1.5
2
2.5
3

13 max-sat


max-sat
Amplitude
Coefficient rank
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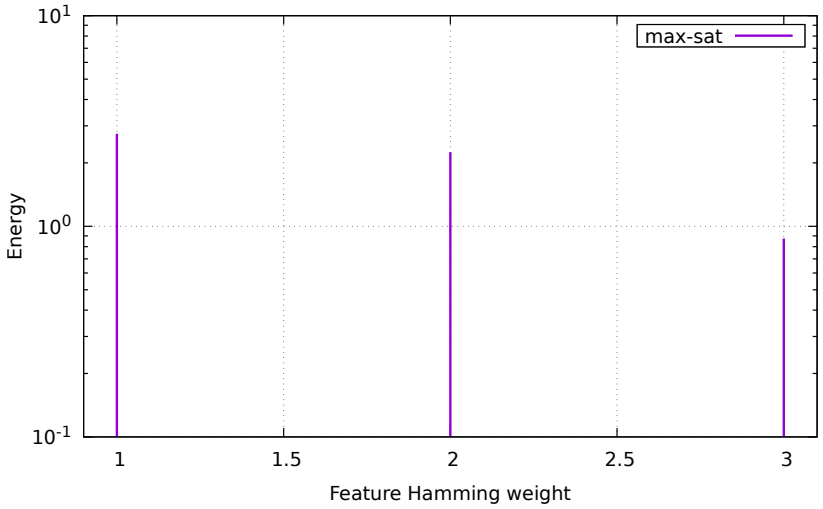
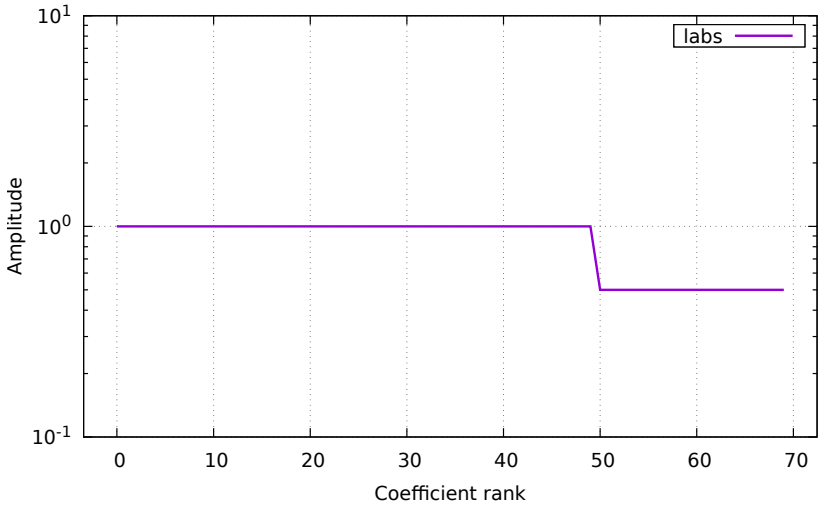
10^0
10^{-1}
0
5
10
15
20
25
30
35


max-sat
Energy
Feature Hamming weight
10^1
10^0
10^{-1}
1
1.5
2
2.5
3

14 labs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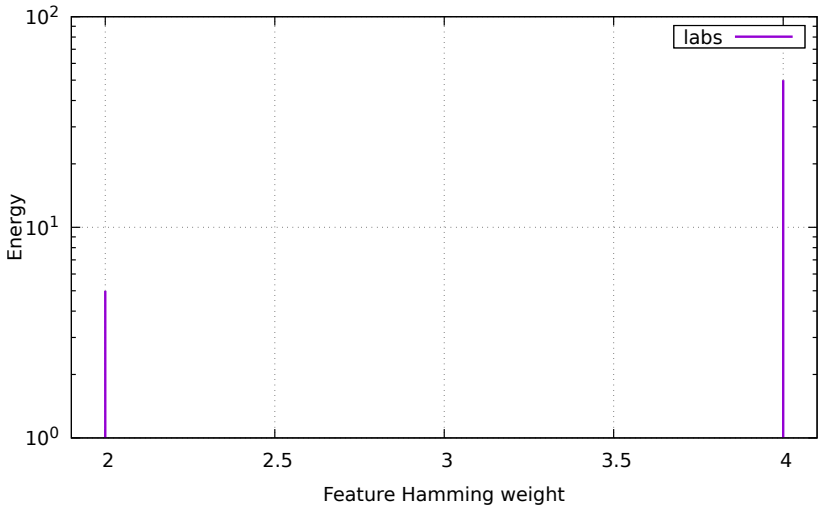
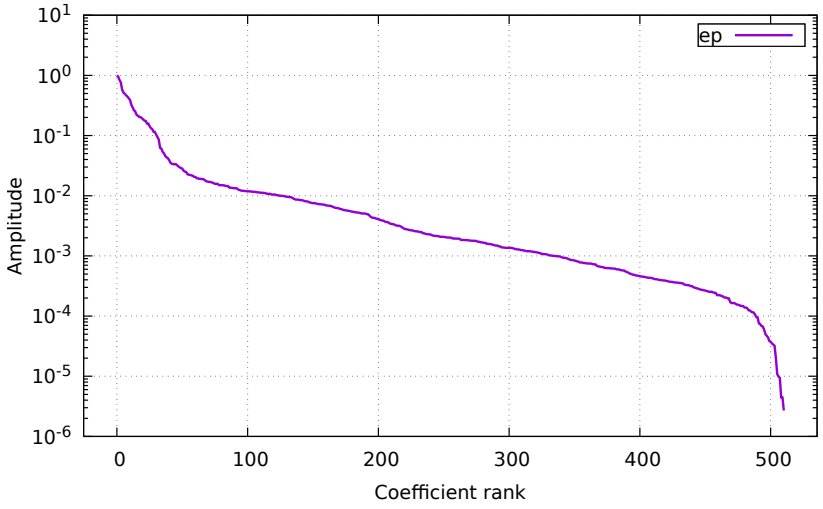
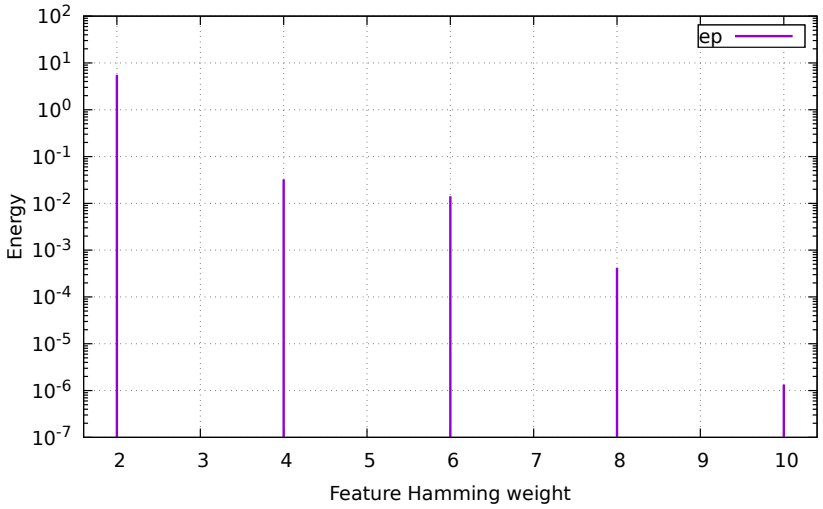
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
0
10
20
30
40
50
60
70


labs
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
2
2.5
3
3.5
4

15 ep


ep
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
100
200
300
400
500


ep
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
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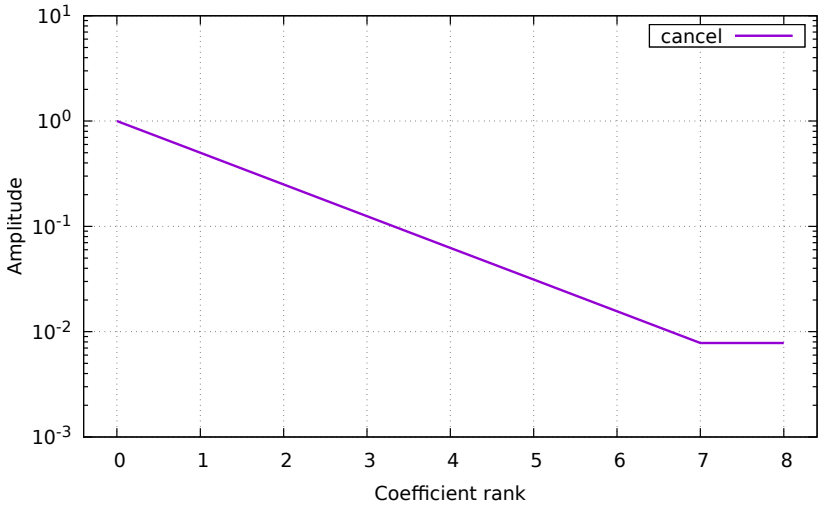
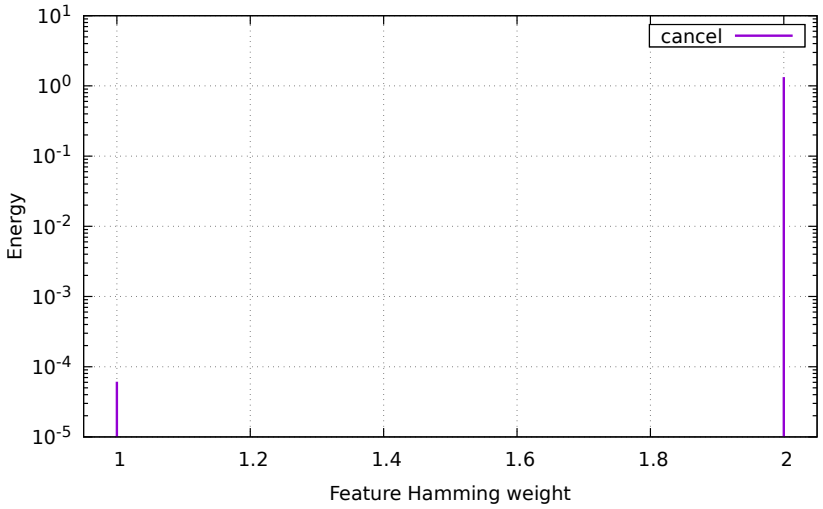
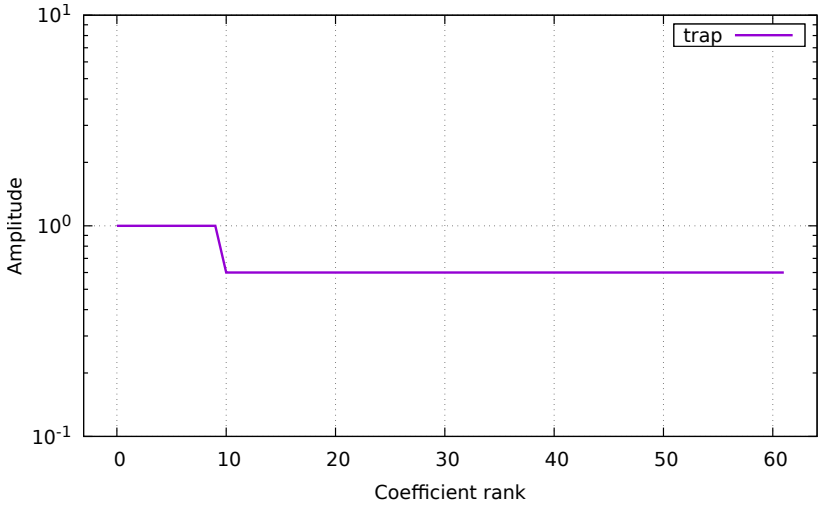
3
4
5
6
7
8
9
10

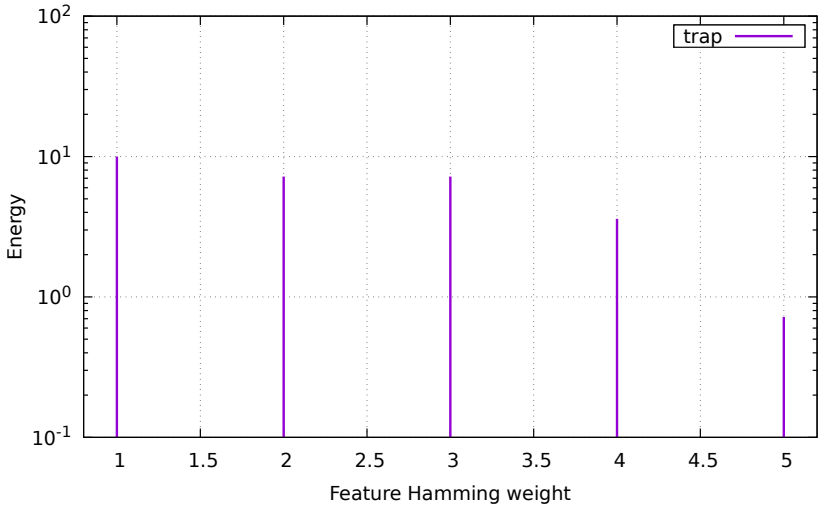
16 cancel


cancel
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
1
2
3
4
5
6
7
8

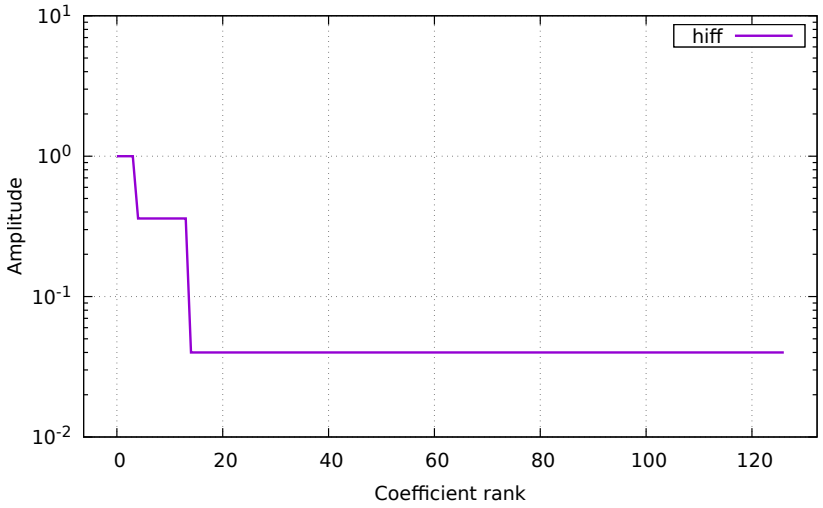

cancel
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
1.2
1.4
1.6
1.8
2

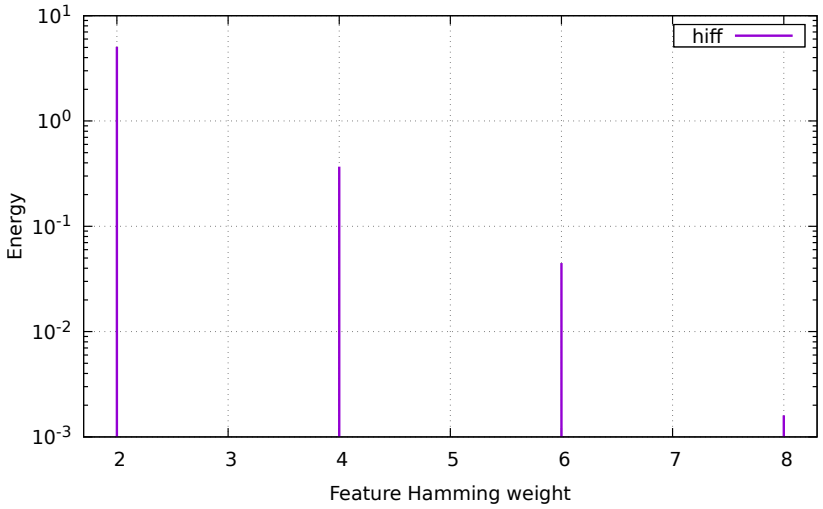
17 trap


trap
Amplitude
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
Coefficient rank

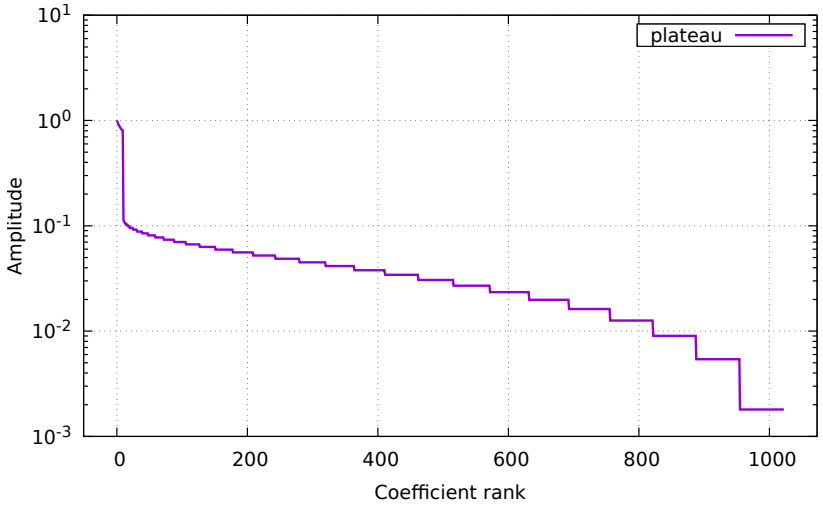

trap
Energy
10^2
10^1
10^0
10^{-1}
1
1.5
2
2.5
3
3.5
4
4.5
5
Feature Hamming weight

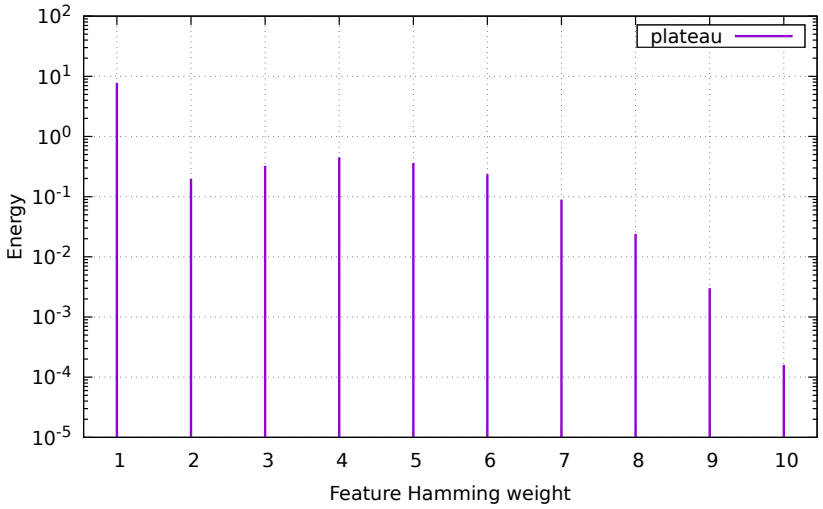
18 hiff


hiff
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
0
20
40
60
80
100
120

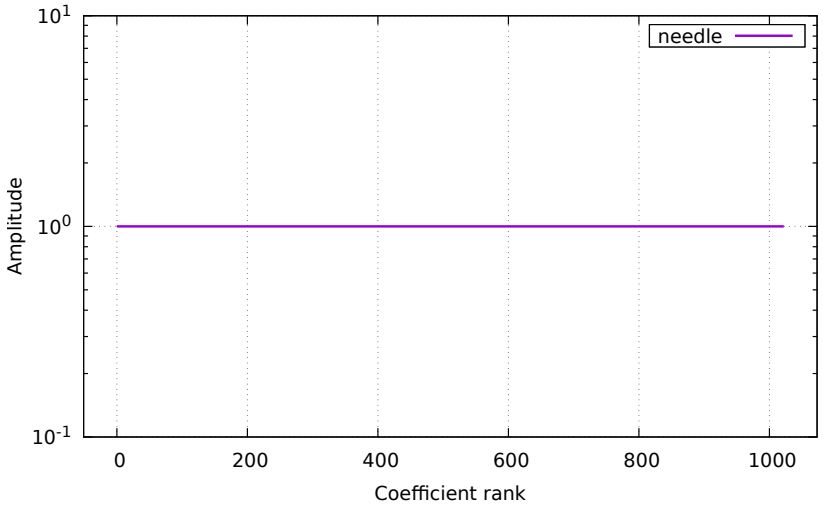

hiff
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
2
3
4
5
6
7
8

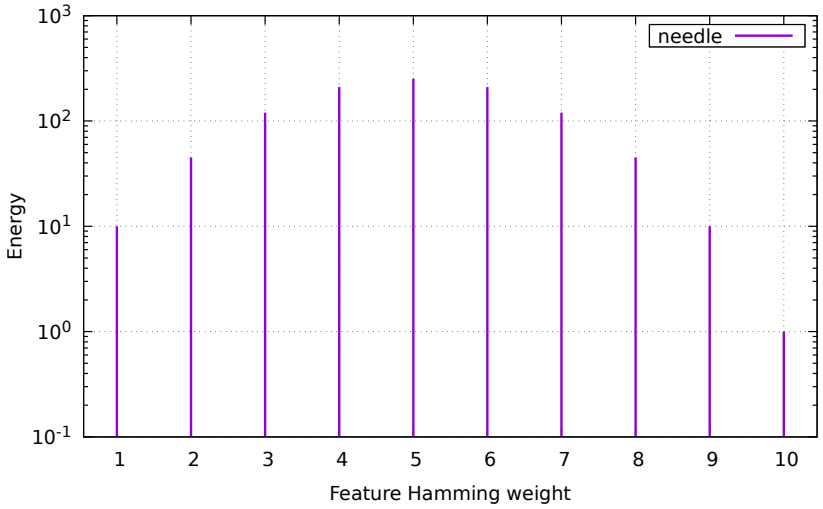
19 plateau


plateau
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
200
400
600
800
1000

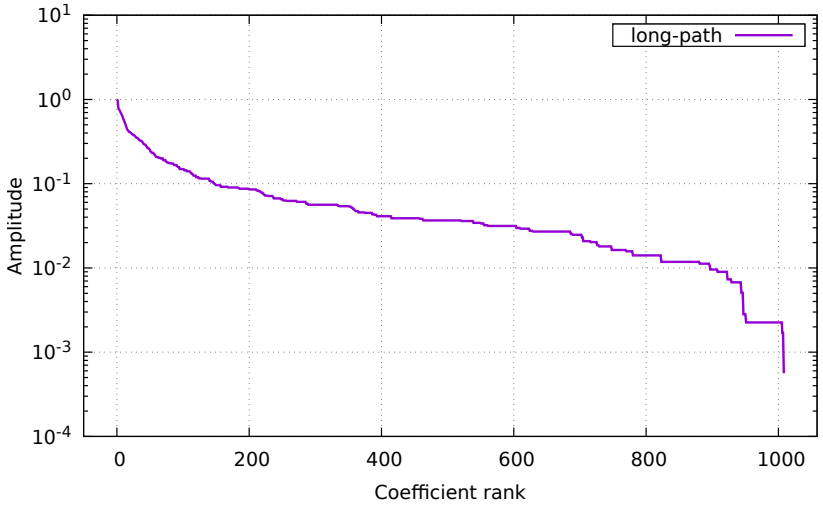

plateau
Energy
Feature Hamming weight
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
2
3
4
5
6
7
8
9
10

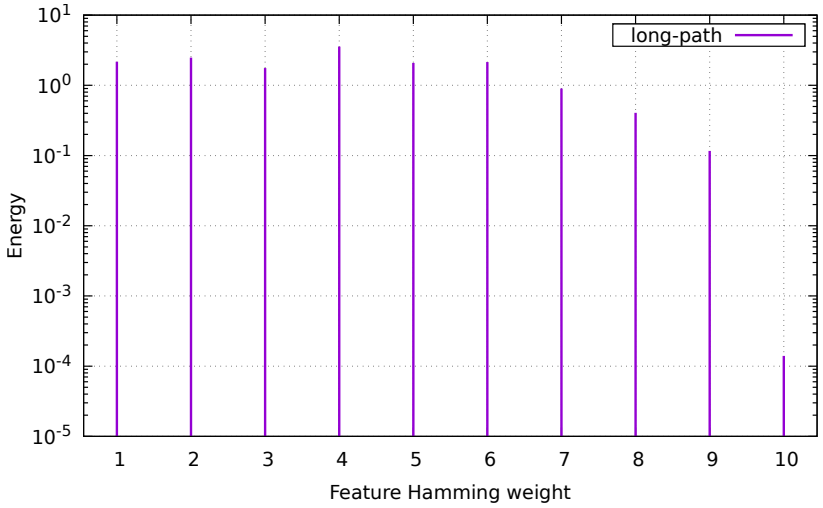
20 needle


needle
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
0
200
400
600
800
1000

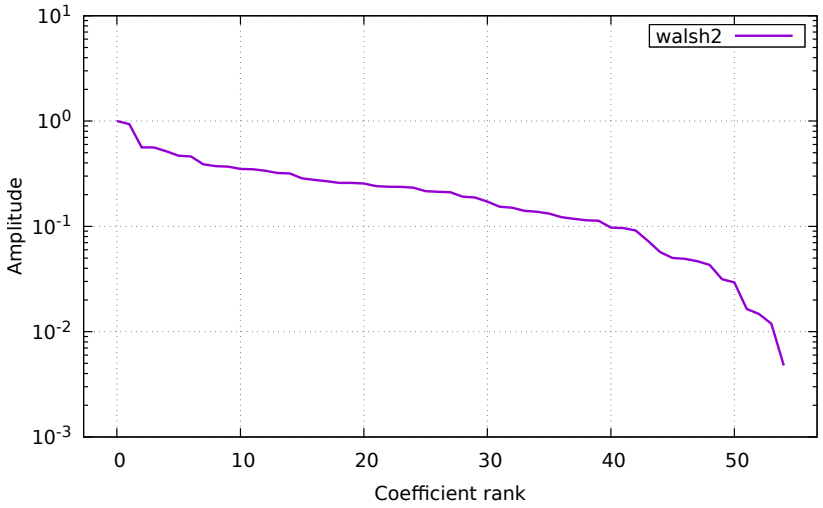

needle
Energy
Feature Hamming weight
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
1
2
3
4
5
6
7
8
9
10

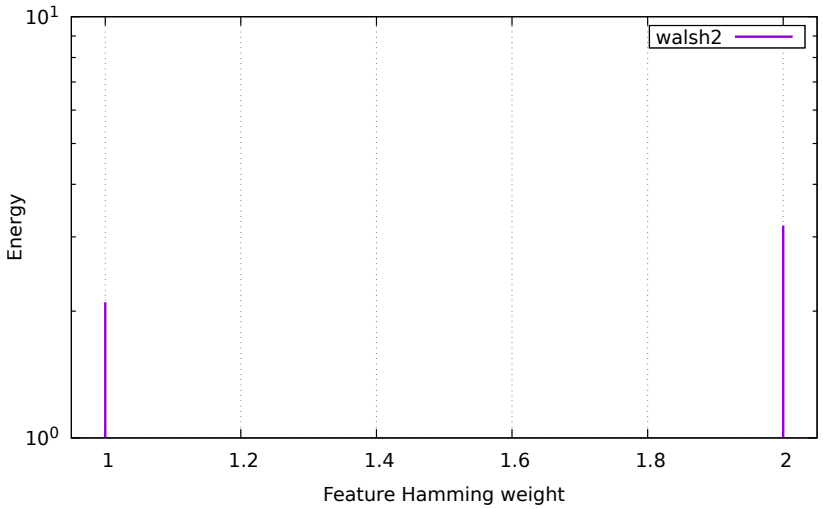
21 long-path


long-path
Amplitude
Coefficient rank
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
200
400
600
800
1000


long-path
Energy
Feature Hamming weight
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
2
3
4
5
6
7
8
9
10

22 walsh2





A Plan


```
{
    "exec": "hnco",
    "opt": "--fn-walsh-transform -b 0 -s 10",
    "parallel": true,
    "results": "results",
    "graphics": "graphics",
    "report": "report",
    "functions": [
        {
            "id": "one-max",
            "opt": "-F 0"
        },
        {
            "id": "lin",
            "opt": "-F 1 -p instances/lin.10"
        },
        {
            "id": "leading-ones",
            "opt": "-F 10"
        },
        {
            "id": "ridge",
            "opt": "-F 11"
        },
        {
```

```
    "id": "jmp-2",
    "opt": "-F 30 -t 2"
},
{
    "id": "jmp-4",
    "opt": "-F 30 -t 4"
},
{
    "id": "djmp-2",
    "opt": "-F 31 -t 2"
},
{
    "id": "djmp-4",
    "opt": "-F 31 -t 4"
},
{
    "id": "fp-2",
    "opt": "-F 40 -t 2"
},
{
    "id": "fp-4",
    "opt": "-F 40 -t 4"
},
{
    "id": "nk",
    "opt": "-F 60 -p instances/nk.10.2"
},
{
    "id": "max-sat",
    "opt": "-F 70 -p instances/ms.10.3.10"
},
{
    "id": "labs",
    "opt": "-F 80"
},
{
    "id": "ep",
    "opt": "-F 90 -p instances/ep.10"
},
{
    "id": "cancel",
    "opt": "-F 100 -s 9"
},
{
    "id": "trap",
    "opt": "-F 110 --fn-num-traps 2"
},
{
    "id": "hiff",
    "opt": "-F 120 -s 8"
},
{
    "id": "plateau",
    "opt": "-F 130"
},
{
    "id": "needle",
    "opt": "-F 20"
},
{
    "id": "long-path",
    "opt": "-F 140"
},
```

```
        {
            "id": "walsh2",
            "opt": "-F 162 -p instances/walsh2.10"
        }
    ]
}
```

B Default parameters

```
# algorithm = 100
# bm_mc_reset_strategy = 1
# bm_num_gs_cycles = 1
# bm_num_gs_steps = 100
# bm_sampling = 1
# budget = 10000
# bv_size = 100
# description_path = description.txt
# ea_lambda = 100
# ea_mu = 10
# expression = x
# fn_name = noname
# fn_num_traps = 10
# fn_prefix_length = 2
# fn_threshold = 10
# fp_expression = (1-x)^2+100*(y-x^2)^2
# fp_lower_bound = -2
# fp_num_bits = 8
# fp_upper_bound = 2
# function = 0
# ga_crossover_bias = 0.5
# ga_crossover_probability = 0.5
# ga_tournament_size = 10
# hea_bit_herding = 0
# hea_num_seq_updates = 100
# hea_reset_period = 0
# hea_sampling_method = 0
# hea_weight = 1
# learning_rate = 0.001
# map = 0
# map_input_size = 100
# map_path = map.txt
# map_ts_length = 10
# map_ts_sampling_mode = 0
# mutation_rate = 1
# neighborhood = 0
# neighborhood_iterator = 0
# noise_stddev = 1
# num_iterations = 0
# num_threads = 1
# path = function.txt
# pn_mutation_rate = 1
# pn_neighborhood = 0
# pn_radius = 2
# population_size = 10
# pv_log_num_components = 5
# radius = 2
# results_path = results.json
# rls_patience = 50
# sa_beta_ratio = 1.2
# sa_initial_acceptance_probability = 0.6
# sa_num_transitions = 50
# sa_num_trials = 100
```

```
# seed = 0
# selection_size = 1
# solution_path = solution.txt
# target = 100
# print_defaults
# last_parameter
# exec_name = hnco
# version = 0.15
# Generated from hnco.json
```