LU Factorization

- We want to decompose a matrix into LU. as U is upper triangle matrix and L is lower triangle matrix

Example. as 2x2

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$$= \underset{\tilde{}}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{D}{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}} \underset{u}{\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}}$$

as 3x3 Matrix

$$E_{32} E_{31} E_{21} A = U \text{ (no row exchanges)}$$

$$A = \left(E_{21}^{-1} E_{31}^{-1} E_{32}^{-1}\right) U = LU.$$

$\boxed{A=LU}$: If no row exchanges, multipliers go directly into L

How many ops on $n \times n$ matr. A?

$$(n-1)^2 + (n-2)^2 + \cdots + 1^2 \approx \frac{1}{3} n^3.$$

Change of rows : $A: \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A^T = A^{-1}$