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§ 1.8 Linear Time Selection

1. One armed QUICKSORT : QUICKSELECT

```
QUICKSELECT (A[1..n], k): // Select kth elem in A
  if n=1
     return A[1]
  else
     choose pivot element A[p]
     r ← Partition (A[1..n], p)
     if k<r
        return QUICKSELECT (A[1..r-1], k)
     else if k>r
        return QUICKSELECT (A[r+1..n], k-r)
     else return A[r].
```

▷ Worst runtime:

$$T(n) \le \max_{1\le r\le n} \{\underbrace{T(r-1)+T(n-r)}_{\ell := \text{length of recursive subprob}}\} + O(n)$$

$$T(n) \le \max_{0\le \ell\le n-1} T(\ell)+O(n)$$, if always smallest,

$$T(n) = O(n^2).$$

2. Choosing good pivots : $\ell \le \alpha n$ $(\alpha<1)$.

Idea : ~~Choose~~ recursively computing the median of a carefully-chosen subset of an array.

How.
- Divide input array to $\lceil n/5 \rceil$ blocks, each containing exactly 5 (possible expept the last).
- Compute these blocks' median by brute force (possibly by a few ifs).
- Collect those medians into a new list, $M[1\ldots\lceil\frac{n}{5}\rceil]$,

- Use median of block medians as quickselect pivot.

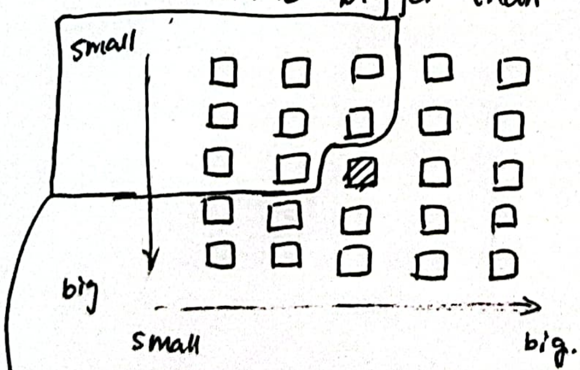
```
MoMSelect(A[1..n], k):
    if n≤25
        use bruteforce
    else
        m ← ⌈n/5⌉
        for i=1 to m
            M[i] ← MedianOfFive(A[5i-4..5i])
        mom ← MomSelect(M[1..m], ⌊m/2⌋)   (Median of Media
        r ← Partition(A[1..n], mom)
        if k<r
            return MomSelect(A[1..r-1], k)
        else if k>r
            return MomSelect(A[r+1...n], k-r)
        else return mom.
```

This is our pivot

- Analysis : MOM = Median Of Median

  1° MOM is a good pivot.

  - MOM is larger than $\lfloor \lceil n/5 \rceil / 2 \rfloor - 1 \approx n/10$ block median,
  - each block median is larger 2 other elems in its block
  - MoM is bigger than at least $\frac{3n}{10}$ elems in input arr.



Assume sorted in both directions. (for demo)

THE ALGO DONT DO THIS!

Approx. $\frac{3n}{10}$ ~~elms~~ elems.

2° The calls. $T(n) \le T(\frac{n}{5}) + T(\frac{7n}{10}) + O(n)$

$\sim \frac{9}{10}n$ of the prev layer,

$T(n) = O(n)$.

About constant 5: First number result in exp. decay.