Learning from data

§ I.2 Multiplying and Factoring matrices

FACTORIZATIONS

$A = LU$ elimination solving linear systems

▸ $A = QR$ othnormal columns (Gram Schmidt).

Symmetric ▸ $S = Q\Lambda Q^T$. Λ is eigenvec $(\lambda_1 \lambda_2 \lambda_3)$ like this.

$A = X\Lambda X^{-1}$ $\left[\begin{matrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{matrix}\right]\left[\begin{matrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_n \end{matrix}\right]\left[\begin{matrix} x_1 \longrightarrow x_n \\ \vdots \\ -x_n- \end{matrix}\right]$ $\lambda_i x_i = Ax_i$

▸ $A = U\Sigma V^T$ (U, V orthnormal, Σ diagnoal) Singular Value Dec.

1. Mult with $Q\Lambda Q^T$. $(Q\Lambda)(Q^T)$.

$S =$ (cols of $Q\Lambda$)(rows of Q^T). $[\,|\,|\,] \times [\,-\,] = [\,\#\,]$ (rk=1).

$=$ sum of rank 1.

$= \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots + \lambda_n q_n q_n^T$. $\left[\begin{matrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{matrix}\right]\left[\begin{matrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{matrix}\right]$

(Spectrum theorem)

Check, check: $Sq_1 = \lambda_1 q_1 \underbrace{(q_1^T q_1)}_{1} + \lambda_2 q_2 \overbrace{(q_2^T q_1)}^{\to 0,\ \text{orthognal}} + \cdots + 0$.

2. Mult with LU

$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \to \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

A U upper thiangular

$A = L \quad U$

$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

substract 2 from here, multiplier

reducing

$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 4 & \boxed{6} \end{bmatrix}}_{\text{rank 1}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \boxed{1} \end{bmatrix}}_{\text{rank 1}} = [l_1][u_1^T] + [l_2][u_2^T]$.

More generally.

$A = [\quad] = (\text{col } 1)(\text{row } 1) + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \boxed{A_2} & \\ 0 & & \end{bmatrix}$

3. The fundamental Theorem of linear Algebra

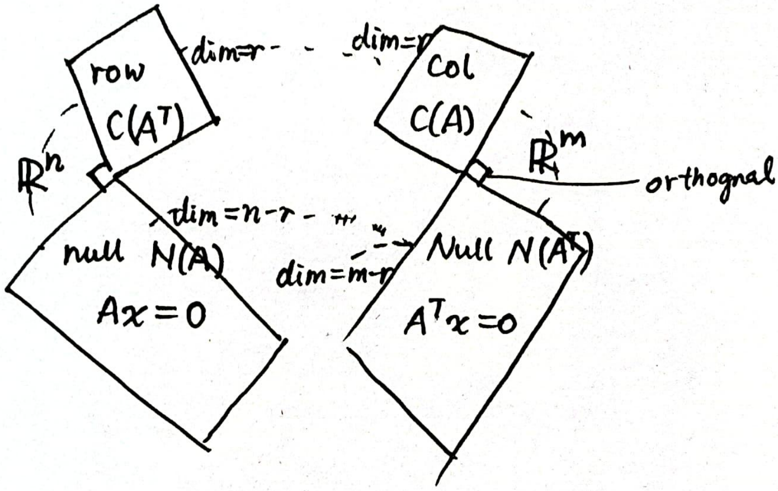
- 4 fundamental supspaces A of $m \times n$ of rank r.

Column Spaces $C(A)$ Row Spaces $C(A^T)$ dim $= r$.

Null space $N(A)$ Null space $N(A^T)$ dim $= n-r$.

all solutions x
$Ax = 0$.

If $Ax=0$
$Ay=0$,
$A(x+y)=0$,
$x+y \in \text{Nul}A$.
$Cx \in \text{Nul}A$
(closed)



Example. $m=2,\ n=3,\ r=1$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

$n-r=2$. We search ~~sever~~ 2 differend independent vecs: $\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$. $Ax=0 \Leftrightarrow$ They are orthognal! for row and col

Homework problem I.2

2. $\left(\begin{smallmatrix} a_1 \\ \vdots \\ a_m \end{smallmatrix}\right) (b_1 \cdots b_p)$ Yes, they can.

$m \times 1 \quad 1 \times p$

ab^T will resuet in $m \times p$. row i, col j get $(ab)^T_{ij} = a_i b_j$. aa^T will get $\|a\|^2$.

6. $A = \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} \quad B = I$

~~That is what is $AD=I$, $D=A^+$~~.

$$A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & * & * \\ a_{13} & * & * \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} [a_{11} \; a_{21} \; a_{31}] + \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_{22}-a_{12}a_{21} & a_{23}-a_{13}a_{31} \\ 0 & a_{32}-a_{31}a_{13} & a_{33}-a_{13}a_{31} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 0 \\ * \\ * \end{bmatrix} [0 \; * \; *] + \begin{bmatrix} 0 \\ *\text{left} \\ *\text{left} \end{bmatrix} [0 \; *\text{left} \; *\text{le}$$