THE UNIVERSITY OF TEXAS AT DALLAS

CS4341 Digital Logic & Computer Design

Lecture Notes 8

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Review: Karnaugh Map (K-Map)

- A K-map is a diagram made of a collection of adjacent squares:
  - Each square represents a minterm
  - The collection of squares is a graphical representation of a Boolean function
  - Adjacent squares differ in the value of **one** variable only
  - Alternative algebraic expressions for the same function are derived by recognizing patterns of squares
- The K-map can be viewed as a reorganized version of the truth table

Review: Importance of K-Map

- K-Maps provide means of:
  - Finding optimum or near optimum
    - SOP and POS standard forms
    - Two-level AND/OR and OR/AND circuits
  - Visualizing concepts related to manipulating Boolean expressions
  - Demonstrating concepts used by computer-aided design programs to simplify large circuits

2-Variable K-Map

- If we represent each minterm as a box, then we have:

$m_0 = x'y'$	$m_1 = x'y$	$m_2 = xy'$	$m_3 = xy$

- We need to assemble the boxes such that adjacent boxes differ in exactly one literal value

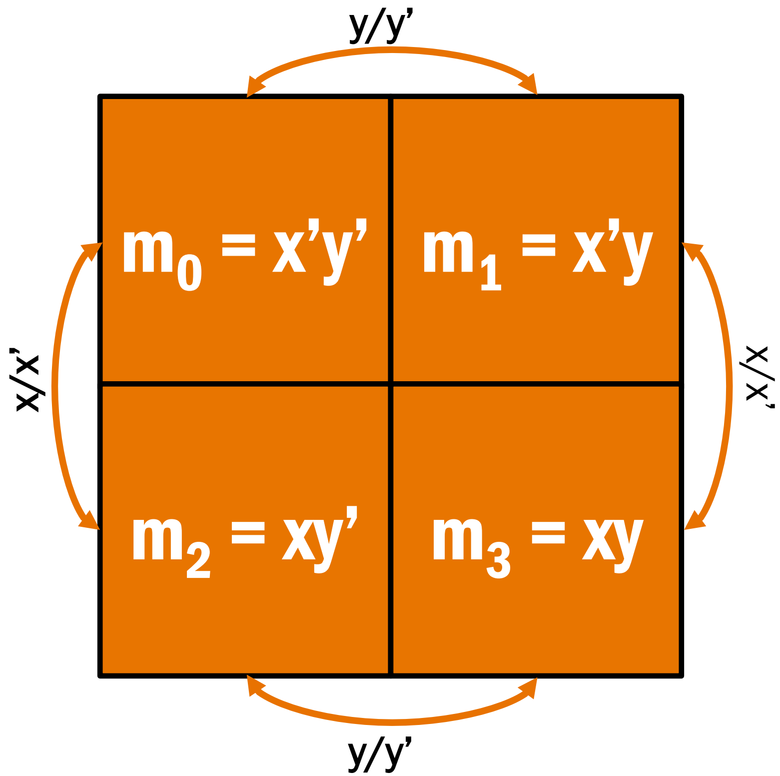
How to connect?

$m_0 = x'y'$	$m_1 = x'y$

$m_2 = xy'$	$m_3 = xy$

How to connect?

2-Variable K-Map



2-Variable K-Map

x \ y	0	1
0	m_0 **x'y'**	m_1 **x'y**
1	m_2 **xy'**	m_3 **xy**

2-Variable K-Map Minimization

- Use K-Map to simplify the Boolean function expressed in the following truth table

Input Values (x,y)	Function Value F(x,y)
0 0	1
0 1	0
1 0	1
1 1	1

Step 1: represent the truth table in a K-Map diagram

x \ y	y = 0	y = 1
x = 0	1	0
x = 1	1	1

2-Variable K-Map Minimization

- Using algebraic simplification:
  - $F(x,y) = m_0 + m_2 + m_3$
  - $F = x'y' + xy' + xy$
  - $F = (x' + x)y' + xy = (y' + x)(y' + y) = x + y'$
- Using K-Map simplification:
  - Two adjacent 1s means one of the two variables is not impacting the output and can be ignored

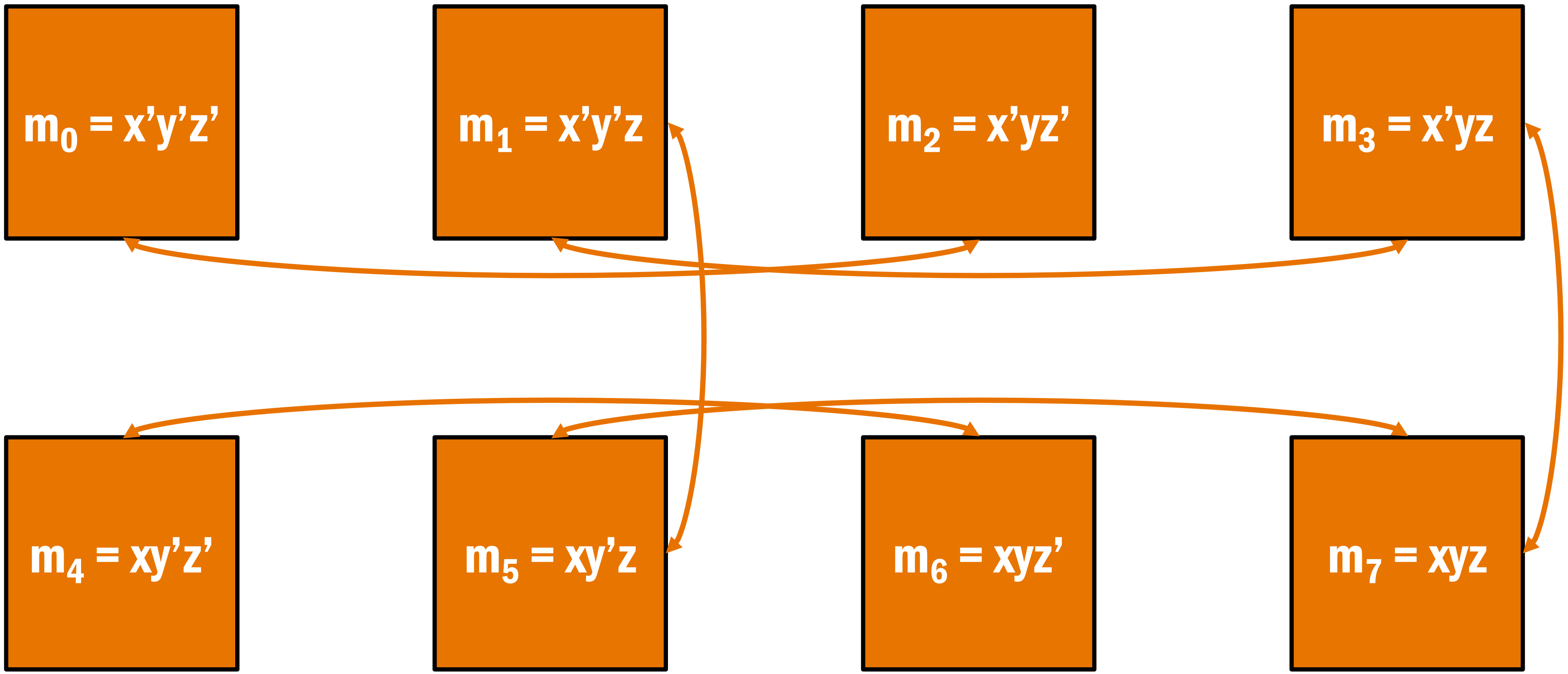
x \ y	y = 0	y = 1
x = 0	1	0
x = 1	1	1

- F= sum of all the simplified terms (prime implicants).
  - $F = x + y'$

x variable can be Ignored. The output = y'

y variable can be Ignored. The output = x

3-Variable Adjacency View



3-Variable K-Map

x \ yz	00	01	11	10
0	m_0 x'y'z'	m_1 x'y'z	m_3 x'yz	m_2 x'yz'
1	m_4 xy'z'	m_5 xy'z	m_7 xyz	m_6 xyz'

Combining Squares

- By finding largest possible 2^n squares and by combining squares, we reduce number of literals in a product term, hence reducing the gate cost
- On a 3-variable K-Map:
  - One square represents a minterm with 3 variables
  - Two adjacent squares represent a term with 2 variables
  - Four adjacent squares represent a term with 1 variable
  - Eight adjacent square is the function 1 (no variables)

Minimization Example

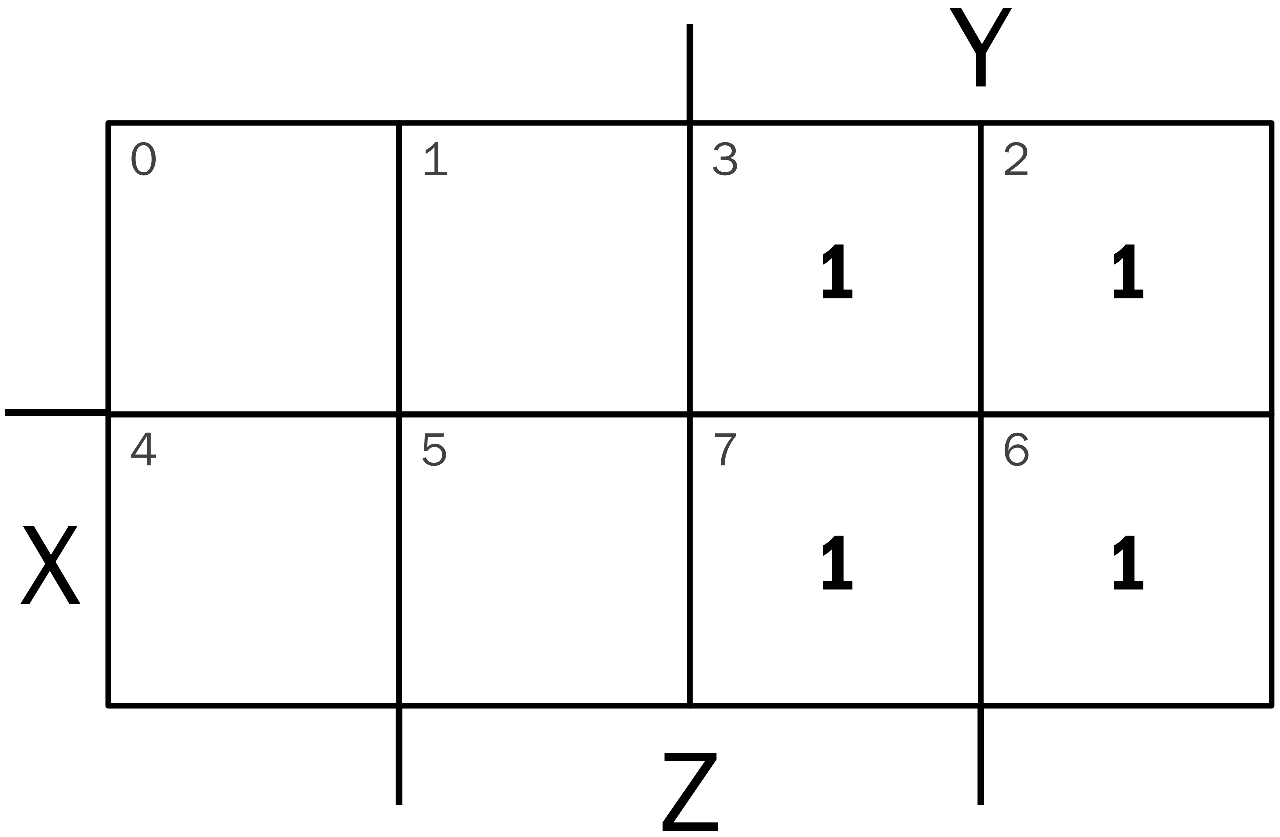
- Minimize (simplify) $F = \sum(2,3,6,7)$

x \ yz	00	01	11	10
0	m_0 **0**	m_1 **0**	m_3 **1**	m_2 **1**
1	m_4 **0**	m_5 **0**	m_7 **1**	m_6 **1**

- $F = \sum(2,3,6,7) = y$
- Using Boolean algebra:
  - F = x’yz’ + x’yz + xyz’ + xyz
  - = x’yz + xyz + x’yz’ + xyz’
  - = yz + yz’ = y

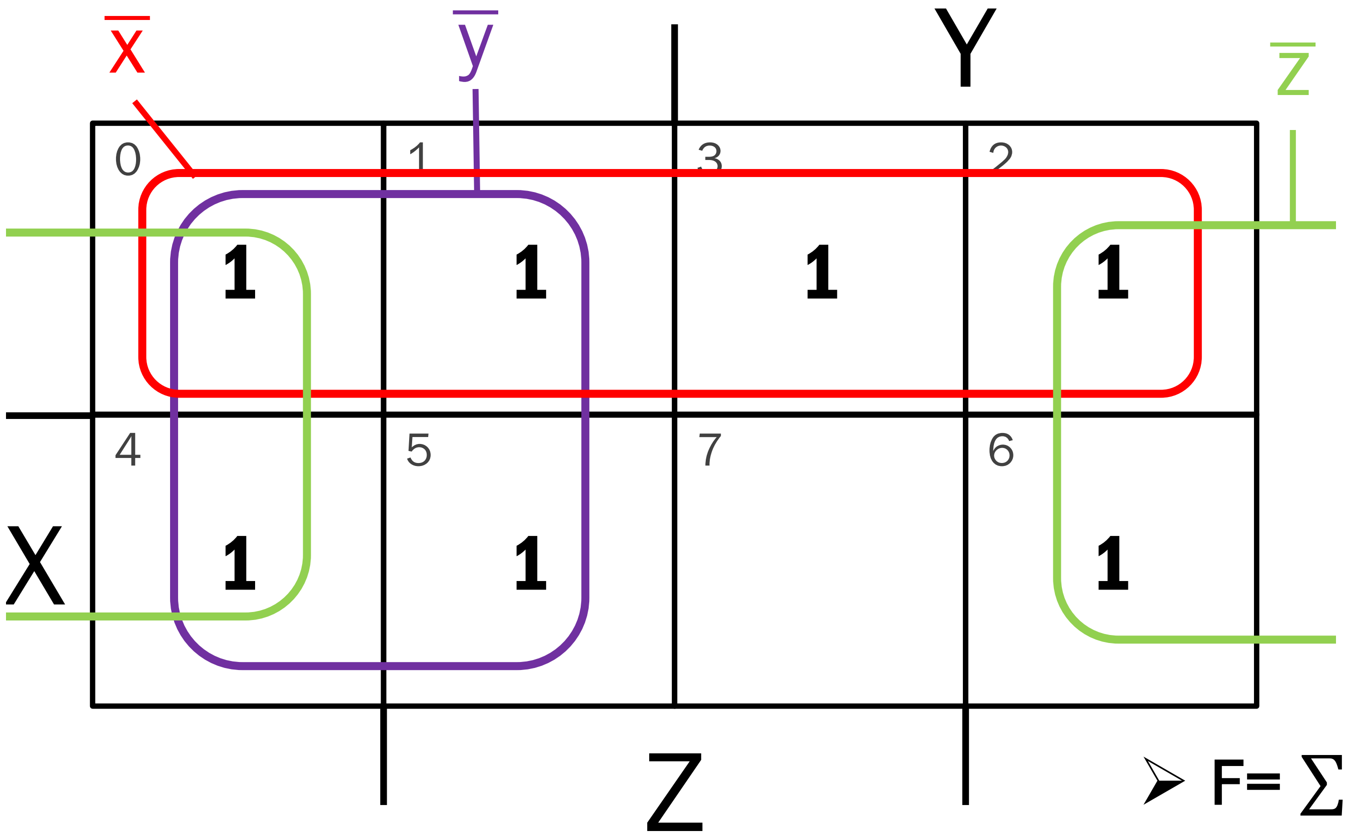
Visual Simplification

- Now we understand how to construct and read a k-map, we can simplify its visual to make it less busy diagram



Minimization Example

- Describe F (x, y, z) in its minimized form



- F= $\sum(0,1,2,3,4,5,6) = x' + y' + z'$

4-Variable K-Map

wx \ yz	00	01	11	10
00	0 $m_0 = \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$	1 $m_1 = \overline{w}\,\overline{x}\,\overline{y}\,z$	3 $m_3 = \overline{w}\,\overline{x}\,y\,z$	2 $m_2 = \overline{w}\,\overline{x}\,y\,\overline{z}$
01	4 $m_4 = \overline{w}\,x\,\overline{y}\,\overline{z}$	5 $m_5 = \overline{w}\,x\,\overline{y}\,z$	7 $m_7 = \overline{w}\,x\,y\,z$	6 $m_6 = \overline{w}\,x\,y\,\overline{z}$
11	12 $m_{12} = w\,x\,\overline{y}\,\overline{z}$	13 $m_{13} = w\,x\,\overline{y}\,z$	15 $m_{15} = w\,x\,y\,z$	14 $m_{14} = w\,x\,y\,\overline{z}$
10	8 $m_8 = w\,\overline{x}\,\overline{y}\,\overline{z}$	9 $m_9 = w\,\overline{x}\,\overline{y}\,z$	11 $m_{11} = w\,\overline{x}\,y\,z$	10 $m_{10} = w\,\overline{x}\,y\,\overline{z}$

$\overline{Y}$ Y $\overline{W}$ W $\overline{X}$ X $\overline{X}$ $\overline{Z}$ Z $\overline{Z}$

To Do List

- Review lecture notes, and try the examples yourself
- Study chapter 2
- Work on assignment 1