THE UNIVERSITY OF TEXAS AT DALLAS

CS4341 Digital Logic & Computer Design

Lecture Notes 5

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Review: Postulates of Boolean Algebra

	Expression		Duality	Postulate
1	X + 0 = X	2	X . 1 = X	Identity Element
3	X + 1 = 1	4	X . 0 = 0	Domination
5	X + X = X	6	X . X = X	Idempotence
7	X + X’ = 1	8	X . X’ = 0	Complement
9	X’’ = X			Involution
10	X + Y = Y + X	11	X . Y = Y . X	Commutative
12	(X + Y) + Z = X + (Y + Z)	13	(X . Y) . Z = X . (Y . Z)	Associative
14	X. (Y + Z) = (X . Y) + (X . Z)	15	X + (Y . Z) = (X + Y) . (X + Z)	Distributive
16	(X + Y)’ = X’ . Y’	17	(X . Y)’ = X’ + Y’	DeMorgan’s

Review: Function Complement

- Complement of *F is F′*
- Complement of a function is obtained by interchange of 0's for 1's and 1's for 0's in the values of *F in the truth table.*
- We can also use DeMorgan's theorem to get *F′.*
- The following identities are very useful:
  - $(A + B)' = A' B'$ $A'+B' = (AB)'$
  - $(A+B+C)' = A'B'C'$ $A'+B'+C' = (ABC)'$

Function Complement Example

- Find the complement of $F = AB + C$
- Answer:
  - $F' = (AB + C)'$
  - Applying DeMorgan's theorem:
  - $= (A' + B') \,.\, C'$
  - $= A'C' + B'C'$

Function Complement From Dual

- We can find the complement of a function by finding its dual then complementing each literal
- Example: $F = x'yz' + x'y'z$
  - $F(dual) = (x' + y + z').(x' + y' + z)$
  - $F(complement) = F' = (x + y' + z)(x + y + z')$

Canonical Forms

- *Minterms and Maxterms*
- *Sum-of-Minterm (SOM) Canonical Form*
- *Product-of-Maxterm (POM) Canonical Form*
- *Representation of Complements of Functions*
- *Conversions between Representations*

Definition: Minterms

- A minterm (or standard product) is the product (AND) of ALL input variables where each variable (literal) is in the normal (x) or the complement (x') state.
- For n variables, there are 2^n minterms (all possible combinations)
- Example: Two variables (x and y) produce 4 combinations (4 minterms): x'y', x'y, xy', xy
- From the truth table, a minterm can be derived with each variable primed if the corresponding bit is 0, or unprimed otherwise.

Definition: Maxterms

- A maxterm (or standard sums) is the sum (OR) of ALL input variables where each variable is in the normal (x) or the complement (x’) state.
- For n variables, there are 2^n maxterms (all possible combinations)
- Example: Two variables (x and y) produce 4 combinations (4 maxterms): x+y, x+y’, x’+y, x’+y’
- From the truth table, a maxterm can be derived with each variable primed if the corresponding bit is 1, or unprimed otherwise.

Minterms & Maxterms for 3 Binary Variables

			Minterms		Maxterms	
x	***y***	***z***	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Sum of Minterm (Product) Representation

- A Boolean function can be expressed algebraically from a given truth table by:
  - Generate all the minterms of the input combinations that produce an output value of 1
  - OR (sum) these minterms
  - This representation of the function is called Sum of Minterms (derived from Sum of product representation) canonical form.

Sum of Minterm Example

- Express the functions represented by the following truth table using Sum of Product canonical form

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Sum of Product Example

- $f_1 = x'y'z + xy'z' + xyz$
  $= m_1 + m_4 + m_7$
- $= \sum(1,4,7)$

- $f_2 = x'yz + xy'z + xyz' + xyz$
  $= m_3 + m_5 + m_6 + m_7$
  $= \sum(3,5,6,7)$

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Product of Sum Representation

- Using the sum of products approach, we can also represent the complement of a function as the sum (OR) of all minterms that produce an output value of 0.
- $f'_1 = x'y'z' + x'yz' + x'yz + xy'z + xyz'$
- Using the involution rule:
- $(f'_1)' = f_1 = (x'y'z' + x'yz' + x'yz + xy'z + xyz')'$
- $f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$
- $= M_0 . M_2 . M_3 . M_5 . M_6 = \prod(0,2,3,5,6)$
- This representation of the function is called Product of Sum (or Product of maxterms) canonical form.

Representing a Boolean Function

- We can express any function by "ORing" the minterms corresponding to the ‘1’ outputs in the truth table.
- We can express any function by "ANDing" the maxterms corresponding to ‘0’ outputs in the truth table
- The same Boolean function can be expressed in two canonical ways: Sum-of-Minterms (SOM) and Product-of-Maxterms (POM).
- If a Boolean function has fewer ‘1’ outputs, then the SOM canonical form will contain fewer terms than POM. However, if it has fewer ‘0’ outputs then the POM form will have fewer terms than SOM.

Examples

- $F(a, b, c, d) = \sum(2,3,6,10,11)$, express F as SOM
  - $F = m_2 + m_3 + m_6 + m_{10} + m_{11}$
  - $= a'b'cd' + a'b'cd + a'bcd' + ab'cd' + ab'cd$

- $G(a, b, c, d) = \prod(0,4,12,15)$, express G as POM
  - $G = M_0 . M_4 . M_{12} . M_{15}$
  - $= (a+b+c+d).(a+b'+c+d).(a'+b'+c+d).(a'+b'+c'+d')$

Standard Order

- All variables should be present in a minterm or maxterm and should be listed in the same order.
- Example: For variables a, b, c:
  - Maxterms (a + b + c), (a + b' + c) are in standard order
  - However, (b + a + c) is NOT in standard order
  - (a + c) does NOT contain all variables
  - Minterms (abc) and (abc') are in standard order
  - However, (bac) is not in standard order
  - (ac) does not contain all variables

To Do List

- Review lecture notes, and try the examples yourself
- Study chapter 2 until 2.7