

# cs4341 Digital Logic & Computer Design

**Lecture Notes 18** 

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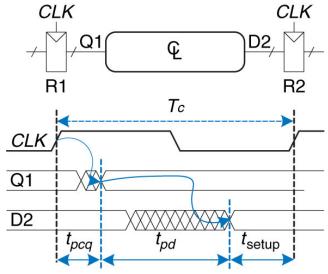
Department of Computer Science

### Review: Setup and Hold Time Constraints

#### > Setup Time Constraint:

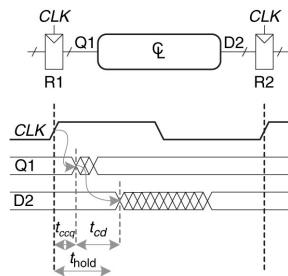
$$ightharpoonup T_c \ge t_{pcq} + t_{pd} + t_{setup}$$

$$\succ$$
  $t_{pd} \le T_c - (t_{pcq} + t_{setup})$ 



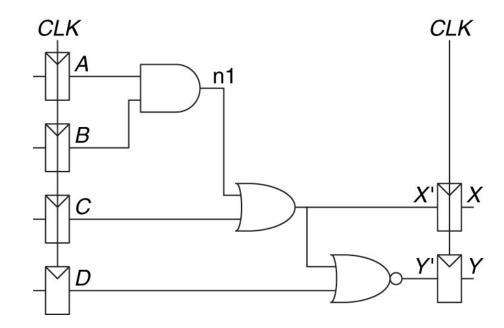
#### ➤ Hold Time Constraint:

$$\succ$$
  $t_{ccq} + t_{cd} \ge t_{hold}$ 



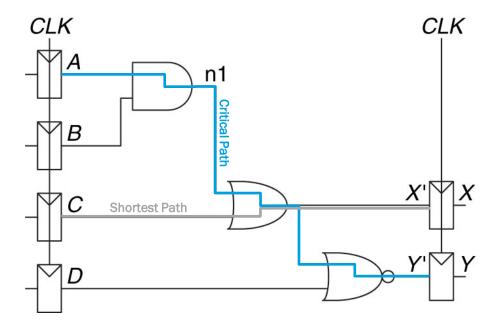
### **Timing Analysis Example**

- The shown circuit has the following time specifications:
  - $\succ$   $t_{ccq} = 30 \text{ ps}$
  - $> t_{pcq} = 80 \text{ ps}$
  - $\succ$   $t_{setup} = 50 \text{ ps}$
  - $\succ$   $t_{hold} = 60 \text{ ps}$
  - $> t_{pd} = 40 \text{ ps}$
  - $> t_{cd} = 25 \text{ ps}$
- What is the maximum clock frequency?
- Is there any hold time violations?

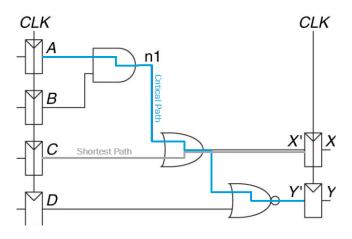


### Solution: Critical and Shortest Paths

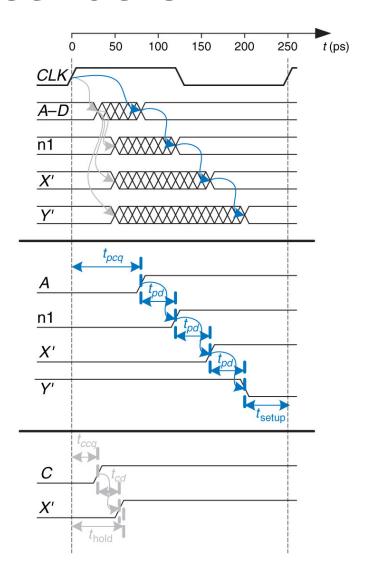
- Determine the critical path (sum of all t<sub>pd</sub> on the longest path)
- Determine the short path (sum of all t<sub>cd</sub> on the shortest path)



### Solution: Critical and Shortest Paths

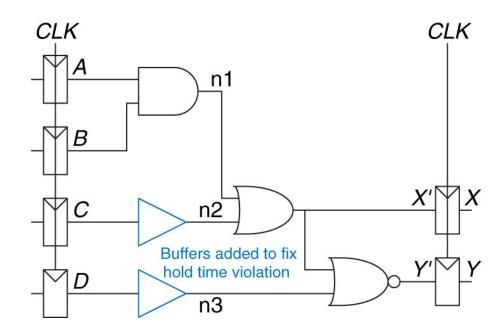


- $\succ$  Calculate  $T_c \ge t_{pcq} + 3t_{pd} + t_{setup}$
- $T_c \ge 80 + 3x40 + 50 = 250 \text{ ps}$
- ightharpoonup Therefore, maximum clock frequency  $f_c = 1/T_c \le 4$  GHz
- $\succ$  For the short path, X' can start changing as early as  $t_{ccq} + t_{cd}$
- $\triangleright$  Hence, X' will start changing as early as 30 + 25 = 55 ps, which violates the flip-flop hold time of 60 ps.



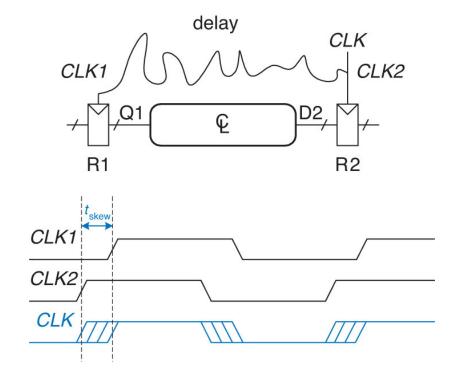
### Timing Analysis Example 2

- ➤ The buffers in this circuit were added to slow down the short path to fix the hold time violation.
- What is the maximum clock frequency?
- Is there still any hold time violations?
- > Solution:
  - Critical path is not affected (why)?
  - $\triangleright$  Hence, clock frequency is the same  $\le 4$  GHz
  - For short path, X' will start changing after 30 + 2 x 25 = 80 ps. This is more than the required 60 ps hold time. Therefore, there is no violation in the circuit.



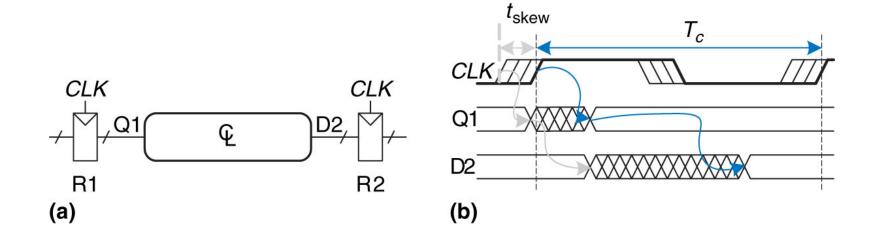
### **Clock Skew**

- ➤ The clock itself can have its own timing problems caused by many reasons.
  - ➤ Wire length, noise, clock gating, etc
  - > This variation in clock edges is called clock skew



#### **Clock Skew Time**

- ➤ The worst-case clock skew timing must be added to the timing calculations.
  - $\succ$  The dark clock line is latest time the clock signal might reach a register. All other lines means the signal might arrive up to  $t_{skew}$  earlier



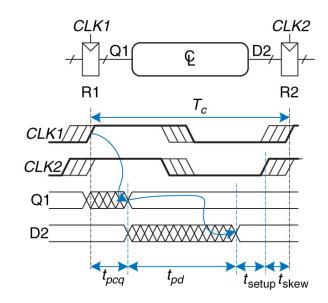
### Accounting for Clock Skew

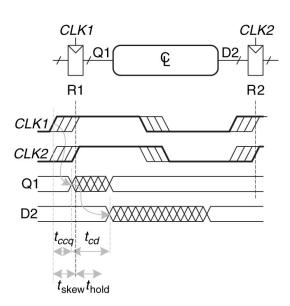
 $\triangleright$  Clock skew needs to be accounted for in  $T_c$ ,  $t_{pd}$ ,  $t_{cd}$  calculations.

$$ightharpoonup T_c \ge t_{pcq} + t_{pd} + t_{setup} + t_{skew}$$

$$\succ$$
  $t_{pd} \le t_c - (t_{pcq} + t_{setup} + t_{skew})$ 

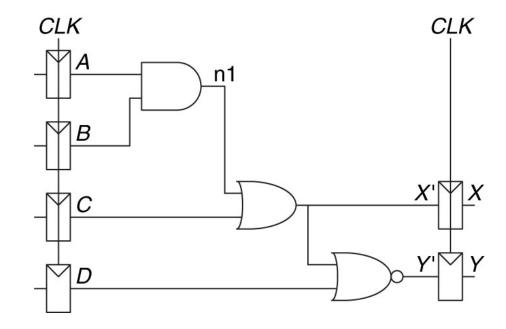
$$\succ$$
  $t_{cd} \ge t_{hold} + t_{skew} - t_{ccq}$ 





## Timing Analysis with Clock Skew Example

- > Study the impact of a 50 ps clock skew on the same circuit analysis
- > Solution
  - Critical path and short path remain the same with only calculation difference
  - $\succ$   $T_c \ge t_{pcq} + 3 \times t_{pd} + t_{setup} + t_{skew}$
  - $> T_c \ge 80 + 3 \times 40 + 50 + 50 = 300 ps$
  - ightharpoonup Therefore,  $f_c = 1/T_c \le 3.33 \, \mathrm{GHz}$
  - $\succ$   $t_{cd}$  is the same as 55 ps, but the hold time is now 60 + 50 = 110 ps. Hence, the hold violation became much worse.
  - ➤ How can we fix this problem?
    - More buffers, or flip-flops with shorter hold time.

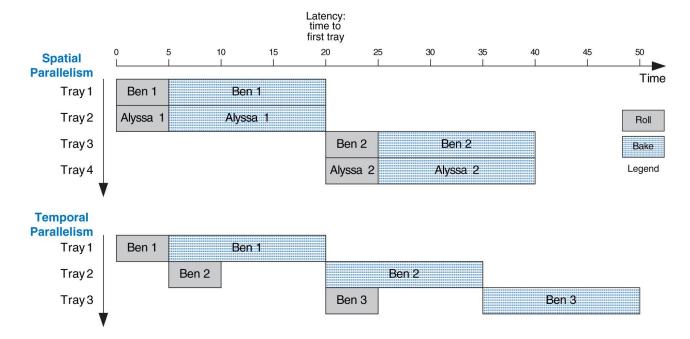


- > We start with some key definitions:
  - > Token: a group of inputs that are processed to produce a group of outputs
  - ➤ Latency: The latency of a system is the time required for one token to pass through the system from start to end.
  - > Throughput: The throughput is the number of tokens that can be produced per unit time
- How can we increase the system throughput?
  - One method is to try to decrease the latency, so more tokens can be processed per unit time
  - > Another method is to try processing multiple tokens at the same time (parallelism)

#### Parallelism has two forms:

- > Spatial parallelism: multiple copies of the hardware are provided so that multiple tasks can be done at the same time.
- Framework Temporal parallelism (pipelining): a task is broken into stages, like an assembly line, and hence multiple tasks can be spread across the stages.
- Although each task must pass through all stages, a different task will be in each stage at any given time so multiple tasks can overlap.
- Simple example: assume preparing a tray of cookies (token) requires 5 minutes rolling in the tray and 15 minutes baking. Study the spatial parallelism if we use two ovens and two helpers, vs a temporal parallelism approach.
- > Use 1 hr as the time unit

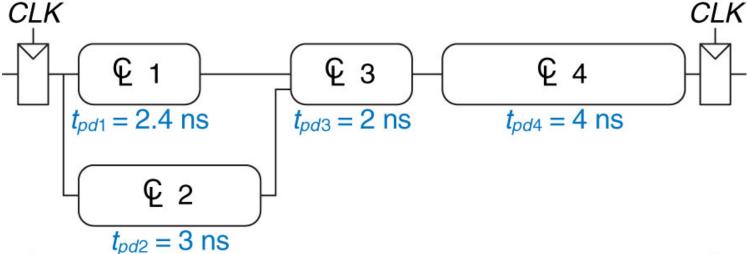
- Single process: latency is 1/3 hr and throughput is 3 trays/hr
- Spatial parallelism: Latency is the same, but throughput is double the single process of 6 trays/hr
- Pipelining: Once a tray is put in the oven, work on rolling the next. Latency is the same, and throughput is 4 trays/hr (why)?



- ➤ In digital circuits, if a task has latency L, in a single process system, the throughput is 1/L
- ➤ In a spatially parallel system with N copies of the hardware, the throughput is N/L
- ➤ In pipelined system, ideally the task is broken into N equal stages, hence throughput is N/L (why)?
- $\triangleright$  However, it is not practical to assume there will be N equal stages. If the longest step takes L<sub>1</sub>, then the throughput is  $1/L_1$  in pipelines system.
- ➤ Pipelining is advantageous over spatial parallelism because it can speed up the performance without duplicating the hardware.

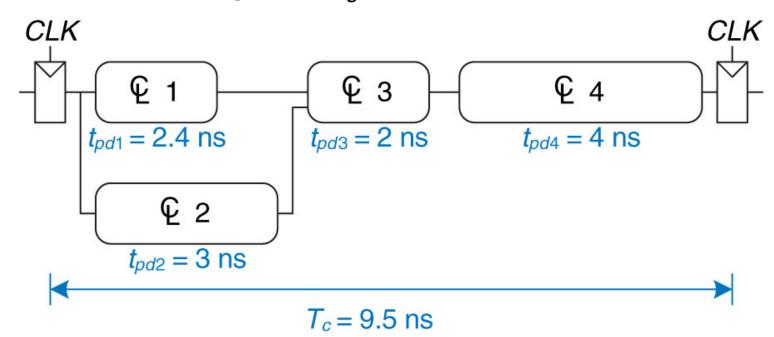
### Pipelining Implementation

- ➤ In pipelining, the combinational circuit is divided into smaller stages by insertion of registers.
- ➤ Registers are important to prevent tokens from catching up with one another.
- > Calculate the throughput of the following single process circuit



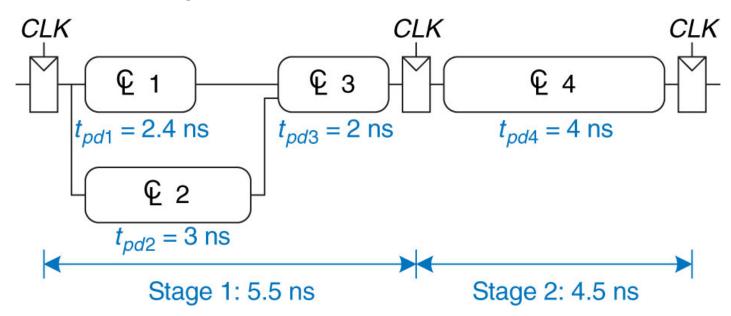
## Throughput Calculation – No Pipelining

- > First, we calculate the latency, which is Tc in this circuit (why)?
  - $\succ$   $T_c = t_{pcq} + t_{pd} + t_{setup}$
  - $\succ$   $t_{pd}$  = sum of all  $t_{pd}$  on the critical path (CL 2, 3 and 4) = 3 + 2 + 4 = 9 ns
  - $\triangleright$  Assume  $t_{pcq}$  = .3 ns and  $t_{setup}$  = 0.2 ns
  - >  $T_c$  = 9 + .5 = 9.5 ns → throughput  $1/T_c$  = 105 MHz



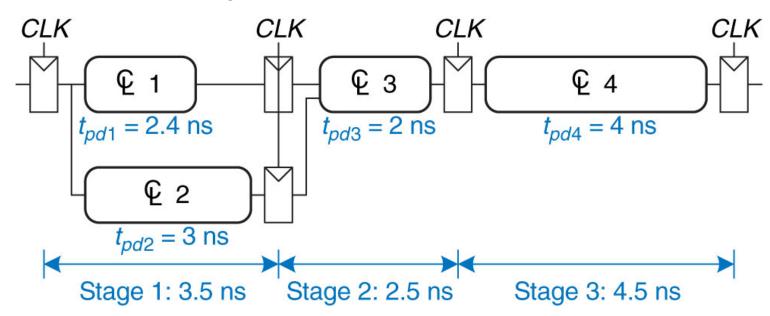
## Throughput Calculation – 2 Stage Pipeline

- $\triangleright$  First, we calculate the latency as T<sub>c</sub> in each stage.
  - $ightharpoonup T_{c1} = 5 + .5 = 5.5 \text{ ns}$
  - $ightharpoonup T_{c2} = 4 + .5 = 4.5 \text{ ns}$
  - $\triangleright$  System clock has to follow the slower  $T_c$ , hence, Circuit  $T_c = 5.5$  ns
  - $\rightarrow$  Therefore, throughput  $1/T_c = 182$  MHz. However, latency L is now 5.5 x 2 = 11 ns (why?)



## Throughput Calculation – 3 Stage Pipeline

- $\triangleright$  Again, we calculate the latency as  $T_c$  in each stage.
  - $ightharpoonup T_{c1} = 3 + .5 = 3.5 \text{ ns}$
  - $ightharpoonup T_{c2} = 2 + .5 = 2.5 \text{ ns}$
  - $ightharpoonup T_{c2} = 4 + .5 = 4.5 \text{ ns}$
  - $\rightarrow$  Therefore, throughput  $1/T_c = 1/4.5$ ns = 222 MHz. However, latency L = 13.5 ns



### **Pipelining Limitation**

- ➤ Pipelining is very powerful technique to increase the throughout. However, it assumes there are no dependencies between tokens (or tasks).
- ➤ In many cases, pipelining is of no significant use due to the dependencies between processes.
- Imagine in the cookie example if we had only one tray to use!!
- ➤ So in some situations, we experience the latency delay with no real throughput enhancement.

#### To Do List

- ➤ Review lecture notes
- Create an account on EDA Playground
- ➤ Study chapter 4