计算机数学概述

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什么是“计算机数学”?

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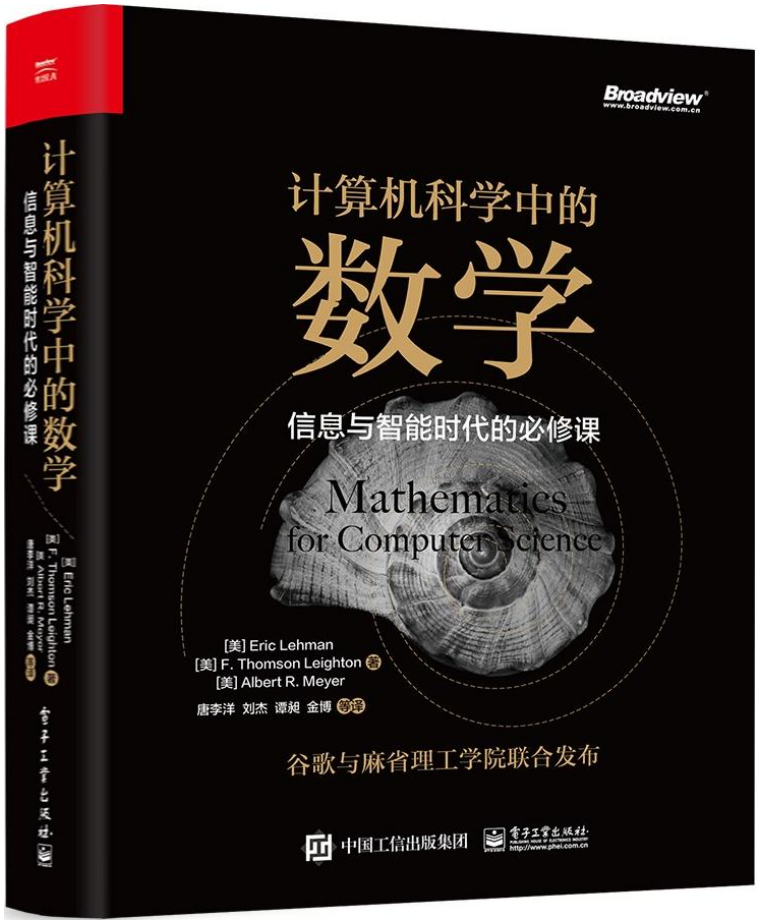
“Mathematics for Computer Science”

(math4cs)



What does “Computer Science” study?

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Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.

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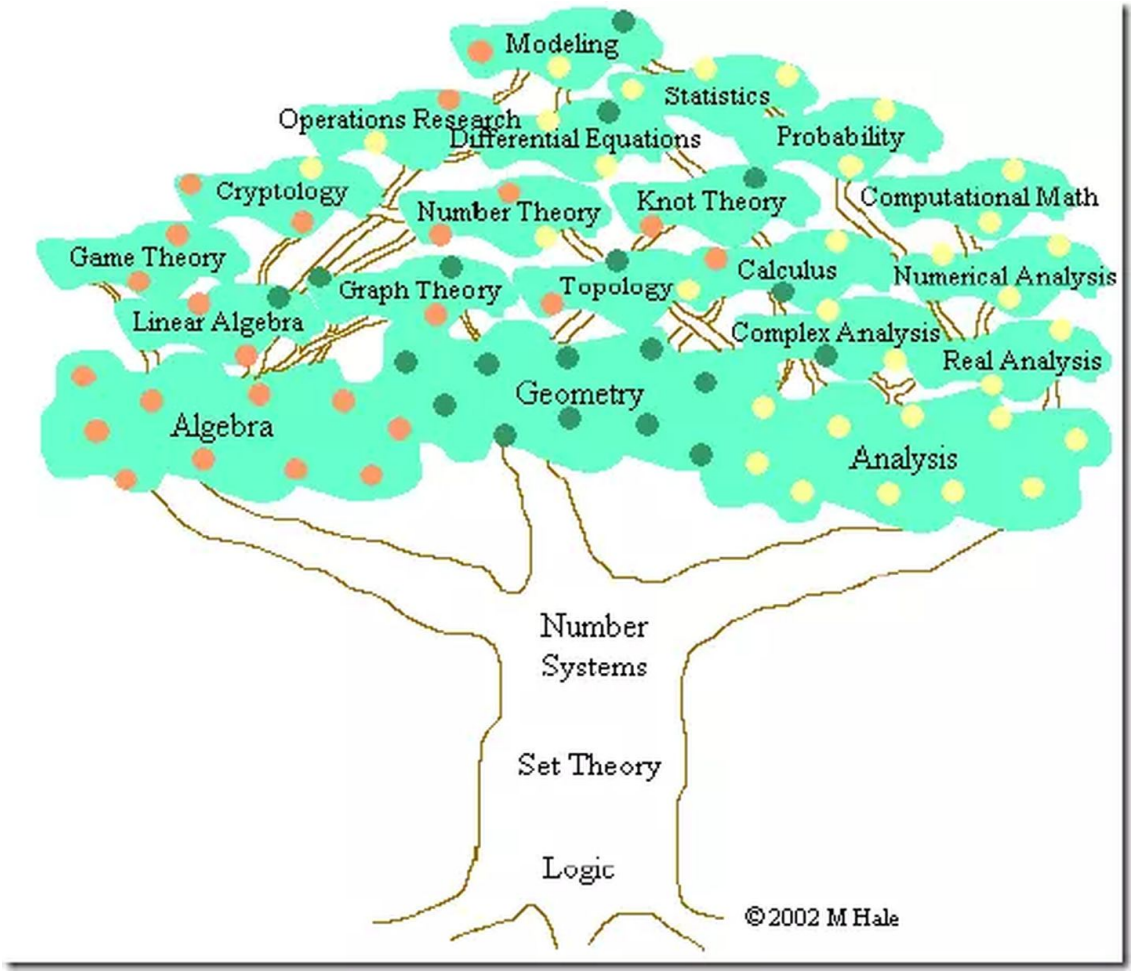
math4cs is **model-and-proof** oriented.

“计算机数学”在哪里？


Modeling
Statistics
Operations Research
Differential Equations
Probability
Cryptology
Number Theory
Knot Theory
Computational Math
Game Theory
Graph Theory
Topology
Calculus
Numerical Analysis
Linear Algebra
Complex Analysis
Real Analysis
Geometry
Algebra
Analysis
Number Systems
Set Theory
Logic
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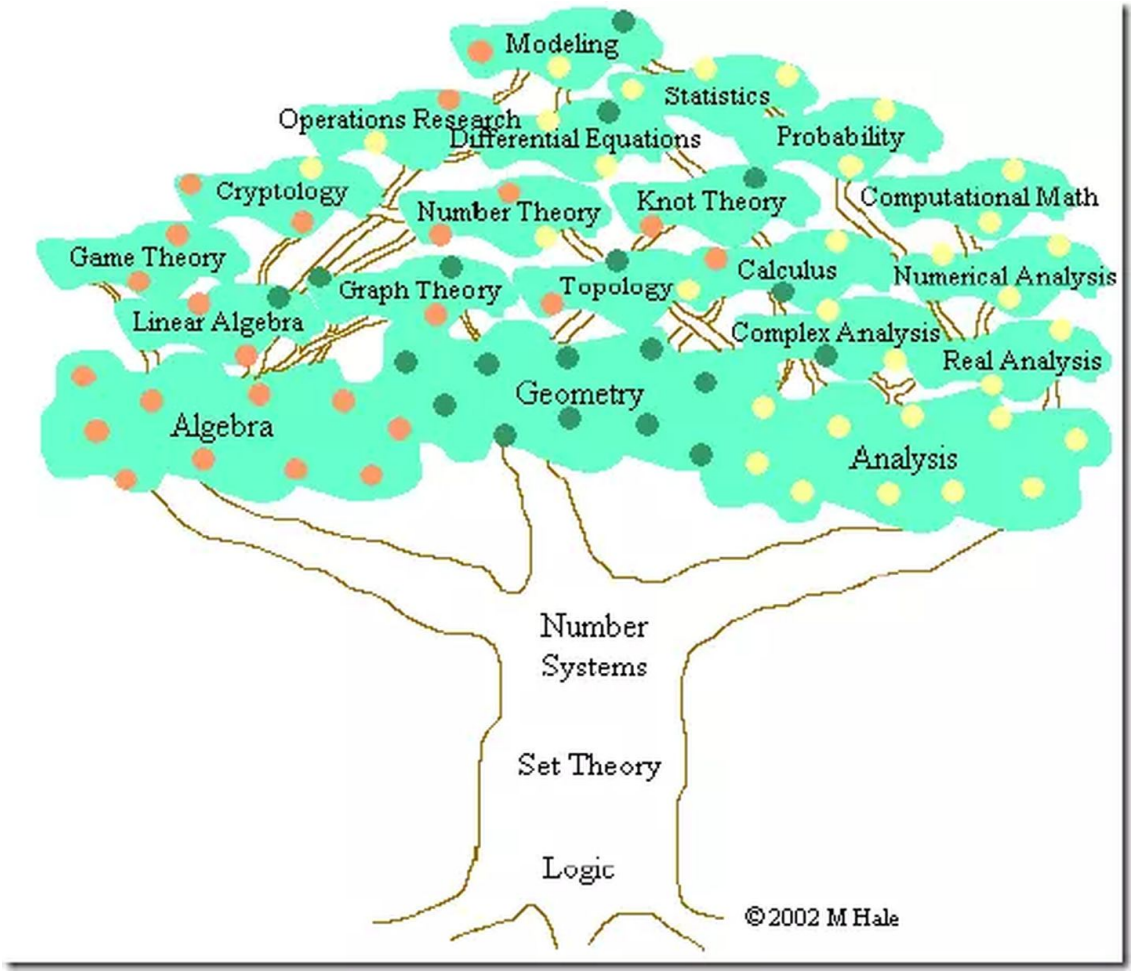
“计算机数学”在哪里？



计算机数学是个大杂烩，啥都学点儿，*啥都没学好*

分班教学
(共 9 个班级)

授课内容与作业可能有出入, 不影响考试与成绩分配

平时作业 *vs.* 期中测试 *vs.* 期末测试

4 : 2 : 4

弹性制

每周二、周四下午 14:00 发布作业　　下周四晚 22:00 前提交作业

每次作业按 **10** 分计算

请按时提交，过时不补，按 0 分计

(助教：、)

“教学立方”课程邀请码：PLD8QKTZ

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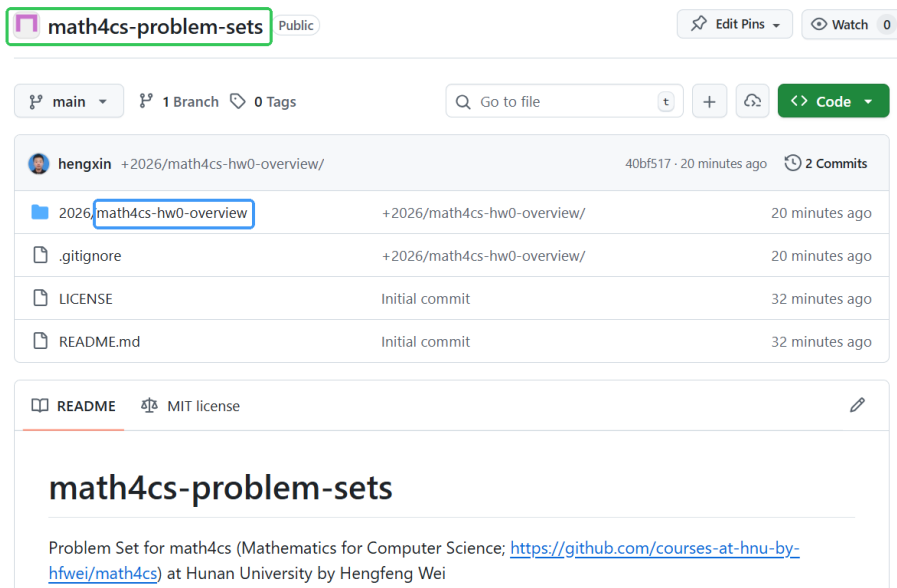


https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets

约法三章

非必要，不点名

非必要，不迟到

非必要，不迟到

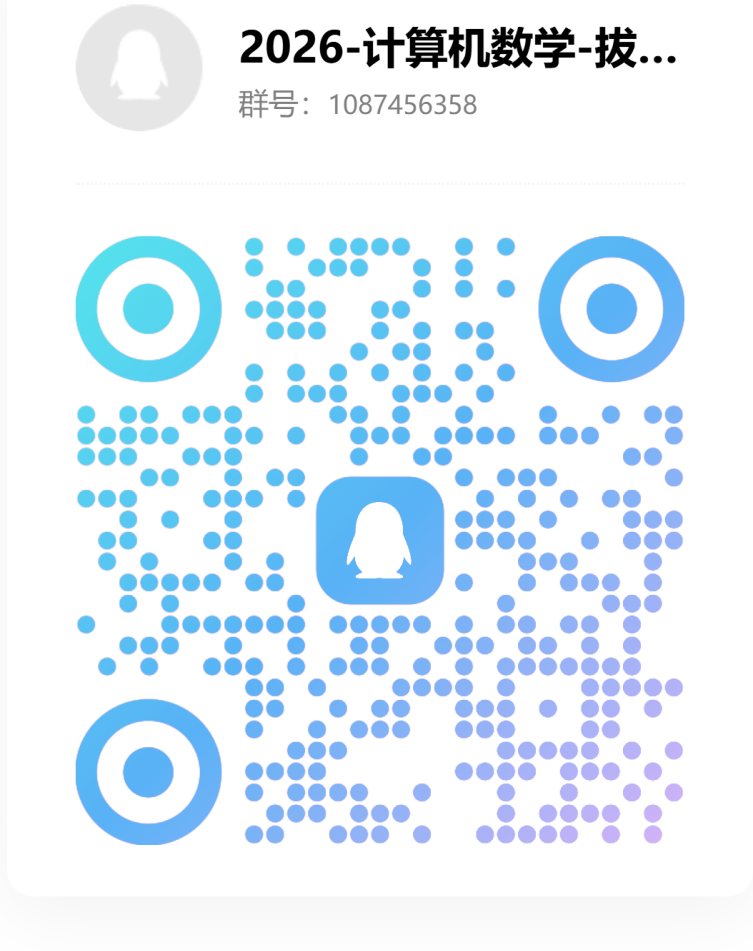
尽量吃早餐，但不可以在教室吃早餐

~~非必要~~，不抄袭；一经发现，后果严重

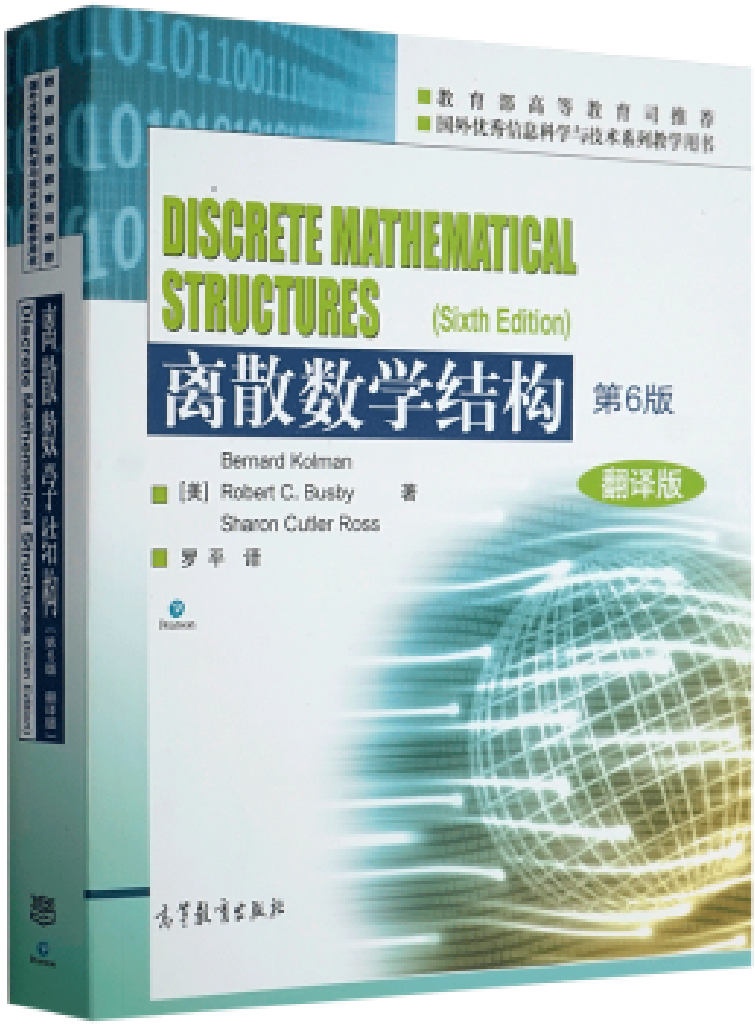
~~非必要~~, 不抄袭; 一经发现, 后果严重

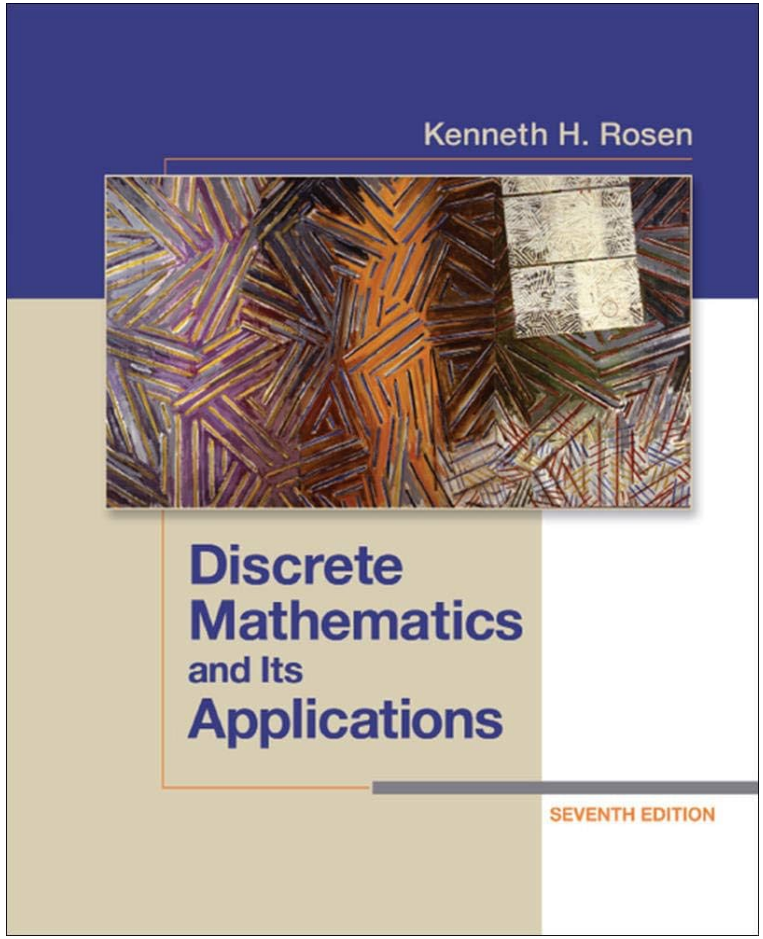
当次作业计 0 分; 总评扣 10 分

QQ 群号：108 745 6358

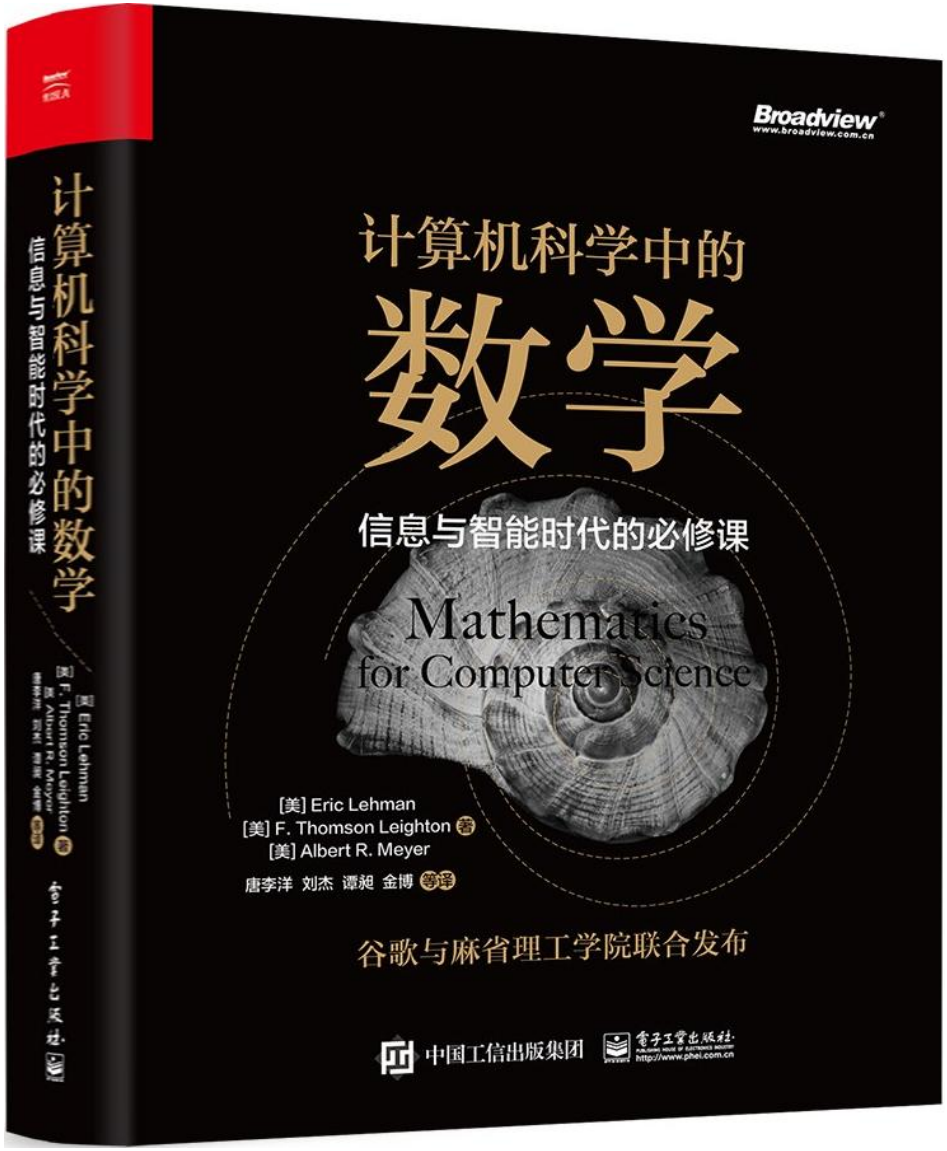


授课内容不局限于教材, 认真听讲很重要

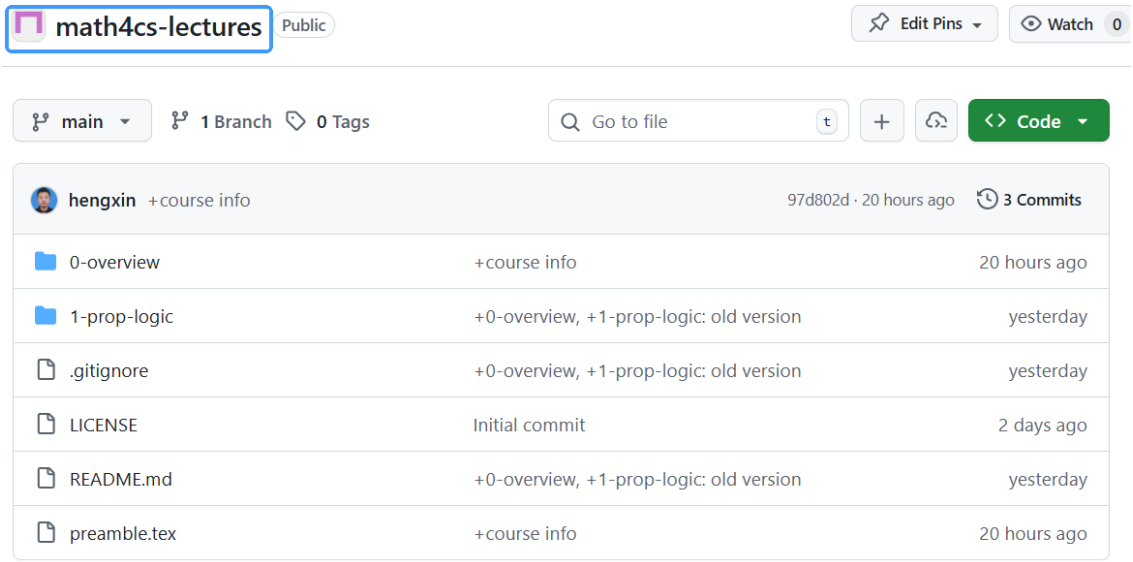




内容与习题偏简单, 略显琐碎



推荐阅读; 其它参考书随课程进度安排



https://github.com/courses-at-hnu-by-hfwei/math4cs-lectures

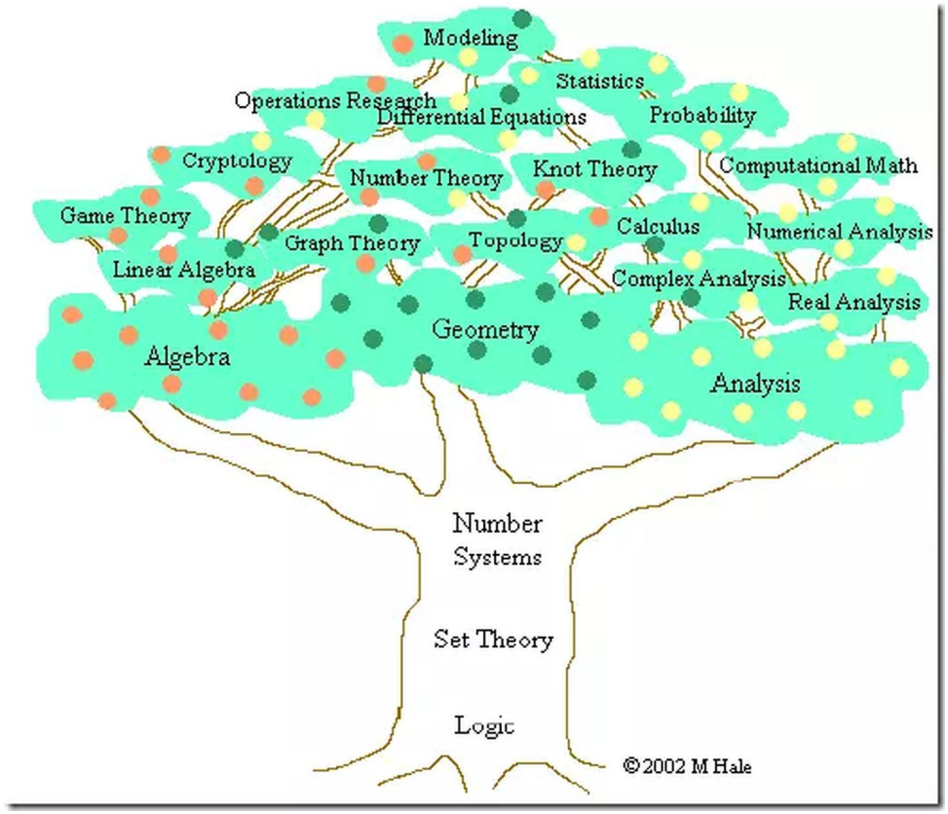

welcome
again

离散数学

离散数学

研究**离散对象的结构、性质、操作等**的数学分支 (**大杂烩**)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、（离散）概率

关于离散数学, 学长纷纷表示:

我太难了

啥用没有

真得有那么难吗?

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力


Show me your proof...
whisper

Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

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Q：这是构造性证明吗?

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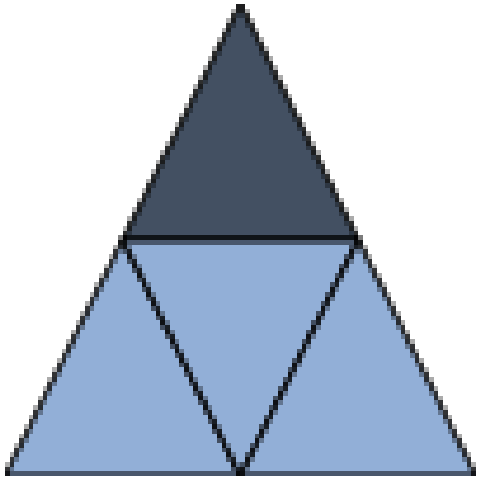
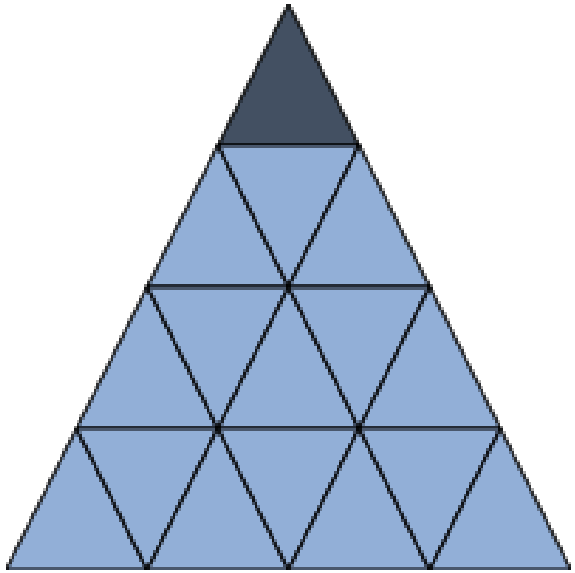
Q：这是构造性证明吗？这是反证法吗？

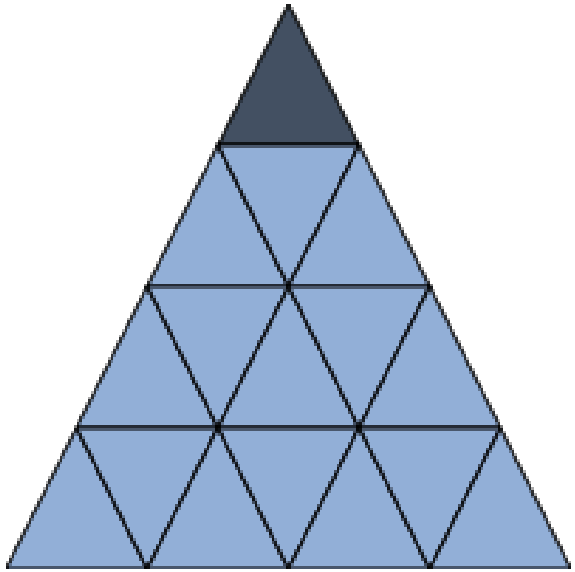
Tiling Puzzle

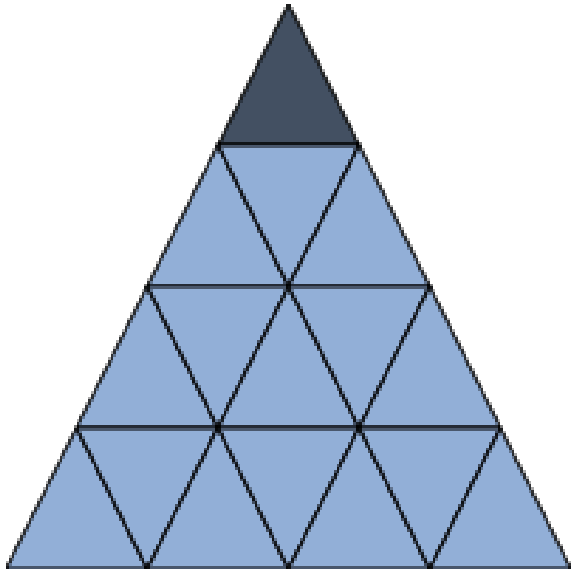
Suppose n is a positive integer. An equilateral triangle is cut into 4^n congruent equilateral triangles, and one corner is removed.

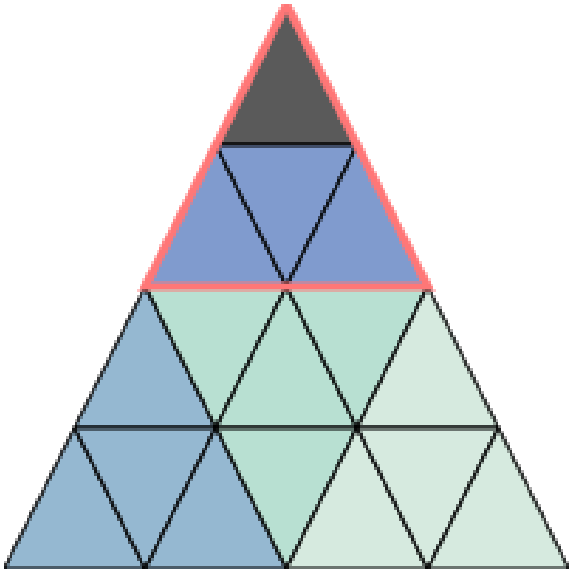
Show that the remaining area can be covered by tiles below

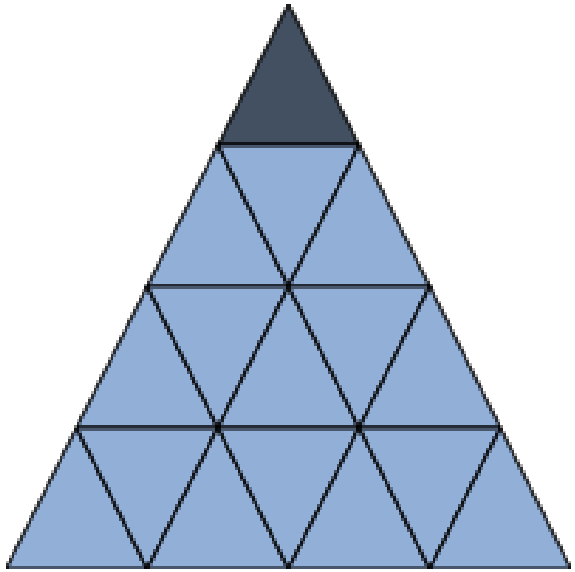


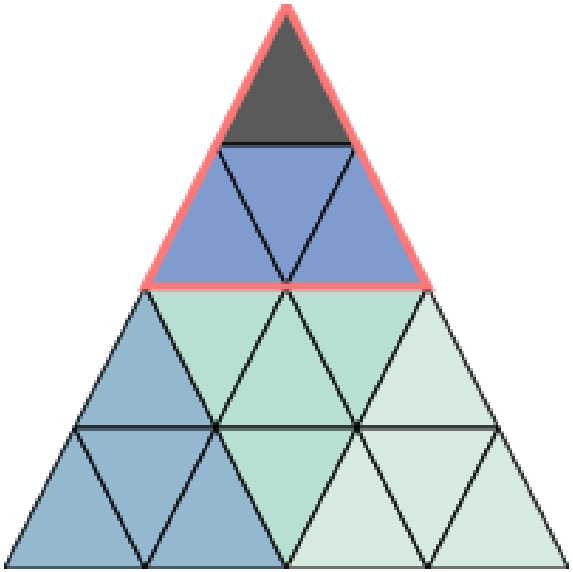


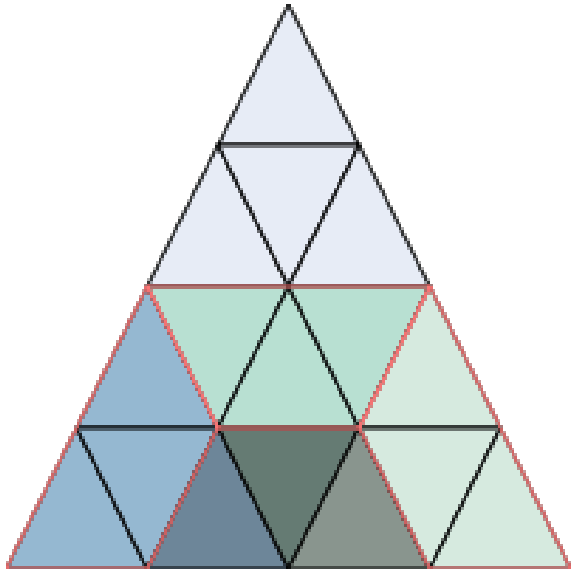












Base Case:

Induction Hypothesis:

Induction Step: . . . by induction hypothesis . . .

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.

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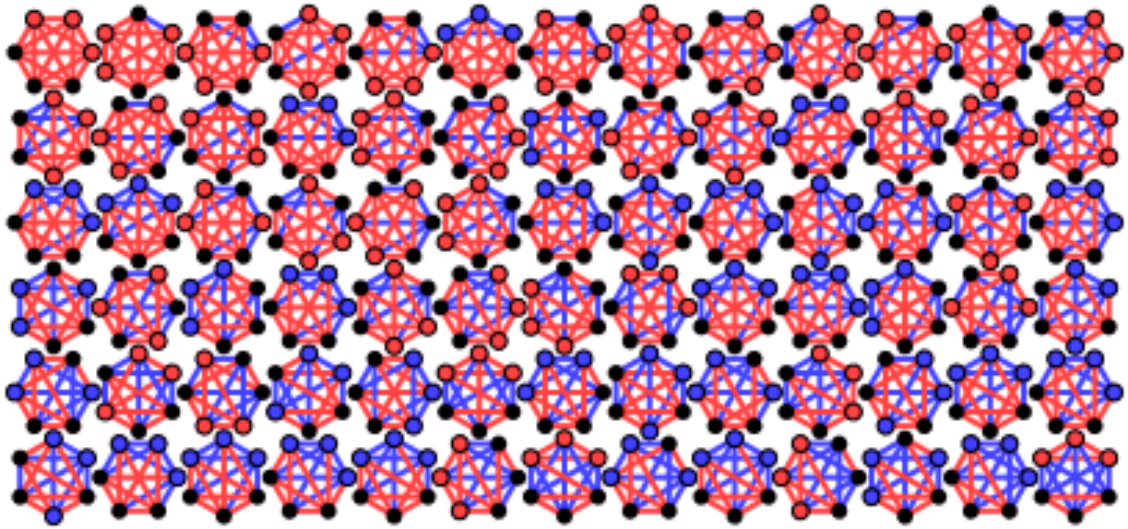


In Terms of **Graph Theory**.

(Is there a monochromatic triangle in any 2-coloring of K_6?)

Theorem on Friends and Strangers

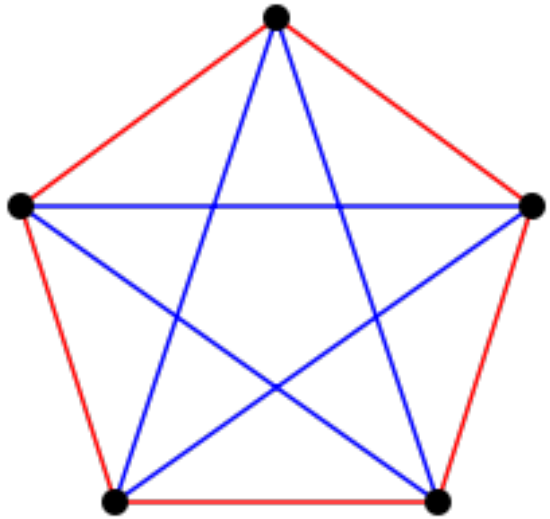
Is there a monochromatic triangle in any 2-coloring of K_6?

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_5?

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_5?



Ramsey theory

How **big** must the **structure** be

to ensure that it has a given interesting **property**?

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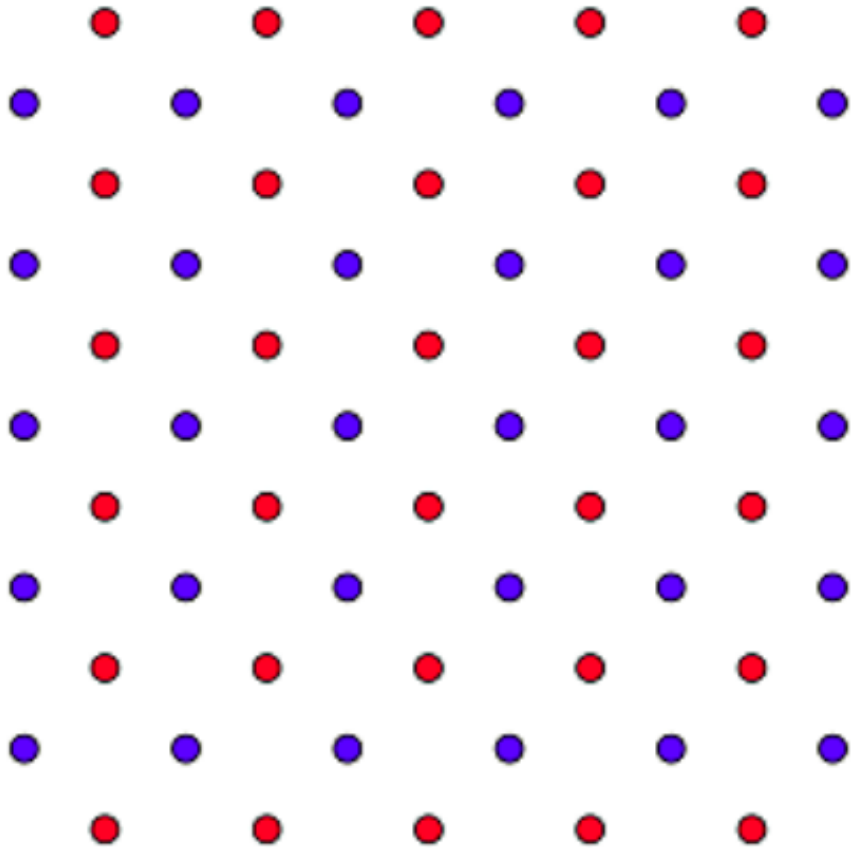
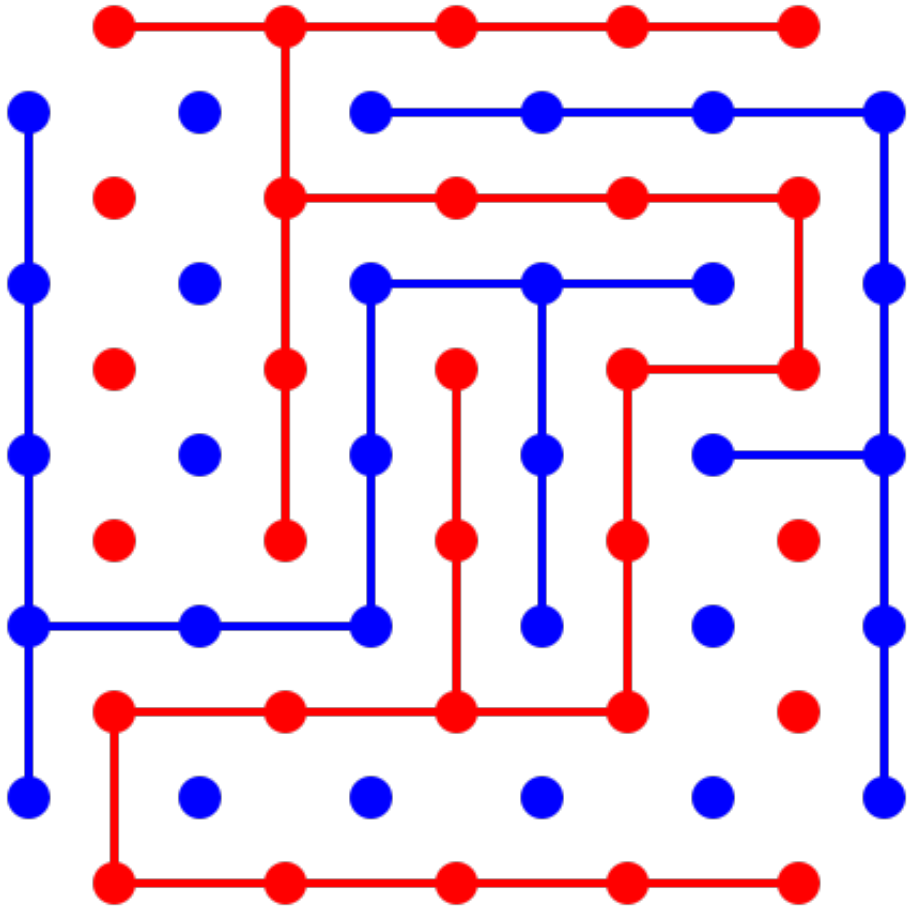
Bridg-It Game (David Gale, 1958)



5×6 *vs.* 6×5



5×6 *vs.* 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie**?

Will Bridg-It **end in a tie**?

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

Does **Player 2** have a **winning strategy**?

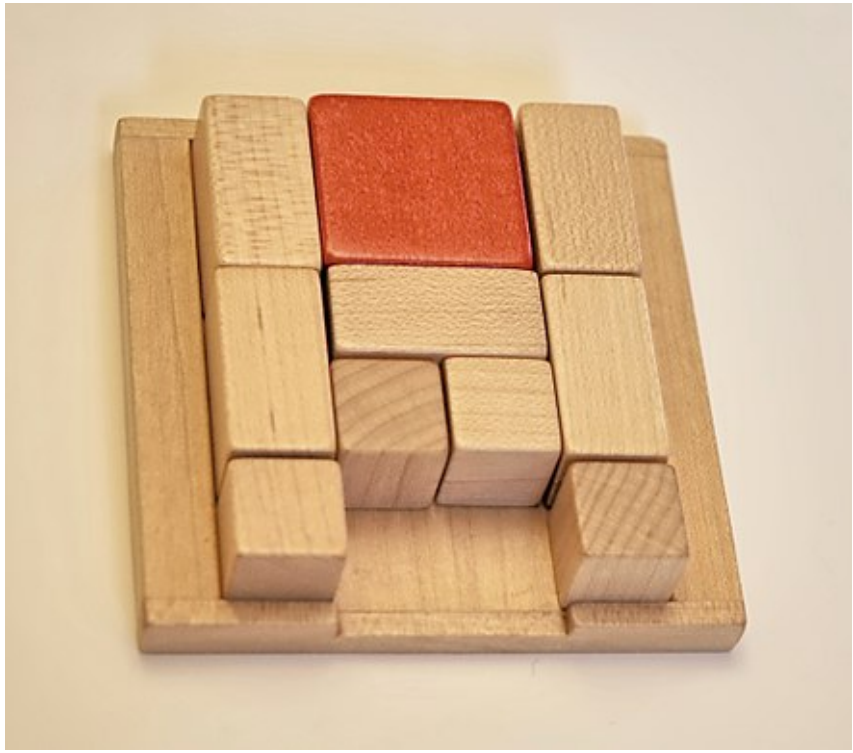
No! By the **strategy stealing argument**.

Does **Player 1** have a **winning strategy**?

Does **Player 1** have a **winning strategy**?

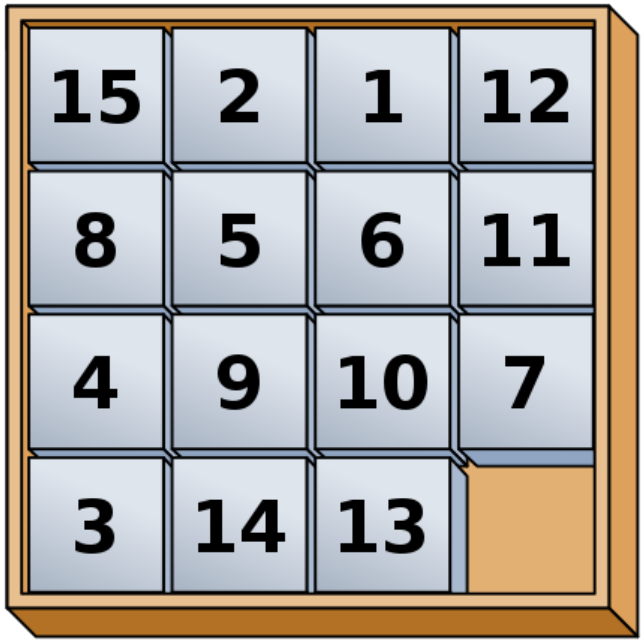
Yes! It uses **spanning trees** in **graph theory**.

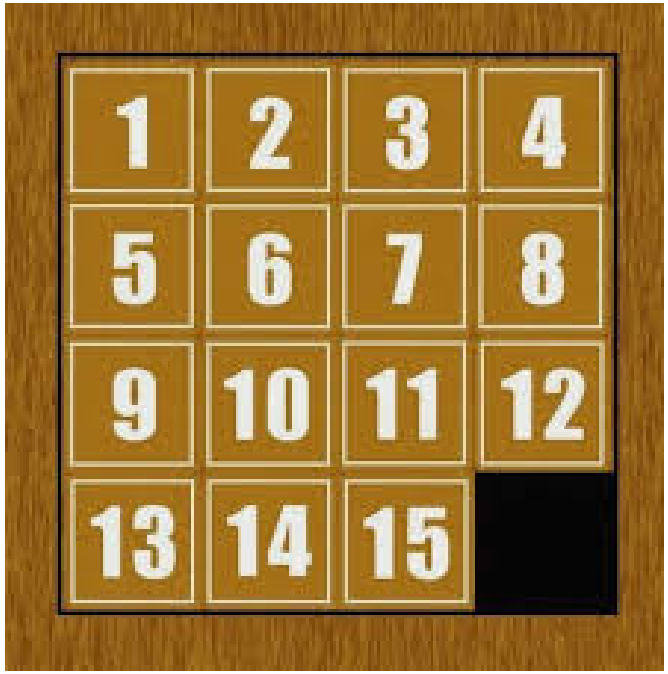
Klotski Puzzle (华容道)

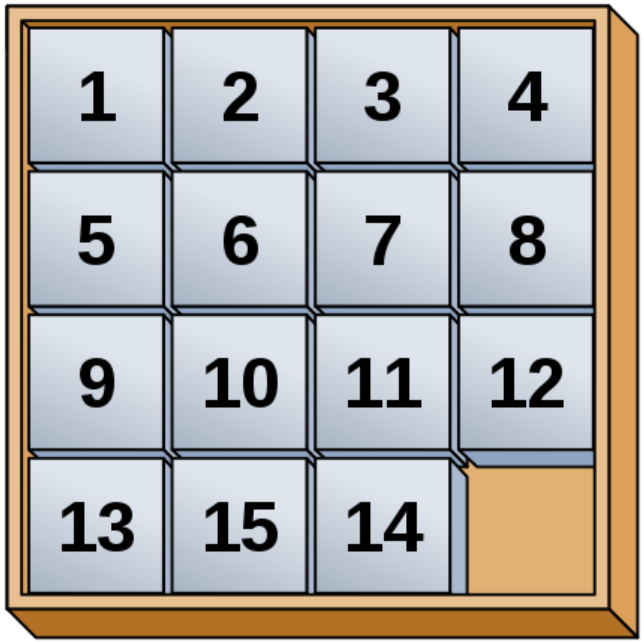


Klotski Puzzle (华容道; 中国版本)

15 Puzzle (数字华容道)







Is it solvable?

How to solve it?

How to solve it?

It uses **permutation groups** in **group theory**.

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	**Women** $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

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$$\{xb, yc, zd, wa\}$$

(x, a) is an unstable pair

$$\{xa, yb, zd, wc\}$$

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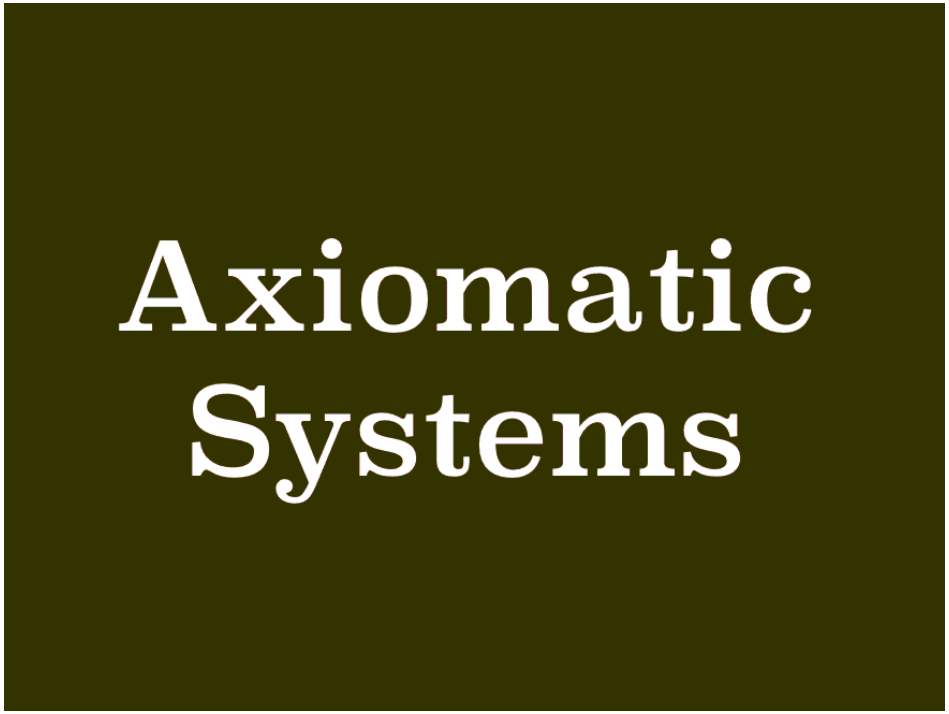
Theorem (The Gale-Shapley Algorithm (1962))

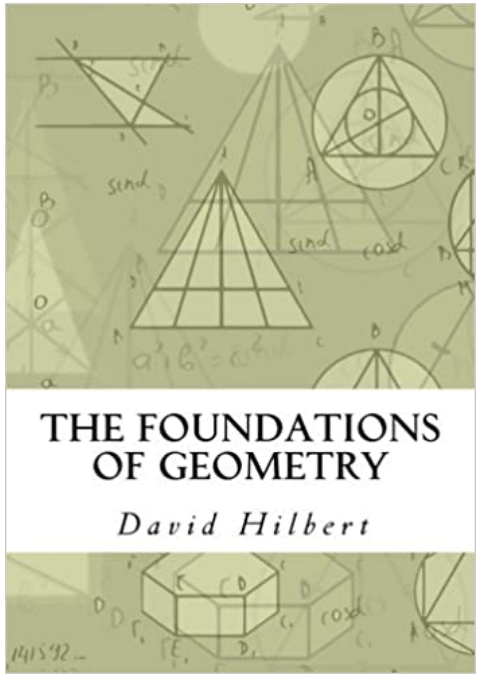
It is always possible to solve SMP.

Self-contained (自包含; 自给自足)



Syntax *vs.* Semantics (语法与语义对立统一)

三个公理系统：逻辑、集合论、图论、抽象代数 (群论)



(1) To draw a straight line from any point to any point.

(2) To extend a finite straight line continuously in a straight line.

(3) To describe a circle with any center and radius.

(4) That all right angles are equal to one another.

(5) The parallel postulate.

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

Axioms:

(1) There are exactly four points.

(2) It is impossible for three points to be on the same line.

(3) For every pair of distinct points x and y, there is a unique line l such that x is on l and y is on l.

(4) Given a line l and a point x that is not on l, there is a unique line m such that x is on m and no point on l is also on m.

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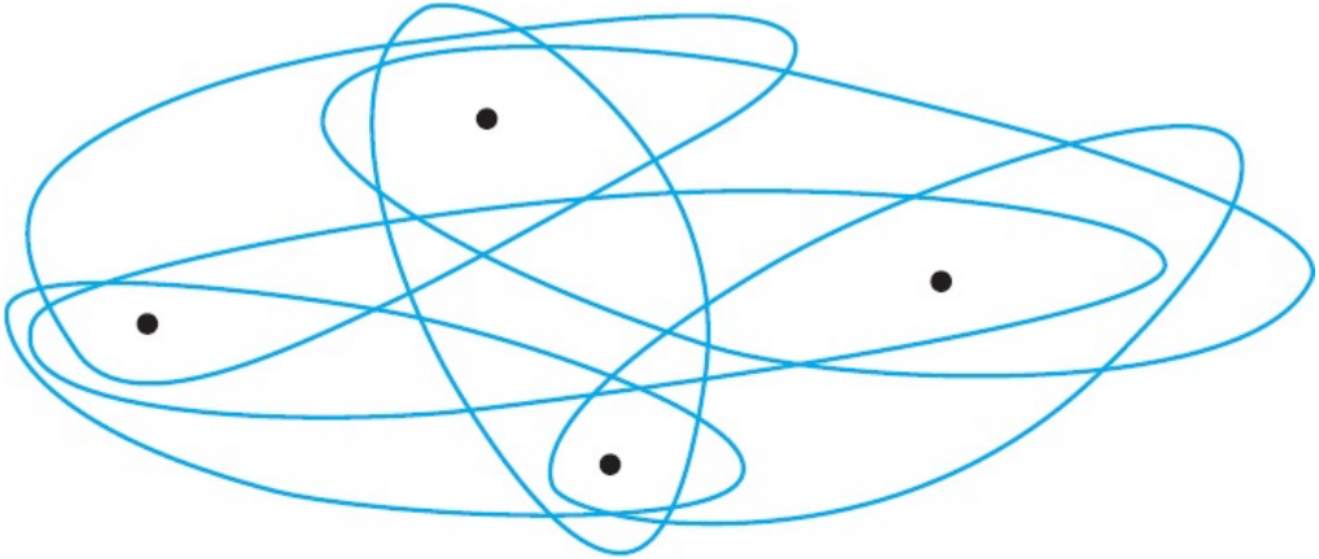
Theorem

There are at least two distinct lines.

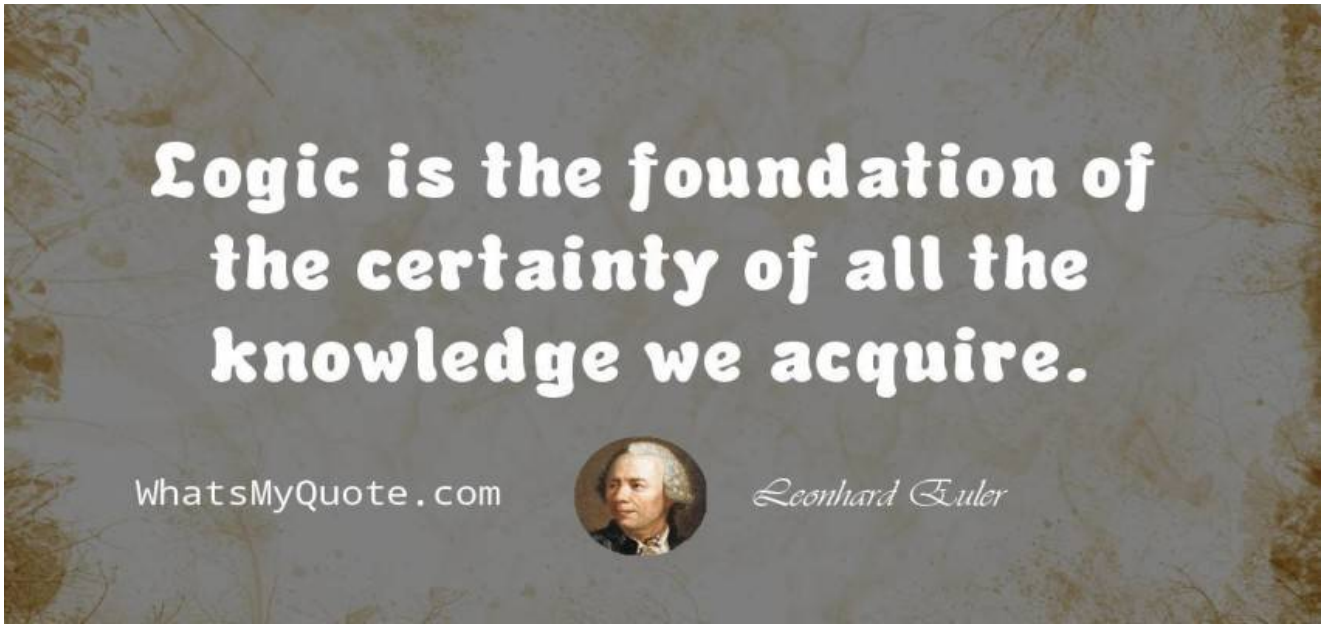
Syntax *vs.* Semantics

Syntax *vs.* Semantics



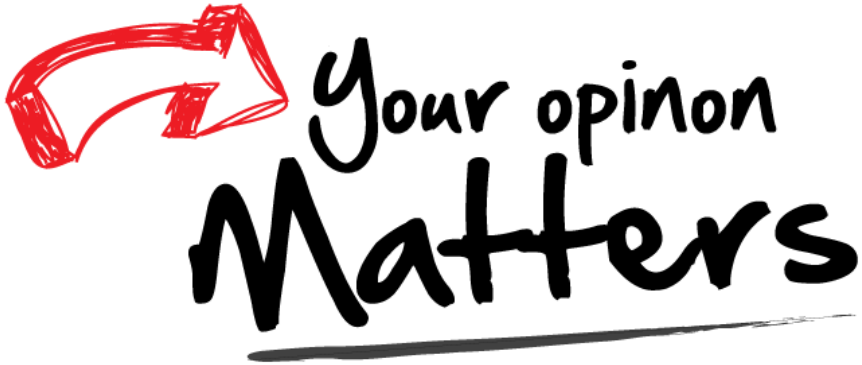
point : · line : ◯ is on : ⊙

什么样的推理是正确的?


Logic is the foundation of the certainty of all the knowledge we acquire.
WhatsMyQuote.com
Leonhard Euler


SEE YOU
NEXT WEEK


Thank
You!



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