

计算机数学概述

魏恒峰

hfwei@hnu.edu.cn

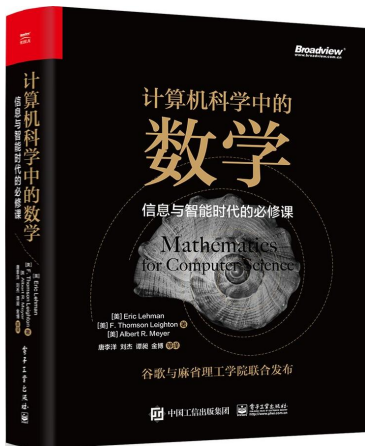
2026 年 03 月 03 日



什么是“计算机数学”？

“Mathematics for Computer Science”

(math4cs)

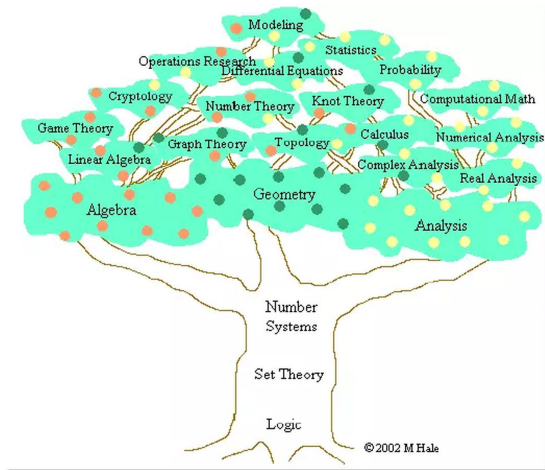


What does “Computer Science” study?

Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.

math4cs is **model-and-proof** oriented.

“计算机数学”在哪里？



计算机数学是个大杂烩，啥都学点儿，~~啥都没学好~~

分班教学 (共 9 个班级)

拔尖班: 独立授课, 独立考核



平时作业 *vs.* 课堂测验 *vs.* 期中测试 *vs.* 期末测试

1 : 3 : 2 : 4

每周四下午 14:00 发布平时作业 **下周四**晚 22:00 前提交作业

取最高的 **12** 次作业成绩计入总评

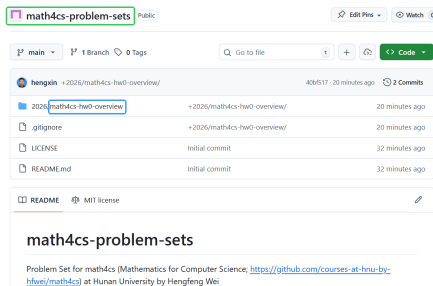
请按时提交, 过时不补

(助教: 罗熙辰)

“教学立方”课程邀请码: PLD8QKTZ



TEX



<https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets>

约**每两周一次**课堂测验 (提前通知时间与测验范围)

取最高的 **6** 次课堂测验成绩计入总评



请准时参加, 不安排补考

约法三章

非必要, 不点名

非必要, 不迟到

尽量吃早餐, 但不可以在教室吃早餐

~~非/必/要/~~, 不抄袭; 一经发现, 后果严重

~~当/次/作/业/计/0/分/~~; 平时作业成绩扣 2 分, 扣完为止

QQ 群号: 108 745 6358



2026-计算机数学-拔...

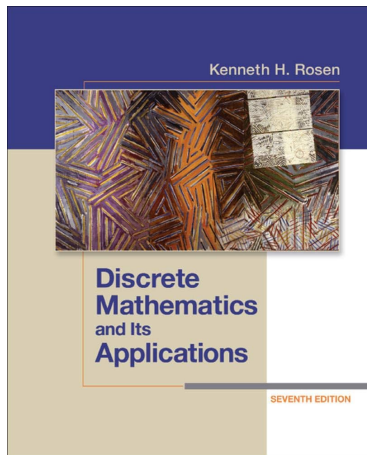
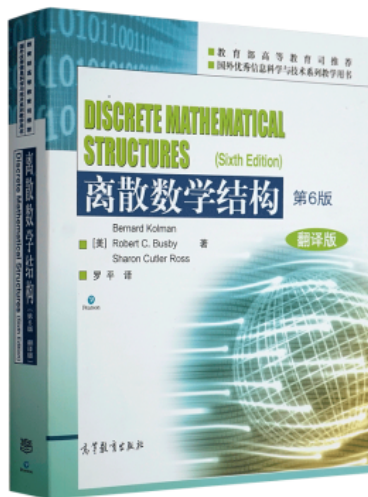
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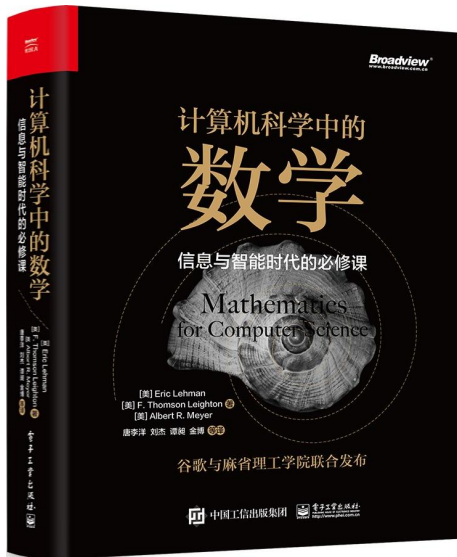
扫一扫二维码，加入群聊



授课内容不局限于教材, 认真听讲很重要



内容与习题偏简单, 略显琐碎; 可用于课前预习及课后基础练习



推荐阅读; 其它参考书随课程进度安排

0-overview	+course info	20 hours ago
1-prop-logic	+0-overview, +1-prop-logic: old version	yesterday
.gitignore	+0-overview, +1-prop-logic: old version	yesterday
LICENSE	Initial commit	2 days ago
README.md	+0-overview, +1-prop-logic: old version	yesterday
preamble.tex	+course info	20 hours ago

<https://github.com/courses-at-hnu-by-hfwei/math4cs-lectures>

0-overview.pdf

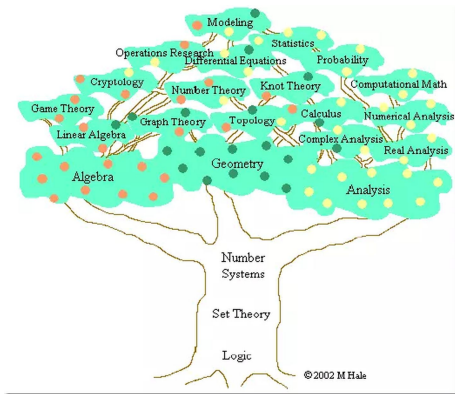
0-overview-**handout**.pdf



计算机（离散）数学

研究离散对象的结构、性质、操作等的数学分支 (大杂烩)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、(离散) 概率

关于计算机 (离散) 数学的古老传说:

我太难了

啥用没有

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力



Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

$$\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$$

$$\sqrt{2}^{\sqrt{2}}$$

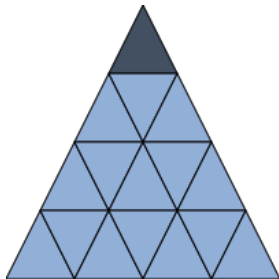
$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$$

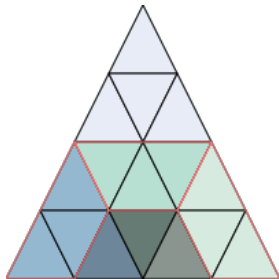
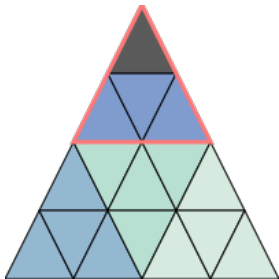
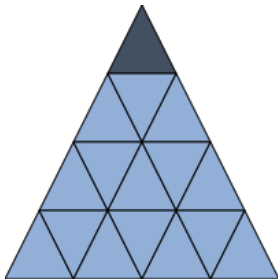
Q : 这是构造性证明吗? 这是反证法吗?

Tiling Puzzle

Suppose n is a positive integer. An equilateral triangle is cut into 4^n congruent equilateral triangles, and one corner is removed.

Show that the remaining area can be covered by tiles below





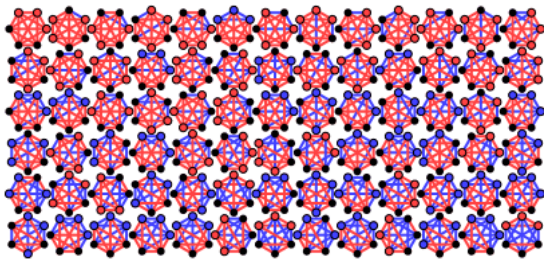
Base Case:

Induction Hypothesis:

Induction Step: ... by induction hypothesis ...

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.



In Terms of **Graph Theory**.

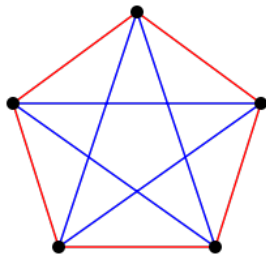
(Is there a **monochromatic** triangle in any 2-coloring of K_6 ?)

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_6 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?

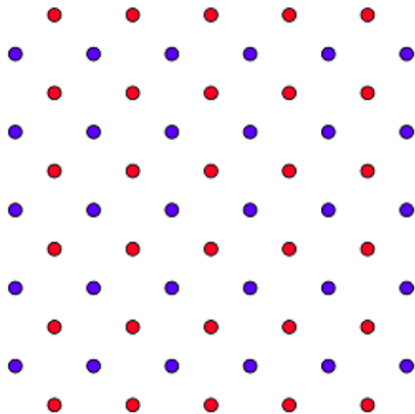


Ramsey theory

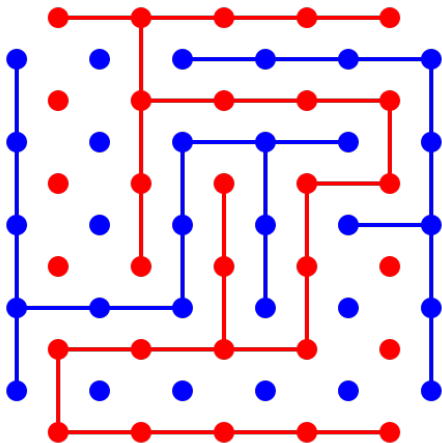
How **big** must the **structure** be
to ensure that it has a given interesting **property**?



Bridg-It Game (David Gale, 1958)



5×6 vs. 6×5



5×6 vs. 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie**?

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

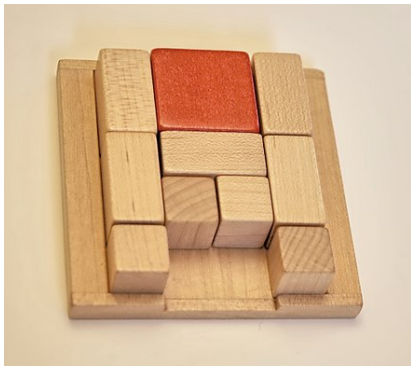
Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.



STAY TUNED

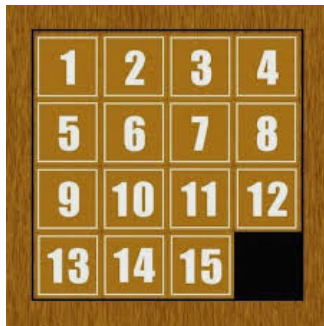
Klotski Puzzle (华容道)



Klotski Puzzle (华容道; 中国版本)



15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

It uses **permutation groups** in **group theory**.



STAY TUNED

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

$$\{xb, yc, zd, wa\}$$

(x, a) is an unstable pair

$$\{xa, yb, zd, wc\}$$

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

Theorem (The Gale-Shapley Algorithm (1962))

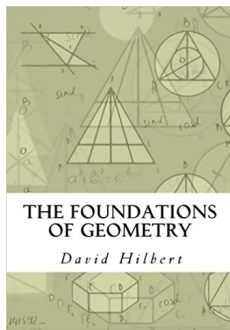
It is always possible to solve SMP.

Self-contained (自包含; 自给自足)

Axiomatic Systems

Syntax *vs.* Semantics (语法与语义对立统一)

三个公理系统: 逻辑、集合论、图论、抽象代数 (群论)



- (1) To draw a straight **line** from any **point** to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) **The parallel postulate.**

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

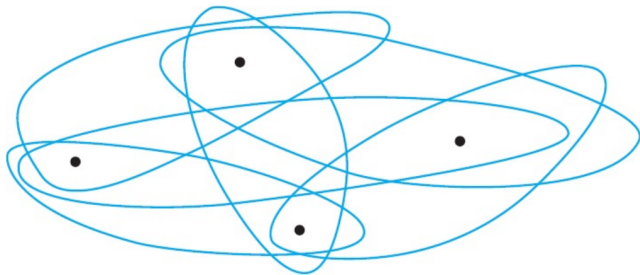
Axioms:

- (1) There are exactly four points.
- (2) It is impossible for three points to be on the same line.
- (3) For every pair of distinct points x and y , there is a unique line l such that x is on l and y is on l .
- (4) Given a line l and a point x that is not on l , there is a unique line m such that x is on m and no point on l is also on m .

Theorem

There are at least two distinct lines.

Syntax *vs.* Semantics



point : \cdot line : \bigcirc is on : $\bigcirc \cdot$

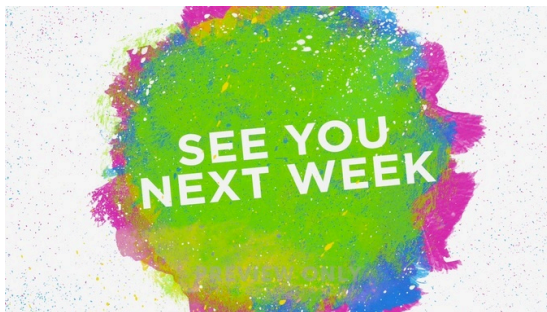
什么样的推理是正确的？

**Logic is the foundation of
the certainty of all the
knowledge we acquire.**

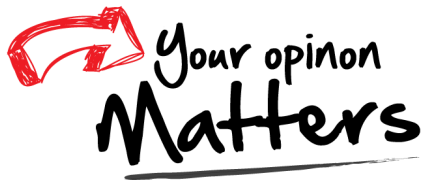
WhatsMyQuote.com



Leonhard Euler



Thank
You!



Office 926

hfwei@nju.edu.cn