计算机数学概述

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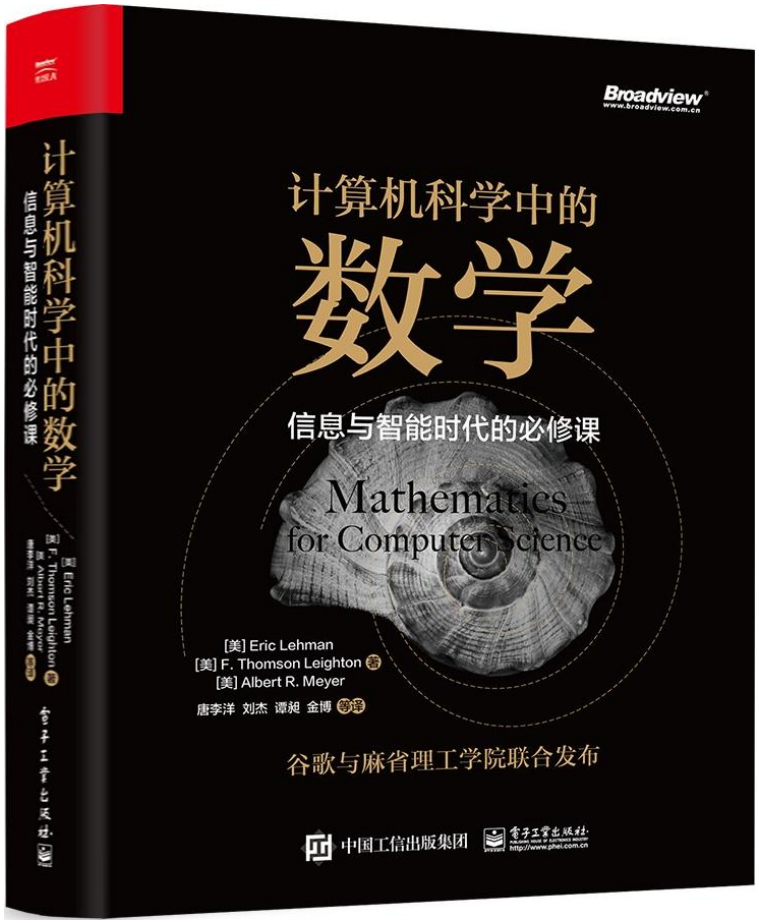
2026 年 03 月 03 日

什么是“计算机数学”?

“Mathematics for Computer Science”

(math4cs)

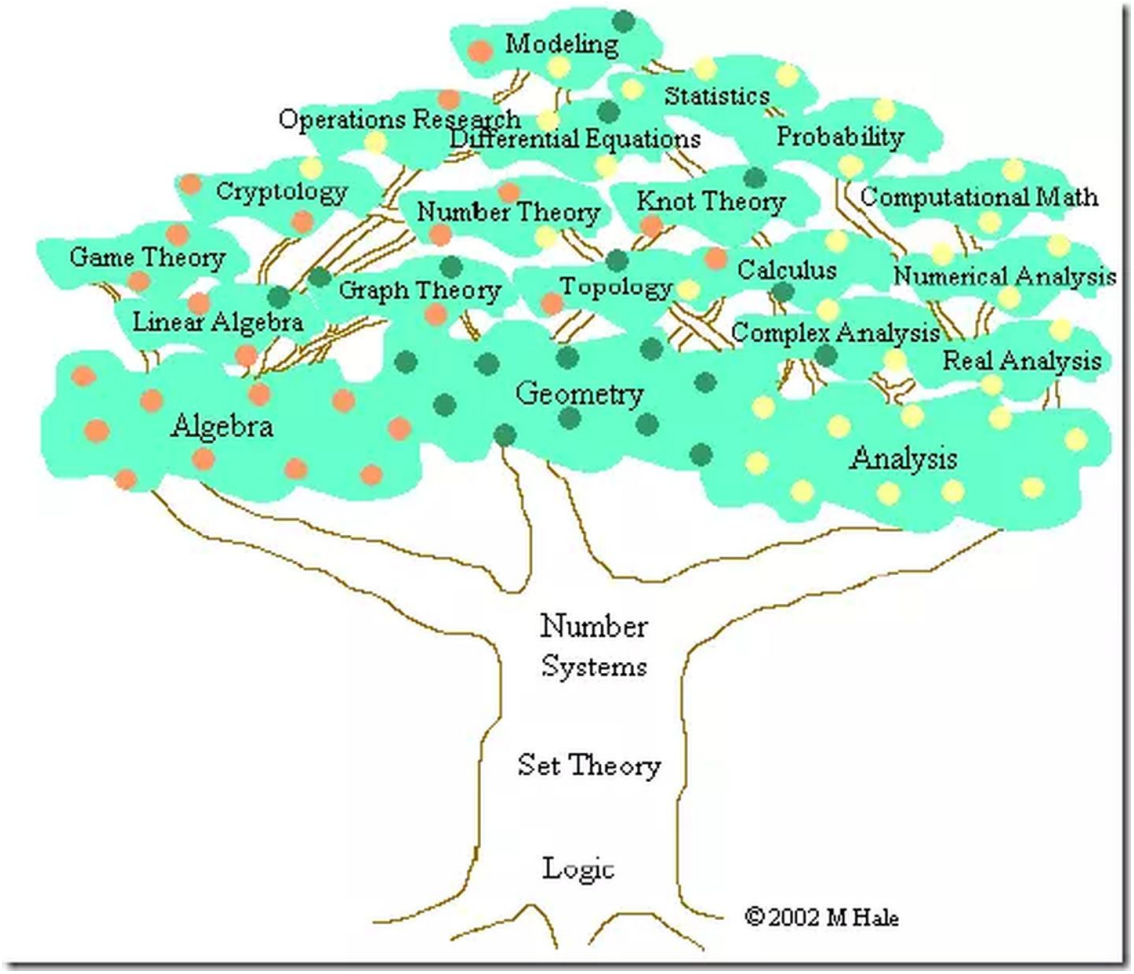


What does "Computer Science" study?

Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.

math4cs is **model-and-proof** oriented.

“计算机数学”在哪里？



计算机数学是个大杂烩，啥都学点儿，*啥都没学好*

分班教学 (共 9 个班级)

拔尖班: 独立授课, 独立考核

平时作业 *vs.* 课堂测验 *vs.* 期中测试 *vs.* 期末测试

1 : 3 : 2 : 4

每周四下午 14:00 发布平时作业　　下周四晚 22:00 前提交作业

取最高的 **12** 次作业成绩计入总评

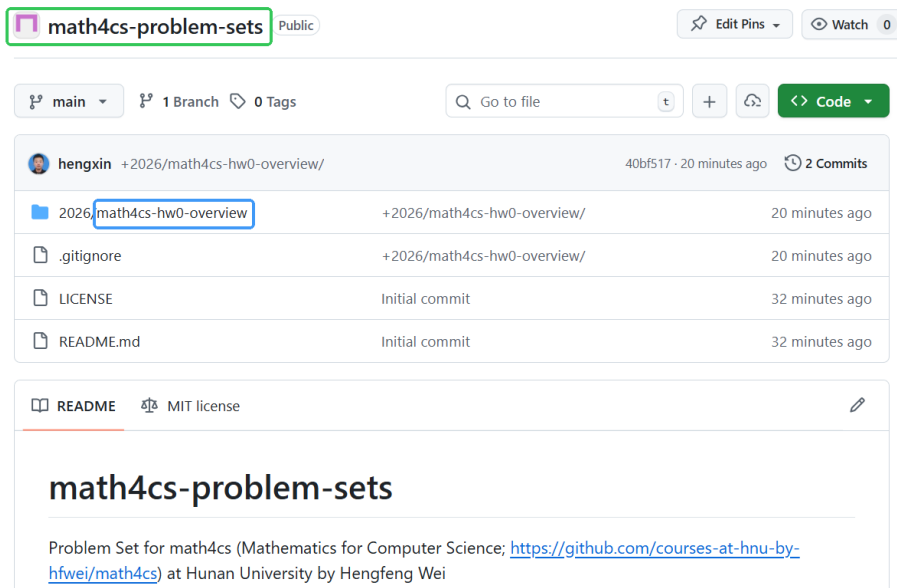
The only way
to learn
mathematics
is to do
mathematics.

PAUL HALMOS

请按时提交，过时不补

(**助教**: 罗熙辰)

“教学立方”课程邀请码：PLD8QKTZ



https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets

约**每两周一次**课堂测验 (提前通知时间与测验范围)

取最高的 **6** 次课堂测验成绩计入总评

请准时参加，不安排补考

约法三章

非必要，不点名

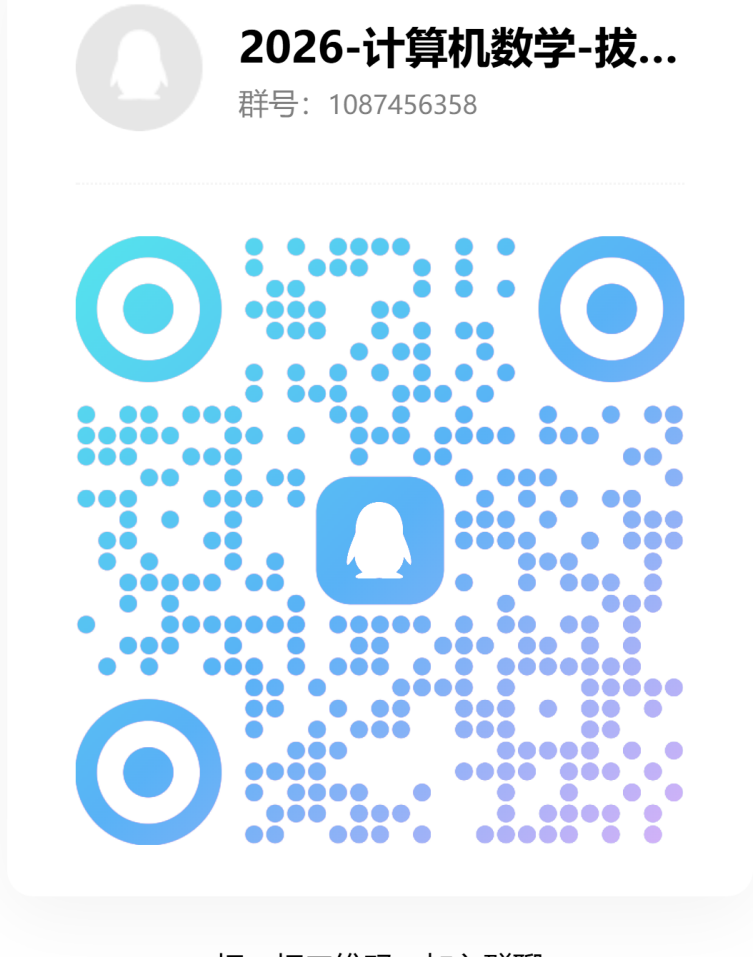
非必要，不迟到

尽量吃早餐，但不可以在教室吃早餐

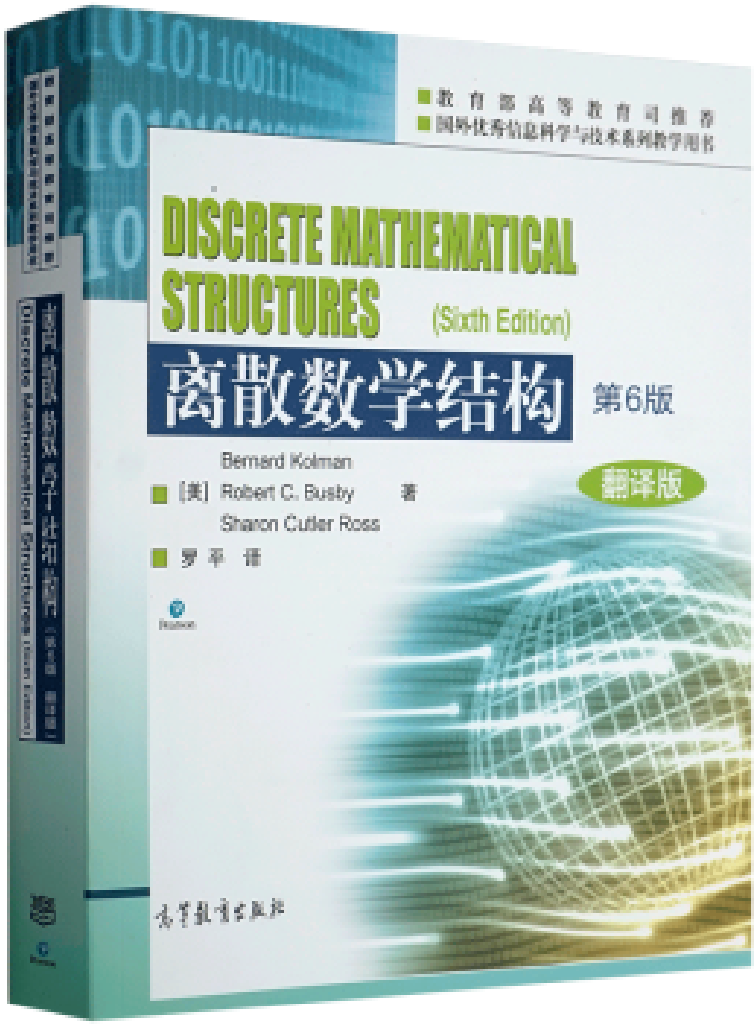
非必要，不抄袭；一经发现，后果严重

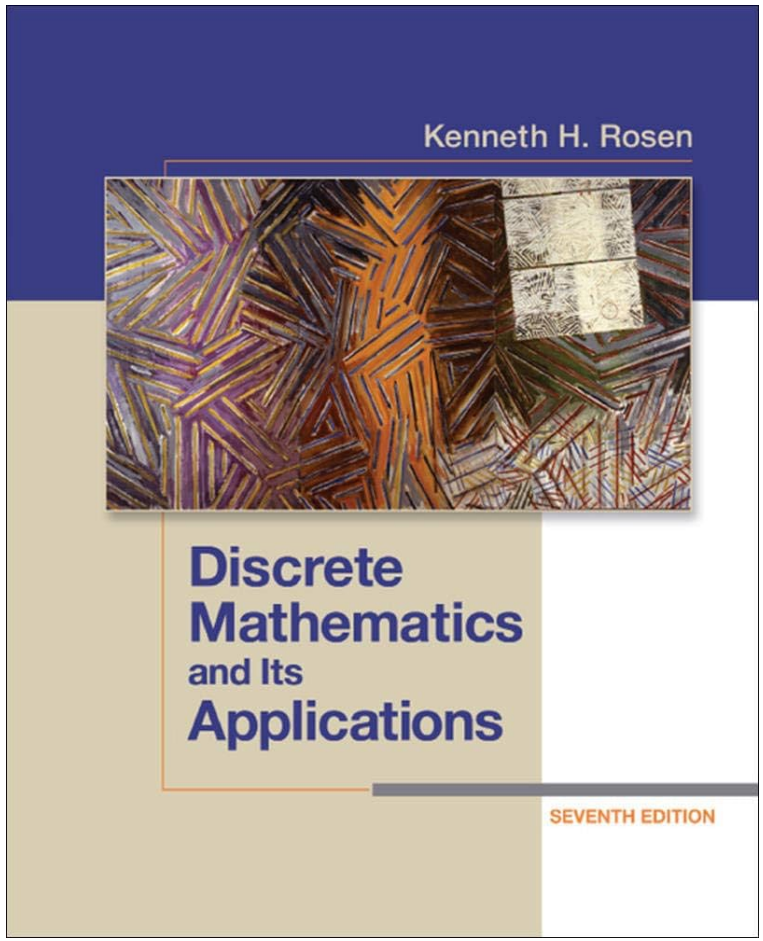
当次作业计 0 分；平时作业成绩扣 2 分，扣完为止

QQ 群号：**108 745 6358**

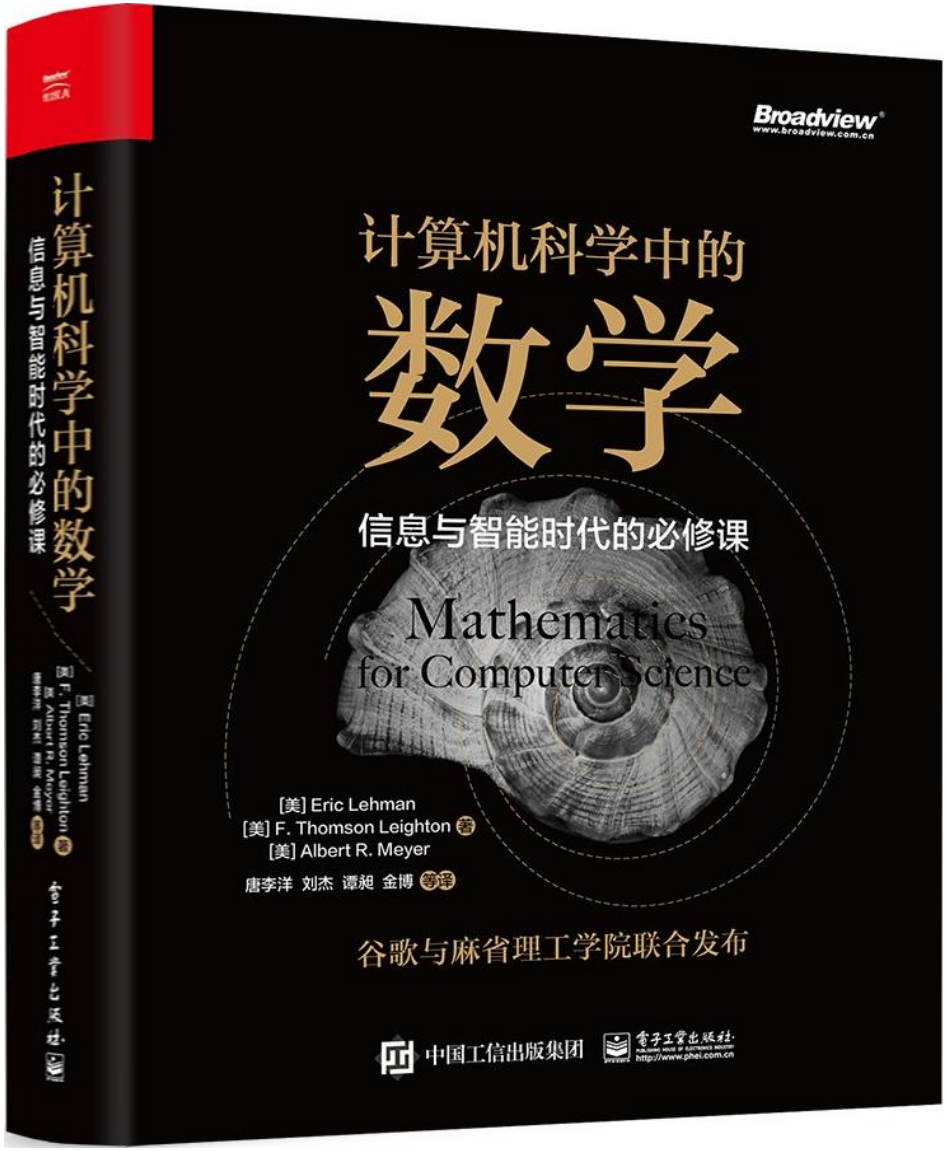


授课内容不局限于教材, 认真听讲很重要

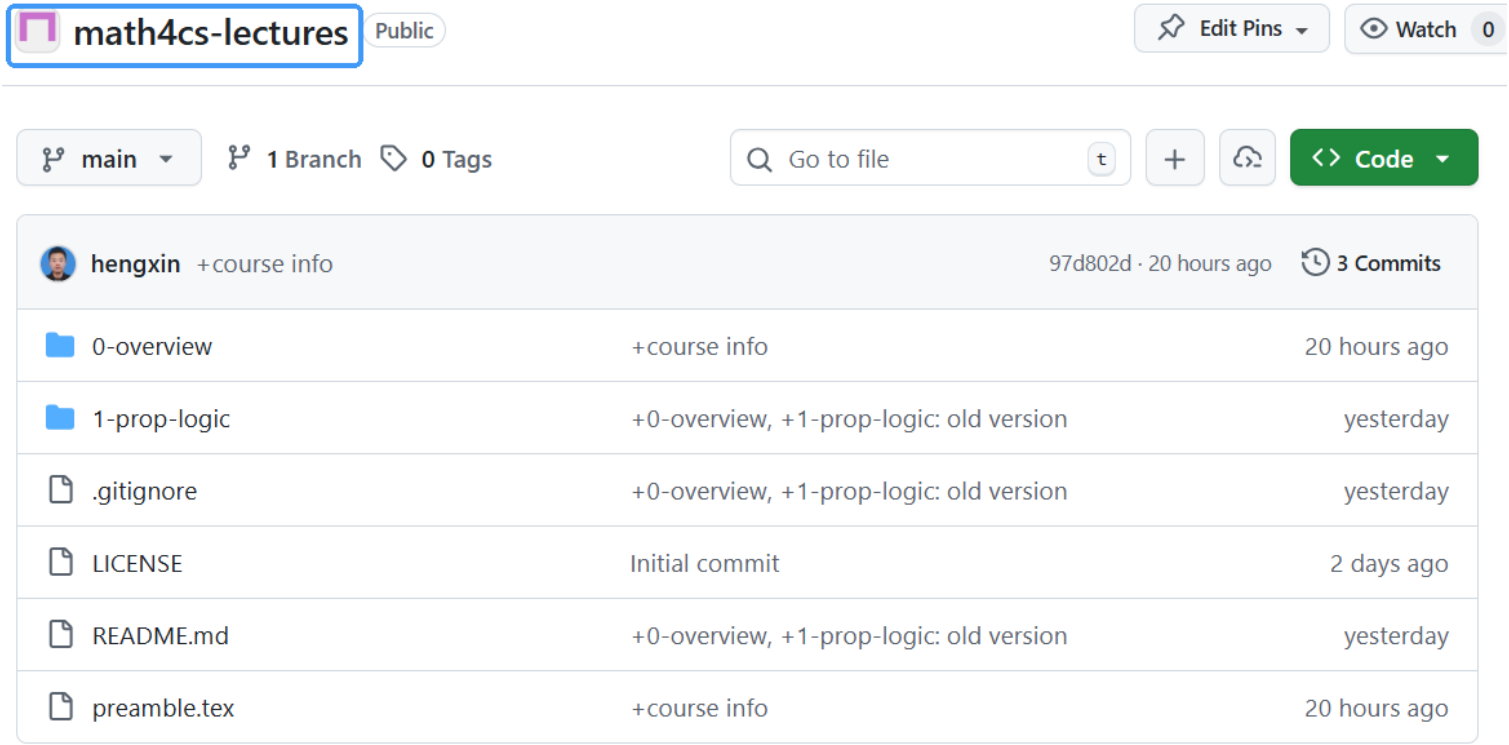




内容与习题偏简单, 略显琐碎; 可用于课前预习及课后基础练习



推荐阅读; 其它参考书随课程进度安排



https://github.com/courses-at-hnu-by-hfwei/math4cs-lectures

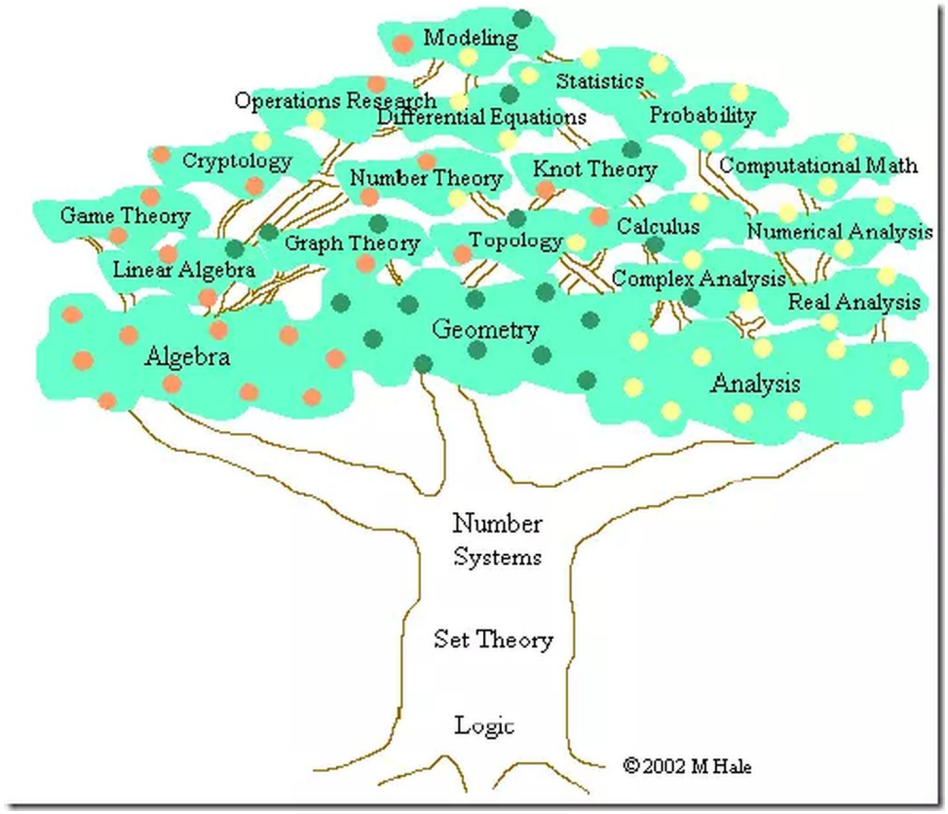
0-overview.pdf 0-overview-**handout**.pdf


welcome
again

计算机（离散）数学

研究**离散对象的结构、性质、操作等**的数学分支 (**大杂烩**)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、（离散）概率

关于计算机 (离散) 数学的古老传说:

我太难了

啥用没有

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

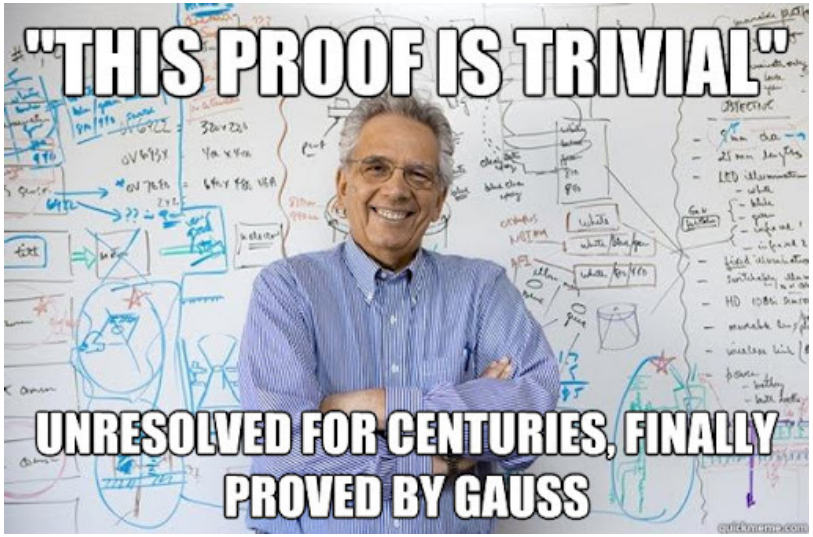
浅尝辄止, 想用但用不上 (群论)

将计算机 (离散) 数学看作一门语言，一套工具

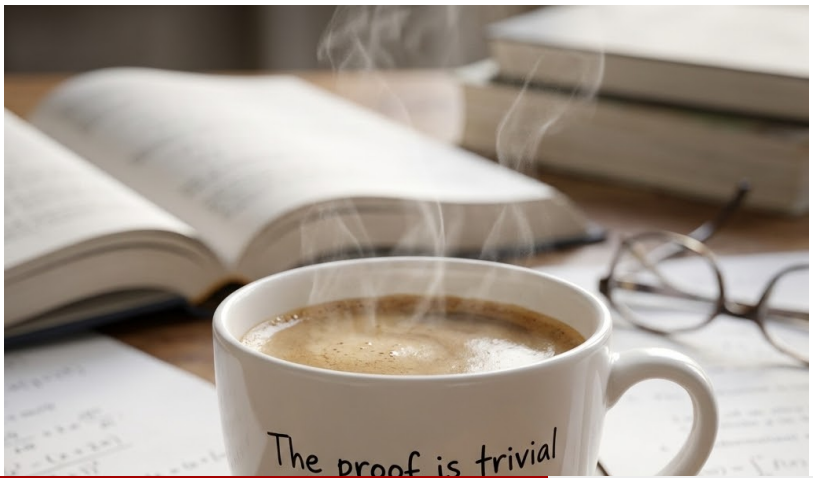
培养形式化描述问题的能力

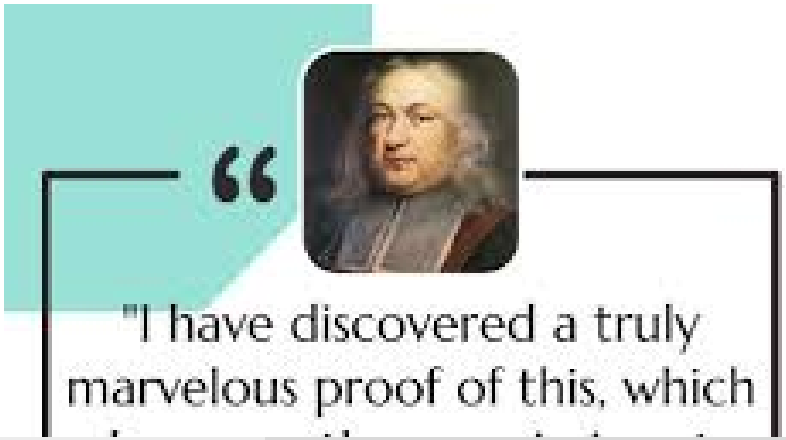
培养做严格证明的能力

本课程最重要的主题: **证明**

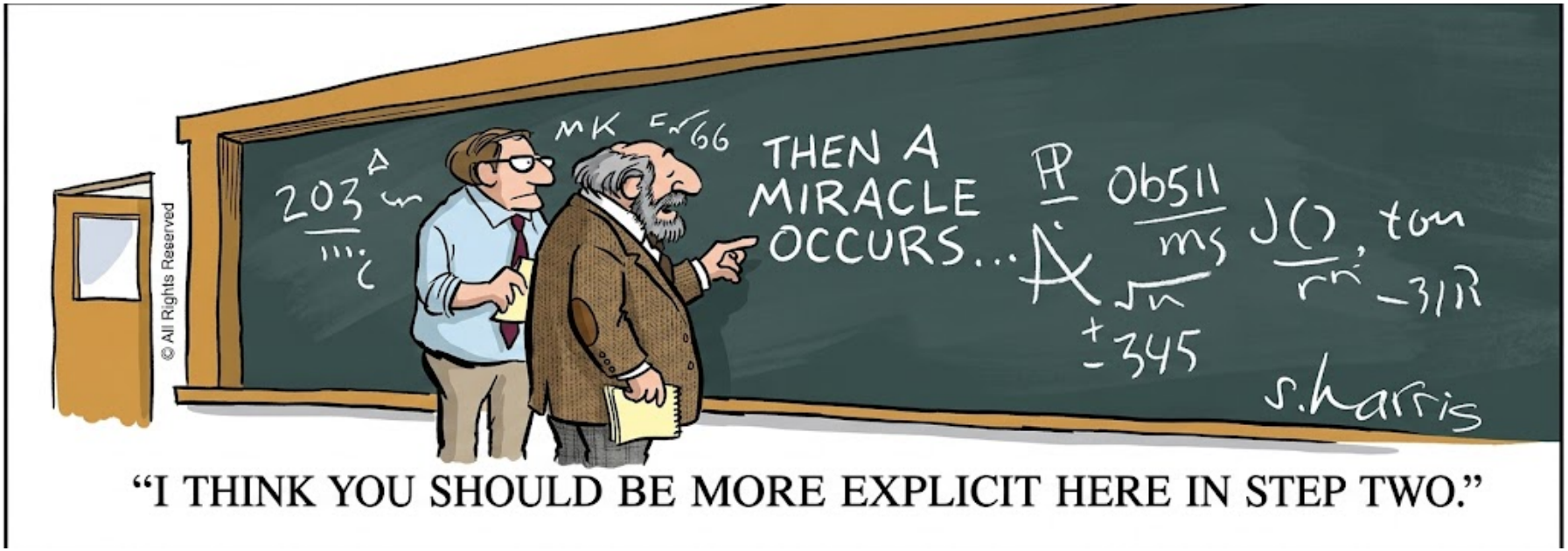

"THIS PROOF IS TRIVIAL"
UNRESOLVED FOR CENTURIES, FINALLY
PROVED BY GAUSS


The proof is left as an
exercise to the reader.


The proof is trivial


"I have discovered a truly
marvelous proof of this, which

凡是“跳步”的证明, 一律默认是错的!!!



Theorem ((第一) 数学归纳法)

设 $P(n)$ 是关于自然数的一个性质。如果

(i) $P(0)$ 成立;

(ii) 对任意自然数 n, 如果 $P(n)$ 成立, 则 $P(n+1)$ 成立。

那么, $P(n)$ 对所有自然数 n 都成立。

Definition (良序原理 (The Well-Ordering Principle))

自然数集的任意非空子集都有一个最小元。

Theorem

良序原理与 (第一) 数学归纳法等价。

Lemma

(第一) 数学归纳法蕴含良序原理。

Proof.

By mathematical induction on the size n of non-empty subsets of $\mathbb{N}$.

$$P(n) : \text{All subsets of size } n \text{ contain a minimum.}$$

Basis Step: $P(1)$

Inductive Hypothesis: $P(n)$

Inductive Step: $P(n) \to P(n+1)$

- $A' \leftarrow A \setminus a$
- $x \leftarrow \min A'$
- Compare x with a

$$\forall n \in \mathbb{N} : P(n) \quad vs. \quad P(\infty)$$

Lemma

(第一) 数学归纳法蕴含良序原理。

By contradiction.

Suppose that there exists$S \neq \emptyset : S$ has no minimum element.

$$A \triangleq \mathbb{N} \setminus S$$

We claim that $A = \mathbb{N}$ by induction, and thus $S = \emptyset$, a contradiction.

Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

$$\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$$

$$\sqrt{2}^{\sqrt{2}} \in \mathbb{Q} \quad \text{or} \quad \sqrt{2}^{\sqrt{2}} \in \mathbb{R} \setminus \mathbb{Q}$$

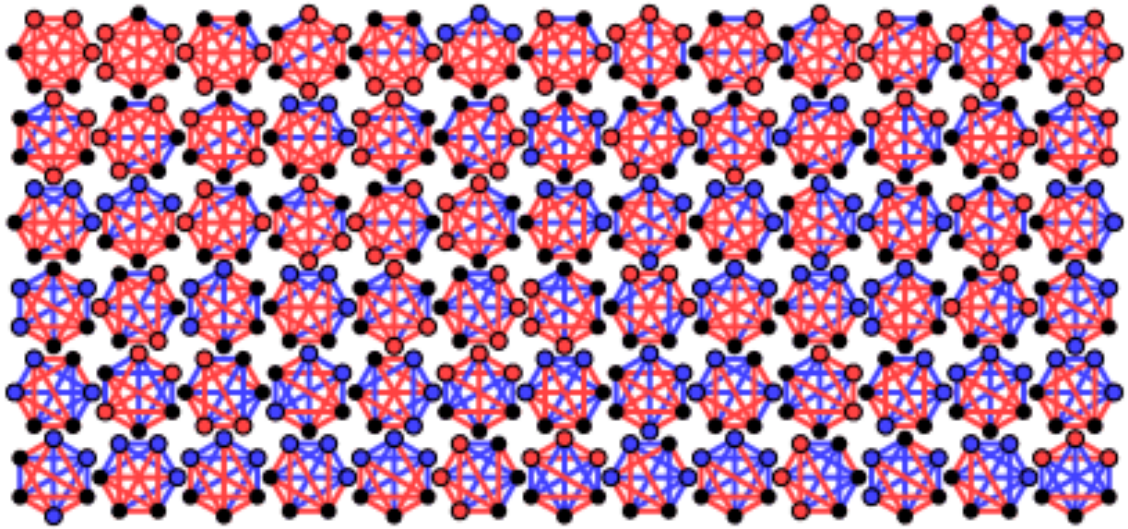
$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} \cdot \sqrt{2}} = \sqrt{2}^2 = 2 \in \mathbb{Q}$$

Q：这是构造性证明吗？这是反证法吗？

$$a = \sqrt{2} \qquad b = 2 \log_2 3 \qquad (a^b = 3)$$

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.



In Terms of **Graph Theory**.

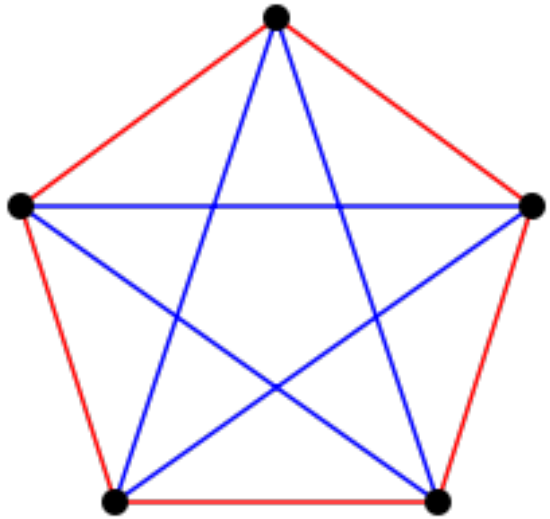
(Is there a monochromatic triangle in any 2-coloring of K_6?)

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_6?

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_5?

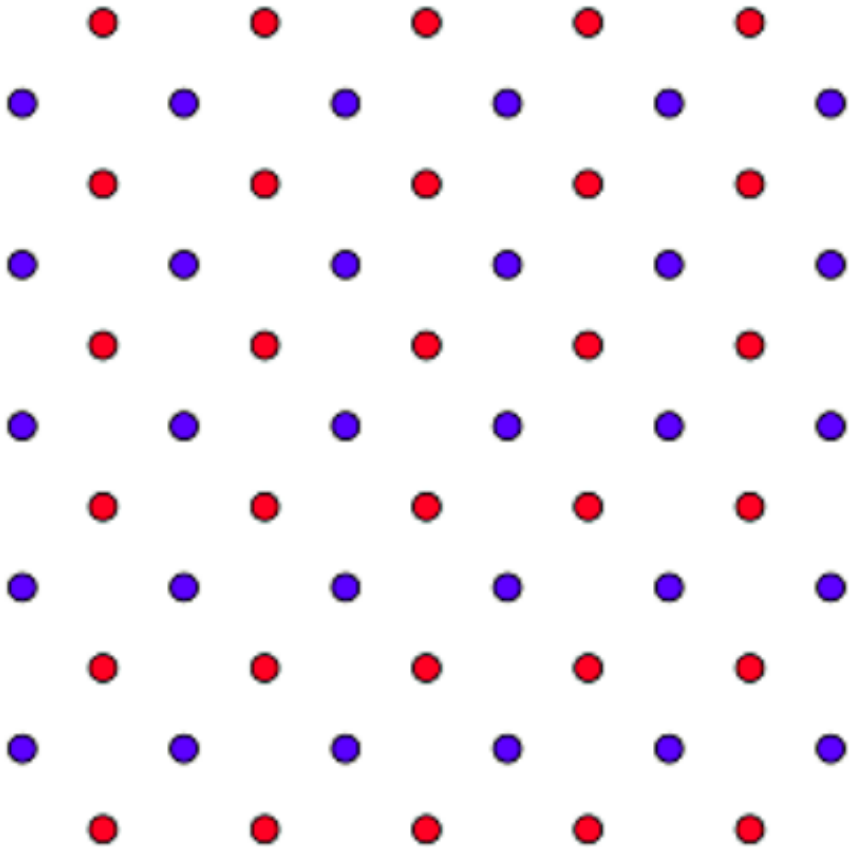


Ramsey theory

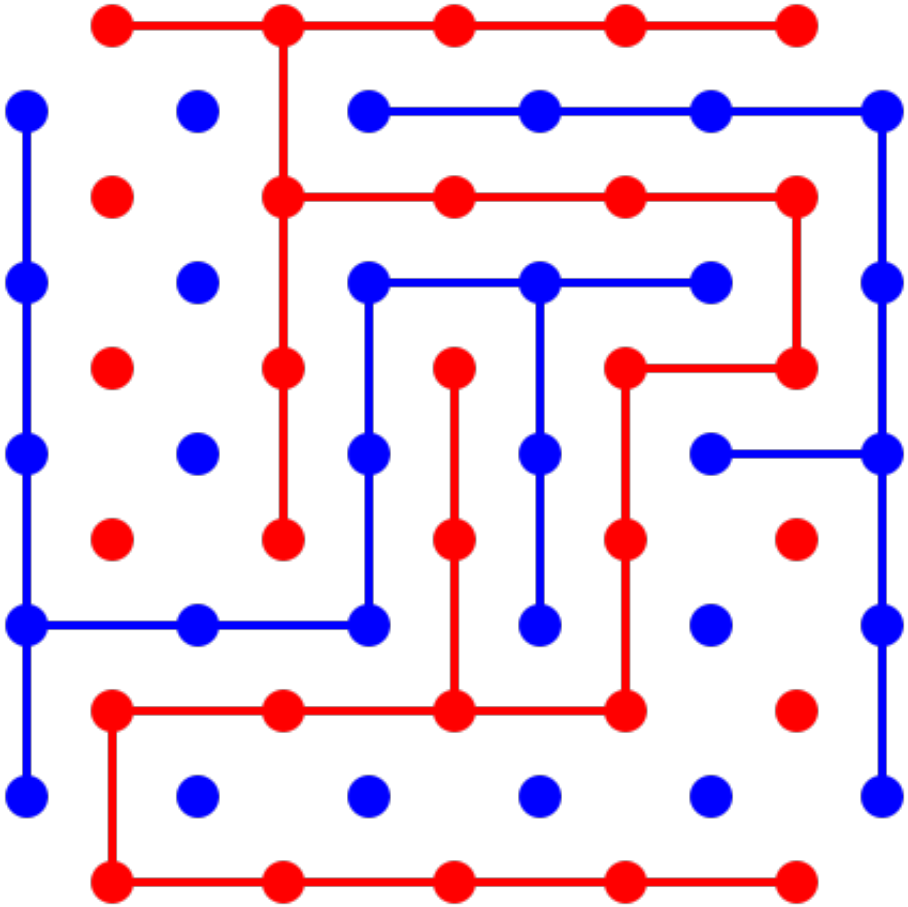
How **big** must the **structure** be

to ensure that it has a given interesting **property**?

Bridg-It Game (David Gale, 1958)



5×6 *vs.* 6×5



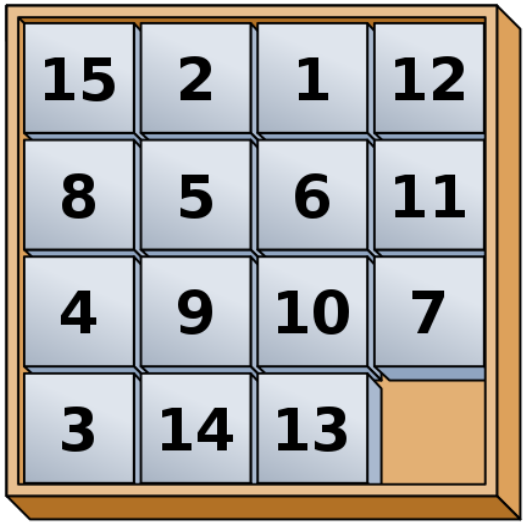

Ask
the right
questions

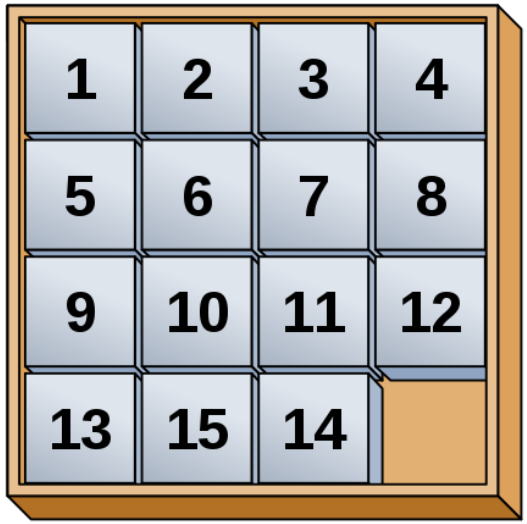
Does **Player 1** have a **winning strategy**?

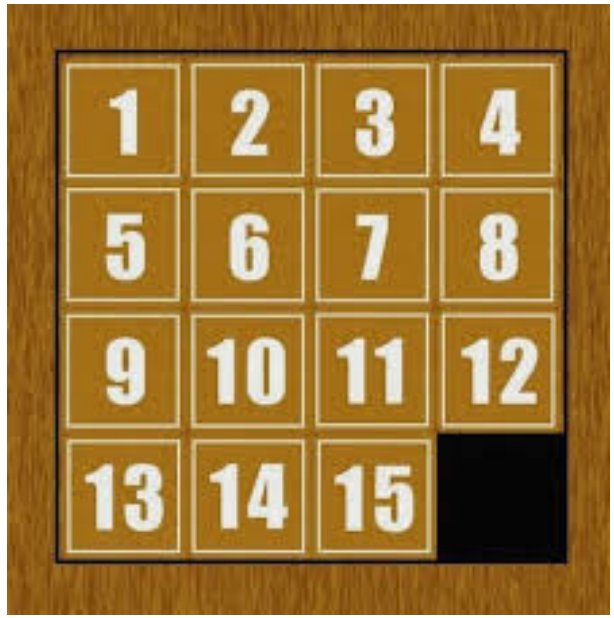
(先手是否有必胜策略?)

Yes! It uses **spanning trees** in **graph theory**.

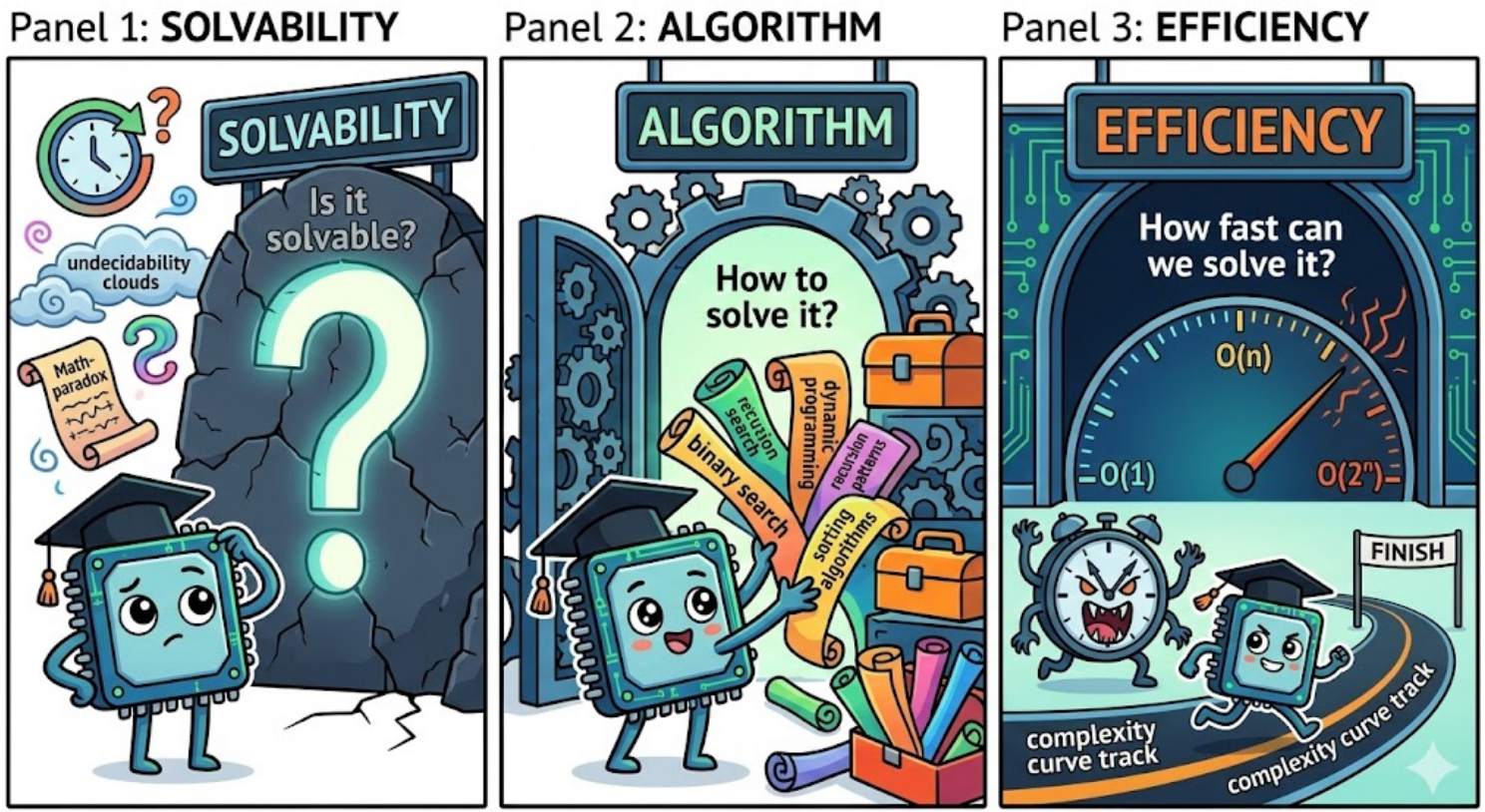
15 Puzzle (数字华容道)







Is it solvable? How to solve it? How fast can we solve it?



计算机科学的“灵魂三问：可解性、算法设计、算法效率

It uses **permutation groups** in **group theory**.

A Modern Treatment of the 15 Puzzle

Aaron F. Archer

1. INTRODUCTION. In the 1870's the impish puzzlemaker Sam Loyd caused quite a stir in the United States, Britain, and Europe with his now-famous 15-puzzle. In its original form, the puzzle consists of fifteen square blocks numbered 1 through 15 but otherwise identical and a square tray large enough to accommodate 16 blocks. The 15 blocks are placed in the tray as shown in Figure 1, with the lower right corner left empty. A legal move consists of sliding a block adjacent to the empty space into the empty space. Thus, from the starting placement, block 12 or 15 may be slid into the empty space. The object of the puzzle is to use a sequence of legal moves to switch the positions of blocks 14 and 15 while returning all other blocks to their original positions.

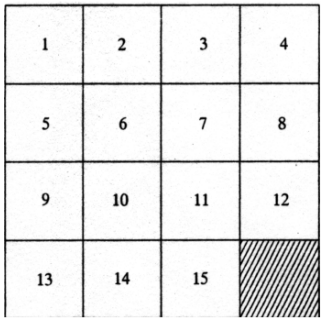


Figure 1. The starting position for the 15-puzzle. The shaded square is left empty.

Why is this 15-Puzzle Impossible? - Numberphile

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a stable marriage.

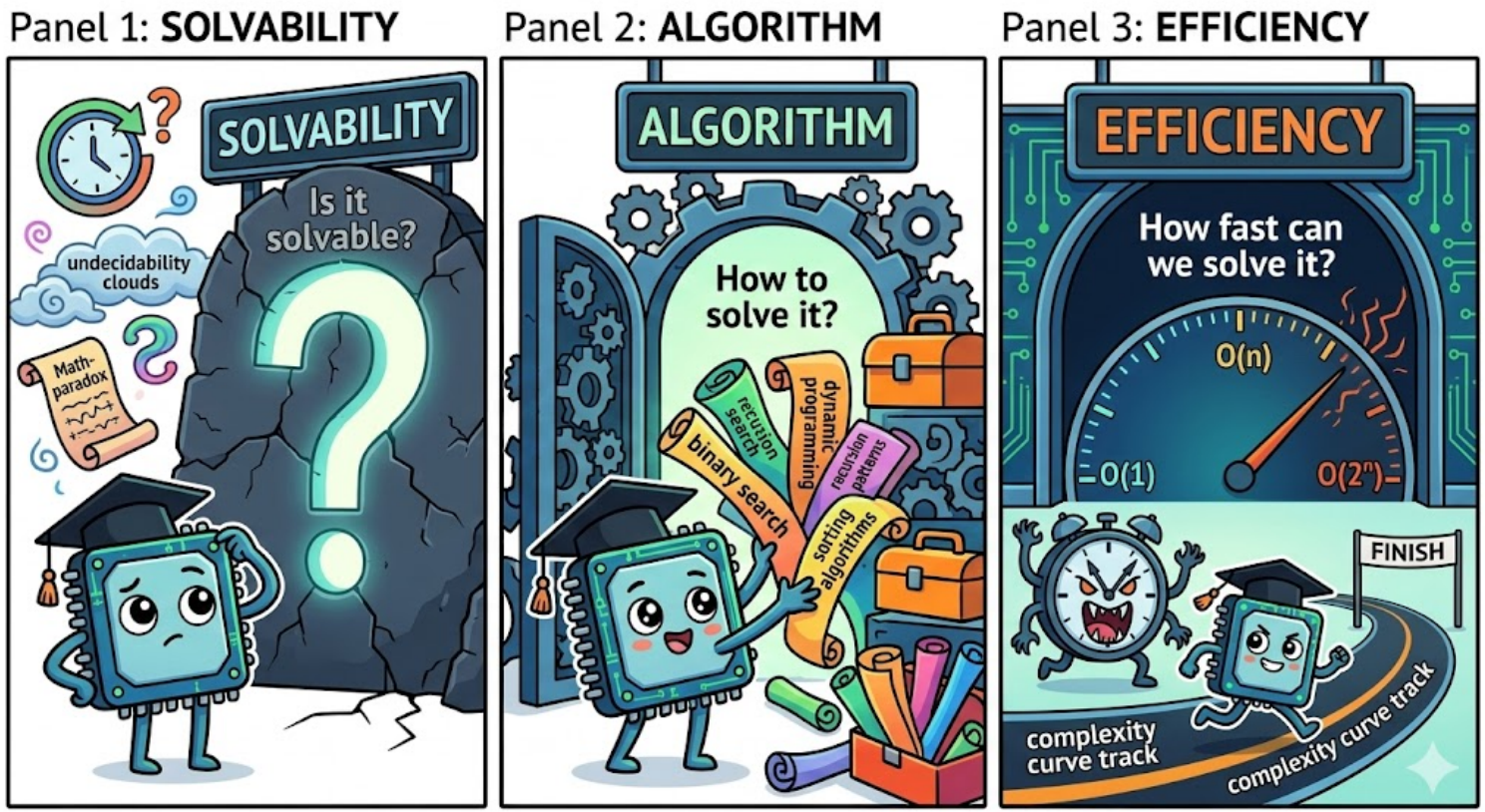
Men $\{x, y, z, w\}$	**Women** $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

$\{xb, yc, zd, wa\}$ is an unstable matching

(x, a) is a (dangerous) unstable unmatched pair

Definition (Stable Matching)

A matching is stable when there does *not* exist any pair (A, B) which both prefer each other to their current partner under the matching.



计算机科学的“灵魂三问：可解性、算法设计、算法效率

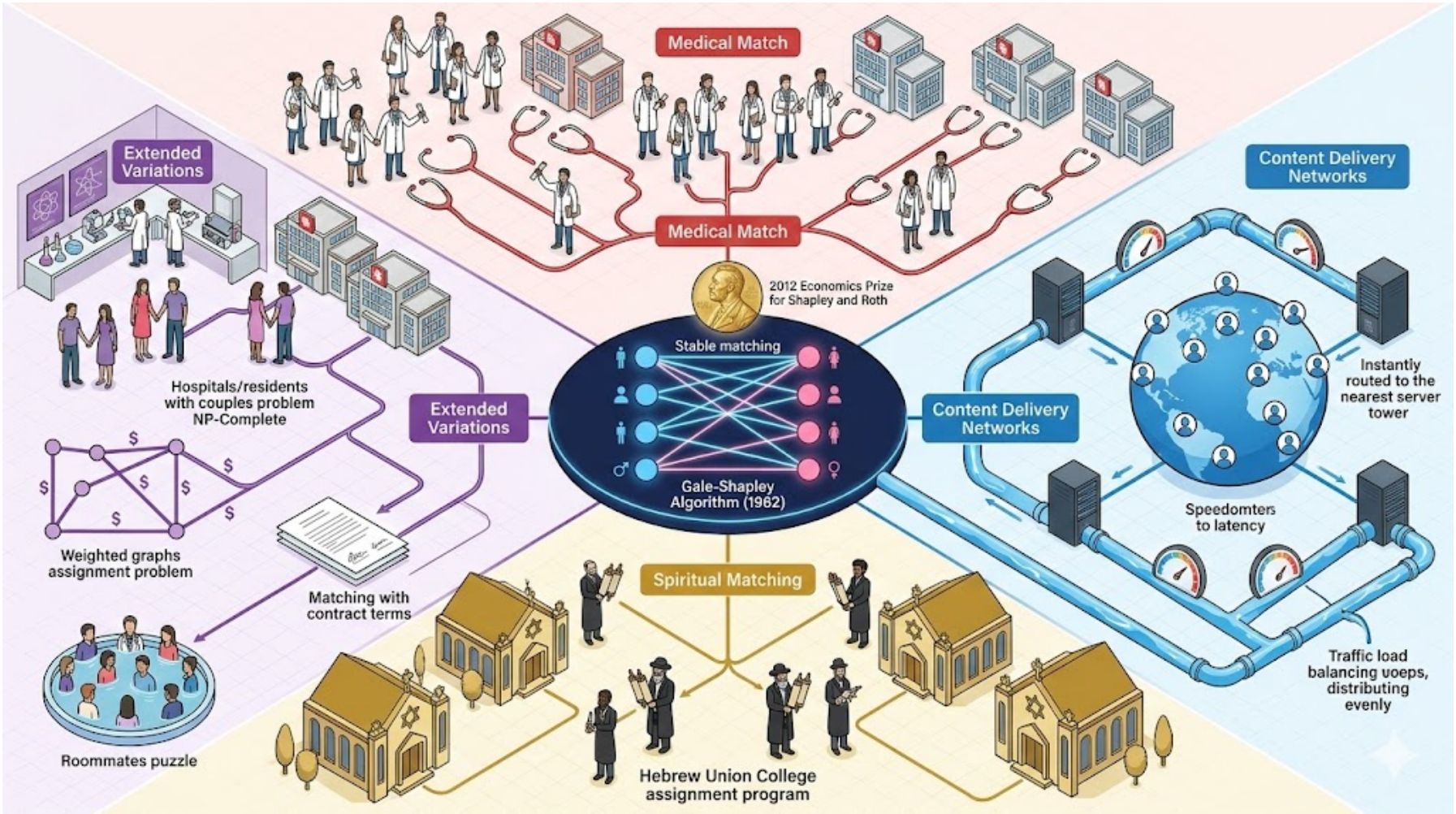
$$\{xa, yb, zd, wc\}$$

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

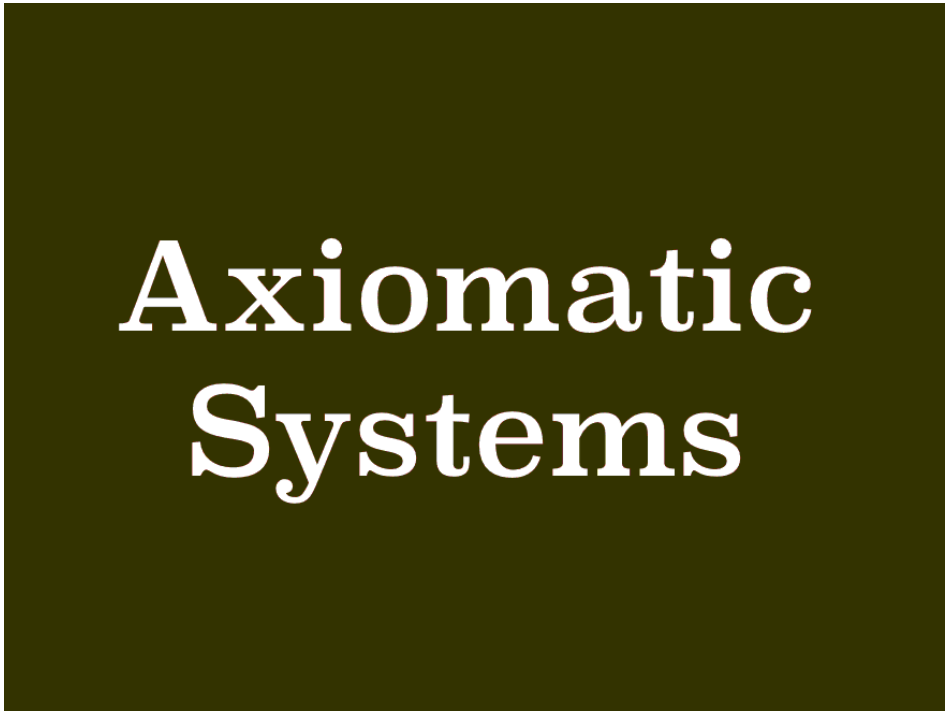
Theorem (The Gale-Shapley Algorithm (1962))

It is always possible to solve SMP*: Men propose + Women accept/reject.*

Applications and Extensions



Self-contained (自包含; 自给自足)



Syntax *vs.* Semantics (语法与语义对立统一)

Peano 公理体系刻画了**自然数的递归结构**

Definition (Peano Axioms)

(1) 0 是自然数;

(2) 如果 n 是自然数, 则它的后继 $\mathbf{S}n$ 也是自然数;

(3) 0 不是任何自然数的后继;

(4) 两个自然数相等当且仅当它们的后继相等;

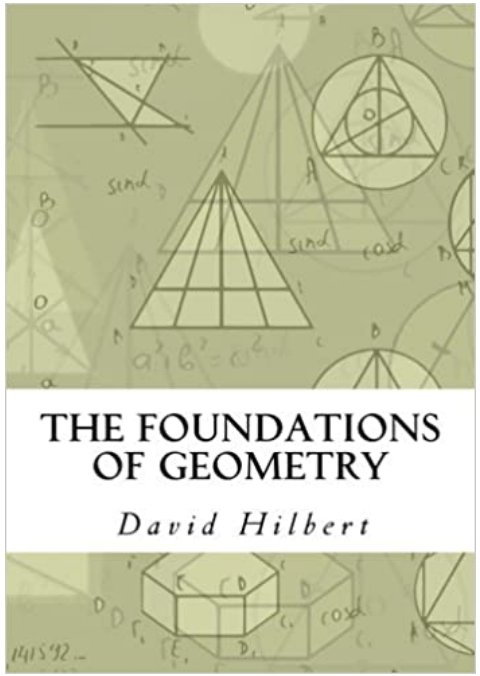
(5) **数学归纳原理:** 如果

(i) $P(0)$ 成立;

(ii) 对任意自然数 n, 如果 $P(n)$ 成立, 则 $P(n+1)$ 成立。

那么, $P(n)$ 对所有自然数 n 都成立。

三个公理系统：逻辑、集合论、图论、抽象代数 (群论)



(1) To draw a straight line from any point to any point.

(2) To extend a finite straight line continuously in a straight line.

(3) To describe a circle with any center and radius.

(4) That all right angles are equal to one another.

(5) The parallel postulate.

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

Axioms:

(1) There are exactly four points.

(2) It is impossible for three points to be on the same line.

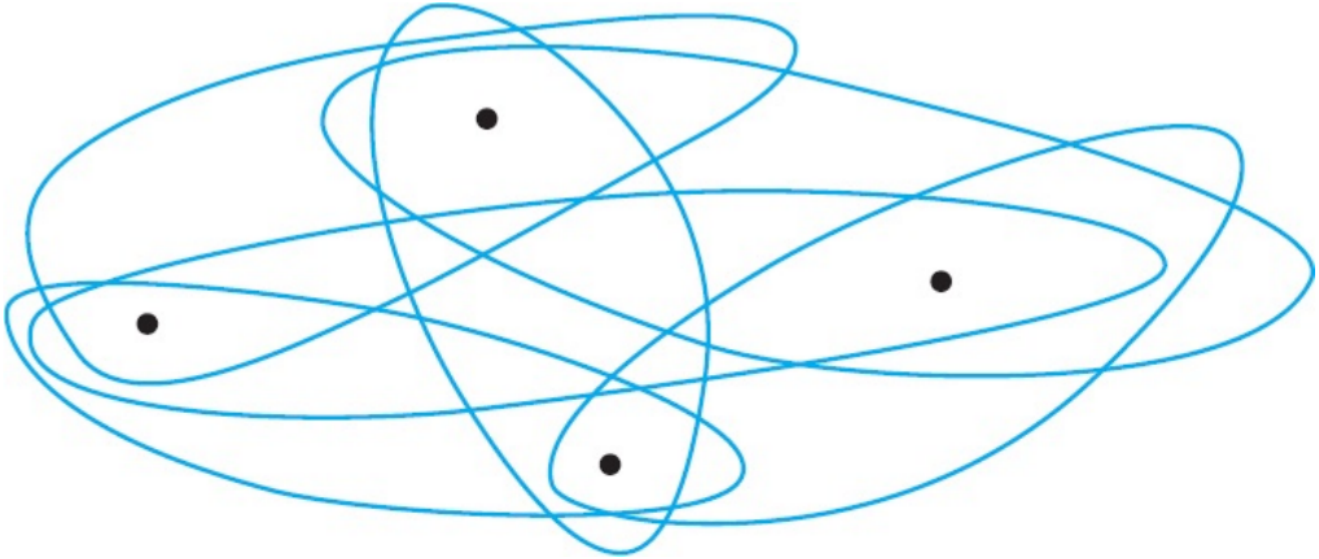
(3) For every pair of distinct points x and y, there is a unique line l such that x is on l and y is on l.

(4) Given a line l and a point x that is not on l, there is a unique line m such that x is on m and no point on l is also on m.

Theorem

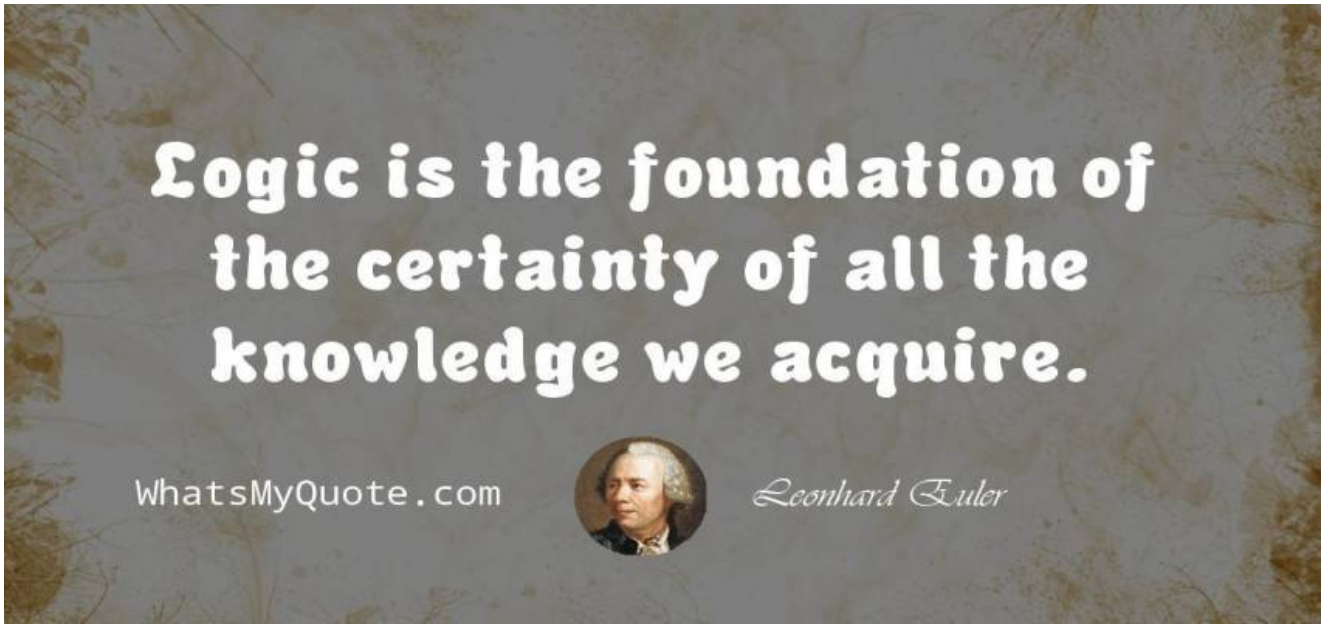
There are at least two distinct lines.

Syntax *vs.* Semantics



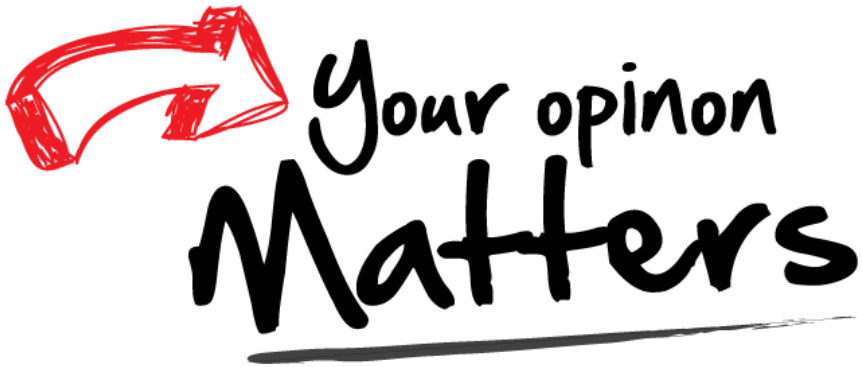
point : · line : ◯ is on : ⊙

什么样的推理是正确的?


Logic is the foundation of the certainty of all the knowledge we acquire.
WhatsMyQuote.com
Leonhard Euler


SEE YOU
NEXT WEEK


Thank
You!



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