

计算机数学概述

魏恒峰

hfwei@hnu.edu.cn

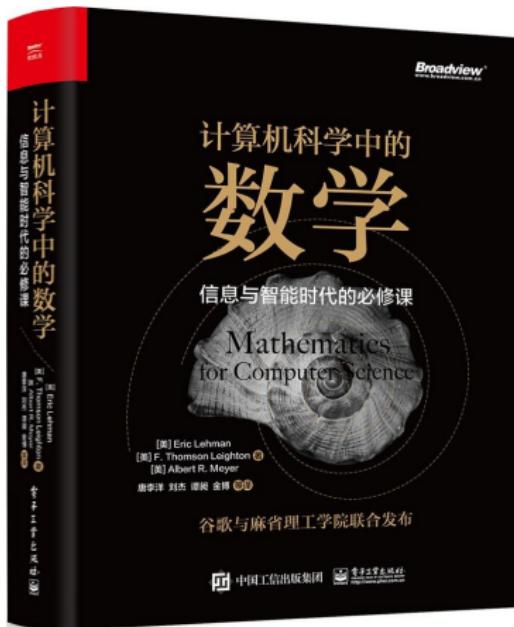
2026 年 03 月 02 日



什么是“计算机数学”？

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“Mathematics for Computer Science”
(math4cs)



What does “Computer Science” study?

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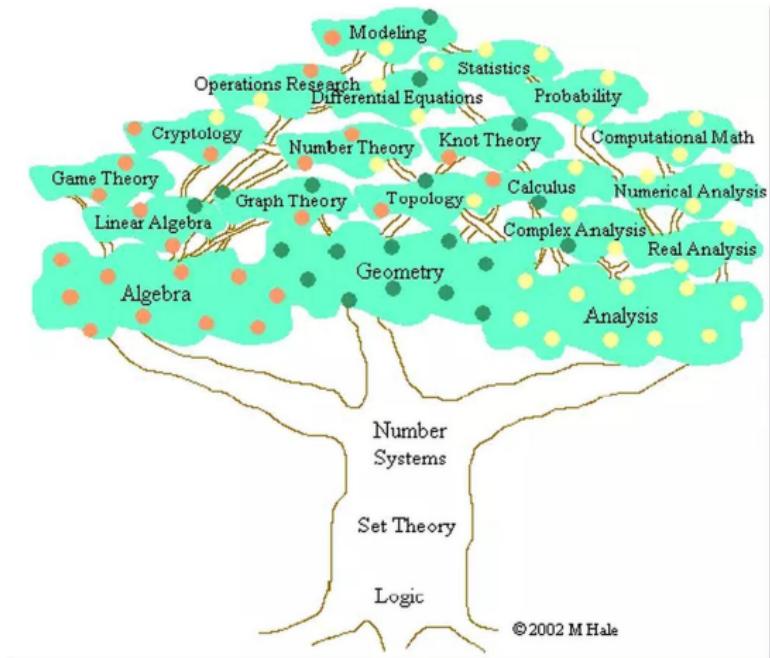
Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.

What does “Computer Science” study?

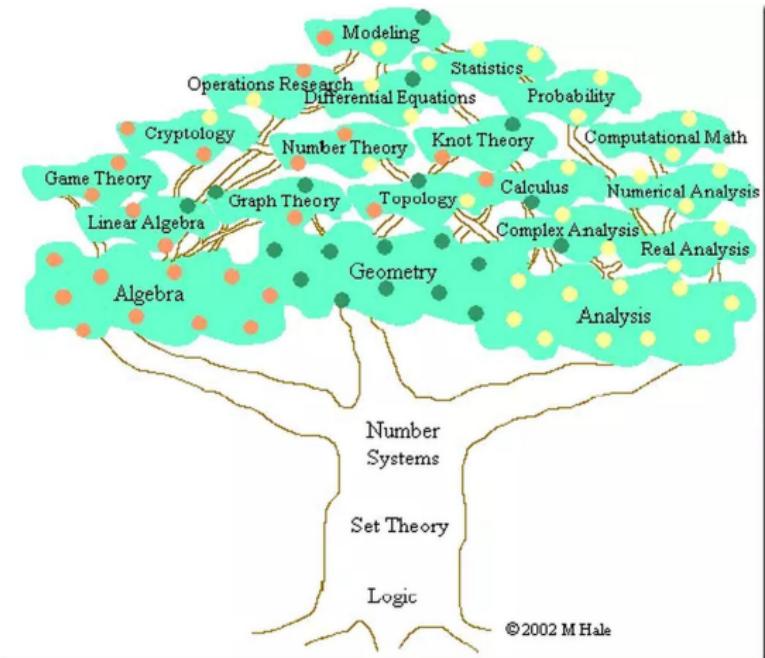
Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.

math4cs is **model-and-proof** oriented.

“计算机数学”在哪里?



“计算机数学”在哪里?



计算机数学是个大杂烩，啥都学点儿，啥都没学好

分班教学 (共 xxx 班)

授课内容与作业可能有出入, 不影响考试与成绩分配

平时作业 *vs.* 期中测试 *vs.* 期末测试

3 : 3 : 4

4 : 3 : 3

弹性制

每周二、周四下午 14:00 发布作业 下周四晚 22:00 前提交作业

每次作业按 10 分计算

迟交: 周四前向助教登记, 可延长两天, 总分 8 分

(助教: 、)

“教学立方”课程邀请码: PLD8QKTZ



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TEX

math4cs-problem-sets Public

main 1 Branch 0 Tags Go to file + ⌂ Code

hengwei · 2026/math4cs-hw0-overview/ 40b517 · 20 minutes ago 2 Commits

2026/math4cs-hw0-overview · +2026/math4cs-hw0-overview/ 20 minutes ago

.gitignore · +2026/math4cs-hw0-overview/ 20 minutes ago

LICENSE · Initial commit 32 minutes ago

README.md · Initial commit 32 minutes ago

README MIT license

math4cs-problem-sets

Problem Set for math4cs (Mathematics for Computer Science; <https://github.com/courses-at-hnu-by-hfwei/math4cs>) at Hunan University by Hengfeng Wei

<https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets>

约法三章

非必要，不点名

非必要，不迟到

非必要，不迟到

尽量吃早餐，但不可以在教室吃早餐

~~非必要~~, 不抄袭; 一经发现, 后果严重

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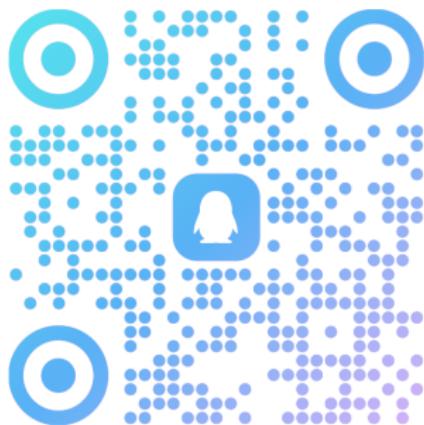
当次作业计 0 分; 总评扣 10 分

QQ 群号: 108 745 6358

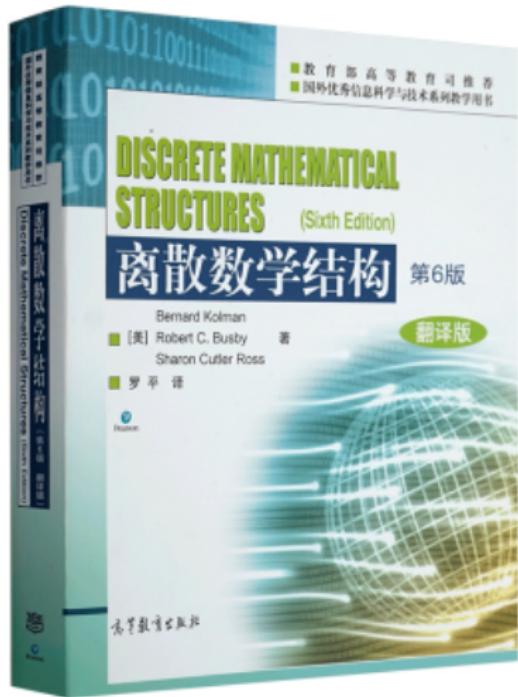


2026-计算机数学-拔...

群号: 1087456358



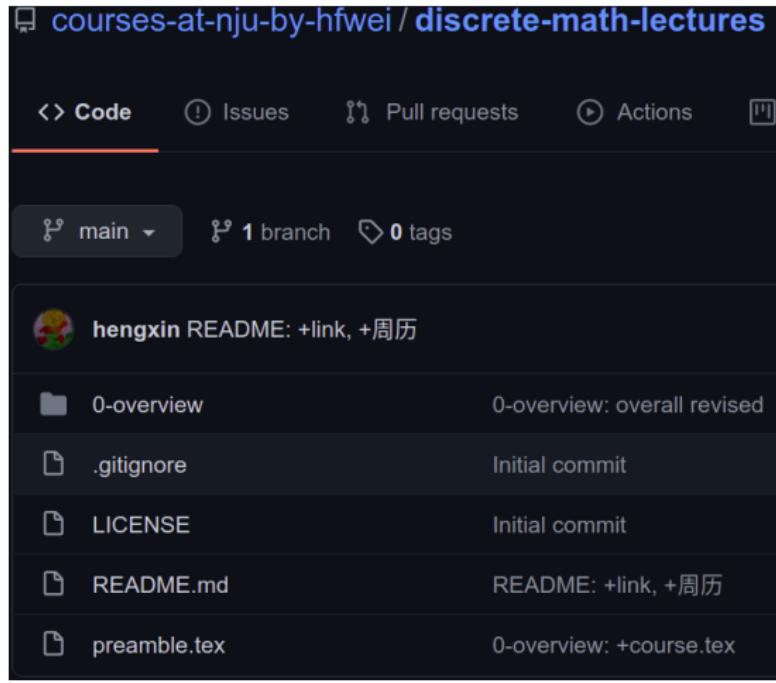
扫一扫二维码, 加入群聊



教材不重要，听讲更重要



其它参考书随课程进度安排

 courses-at-nju-by-hfwei / discrete-math-lectures

Code Issues Pull requests Actions

main ▾ 1 branch 0 tags

 hengxin README: +link, +周历

	0-overview
	.gitignore
	LICENSE
	README.md
	preamble.tex
	0-overview: overall revised
	Initial commit
	Initial commit
	README: +link, +周历
	0-overview: +course.tex

https:
[//github.com/courses-at-nju-by-hfwei/discrete-math-lectures](https://github.com/courses-at-nju-by-hfwei/discrete-math-lectures)

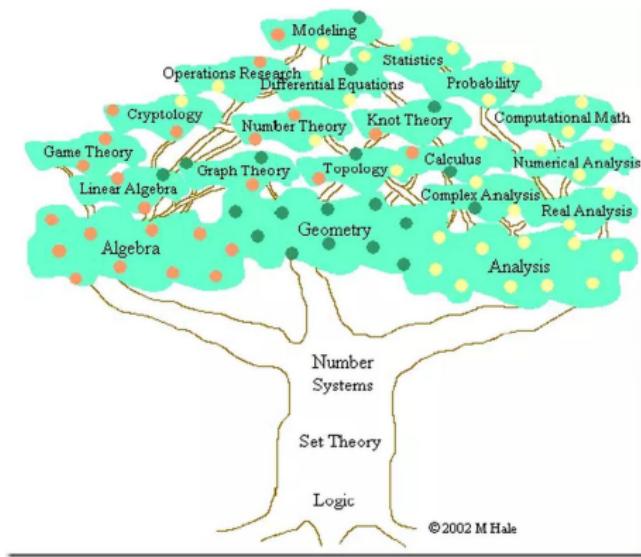


离散数学

离散数学

研究**离散对象的结构、性质、操作等**的数学分支 (**大杂烩**)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、（离散）概率

关于离散数学，学长纷纷表示：

我太难了

啥用没有

真得有那么难吗？

真得有那么难吗？

确实蛮难的：知识点多而分散、概念抽象

真得没啥用吗？

真得没啥用吗?

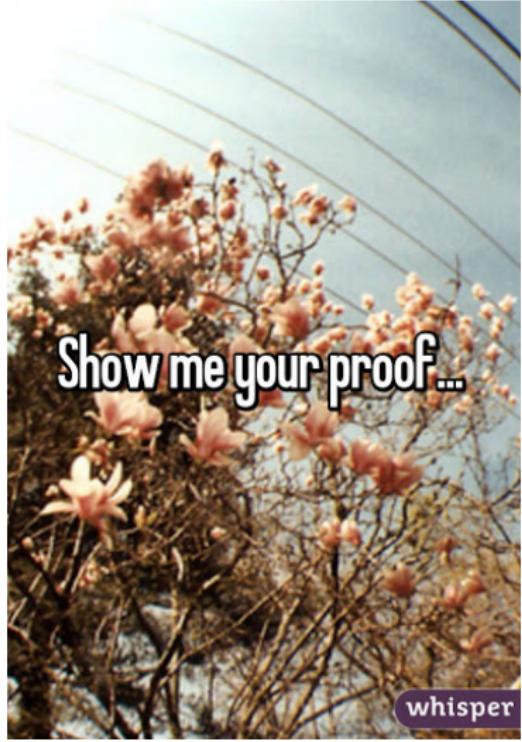
太基础, 用了但不自觉 ([逻辑](#))

浅尝辄止, 想用但用不上 ([群论](#))

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力



Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

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Q : 这是构造性证明吗?

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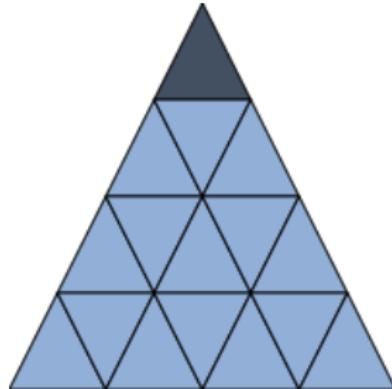
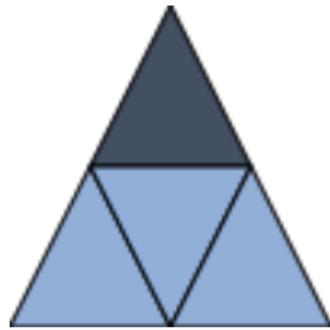
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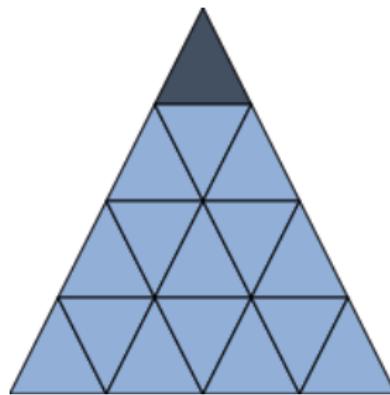
Q : 这是构造性证明吗？这是反证法吗？

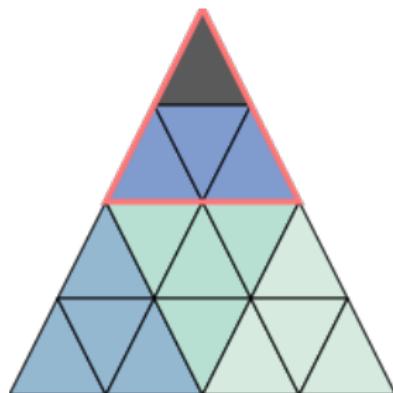
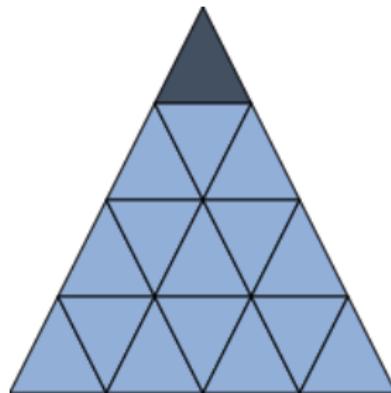
Tiling Puzzle

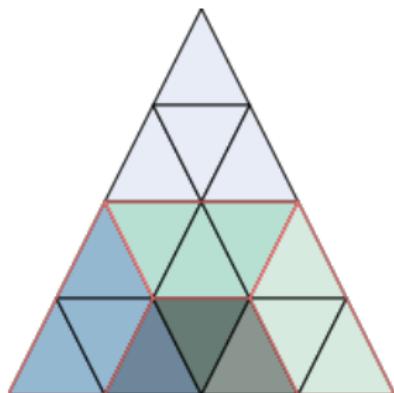
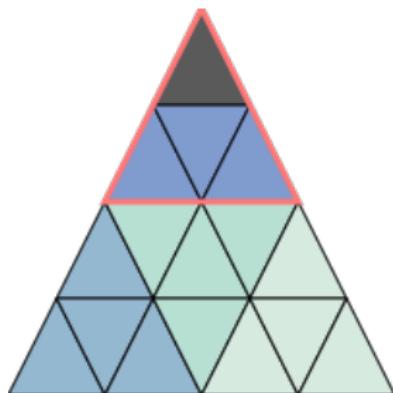
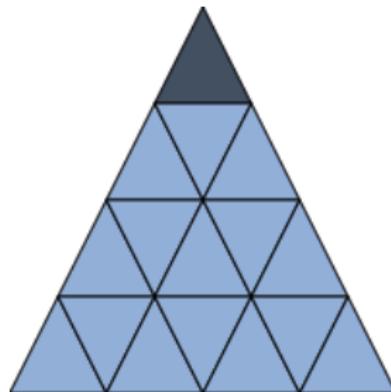
Suppose n is a positive integer. An equilateral triangle is cut into 4^n congruent equilateral triangles, and one corner is removed.

Show that the remaining area can be **covered** by tiles below









Base Case:

Induction Hypothesis:

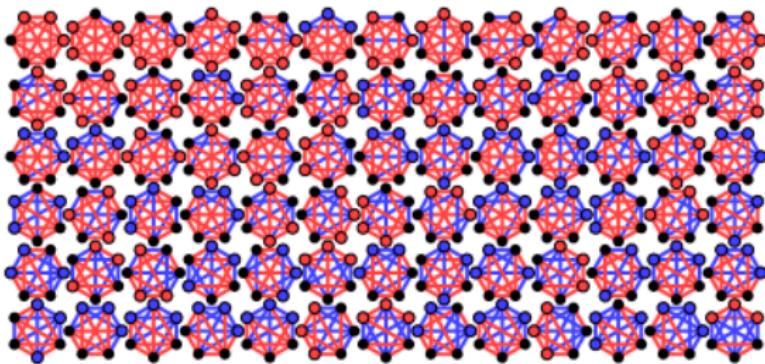
Induction Step: ... by induction hypothesis ...

Theorem on Friends and Strangers

At any party with at least **6** people, there are **3** people who are all either **mutual acquaintances** or **mutual strangers**.

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In Terms of **Graph Theory**.

(Is there a **monochromatic** triangle in any 2-coloring of K_6 ?)

Theorem on Friends and Strangers

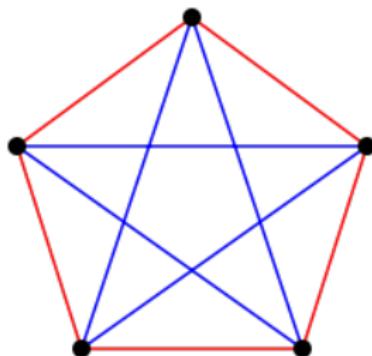
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Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?



Ramsey theory

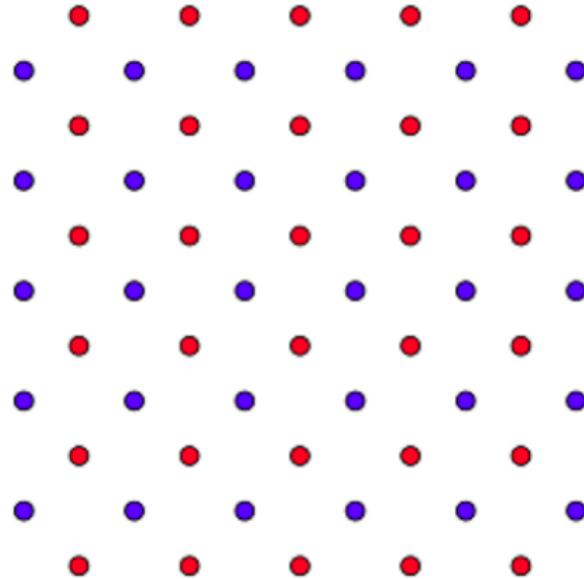
How **big** must the **structure** be
to ensure that it has a given interesting **property**?

Ramsey theory

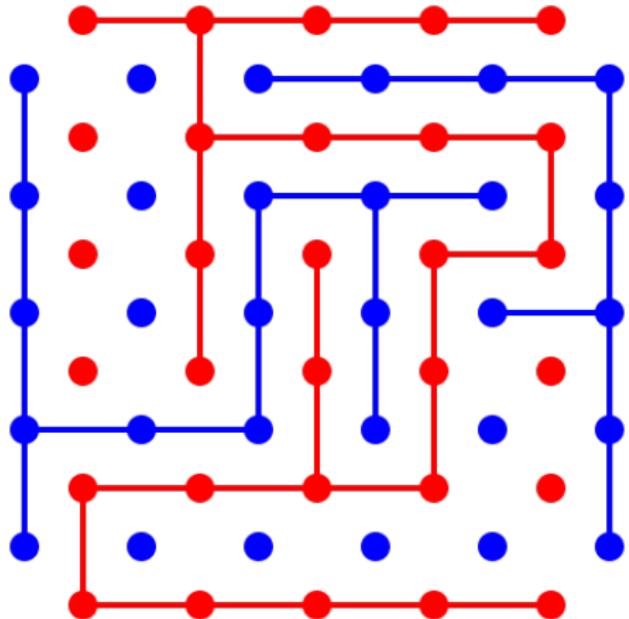
How **big** must the **structure** be
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Bridg-It Game (David Gale, 1958)



5×6 vs. 6×5



5×6 vs. 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie?**

Will Bridg-It **end in a tie?**

No! By **contradiction.**

Does **Player 2** have a **winning strategy**?

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

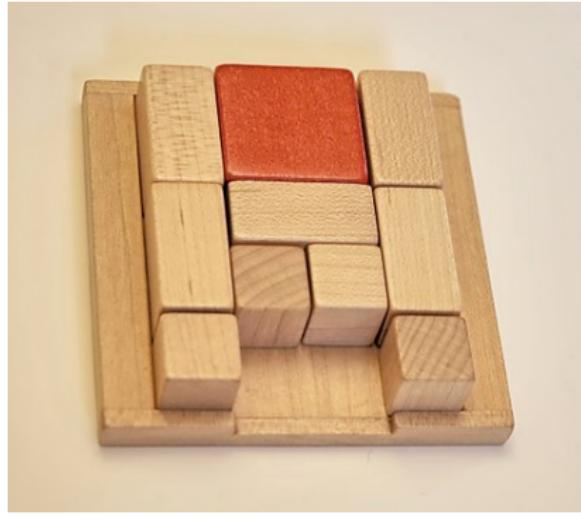
Does **Player 1** have a **winning strategy**?

Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.

STAY TUNED

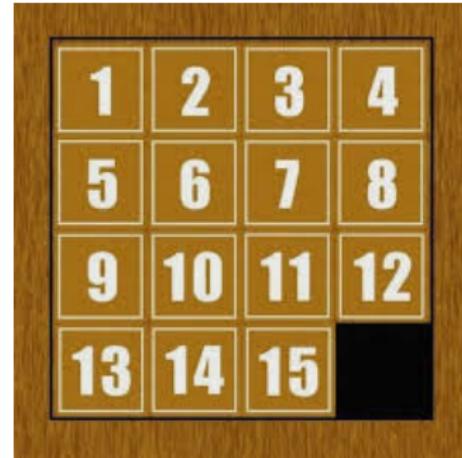
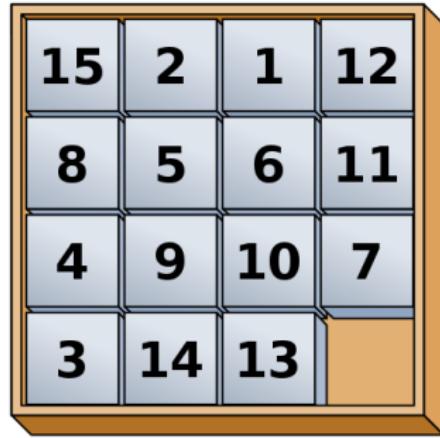
Klotski Puzzle (华容道)



Klotski Puzzle (华容道; 中国版本)



15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

How to solve it?

It uses **permutation groups** in **group theory**.



STAY TUNED

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a **stable** marriage.

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
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$\{xb, yc, zd, wa\}$

(x, a) is an unstable pair

$$\{xa, yb, zd, wc\}$$

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Theorem (The Gale-Shapley Algorithm (1962))

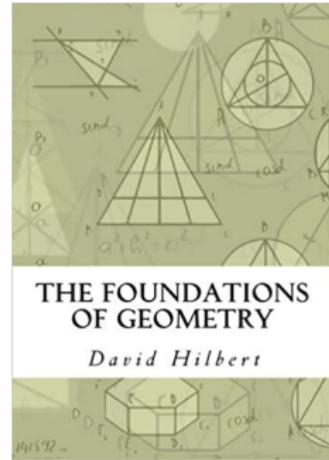
It is always possible to solve SMP.

Self-contained (自包含; 自给自足)

Axiomatic Systems

Syntax *vs.* Semantics (语法与语义对立统一)

三个公理系统：逻辑、集合论、图论、抽象代数（群论）



- (1) To draw a straight **line** from any **point** to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) **The parallel postulate.**

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

Axioms:

- (1) There are exactly four points.
- (2) It is impossible for three points to be on the same line.
- (3) For every pair of distinct points x and y , there is a unique line l such that x is on l and y is on l .
- (4) Given a line l and a point x that is not on l , there is a unique line m such that x is on m and no point on l is also on m .

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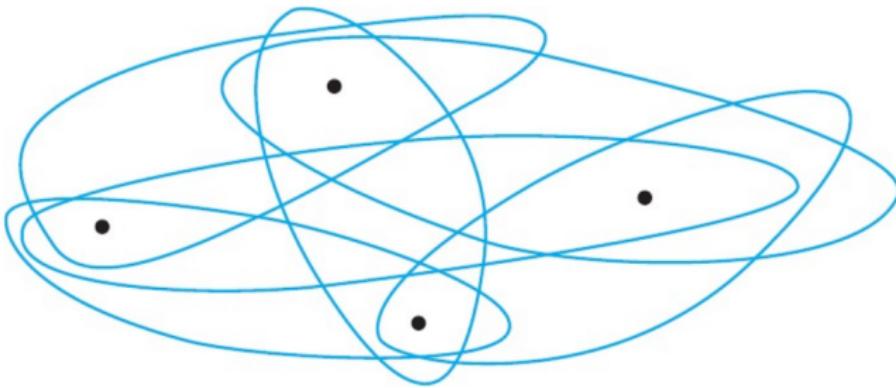
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Theorem

There are at least two distinct lines.

Syntax *vs.* Semantics

Syntax *vs.* Semantics

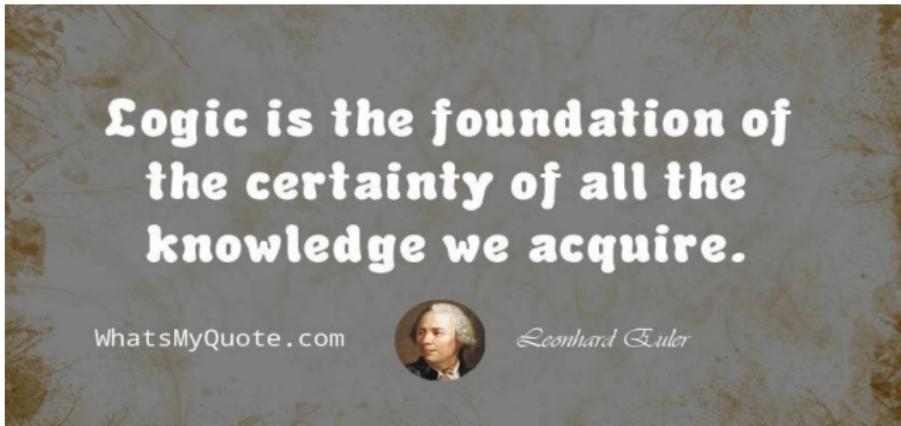


point : •

line : ○

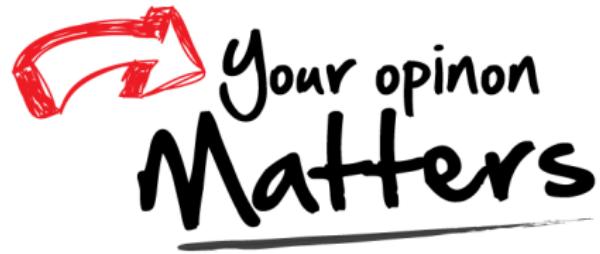
is on : ○•

什么样的推理是正确的？





Thank You!



Office 926

hfwei@nju.edu.cn