

# 计算机数学概述

魏恒峰

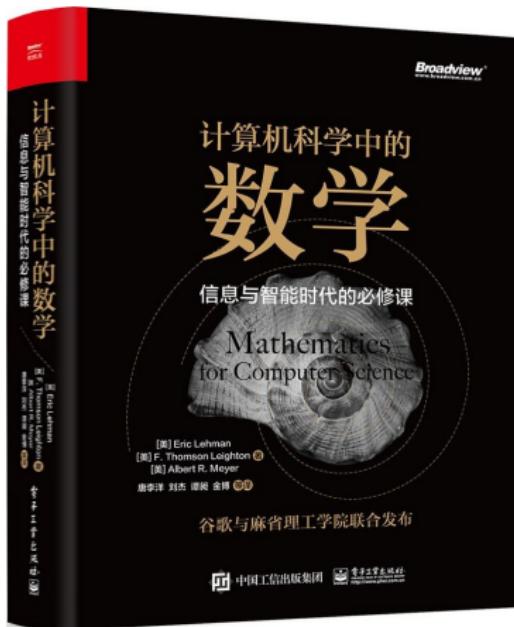
hfwei@hnu.edu.cn

2026 年 03 月 03 日



# 什么是“计算机数学”？

“Mathematics for Computer Science”  
(math4cs)

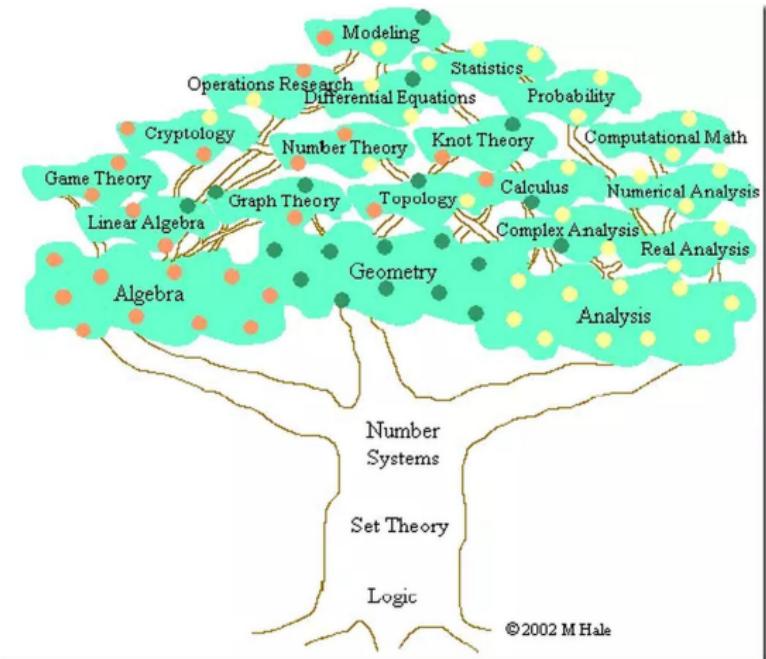


## What does “Computer Science” study?

*Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.*

math4cs is **model-and-proof** oriented.

# “计算机数学”在哪里?



计算机数学是个大杂烩，啥都学点儿，~~啥都没学好~~

# 分班教学 (共 9 个班级)

拔尖班: 独立授课, 独立考核



平时作业 *vs.* 课堂测验 *vs.* 期中测试 *vs.* 期末测试

1 : 3 : 2 : 4

每周四下午 14:00 发布平时作业

下下周四晚 22:00 前提交作业

取最高的 **12** 次作业成绩计人总评

The only way  
to learn  
**mathematics**  
is to do  
**mathematics.**

PAUL HALMOS

请按时提交, 过时不补

(助教: 罗熙辰)

# “教学立方”课程邀请码: PLD8QKTZ



TEX

math4cs-problem-sets Public

main 1 Branch 0 Tags Go to file + ⌂ Code

hengxian · 2026/math4cs-hw0-overview/ 40b517 · 20 minutes ago 2 Commits

2026/math4cs-hw0-overview · 2026/math4cs-hw0-overview/ · 20 minutes ago  
.gitignore · .gitignore · 20 minutes ago  
LICENSE · LICENSE · Initial commit · 32 minutes ago  
README.md · README.md · Initial commit · 32 minutes ago

README MIT license

**math4cs-problem-sets**

Problem Set for math4cs (Mathematics for Computer Science; <https://github.com/courses-at-hnu-by-hfwei/math4cs>) at Hunan University by Hengfeng Wei

<https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets>

约每两周一次课堂测验 (提前通知时间与测验范围)

取最高的 **6** 次课堂测验成绩计入总评



请准时参加, 不安排补考

# 约法三章

非必要，不点名

**非必要，不迟到**

**尽量吃早餐，但不可以在教室吃早餐**

~~非必要~~, 不抄袭; 一经发现, 后果严重

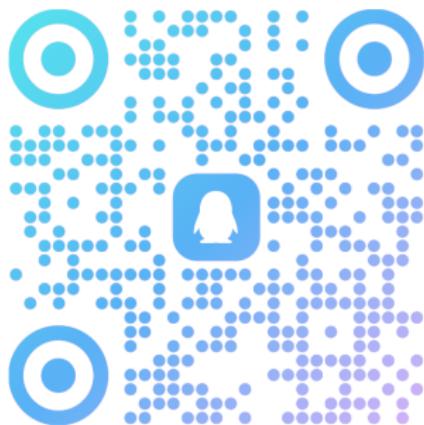
~~当次作业扣 0 分~~; 平时作业成绩扣 2 分, 扣完为止

QQ 群号: 108 745 6358



2026-计算机数学-拔...

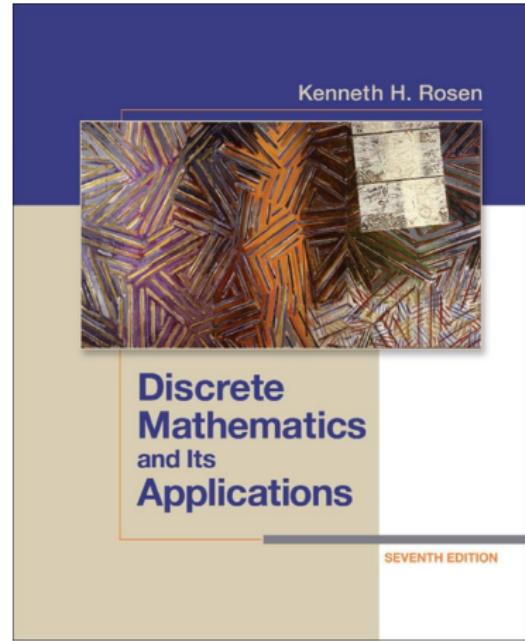
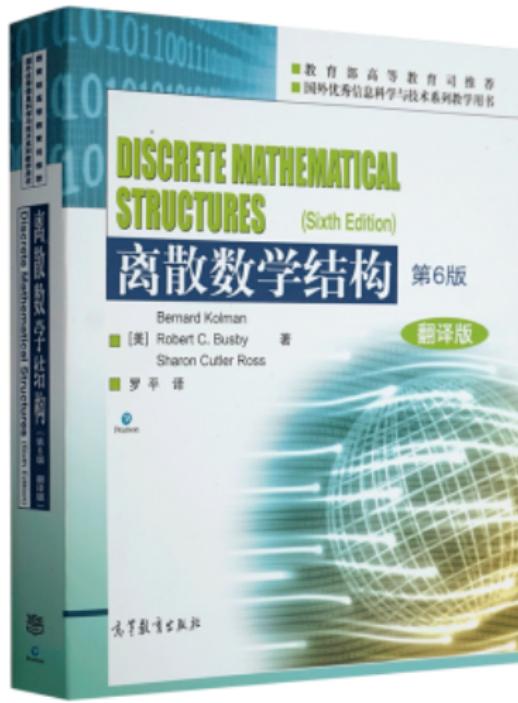
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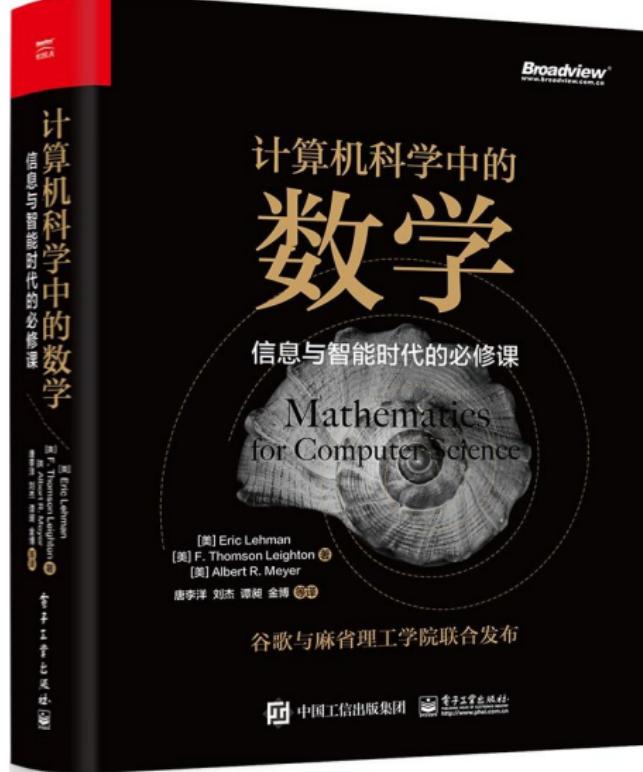
扫一扫二维码, 加入群聊



授课内容不局限于教材，认真听讲很重要



内容与习题偏简单，略显琐碎；可用于课前预习及课后基础练习



推荐阅读; 其它参考书随课程进度安排

 **math4cs-lectures** Public

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main ▾ 1 Branch 0 Tags Go to file t + ⚡ Code ▾

 hengxin	+course info	97d802d · 20 hours ago
 0-overview	+course info	20 hours ago
 1-prop-logic	+0-overview, +1-prop-logic: old version	yesterday
 .gitignore	+0-overview, +1-prop-logic: old version	yesterday
 LICENSE	Initial commit	2 days ago
 README.md	+0-overview, +1-prop-logic: old version	yesterday
 preamble.tex	+course info	20 hours ago

<https://github.com/courses-at-hnu-by-hfwei/math4cs-lectures>

0-overview.pdf

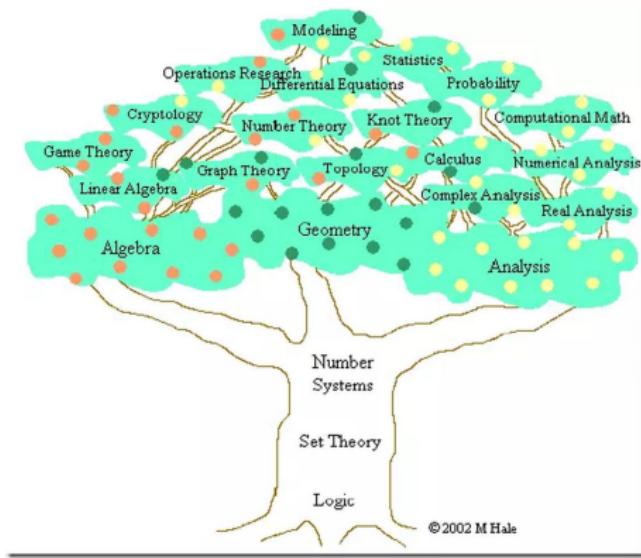
0-overview-handout.pdf



# 计算机（离散）数学

研究**离散对象的结构、性质、操作等**的数学分支 (**大杂烩**)

# 四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、（离散）概率

关于计算机 (离散) 数学的古老传说:

我太难了

啥用没有

真得有那么难吗？

确实蛮难的：知识点多而分散、概念抽象

真得没啥用吗?

太基础, 用了但不自觉 ([逻辑](#))

浅尝辄止, 想用但用不上 ([群论](#))

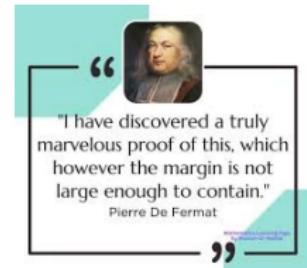
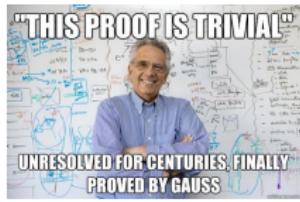
将计算机（离散）数学看作一门语言，一套工具

培养形式化描述问题的能力

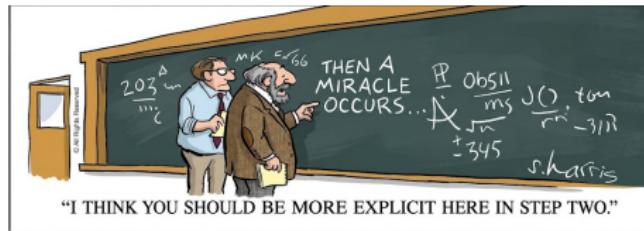
培养做严格证明的能力

# 本课程最重要的主题: 证明





凡是“跳步”的证明，默认都是错误的!!!



Theorem (第一数学归纳法 (The First Mathematical Induction))

设  $P(n)$  是关于自然数的一个性质。如果

(i)  $P(0)$  成立；

(ii) 对任意自然数  $n$ , 如果  $P(n)$  成立, 则  $P(n + 1)$  成立。

那么,  $P(n)$  对所有自然数  $n$  都成立。

## Peano 公理体系刻画了**自然数的递归结构**

### Definition (Peano Axioms)

- (1) 0 是自然数;
- (2) 如果  $n$  是自然数, 则它的后继  $Sn$  也是自然数;
- (3) 0 不是任何自然数的后继;
- (4) 两个自然数相等当且仅当它们的后继相等;
- (5) **数学归纳原理:** 如果
  - (i)  $P(0)$  成立;
  - (ii) 对任意自然数  $n$ , 如果  $P(n)$  成立, 则  $P(n + 1)$  成立。

那么,  $P(n)$  对所有自然数  $n$  都成立。

Definition (良序原理 (The Well-Ordering Principle))

自然数集的任意非空子集都有一个最小元。

Theorem

良序原理与 (第一) 数学归纳法等价。

## Lemma

(第一) 数学归纳法蕴含良序原理。

Proof.

By mathematical induction on the size  $n$  of non-empty subsets of  $\mathbb{N}$ .

$P(n)$  : All subsets of size  $n$  contain a minimum.

Basis Step:  $P(1)$

Inductive Hypothesis:  $P(n)$

Inductive Step:  $P(n) \rightarrow P(n + 1)$

- ▶  $A' \leftarrow A \setminus a$
- ▶  $x \leftarrow \min A'$
- ▶ Compare  $x$  with  $a$

$\forall n \in \mathbb{N} : P(n) \quad vs. \quad P(\infty)$

## Lemma

(第一) 数学归纳法蕴含良序原理。

By contradiction.

$\exists S \neq \emptyset : S \text{ has no minimum element.}$

$$A \triangleq \mathbb{N} \setminus S$$

We claim that  $A = \mathbb{N}$ , and thus  $S = \emptyset$ , a contradiction.

Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

$$\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$$

$$\sqrt{2}^{\sqrt{2}}$$

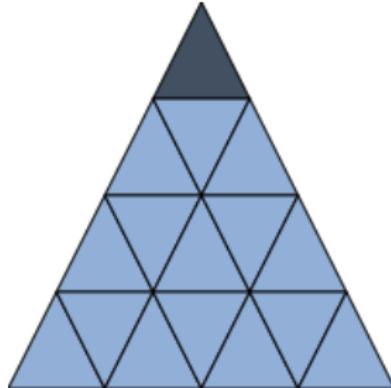
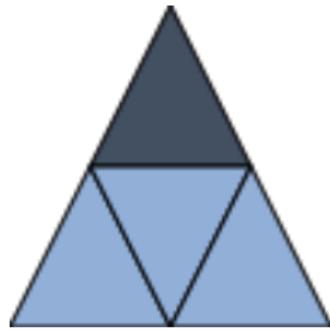
$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$$

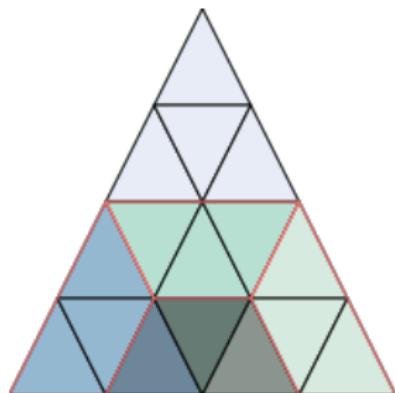
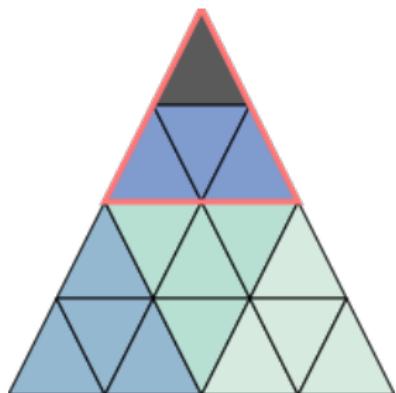
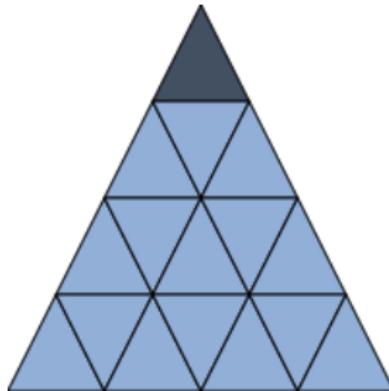
Q：这是构造性证明吗？这是反证法吗？

## Tiling Puzzle

Suppose  $n$  is a positive integer. An equilateral triangle is cut into  $4^n$  congruent equilateral triangles, and one corner is removed.

Show that the remaining area can be **covered** by tiles below





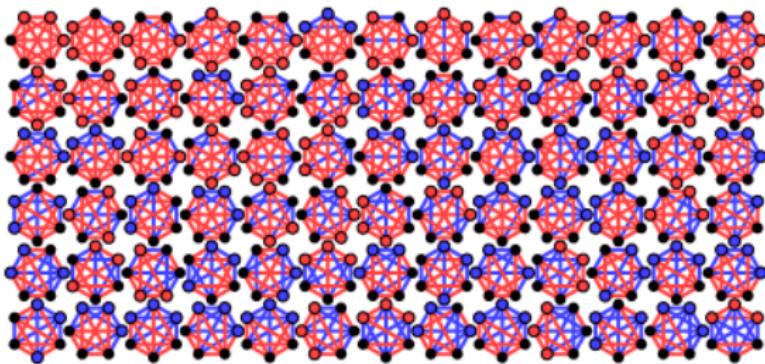
Base Case:

Induction Hypothesis:

Induction Step: ... by induction hypothesis ...

## Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.



In Terms of **Graph Theory**.

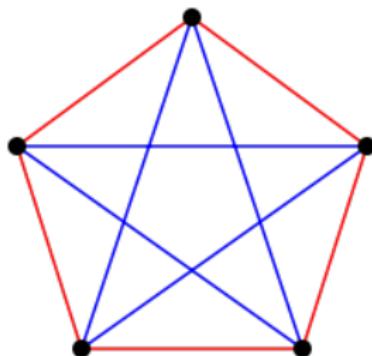
(Is there a **monochromatic** triangle in any 2-coloring of  $K_6$ ?)

## Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of  $K_6$ ?

## Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of  $K_5$ ?

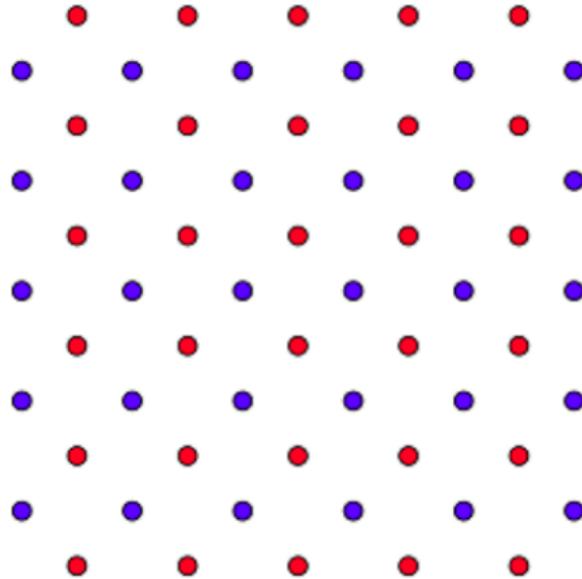


## Ramsey theory

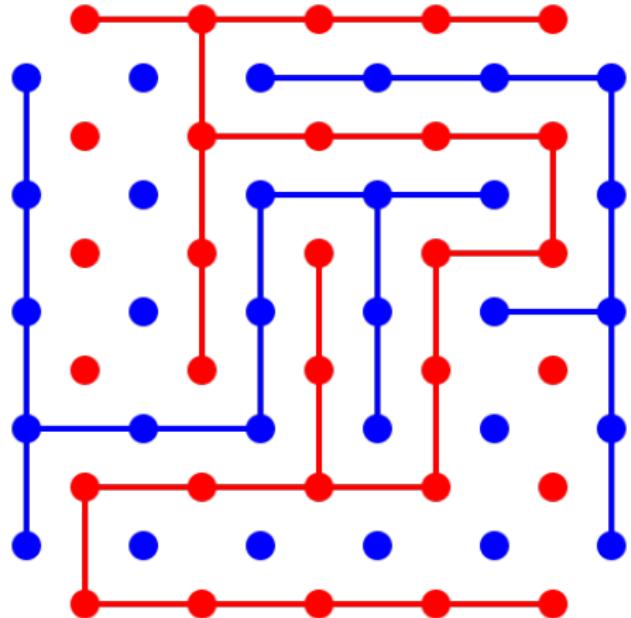
How **big** must the **structure** be  
to ensure that it has a given interesting **property**?



## Bridg-It Game (David Gale, 1958)



$5 \times 6$  vs.  $6 \times 5$



$5 \times 6$  vs.  $6 \times 5$

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie?**

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

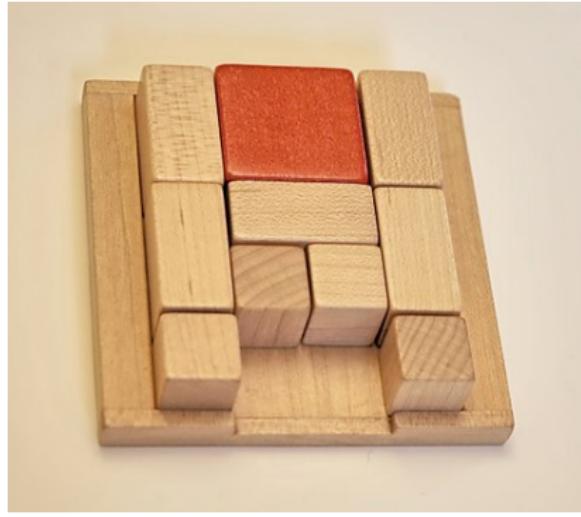
Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.



STAY TUNED

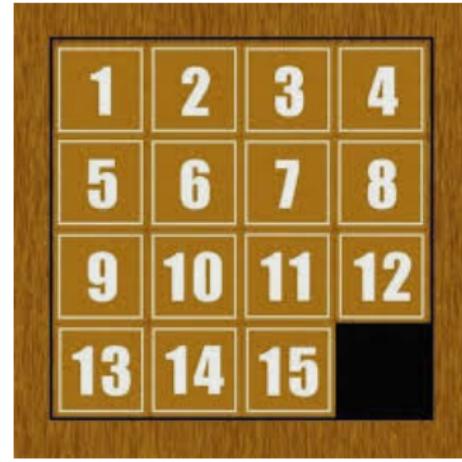
## Klotski Puzzle (华容道)



## Klotski Puzzle (华容道; 中国版本)



## 15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

It uses **permutation groups** in **group theory**.



STAY TUNED

## The Stable Marriage Problem (SMP)

Given  $n$  men and  $n$  women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

$$\{\textcolor{red}{xb}, yc, zd, \textcolor{red}{wa}\}$$

$(x, a)$  is an unstable pair

$$\{xa, yb, zd, wc\}$$

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

Theorem (The Gale-Shapley Algorithm (1962))

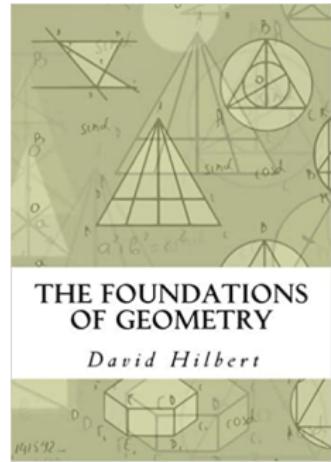
*It is always possible to solve SMP.*

Self-contained (自包含; 自给自足)

# Axiomatic Systems

Syntax *vs.* Semantics (语法与语义对立统一)

# 三个公理系统：逻辑、集合论、图论、抽象代数（群论）



- (1) To draw a straight **line** from any **point** to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) **The parallel postulate.**

## Axiomatic System for a Four-point Geometry

***Undefined terms:*** point, line, is on

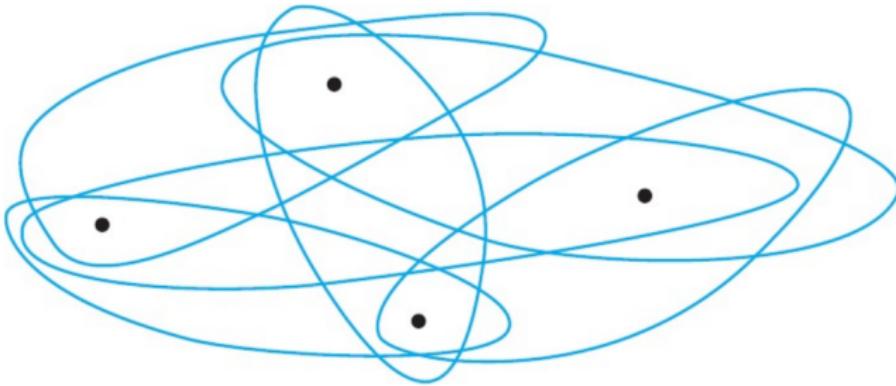
### ***Axioms:***

- (1) There are exactly four points.
- (2) It is impossible for three points to be on the same line.
- (3) For every pair of distinct points  $x$  and  $y$ , there is a unique line  $l$  such that  $x$  is on  $l$  and  $y$  is on  $l$ .
- (4) Given a line  $l$  and a point  $x$  that is not on  $l$ , there is a unique line  $m$  such that  $x$  is on  $m$  and no point on  $l$  is also on  $m$ .

### ***Theorem***

*There are at least two distinct lines.*

## Syntax *vs.* Semantics

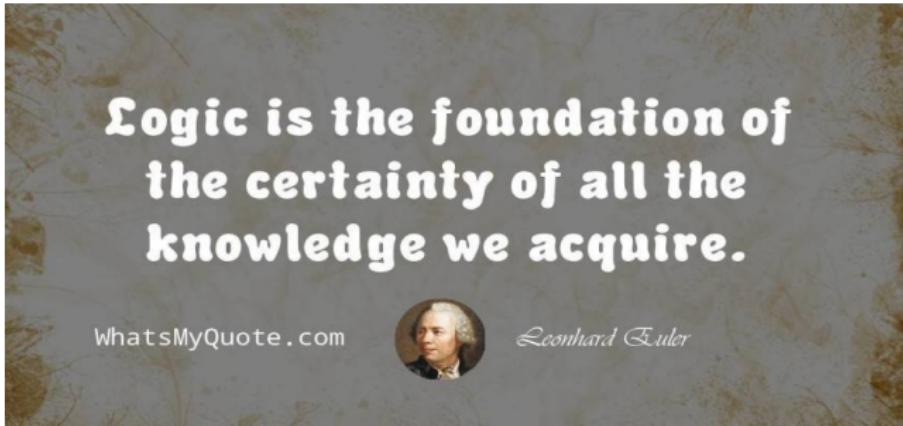


point : •

line : ○

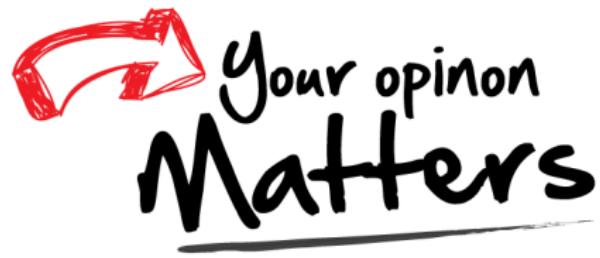
is on : ○•

# 什么样的推理是正确的?





# Thank You!



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