

# 计算机数学概述

魏恒峰

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2026 年 03 月 02 日

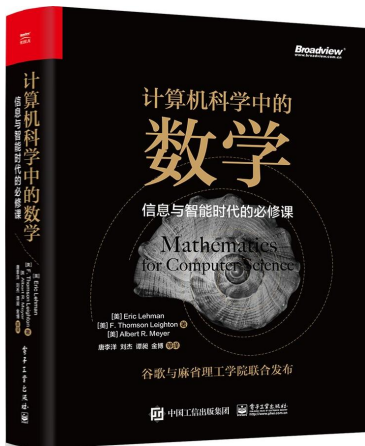


# 什么是“计算机数学”？

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“Mathematics for Computer Science”

(math4cs)



# What does “Computer Science” study?

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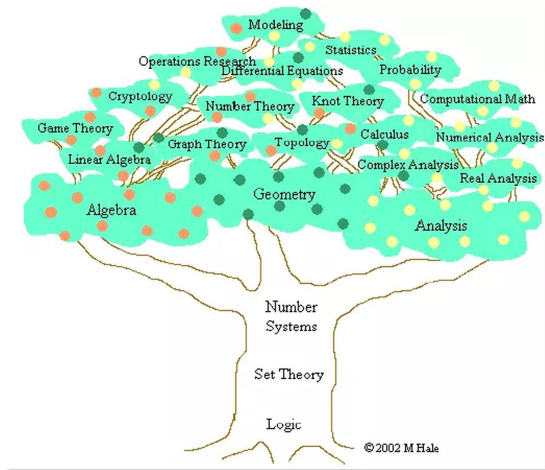
*Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.*

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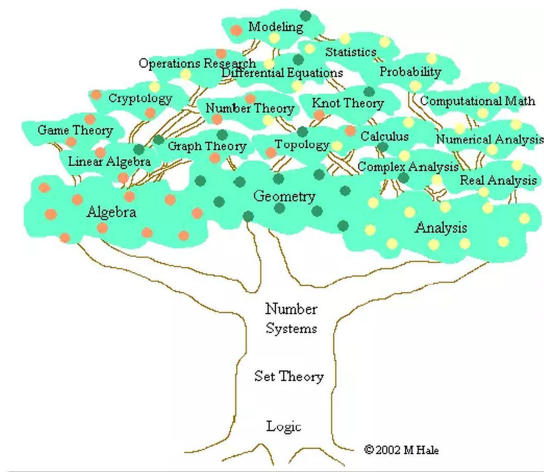
*Computer science focuses on methods involved in design, specification, programming, verification, implementation and testing of human-made computing systems.*

math4cs is **model-and-proof** oriented.

# “计算机数学”在哪里？



# “计算机数学”在哪里？



计算机数学是个大杂烩, 啥都学点儿, 啥都没学好



## 分班教学 (共 xxx 班)

授课内容与作业可能有出入, 不影响考试与成绩分配

平时作业 *vs.* 期中测试 *vs.* 期末测试

3 : 3 : 4

4 : 3 : 3

弹性制

每周二、周四下午 14:00 发布作业      下周四晚 22:00 前提交作业

每次作业按 10 分计算

**迟交:** 周四**前**向助教登记, 可延长两天, 总分 8 分

(助教: 、 、 )

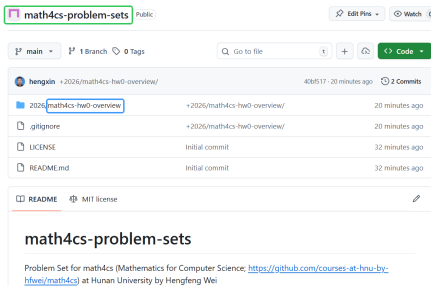
“教学立方” 课程邀请码: PLD8QKTZ



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TEX



<https://github.com/courses-at-hnu-by-hfwei/math4cs-problem-sets>

# 约法三章

非必要, 不点名

非必要, 不迟到



**非必要, 不迟到**

**尽量吃早餐, 但不可以在教室吃早餐**

~~非/必/要/~~ 不抄袭; 一经发现, 后果严重

~~非/必/要/~~，不抄袭；一经发现，后果严重

当次作业计 0 分；总评扣 10 分

QQ 群号: 108 745 6358

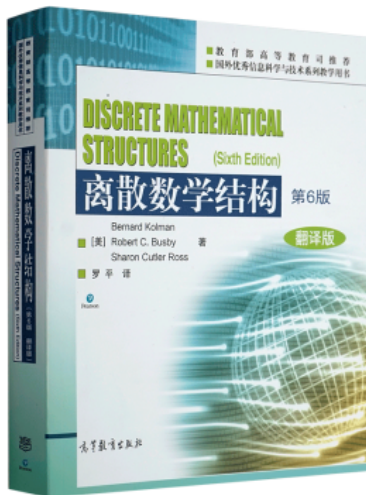


**2026-计算机数学-拔...**

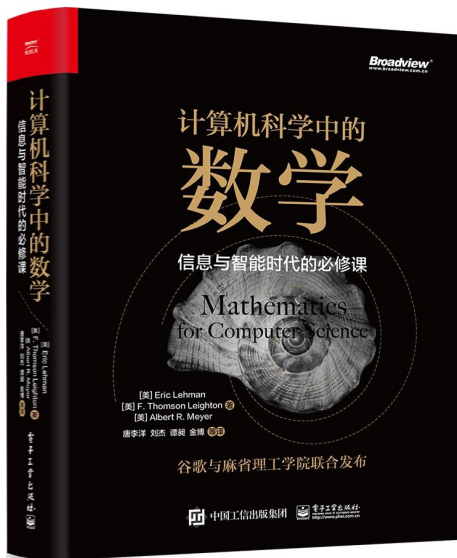
群号: 1087456358



扫一扫二维码, 加入群聊



教材不重要, 听讲更重要



## 其它参考书随课程进度安排



<https://github.com/courses-at-nju-by-hfwei/discrete-math-lectures>



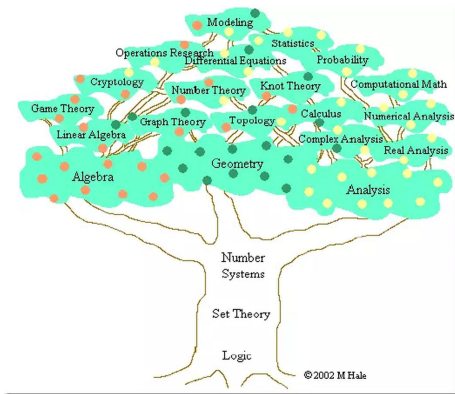


# 离散数学

# 离散数学

研究离散对象的结构、性质、操作等的数学分支 (大杂烩)

## 四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、(离散) 概率

关于离散数学, 学长纷纷表示:

我太难了

啥用没有

真得有那么难吗?

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)



将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力



Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

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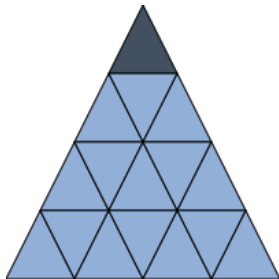
$Q$ : 这是构造性证明吗? 这是反证法吗?

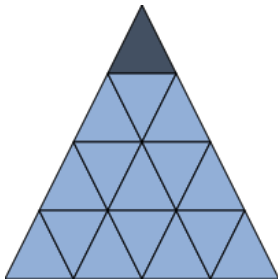


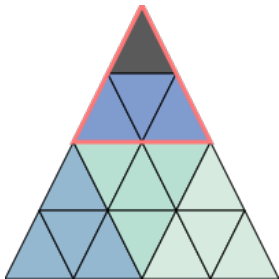
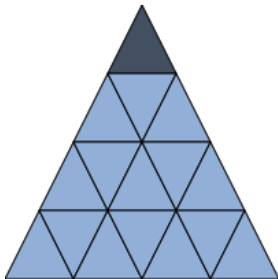
## Tiling Puzzle

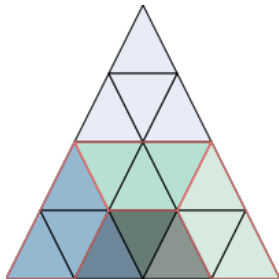
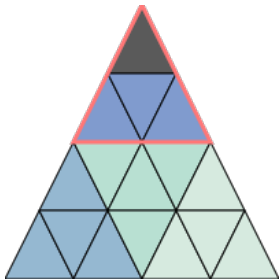
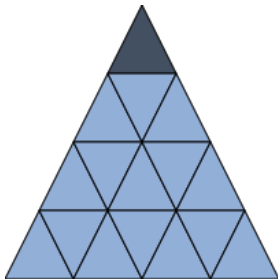
Suppose  $n$  is a positive integer. An equilateral triangle is cut into  $4^n$  congruent equilateral triangles, and one corner is removed.

Show that the remaining area can be covered by tiles below









Base Case:

Induction Hypothesis:

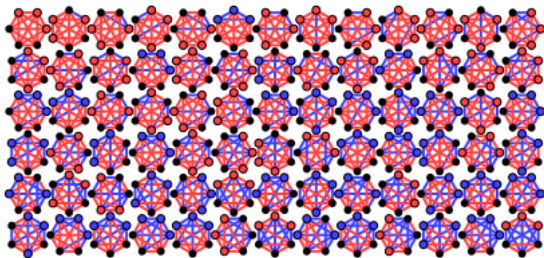
Induction Step: ... by induction hypothesis ...

## Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.

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In Terms of **Graph Theory**.

(Is there a **monochromatic** triangle in any 2-coloring of  $K_6$ ?)

## Theorem on Friends and Strangers

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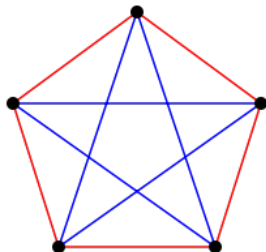


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## Ramsey theory

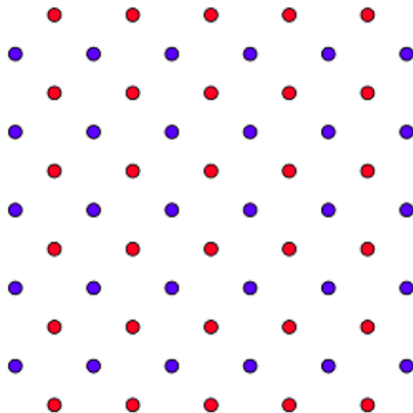
How **big** must the **structure** be  
to ensure that it has a given interesting **property**?

## Ramsey theory

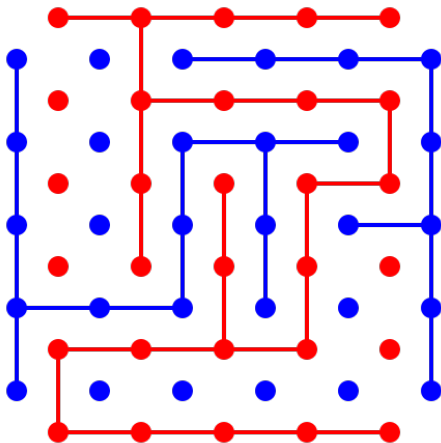
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## Bridg-It Game (David Gale, 1958)



$5 \times 6$  vs.  $6 \times 5$



$5 \times 6$  vs.  $6 \times 5$

Let's Play with it!

Let's Analyze it!



Will Bridg-It **end in a tie**?

Will Bridg-It **end in a tie**?

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

Does **Player 1** have a **winning strategy**?

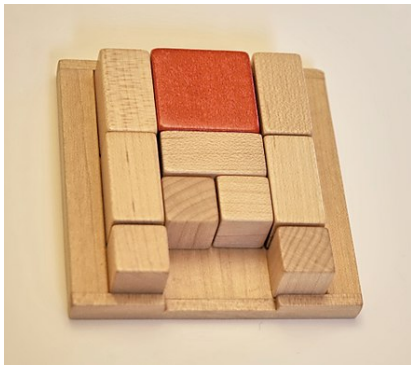
Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.



**STAY TUNED**

## Klotski Puzzle (华容道)

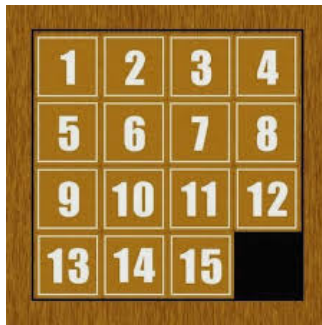
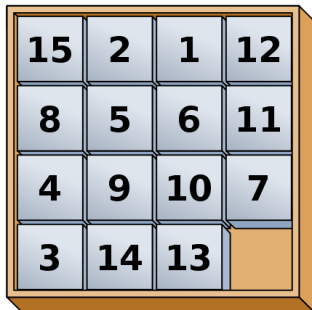


## Klotski Puzzle (华容道; 中国版本)





## 15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

How to solve it?

It uses **permutation groups** in **group theory**.



**STAY TUNED**

## The Stable Marriage Problem (SMP)

Given  $n$  men and  $n$  women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
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$$\{xb, yc, zd, wa\}$$

$(x, a)$  is an unstable pair

$$\{xa, yb, zd, wc\}$$

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Theorem (The Gale-Shapley Algorithm (1962))

*It is always possible to solve SMP.*

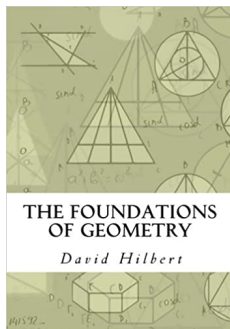
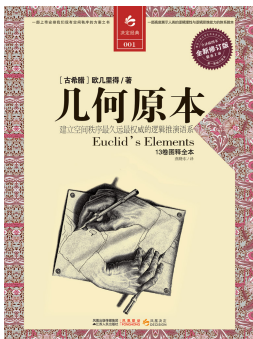
Self-contained (自包含; 自给自足)

# Axiomatic Systems

Syntax *vs.* Semantics (语法与语义对立统一)



## 三个公理系统: 逻辑、集合论、图论、抽象代数 (群论)



- (1) To draw a straight **line** from any **point** to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) **The parallel postulate.**

## Axiomatic System for a Four-point Geometry

*Undefined terms:* point, line, is on

### *Axioms:*

- (1) There are exactly four points.
- (2) It is impossible for three points to be on the same line.
- (3) For every pair of distinct points  $x$  and  $y$ , there is a unique line  $l$  such that  $x$  is on  $l$  and  $y$  is on  $l$ .
- (4) Given a line  $l$  and a point  $x$  that is not on  $l$ , there is a unique line  $m$  such that  $x$  is on  $m$  and no point on  $l$  is also on  $m$ .

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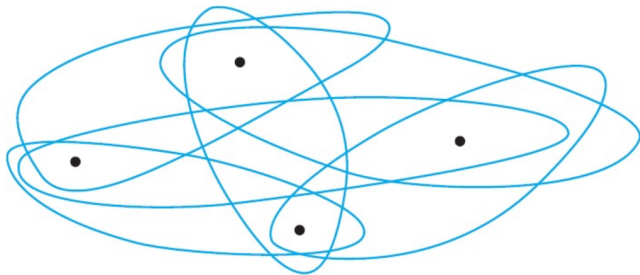
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### Theorem

*There are at least two distinct lines.*

# Syntax *vs.* Semantics

## Syntax *vs.* Semantics



point :  $\cdot$       line :  $\bigcirc$       is on :  $\bigcirc \cdot$

## 什么样的推理是正确的？

**Logic is the foundation of  
the certainty of all the  
knowledge we acquire.**

WhatsMyQuote.com

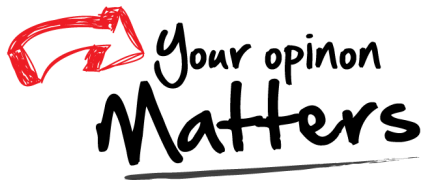


*Leonhard Euler*





Thank  
You!



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