离散数学概述

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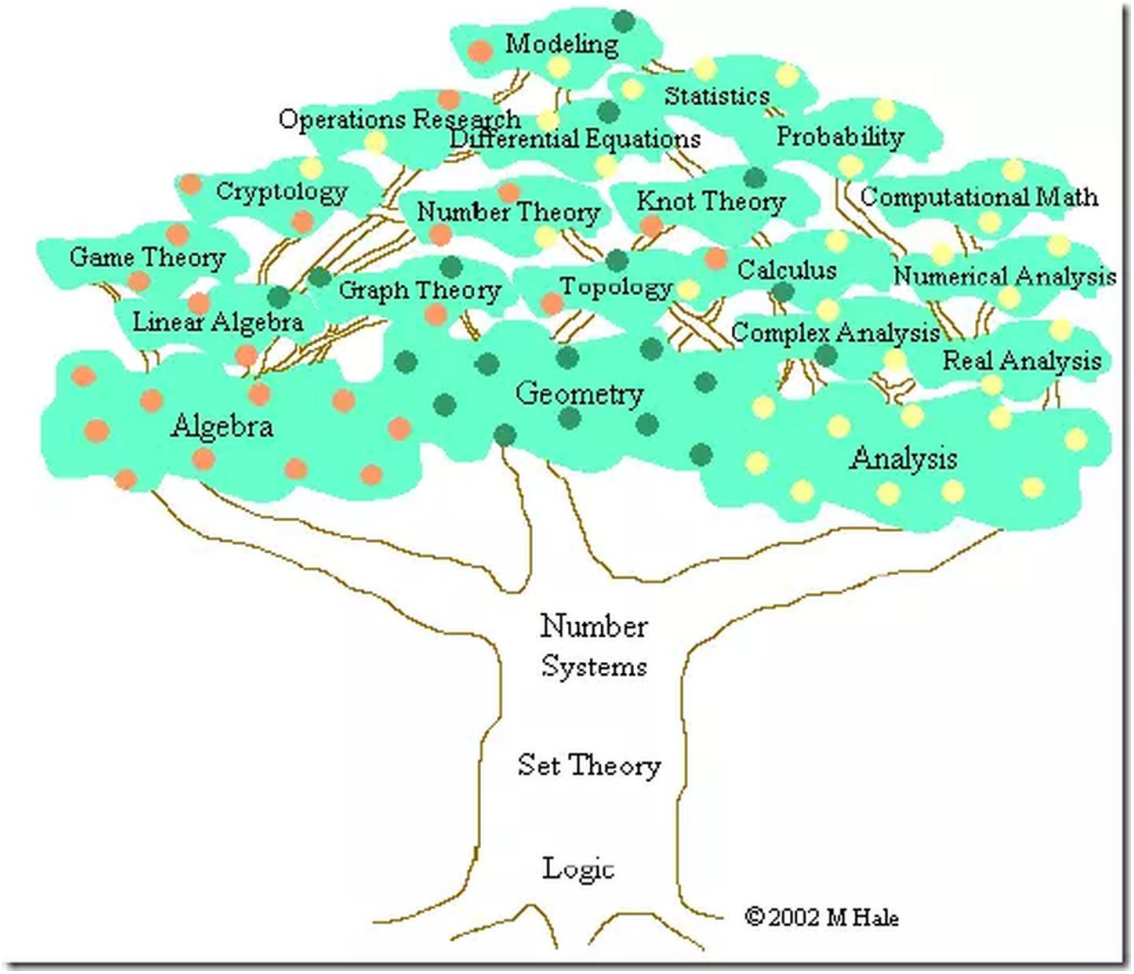
“离散数学”在哪里？



离散数学是个大杂烩，啥都学点儿，啥都没学好

分班教学
(与计算机系赵建华老师)

授课内容与作业可能有出入, 不影响考试与成绩分配

平时作业 *vs.* 期中测试 *vs.* 期末测试

$$3 : 3 : 4$$

$$4 : 3 : 3$$

弹性制

每周四晚上发布作业　　下周四 23:55 前提交作业

每次作业按 10 分计算

迟交: 周四前向助教登记, 可延长两天, 总分 8 分

(**作业助教:** 裴一凡、戴若石、肖江)

“教学立方”课程邀请码：PLD8QKTZ

https://github.com/courses-at-nju-by-hfwei/discrete-math-problem-sets

约法三章

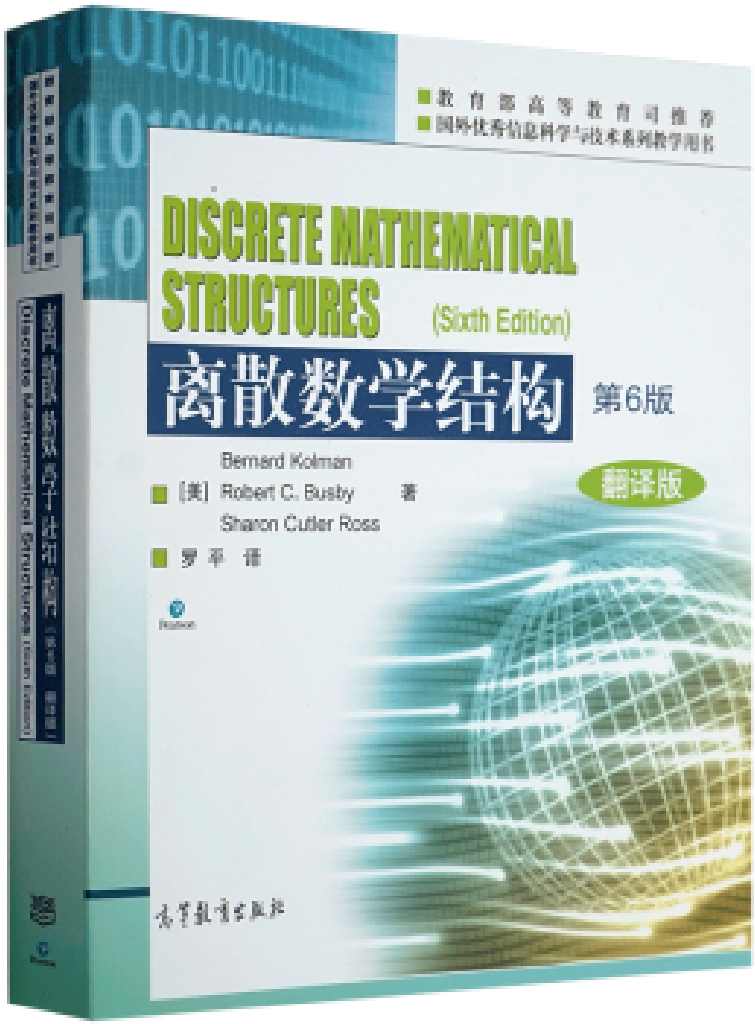
非必要，不点名

非必要，不迟到

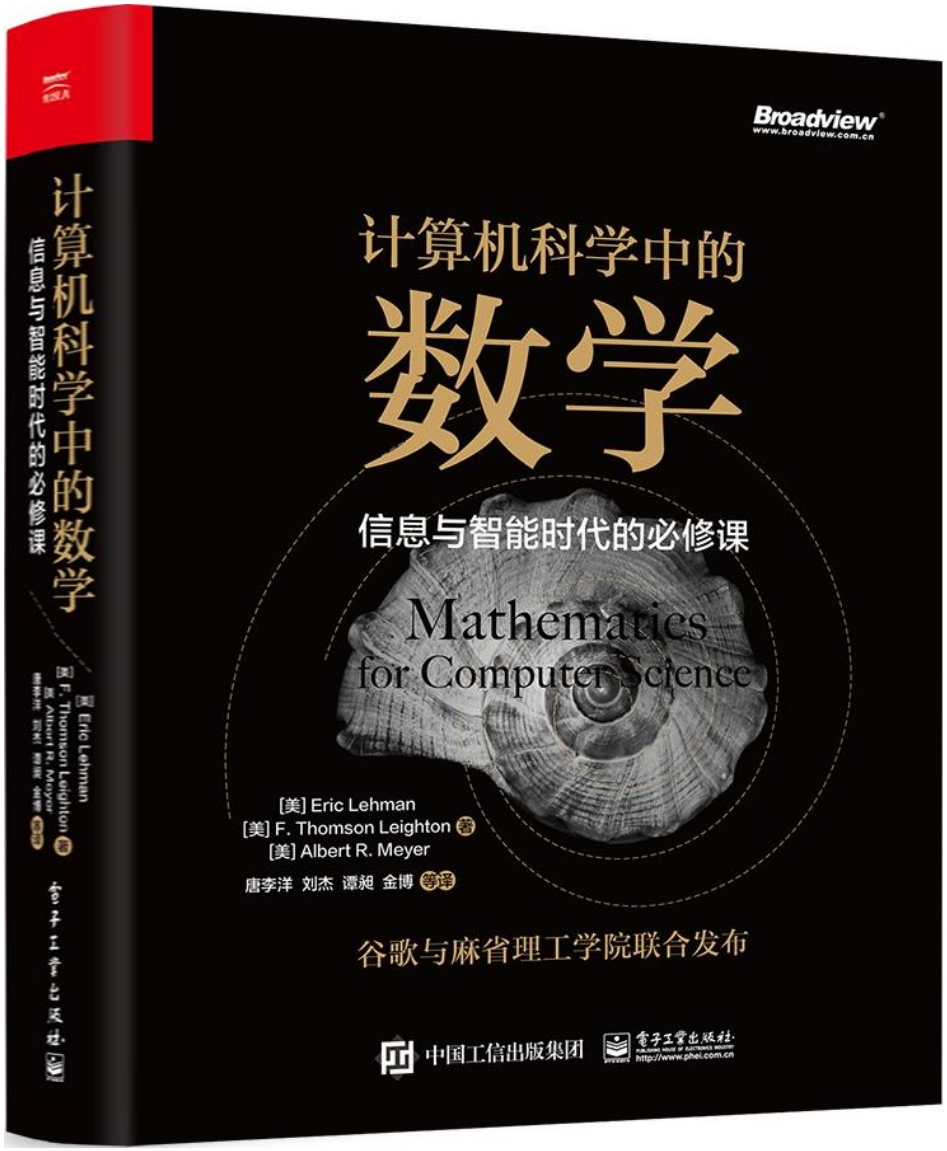
尽量吃早餐，但不可以在教室吃早餐

~~非必要~~, 不抄袭; 一经发现, 后果严重

当次作业计 0 分; 总评扣 10 分


教育部高等教育司推荐
国外优秀信息科学与技术系列教学用书
DISCRETE MATHEMATICAL STRUCTURES
(Sixth Edition)
离散数学结构
第6版
翻译版
Bernard Kolman
[美] Robert C. Busby 著
Sharon Cutler Ross
罗平 译
高等教育出版社

教材不重要, 听讲更重要



其它参考书随课程进度安排

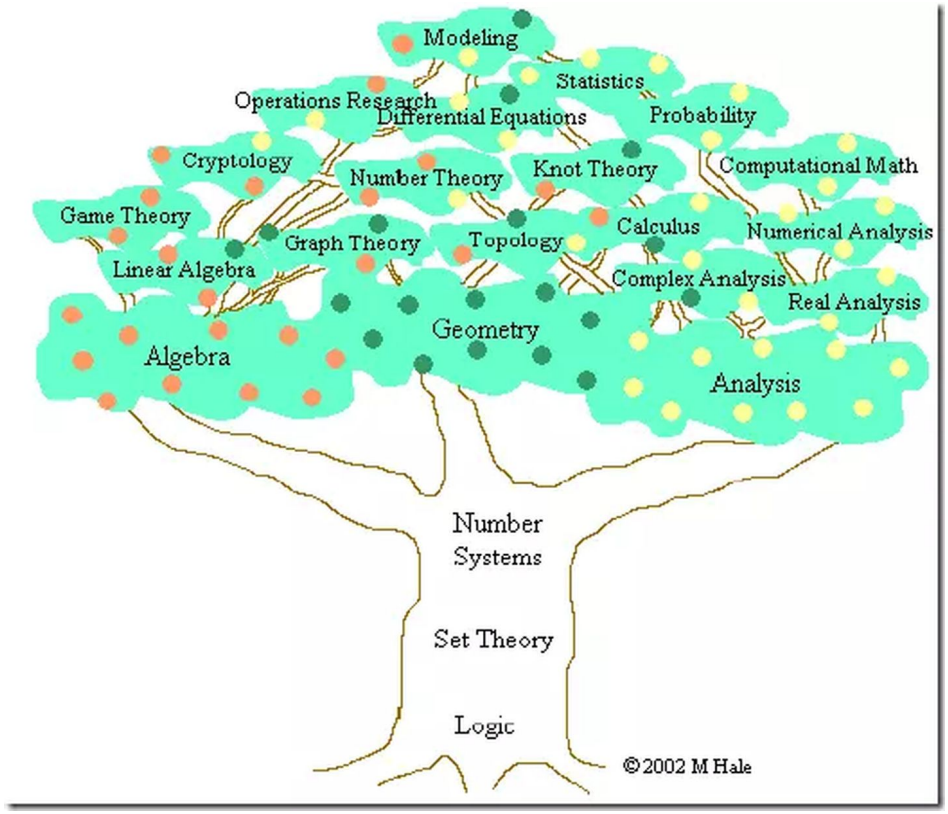
https://github.com/courses-at-nju-by-hfwei/discrete-math-lectures

welcome
again

离散数学

研究**离散对象的结构、性质、操作等**的数学分支 (**大杂烩**)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、（离散）概率

关于离散数学, 学长纷纷表示:

我太难了

啥用没有

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力

Show me your proof...
whisper

Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

$$\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$$

$$\sqrt{2}^{\sqrt{2}}$$

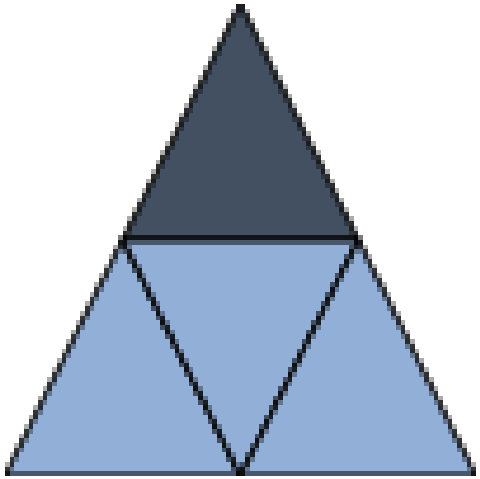
$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$$

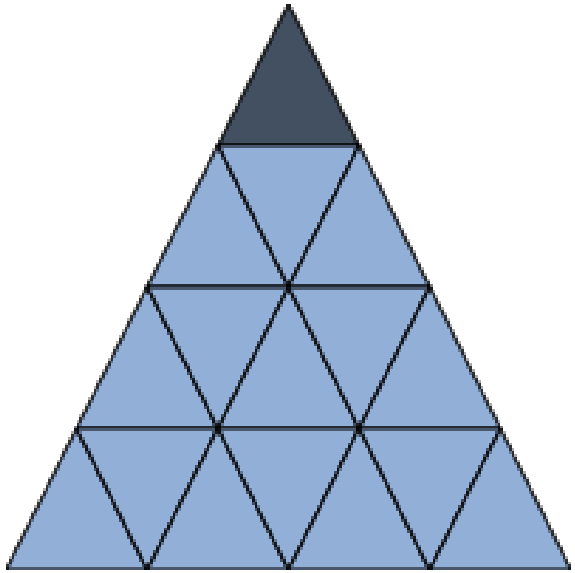
Q：这是构造性证明吗？这是反证法吗？

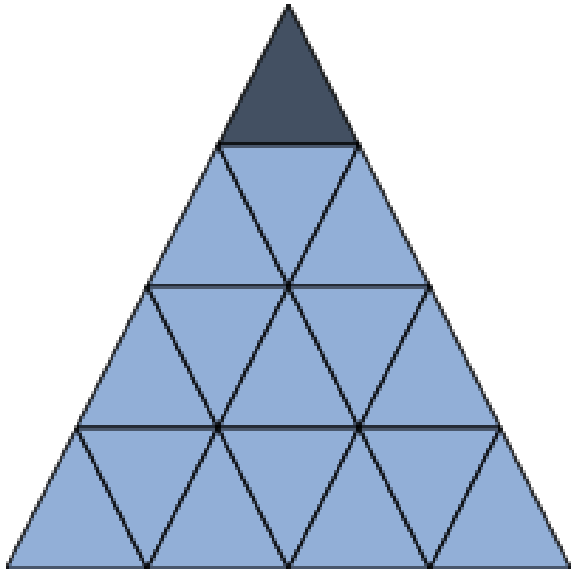
Tiling Puzzle

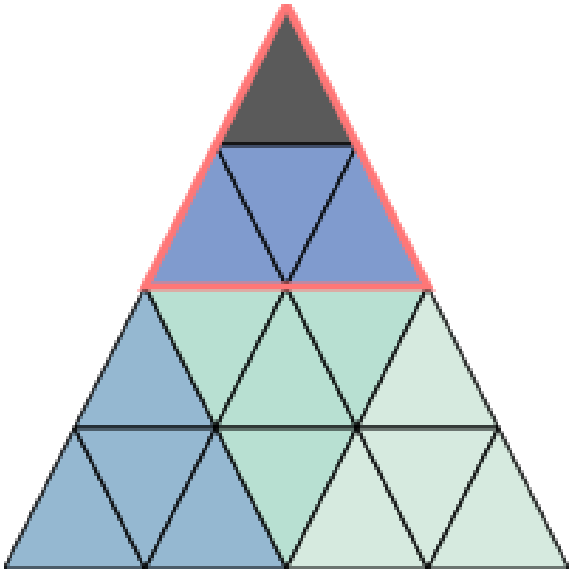
Suppose n is a positive integer. An equilateral triangle is cut into 4^n congruent equilateral triangles, and one corner is removed.

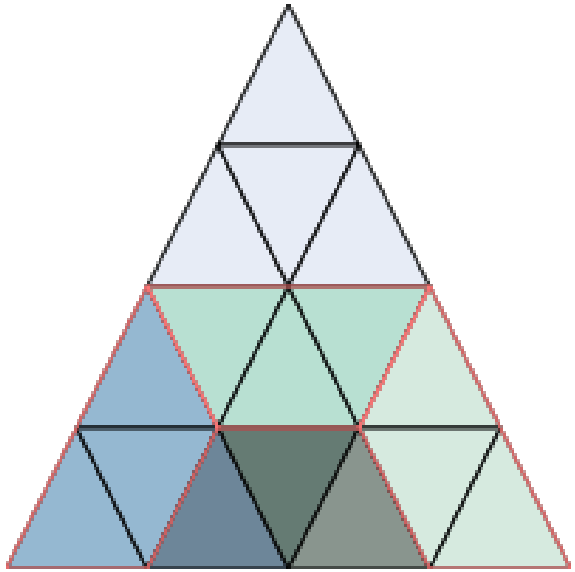
Show that the remaining area can be covered by tiles below











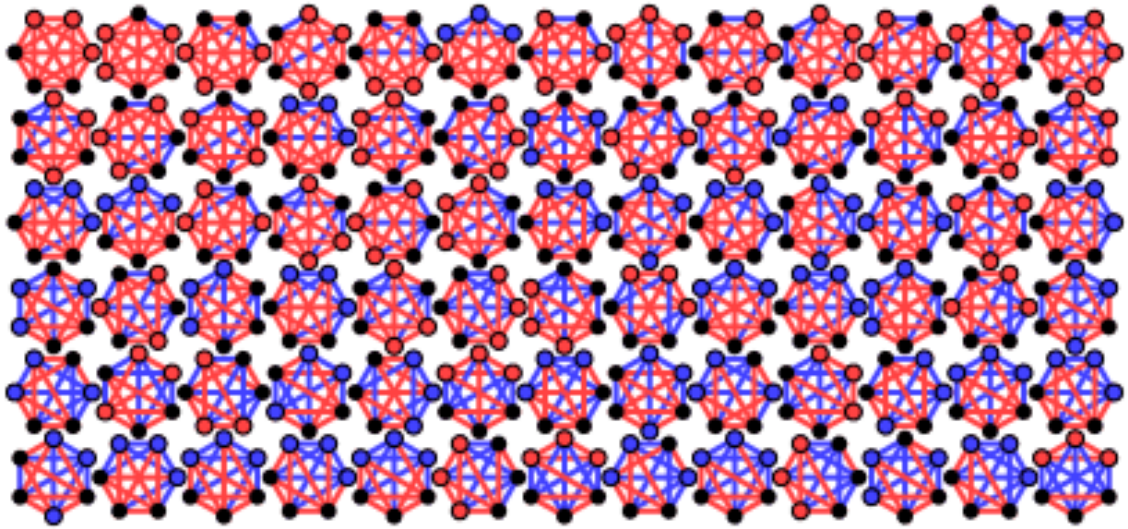
Base Case:

Induction Hypothesis:

Induction Step: . . . by induction hypothesis . . .

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.



In Terms of **Graph Theory**.

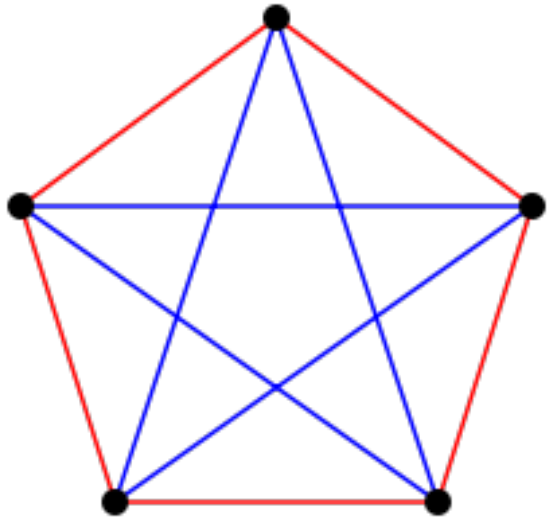
(Is there a monochromatic triangle in any 2-coloring of K_6?)

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_6?

Theorem on Friends and Strangers

Is there a monochromatic triangle in any 2-coloring of K_5?

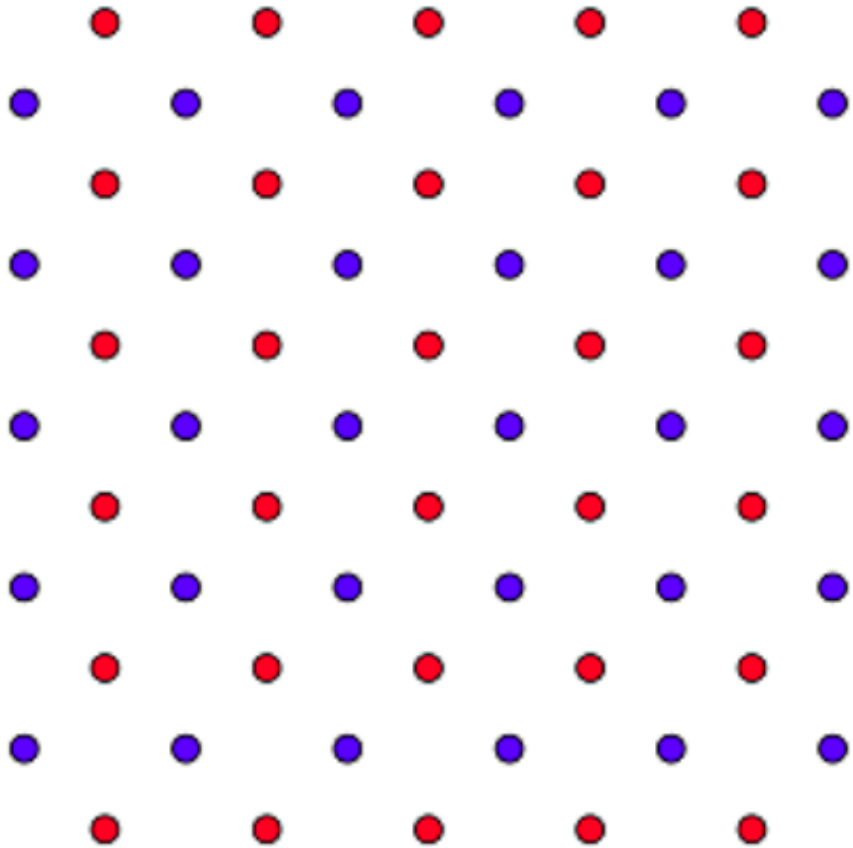


Ramsey theory

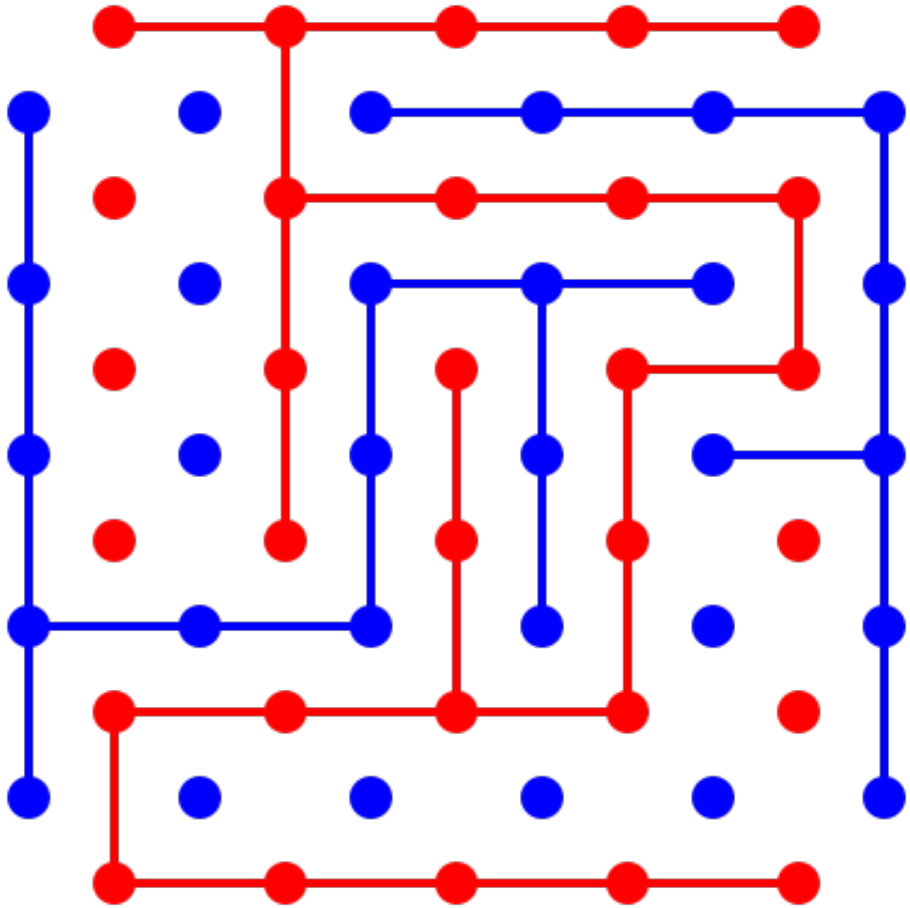
How **big** must the **structure** be

to ensure that it has a given interesting **property**?

Bridg-It Game (David Gale, 1958)



5×6 *vs.* 6×5



5×6 *vs.* 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie**?

No! By **contradiction**.

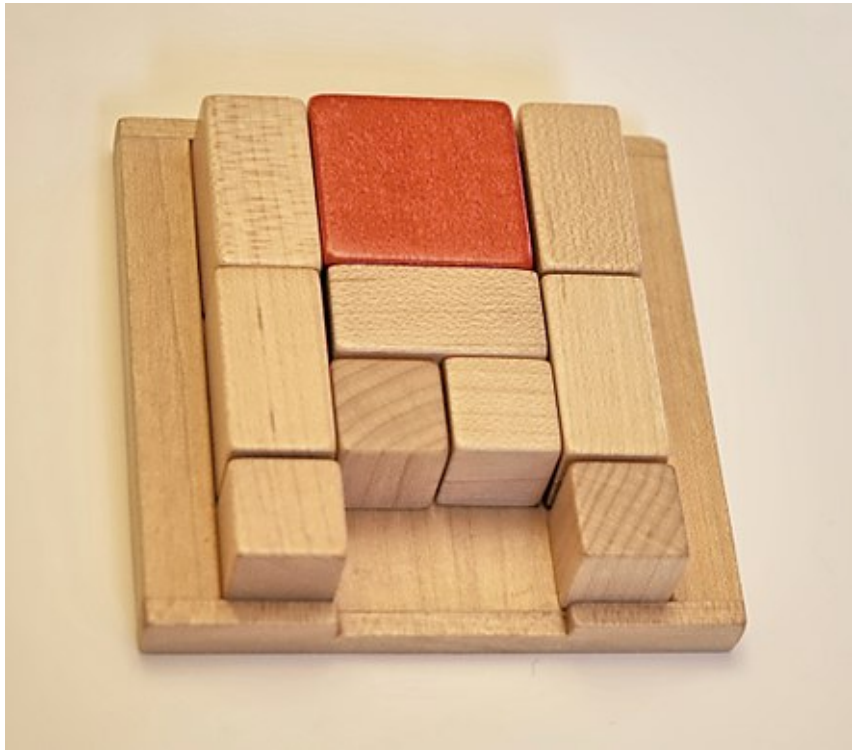
Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

Does **Player 1** have a **winning strategy**?

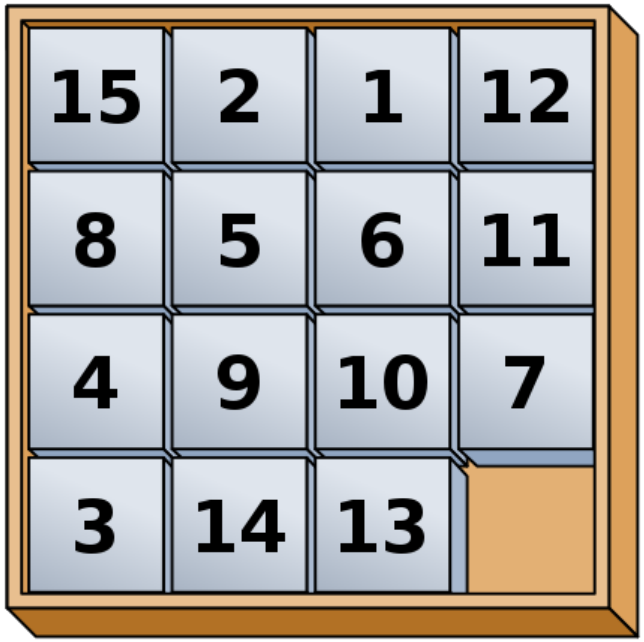
Yes! It uses **spanning trees** in **graph theory**.

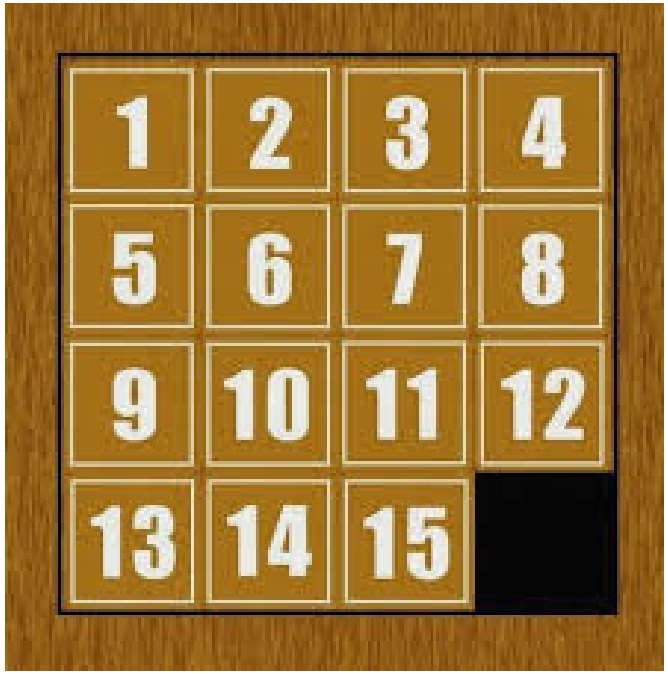
Klotski Puzzle (华容道)

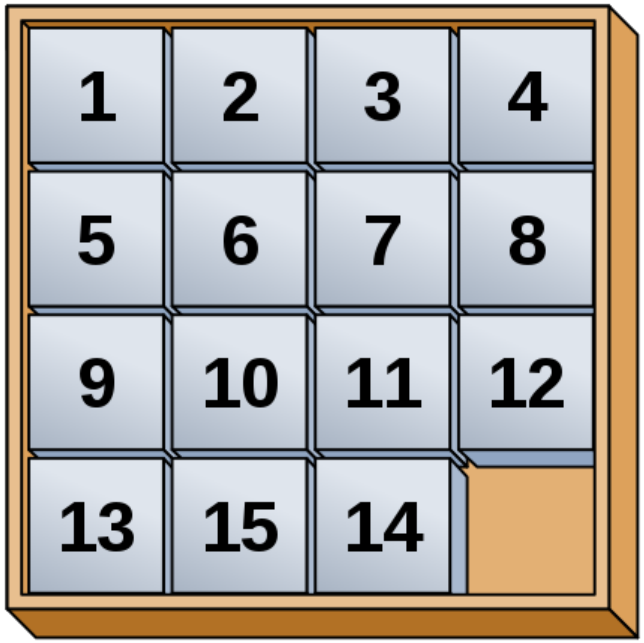


Klotski Puzzle (华容道; 中国版本)

15 Puzzle (数字华容道)







Is it solvable?

How to solve it?

It uses **permutation groups** in **group theory**.

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	**Women** $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

$$\{xb, yc, zd, wa\}$$

(x, a) is an unstable pair

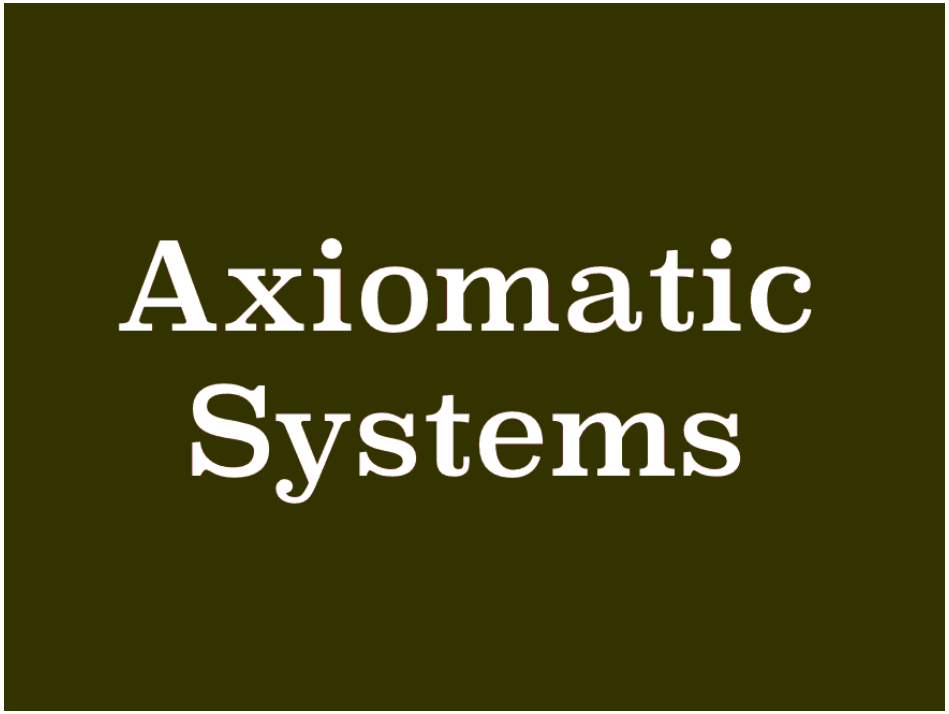
$$\{xa, yb, zd, wc\}$$

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
$w : c > b > a > d$	$d : x > y > z > w$

Theorem (The Gale-Shapley Algorithm (1962))

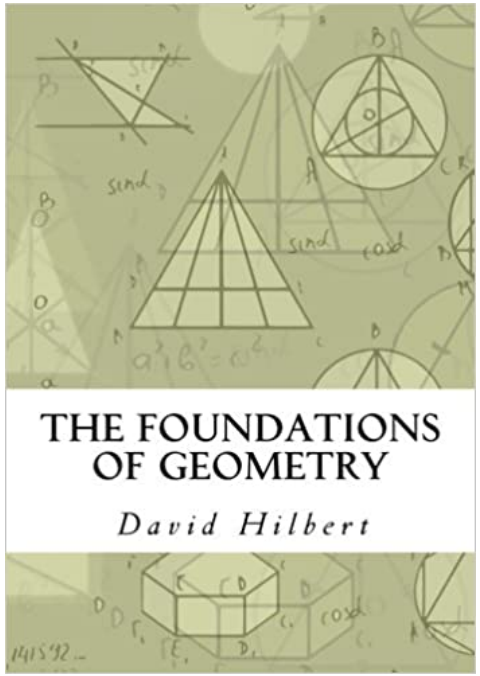
It is always possible to solve SMP.

Self-contained (自包含; 自给自足)



Syntax *vs.* Semantics (语法与语义对立统一)

三个公理系统: 逻辑、集合论、图论、抽象代数 (群论)



(1) To draw a straight line from any point to any point.

(2) To extend a finite straight line continuously in a straight line.

(3) To describe a circle with any center and radius.

(4) That all right angles are equal to one another.

(5) The parallel postulate.

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

Axioms:

(1) There are exactly four points.

(2) It is impossible for three points to be on the same line.

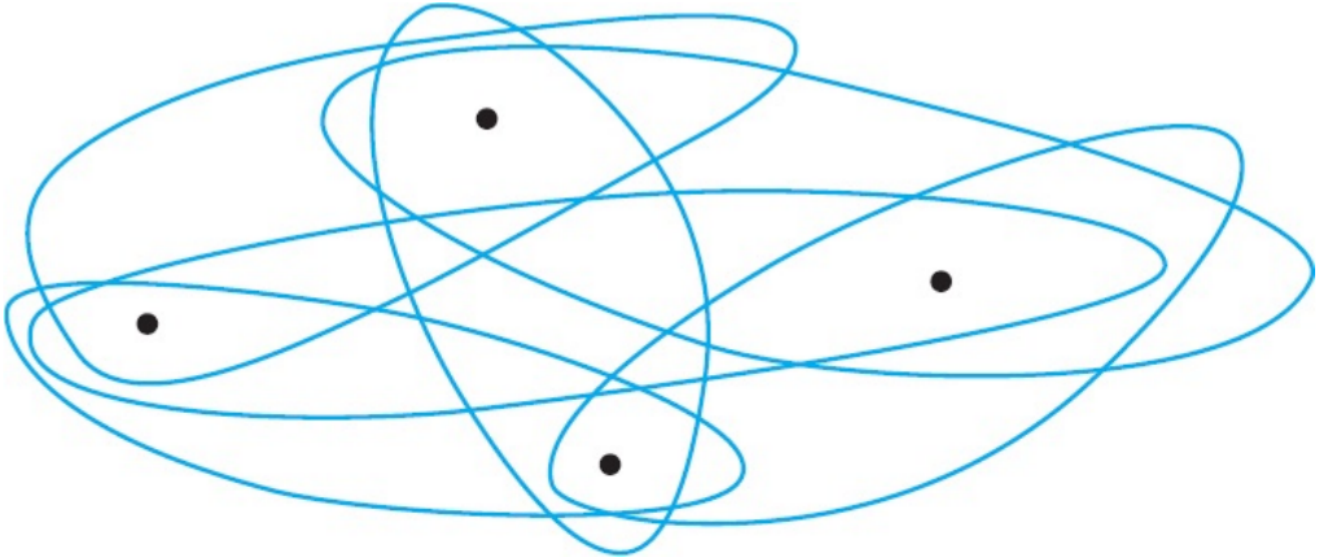
(3) For every pair of distinct points x and y, there is a unique line l such that x is on l and y is on l.

(4) Given a line l and a point x that is not on l, there is a unique line m such that x is on m and no point on l is also on m.

Theorem

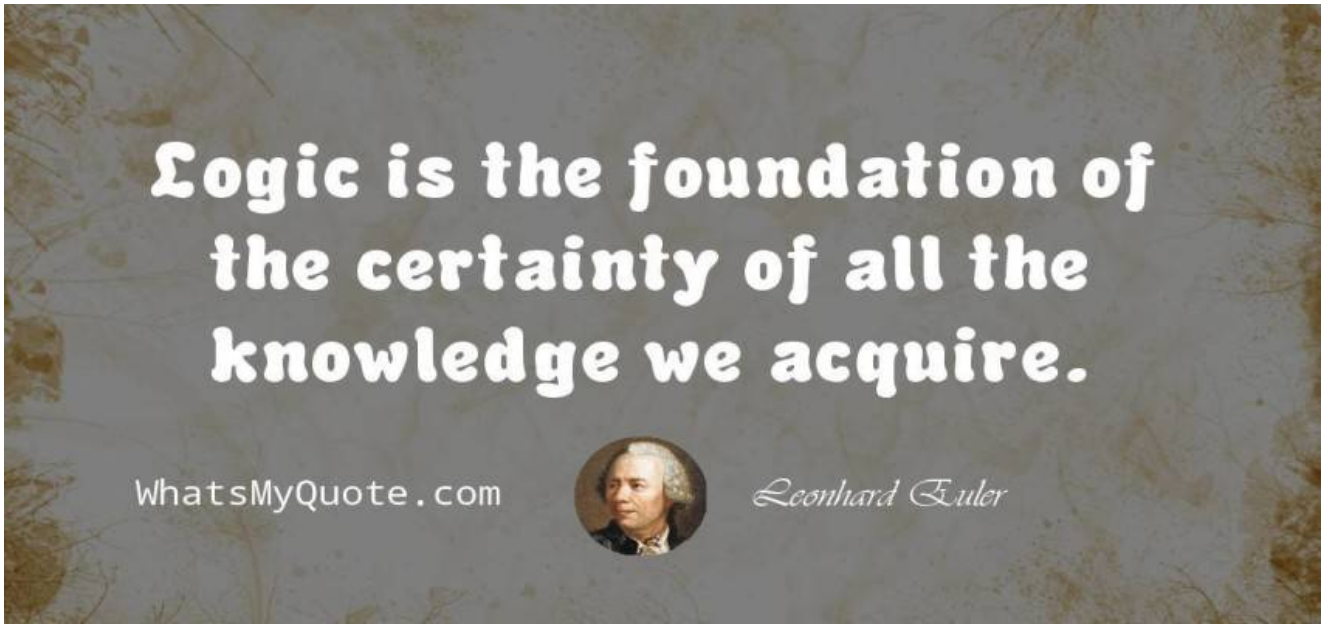
There are at least two distinct lines.

Syntax *vs.* Semantics



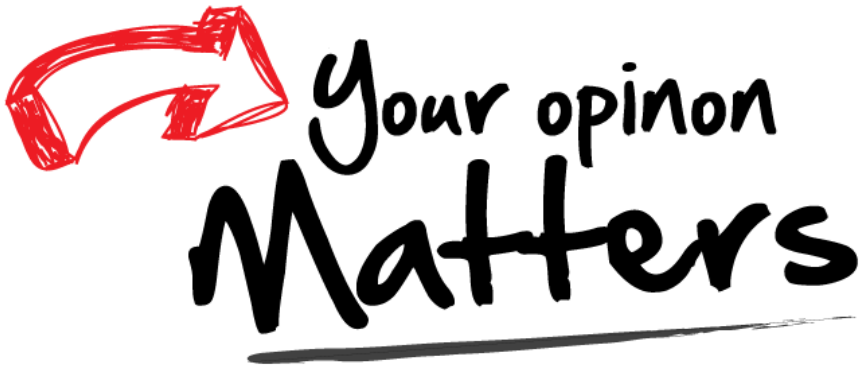
point : ·　　line : ◯　　is on : ⊙

什么样的推理是正确的?


Logic is the foundation of the certainty of all the knowledge we acquire.
WhatsMyQuote.com
Leonhard Euler

SEE YOU
NEXT WEEK

Thank
You!



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