(十) 图论: 树 (Trees)

魏恒峰

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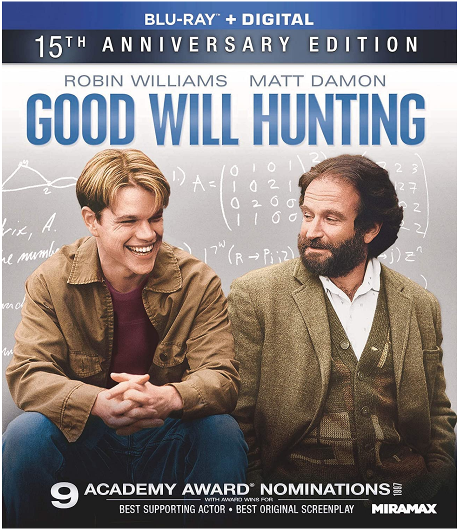
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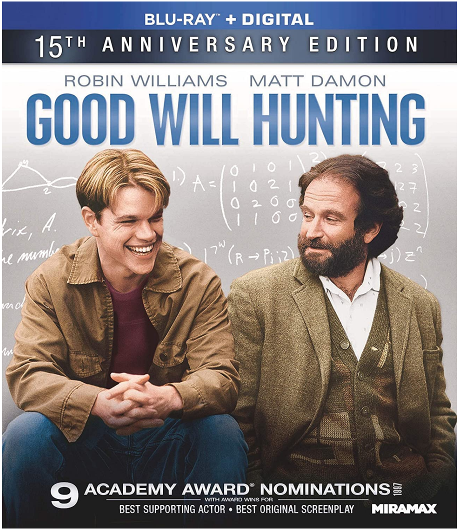
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你, 真得, 看懂了吗?

Definition (Tree (树))

A tree is a connected acyclic undirected graph.

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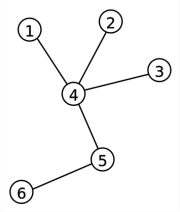

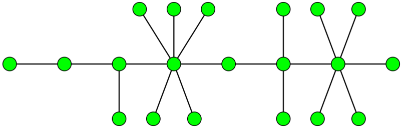

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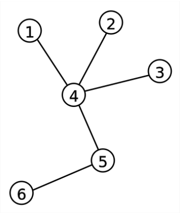

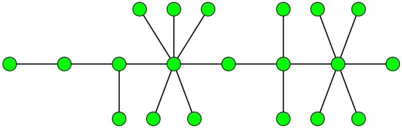

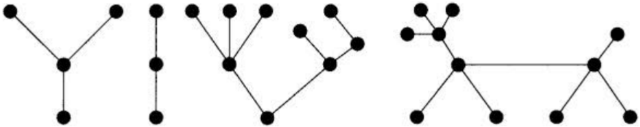

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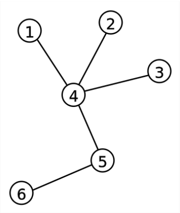

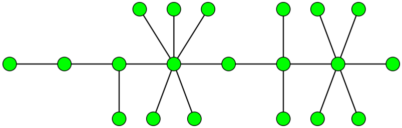

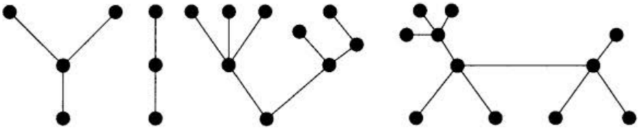

Definition (Forest (森林))

A forest is a acyclic undirected graph.

Definition (Internal Vertex (内部顶点); Leaf (叶子))

In a tree T with ≥ 2 vertices, for a vertex v in T, if

$$\deg(v) = 1$$

then v is called a leaf; otherwise, v is an internal vertex.

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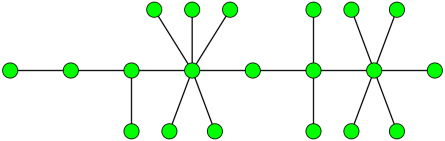

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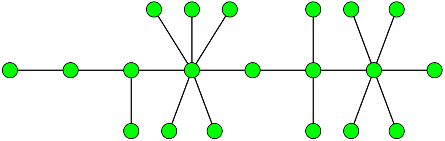

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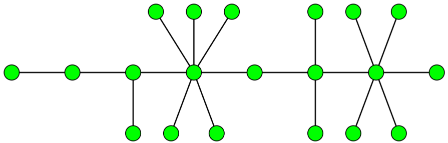

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$$\text{Otherwise, } \forall v \in V.\ \deg(v) \geq 2 \implies T \text{ has cycles.}$$

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Consider the two endpoints of any maximal (nontrivial) path in T.

They are leaves of T.

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Deleting a leaf from a tree T with n vertices produces a tree with $n-1$ vertices.

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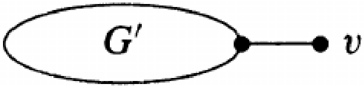

$G' = G - v$ is connected and acyclic.

A leaf does *not* belong to any paths connecting two other vertices.

Definition (Irreducible Tree)

An irreducible tree is a tree T where

$$\forall v \in V(T).\ \deg(v) \neq 2.$$

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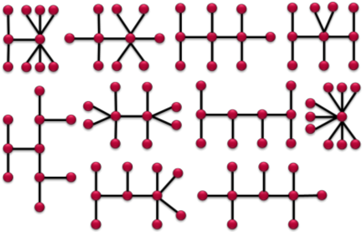

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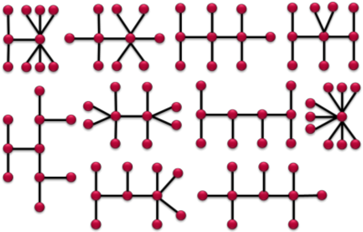

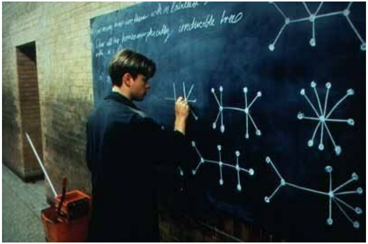

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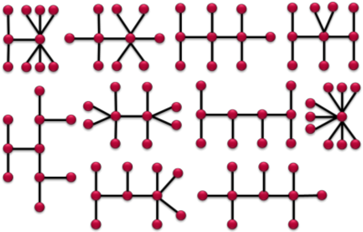

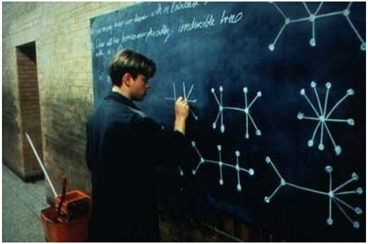

Homeomorphically Irreducible Trees of size $n = 10$

Theorem ((We call it) Tree Theorem)

Let T be an undirected graph with n vertices.

Then the following statements are equivalent:

(1) T *is a tree;*

(2) T *is acyclic, and has* $n-1$ *edges;*

(3) T *is connected, and has* $n-1$ *edges;*

(4) T *is connected, and each edge is a bridge;*

(5) *Any two vertices of* T *are connected by exactly one path;*

(6) T *is acyclic, but the addition of any edge creates exactly one cycle.*


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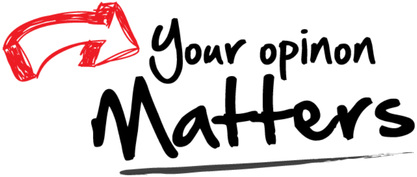

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