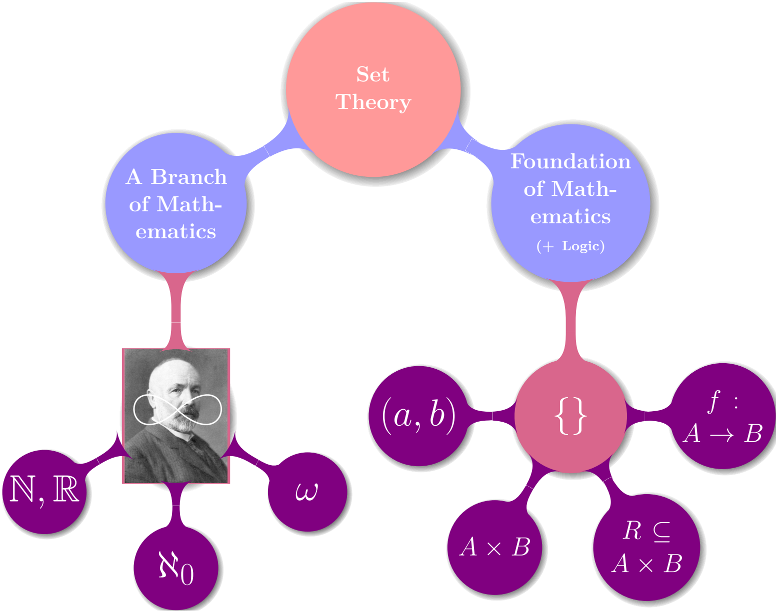
(四) 集合: 基本概念与运算 (Naive Set Theory)

魏恒峰

hfwei@nju.edu.cn

2021 年 04 月 01 日



Set Theory
A Branch of Math-ematics
Foundation of Math-ematics
(+ Logic)
$\mathbb{N}, \mathbb{R}$
$\aleph_0$
ω
(a, b)
$\{\}$
$f : A \to B$
$A \times B$
$R \subseteq A \times B$

Definition (集合)

集合就是任何一个有明确定义的对象的整体。

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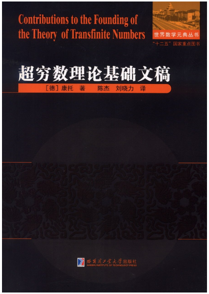

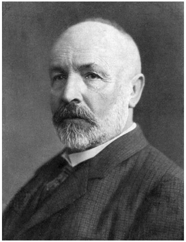

Georg Cantor (1845–1918)

Definition (集合)

我们将集合理解为任何将我们思想中那些确定而彼此独立的对象放在一起而形成的聚合。

Theorem (概括原则)

对于任意性质/谓词 $P(x)$, 都存在一个集合 X:

$$X = \{x \mid P(x)\}$$

$$A = \{2, 3, 5, 7\}$$

$$B = \{x \mid x < 10 \land \text{Prime}(x)\}$$

$$C = \{x \mid x \text{ 是方程 } x^4 - 17x^3 + 101x^2 - 247x + 210 = 0 \text{ 的根}\}$$

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$$3 \in A$$

Definition (外延性原理 (Extensionality))

两个集合相等当且仅当它们有相同的元素。

$$\forall A.\, \forall B.\, \Big(\forall x.\, (x \in A \leftrightarrow x \in B) \leftrightarrow A = B\Big)$$

集合完全由它的元素决定

Definition (子集)

设 A、B 是任意两个集合。

$A \subseteq B$ 表示 A 是 B 的子集 (subset)

$$A \subseteq B \iff \forall x \in A. (x \in A \to x \in B)$$

$A \subset B$ 表示 A 是 B 的真子集 (proper subset)

$$A \subset B \iff A \subseteq B \land A \neq B$$

$$\{1,2\} \subseteq \{1,2,3\} \qquad \{1,2\} \subset \{1,2,3\} \qquad \{1,4\} \neg \subseteq \{1,2,3\}$$

Theorem

两个集合相等当且仅当它们互为子集。

$$A = B \iff A \subseteq B \land B \subseteq A$$

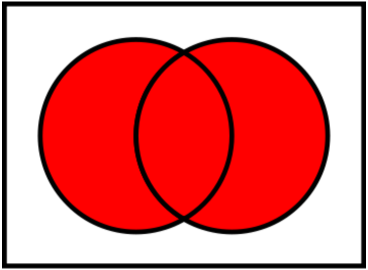
这是证明两个集合相等的常用方法

集合的运算 (I)

$$\cap \qquad \cup \qquad \setminus \qquad \Delta$$

Definition (集合的并 (Union))

$$A \cup B \triangleq \{x \mid x \in A \vee x \in B\}$$



$$A \cup \emptyset = A$$

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$$A \cup B = B \cup A$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

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$$A \cup A = A$$

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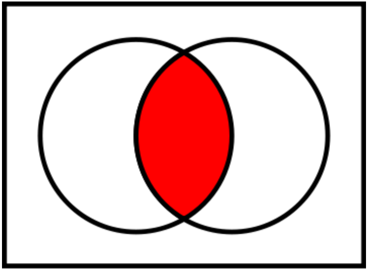
$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$A \subseteq A \cup B$$

$$B \subseteq A \cup B$$

Definition (集合的交 (Intersection))

$$A \cup B \triangleq \{x \mid x \in A \wedge x \in B\}$$



$$A \cap \emptyset = \emptyset$$

$$A \cap \emptyset = \emptyset$$

$$A \cap A = A$$

$$A \cap B = B \cap A$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$A \cap \emptyset = \emptyset$$

$$A \cap A = A$$

$$A \cap B = B \cap A$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$A \cap B \subseteq A$$

$$A \cap B \subseteq B$$

Theorem (分配律 (Distributive Law))

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Theorem (分配律 (Distributive Law))

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Proof.

If $x \in A \cup (B \cap C)$, then $x \in A$ or $x \in B \cap C$. Suppose first that $x \in A$. Then $x \in A \cup B$ and $x \in A \cup C$. In this first case, we see that $x \in (A \cup B) \cap (A \cup C)$. Now suppose that $x \in B \cap C$. Then $x \in B$ and $x \in C$. Since $x \in B$, we see that $x \in A \cup B$. Since we also have $x \in C$, we see that $x \in A \cup C$. Therefore, $x \in (A \cup B) \cap (A \cup C)$ in this case as well. In either case $x \in (A \cup B) \cap (A \cup C)$ and we may conclude that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

To complete the proof, we must now show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. So if $x \in (A \cup B) \cap (A \cup C)$, then $x \in A \cup B$ and $x \in A \cup C$. It is, once again, helpful to break this into two cases, since we know that either $x \in A$ or $x \notin A$. Now if $x \in A$, then $x \in A \cup (B \cap C)$. If $x \notin A$, then the fact that $x \in A \cup B$ implies that x must be in B. Similarly, the fact that $x \in A \cup C$ implies that x must be in C. Therefore, $x \in B \cap C$. Hence $x \in A \cup (B \cap C)$. In either case $x \in A \cup (B \cap C)$ and we may conclude that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

Since we proved containment in both directions we may conclude that the two sets are equal. ■

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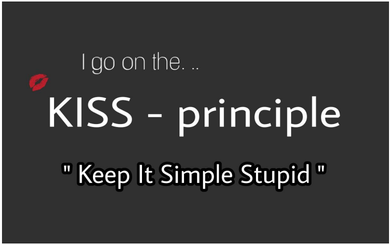


Theorem (分配律 (Distributive Law))

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proof.

对于任意 x,

$$x \in A \cup (B \cap C) \tag{1}$$

□

Theorem

吸收律 *(Absorption Law)*

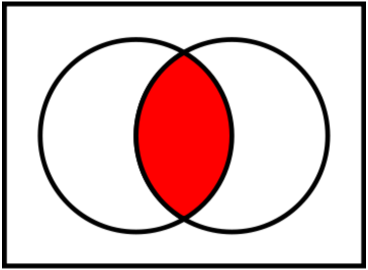
$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Theorem

$$A \subseteq B \iff A \cup B = B \iff A \cap B = A$$

Definition (集合的差 (Set Difference))



Theorem (DeMorgan's Law (Theorem 7.4 (15)))

Let X denote a set, and $A, B \subseteq X$.

$$X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$$

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$$X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$$

$$Q : A, B \subseteq X?$$

Theorem (DeMorgan's Law)

Let A, B, C be three sets.

$$C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$$

Definition (对称差 (Symmetric Difference))

$$A \,\Delta\, B = (A \setminus B) \cup (B \setminus A)$$

$$A \,\Delta\, B = \{x \mid (x \in A) \oplus (x \in B)\}$$

Definition (对称差 (Symmetric Difference))

$$A \mathbin{\Delta} B = (A \setminus B) \cup (B \setminus A)$$

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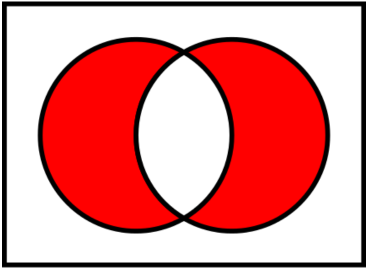

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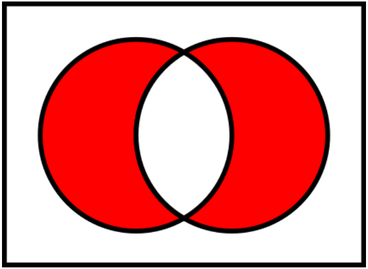
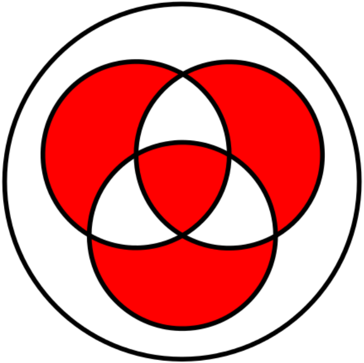


$$A \,\Delta\, B = (A \cup B) \setminus (A \cap B)$$

$$A \mathbin{\Delta} B \mathbin{\Delta} C$$

$$A \,\Delta\, B \,\Delta\, C$$



$$A \oplus \emptyset = A$$

$$A \oplus \emptyset = A$$

$$A \oplus A = \emptyset$$

$$A \oplus B = B \oplus A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

Proof.

$\square$

$$A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$$

$$A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$$

$$A \cup (B \oplus C) \neq (A \cup B) \oplus (A \cup C)$$

$$A \cup B = A \cup C \implies B = C$$

$$A \cap B = A \cap C \implies B = C$$

$$A \oplus B = A \oplus C \implies B = C$$

Theorem 7.4. *Let X denote a set, and A, B, and C denote subsets of X. Then*

1. $\emptyset \subseteq A$ and $A \subseteq A$.
2. $(A^c)^c = A$.
3. $A \cup \emptyset = A$.
4. $A \cap \emptyset = \emptyset$.
5. $A \cap A = A$.
6. $A \cup A = A$.
7. $A \cap B = B \cap A$. (Commutative property)
8. $A \cup B = B \cup A$. (Commutative property)
9. $(A \cup B) \cup C = A \cup (B \cup C)$. (Associative property)
10. $(A \cap B) \cap C = A \cap (B \cap C)$. (Associative property)
11. $A \cap B \subseteq A$.
12. $A \subseteq A \cup B$.
13. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (Distributive property)
14. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. (Distributive property)
15. $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$. (DeMorgan's law)
    (When X is the universe we also write $(A \cup B)^c = A^c \cap B^c$.)
16. $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$. (DeMorgan's law)
    (When X is the universe we also write $(A \cap B)^c = A^c \cup B^c$.)
17. $A \setminus B = A \cap B^c$.
18. $A \subseteq B$ if and only if $(X \setminus B) \subseteq (X \setminus A)$.
    (When X is the universe we also write $A \subseteq B$ if and only if $B^c \subseteq A^c$.)
19. $A \subseteq C$ and $B \subseteq C$ if and only if $A \cup B \subseteq C$.
20. $C \subseteq A$ and $C \subseteq B$ if and only if $C \subseteq A \cap B$.
21. $A \cup B = A$ if and only if $B \subseteq A$.
22. $A \cap B = B$ if and only if $B \subseteq A$.

集合的运算 (II)

$\cap \quad \cup$

$$\bigcup_{j=1}^{n} A_j \triangleq A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{j=1}^{n} A_j \triangleq A_1 \cap A_2 \cap \cdots \cap A_n$$

$$\bigcup_{j=1}^{n} A_j \triangleq A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{j=1}^{n} A_j \triangleq A_1 \cap A_2 \cap \cdots \cap A_n$$

$$\bigcup_{j=1}^{\infty} A_j \triangleq A_1 \cup A_2 \cup \cdots \qquad \bigcap_{j=1}^{\infty} A_j \triangleq A_1 \cap A_2 \cap \cdots$$

$$\bigcup_{j=1}^{n} A_j \triangleq A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{j=1}^{n} A_j \triangleq A_1 \cap A_2 \cap \cdots \cap A_n$$

$$\bigcup_{j=1}^{\infty} A_j \triangleq A_1 \cup A_2 \cup \cdots \qquad \bigcap_{j=1}^{\infty} A_j \triangleq A_1 \cap A_2 \cap \cdots$$

$$\bigcup_{\alpha \in I} A_\alpha \triangleq \Big\{x \mid \exists \alpha \in I : x \in A_\alpha\Big\} \qquad \bigcap_{\alpha \in I} A_\alpha \triangleq \Big\{x \mid \forall \alpha \in I : x \in A_\alpha\Big\}$$

Theorem (DeMorgan's Law (UD Exercise 8.9))

$$X \setminus \bigcup_{\alpha \in I} A_\alpha = \bigcap_{\alpha \in I} (X \setminus A_\alpha)$$

$$X \setminus \bigcap_{\alpha \in I} A_\alpha = \bigcup_{\alpha \in I} (X \setminus A_\alpha)$$

Theorem (DeMorgan's Law (UD Exercise 8.9))

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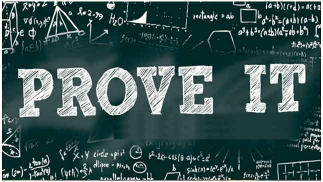

DeMorgan's Law (UD Problem 8.14)

$$A = \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus \{-n, -n+1, \cdots, 0, \cdots, n-1, n\})$$

DeMorgan's Law (UD Problem 8.14)

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$$X_n = \{-n, -n+1, \cdots, 0, \cdots, n-1, n\}$$

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DeMorgan's Law (UD Problem 8.14)

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$$X_n = \{-n, -n+1, \cdots, 0, \cdots, n-1, n\}$$

$$\begin{aligned} A &= \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus X_n) \\ &= \mathbb{R} \setminus \left(\mathbb{R} \setminus \bigcup_{n \in \mathbb{Z}^+} X_n\right) \end{aligned}$$

DeMorgan's Law (UD Problem 8.14)

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集合的运算 (III)

$\mathcal{P}(X)$

$$\boxed{S \in \mathcal{P}(X) \iff S \subseteq X}$$

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))$$

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))$$

$$\{\emptyset\} \in \mathcal{P}(\mathcal{P}(S))$$

$$\emptyset \in \mathcal{P}(\mathcal{P}(S))$$

□

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))$$

$$\{\emptyset\} \in \mathcal{P}(\mathcal{P}(S))$$

$$\emptyset \in \mathcal{P}(\mathcal{P}(S))$$

$$\iff \emptyset \subseteq \mathcal{P}(S)$$

$\square$

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))$$

$$\{\emptyset\} \in \mathcal{P}(\mathcal{P}(S))$$

$$\emptyset \in \mathcal{P}(\mathcal{P}(S)) \qquad \iff \{\emptyset\} \subseteq \mathcal{P}(S)$$

$$\iff \emptyset \subseteq \mathcal{P}(S)$$

□

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\frac{\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))}{\{\emptyset\} \in \mathcal{P}(\mathcal{P}(S))}$$

$$\emptyset \in \mathcal{P}(\mathcal{P}(S)) \qquad \iff \{\emptyset\} \subseteq \mathcal{P}(S)$$

$$\iff \emptyset \subseteq \mathcal{P}(S) \qquad \iff \emptyset \in \mathcal{P}(S)$$

□

Prove that for any set S:

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$$

Proof.

$$\{\emptyset, \{\emptyset\}\} \in \mathcal{P}(\mathcal{P}(\mathcal{P}(S))) \iff \{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\mathcal{P}(S))$$

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$$\iff \emptyset \subseteq \mathcal{P}(S) \qquad \iff \emptyset \in \mathcal{P}(S)$$

$$\iff \emptyset \subseteq S$$

□

UD Exercise 9.4

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

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$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

Proof.

$$x \in \mathcal{P}(A) \cap \mathcal{P}(B)$$

□

UD Exercise 9.4

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

Proof.

$$\begin{aligned} & x \in \mathcal{P}(A) \cap \mathcal{P}(B) \\ \iff & x \in \mathcal{P}(A) \wedge x \in \mathcal{P}(B) \end{aligned}$$

□

UD Exercise 9.4

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

Proof.

$$\begin{aligned} & x \in \mathcal{P}(A) \cap \mathcal{P}(B) \\ \iff & x \in \mathcal{P}(A) \land x \in \mathcal{P}(B) \\ \iff & x \subseteq A \land x \subseteq B \end{aligned}$$

□

UD Exercise 9.4

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

Proof.

$$\begin{aligned}
& x \in \mathcal{P}(A) \cap \mathcal{P}(B) \\
\iff & x \in \mathcal{P}(A) \land x \in \mathcal{P}(B) \\
\iff & x \subseteq A \land x \subseteq B \\
\iff & x \subseteq A \cap B
\end{aligned}$$

□

UD Exercise 9.4

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$

Proof.

$$\begin{aligned}
& x \in \mathcal{P}(A) \cap \mathcal{P}(B) \\
\iff & x \in \mathcal{P}(A) \land x \in \mathcal{P}(B) \\
\iff & x \subseteq A \land x \subseteq B \\
\iff & x \subseteq A \cap B \\
\iff & x \in \mathcal{P}(A \cap B)
\end{aligned}$$

□

UD Problem 9.9

$$\bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) = \mathcal{P}\Big(\bigcap_{\alpha \in I} A_\alpha\Big)$$

Proof.

□

UD Problem 9.9

$$\bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) = \mathcal{P}(\bigcap_{\alpha \in I} A_\alpha)$$

Proof.

$$x \in \bigcap_{\alpha \in I} \mathcal{P}(A_\alpha)$$

□

UD Problem 9.9

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Proof.

$$x \in \bigcap_{\alpha \in I} \mathcal{P}(A_\alpha)$$

$$\iff \forall \alpha \in I : x \in \mathcal{P}(A_\alpha)$$

$\square$

UD Problem 9.9

$$\bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) = \mathcal{P}(\bigcap_{\alpha \in I} A_\alpha)$$

Proof.

$$\begin{aligned} & x \in \bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) \\ \iff & \forall \alpha \in I : x \in \mathcal{P}(A_\alpha) \\ \iff & \forall \alpha \in I : x \subseteq A_\alpha \end{aligned}$$

□

UD Problem 9.9

$$\bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) = \mathcal{P}(\bigcap_{\alpha \in I} A_\alpha)$$

Proof.

$$\begin{aligned}
& x \in \bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) \\
\iff & \forall \alpha \in I : x \in \mathcal{P}(A_\alpha) \\
\iff & \forall \alpha \in I : x \subseteq A_\alpha \\
\iff & x \subseteq \bigcap_{\alpha \in I} A_\alpha
\end{aligned}$$

□

UD Problem 9.9

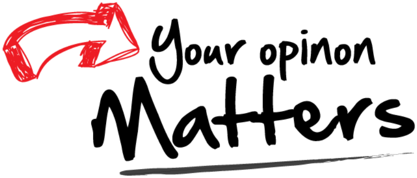
$$\bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) = \mathcal{P}(\bigcap_{\alpha \in I} A_\alpha)$$

Proof.

$$\begin{aligned}
& x \in \bigcap_{\alpha \in I} \mathcal{P}(A_\alpha) \\
\iff & \forall \alpha \in I : x \in \mathcal{P}(A_\alpha) \\
\iff & \forall \alpha \in I : x \subseteq A_\alpha \\
\iff & x \subseteq \bigcap_{\alpha \in I} A_\alpha \\
\iff & x \in \mathcal{P}(\bigcap_{\alpha \in I} A_\alpha)
\end{aligned}$$

□


Thank
You!



Office 926

hfwei@nju.edu.cn