(十一) 图论: 平面图与图着色
(Planarity and Coloring)

魏恒峰

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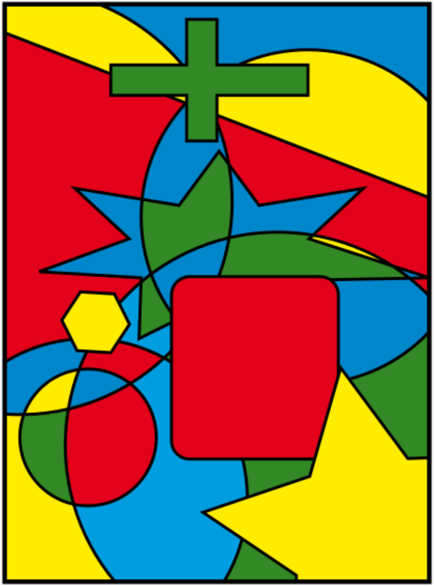
2021 年 05 月 20 日



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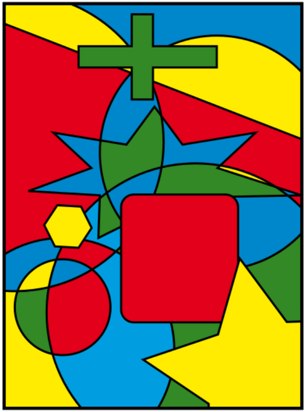

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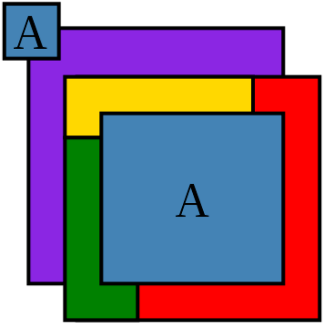

Regions should be contiguous.

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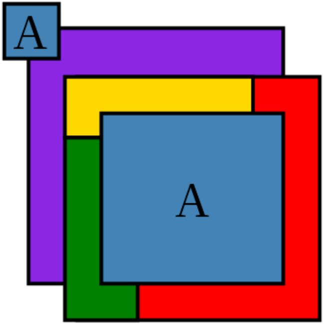

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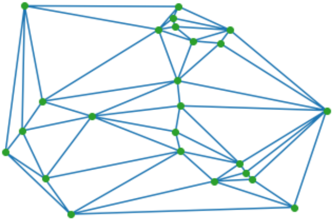

Adjacent regions share a segment.

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What if we have a map in which
every region is adjacent to ≥ 5 other regions?

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What does it to do with GRAPH THEORY?

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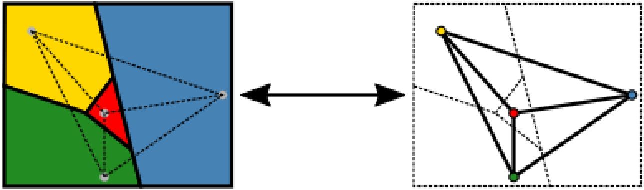

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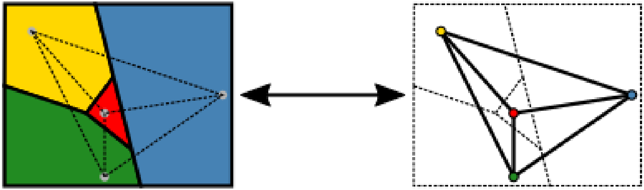

Theorem (Four Color Theorem (Appel and Haken, 1976))

Every simple planar graph is 4-colorable.

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I will *not* show its proof (which I don't understand either)!

Theorem

Every simple planar graph is 6-colorable.

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Theorem (Percy John Heawood)

Every simple planar graph is 5-colorable.

Definition (Planar Graph (平面图))

A planar graph is a graph that can be drawn in the plane without edge crossings.

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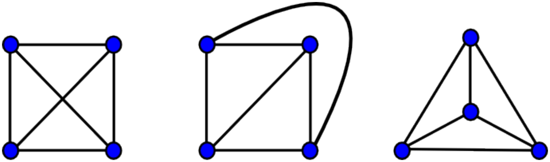

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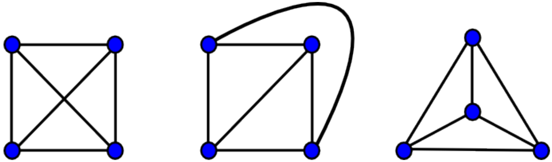
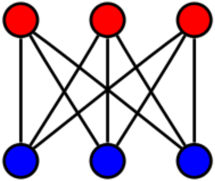


Theorem (K. Wagner (1936); I. Fáry (1948))

Every simple planar graph can be drawn with straight lines.

Theorem (Kazimierz Kuratowski, 1930)

The utility graph $K_{3,3}$ is non-planar.



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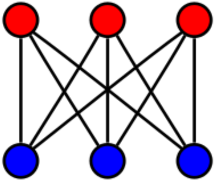

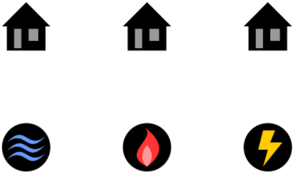

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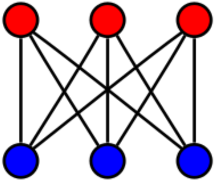

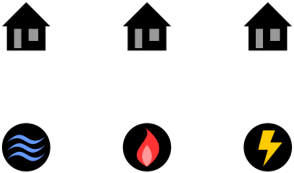

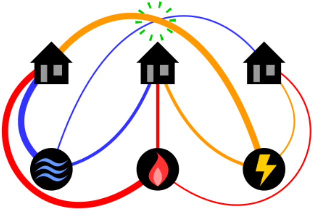

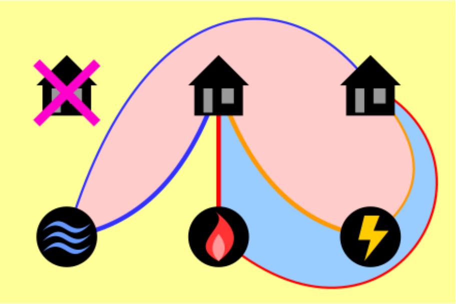

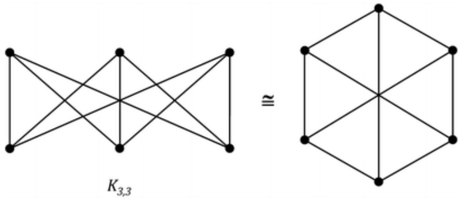
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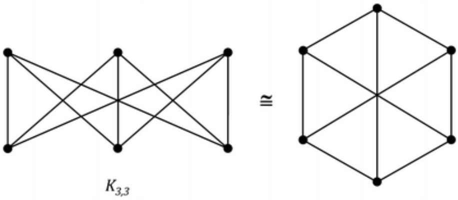
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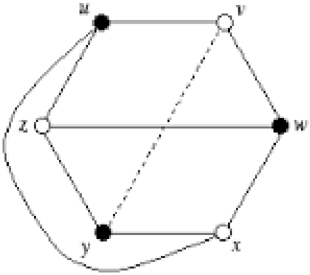
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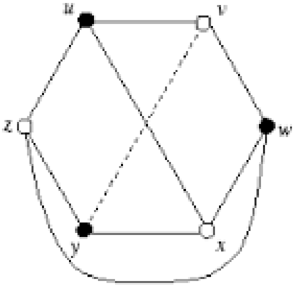
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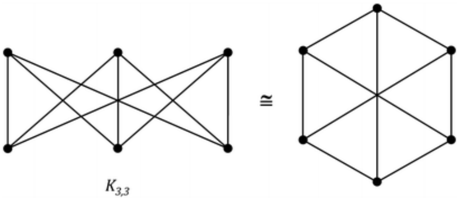
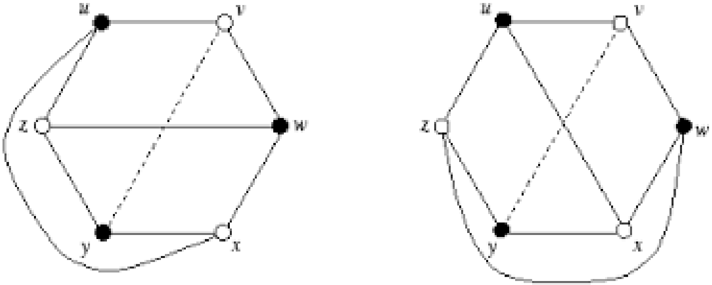
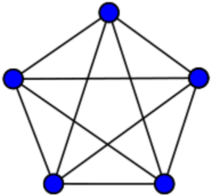




$$\textcolor{red}{\mathrm{cr}}(K_{3,3}) = 1$$

Theorem

K_5 is non-planar.



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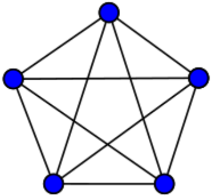

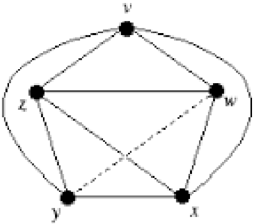

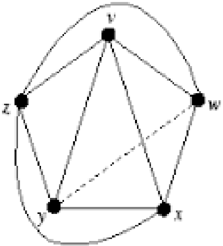

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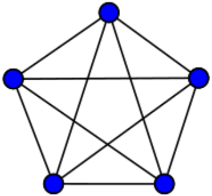

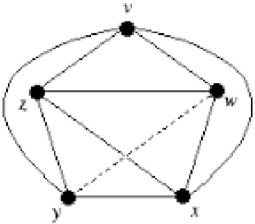

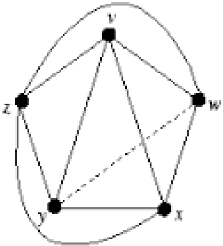

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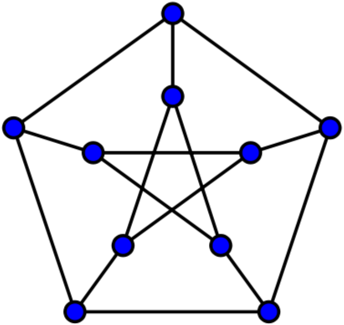
The Petersen graph is non-planar.



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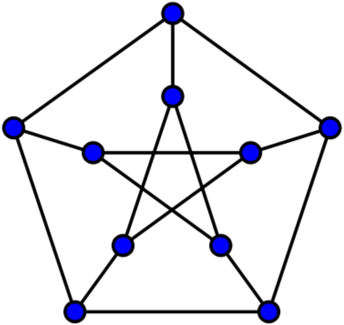

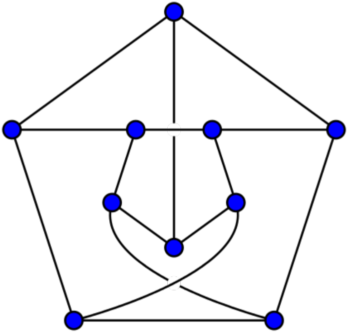

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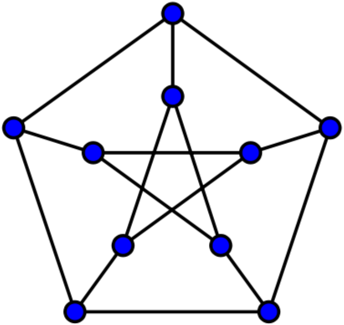

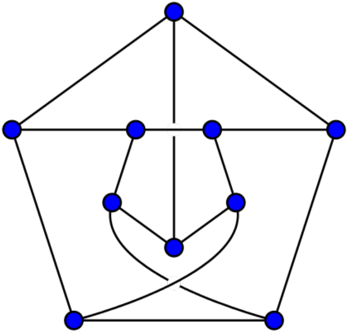

$$\textcolor{red}{\mathrm{cr}}(\text{Petersen Graph}) = 2$$

Theorem (Kazimierz Kuratowski, 1930)

A graph is planar iff it contains no subgraph homeomorphic to K_5 *or* $K_{3,3}$.

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"The K in K_5 *stands for Kazimierz,*
and the K in $K_{3,3}$ *stands for Kuratowski."*

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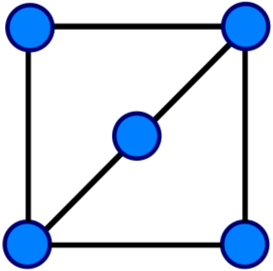

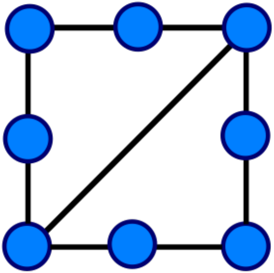

Definition (Homeomorphic)

Two graphs are homeomorphic if one can be obtained from another by inserting or contracting vertices of degree 2.

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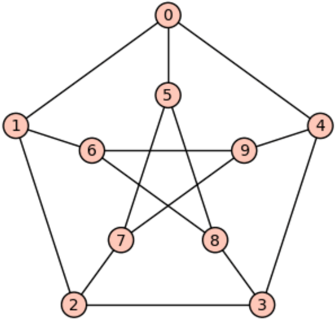

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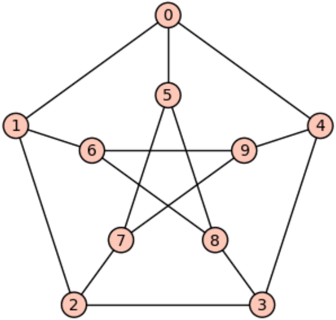

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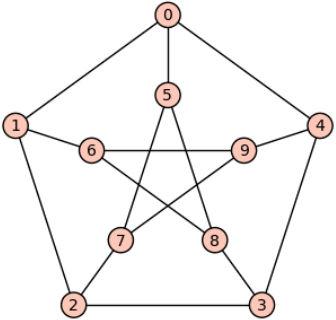

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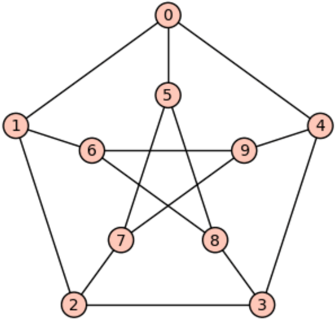

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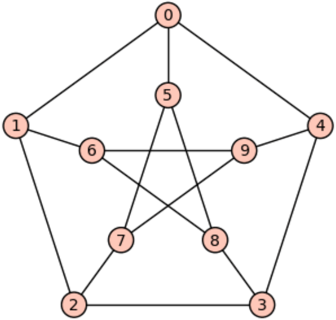

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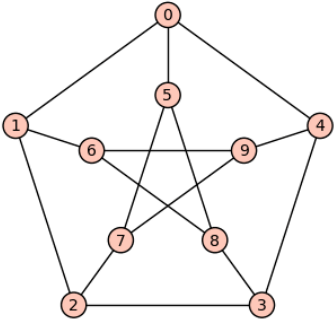

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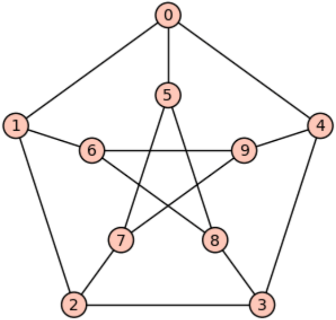

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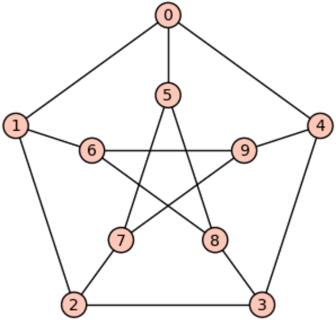

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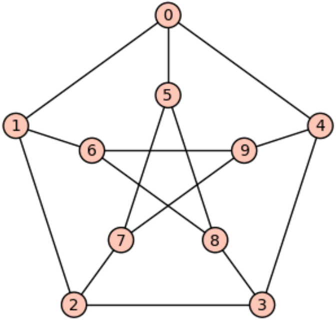

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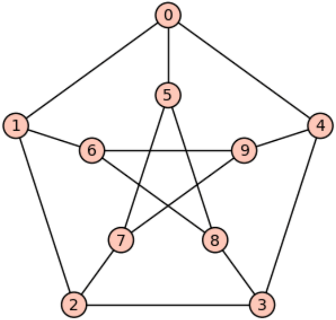

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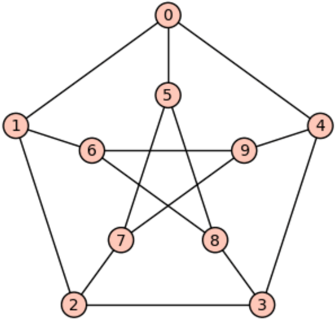

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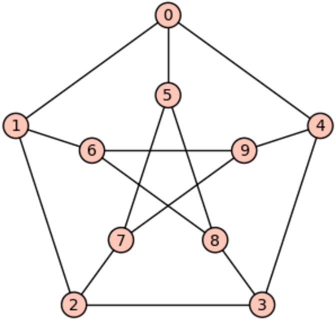

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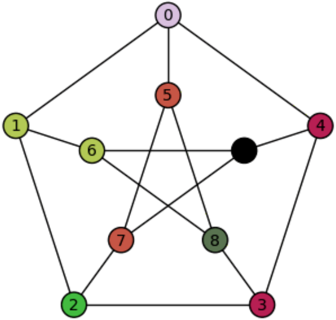

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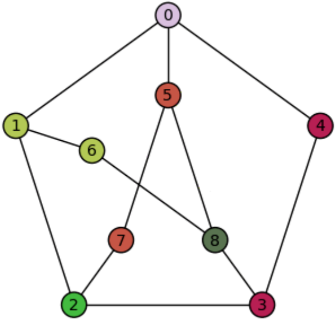

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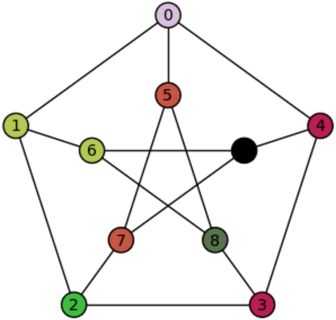

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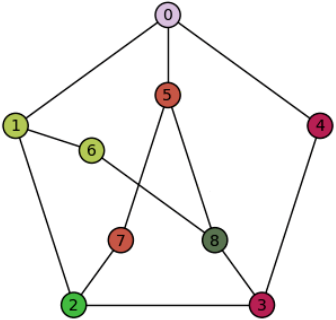

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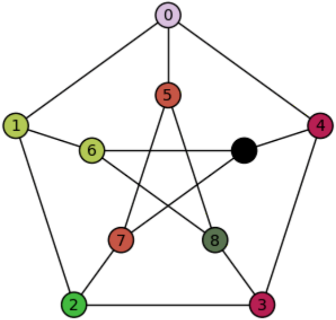

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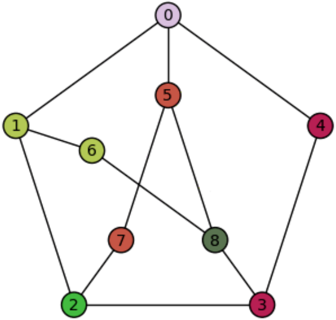

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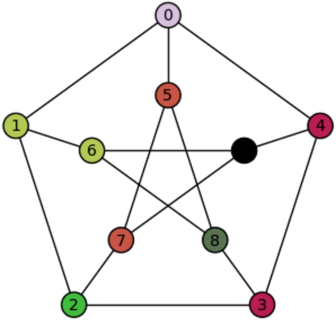

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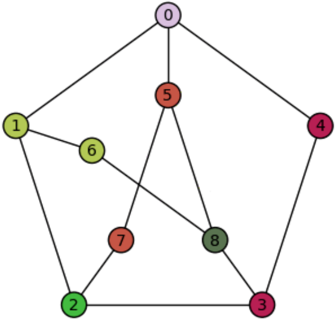

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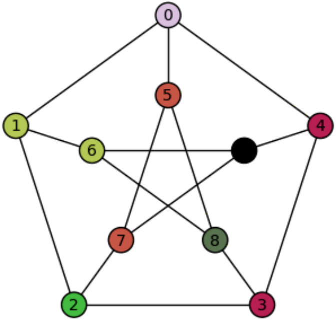

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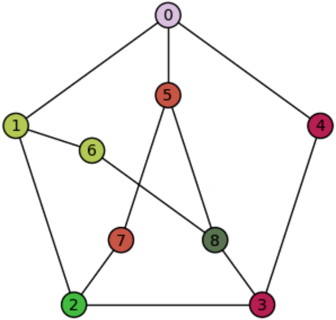

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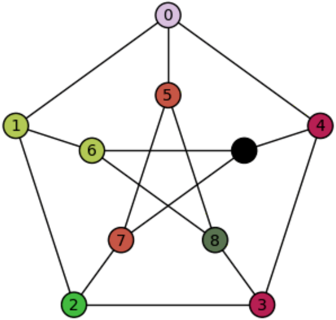

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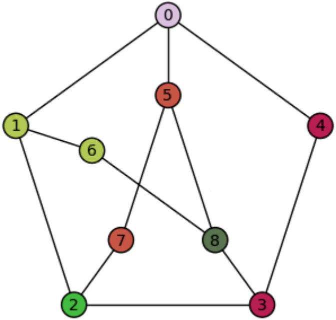

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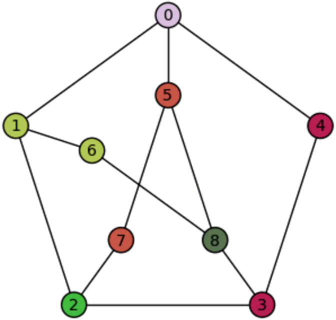

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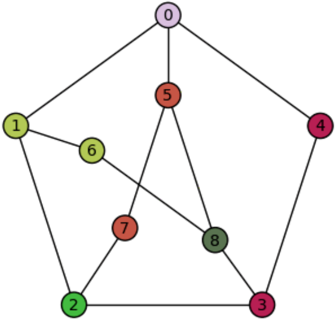

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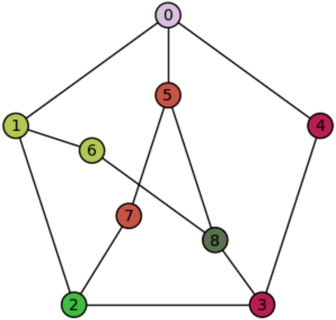

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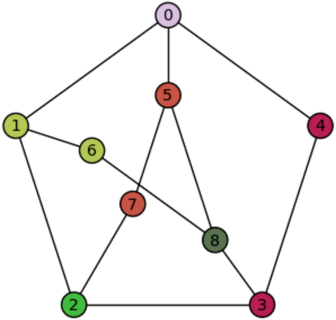

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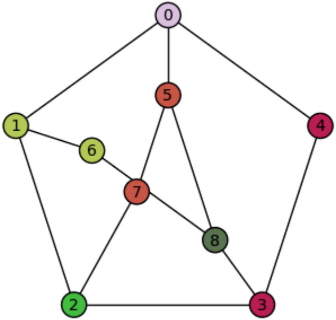

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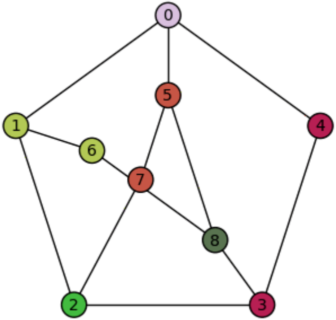

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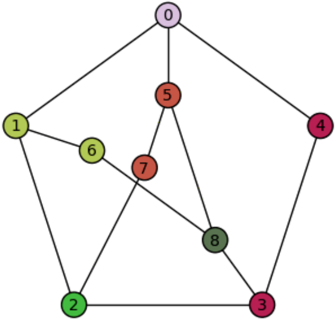

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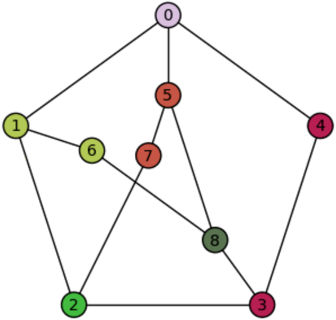

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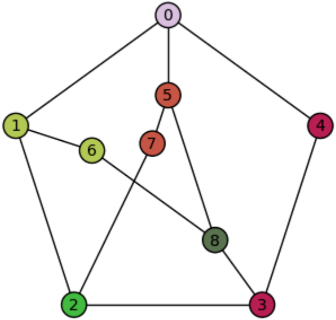

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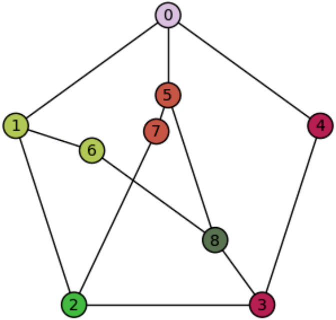

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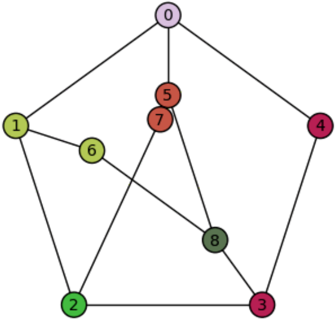

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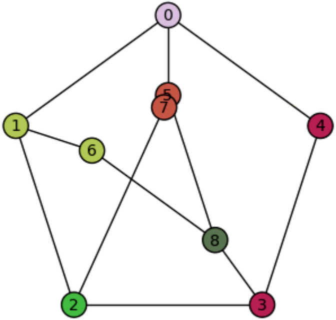

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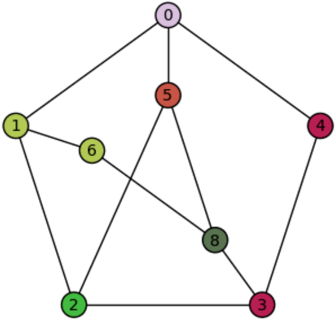

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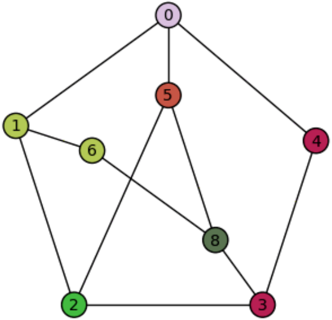

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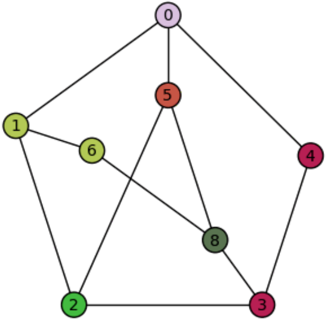

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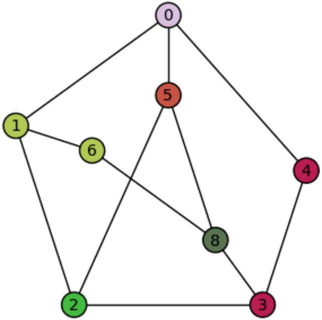

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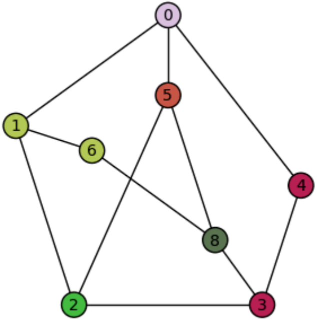

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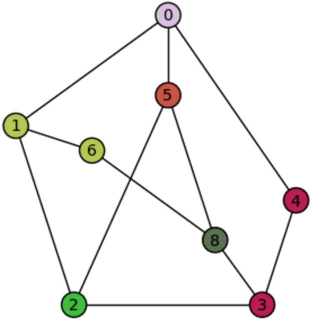

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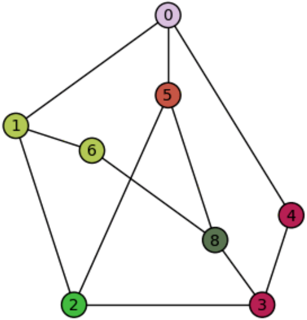

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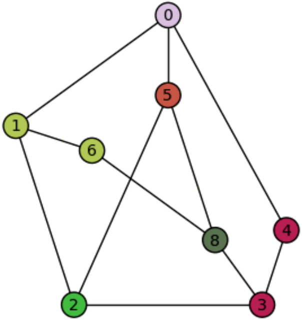

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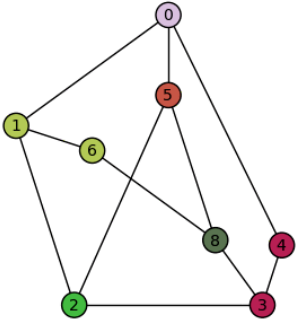

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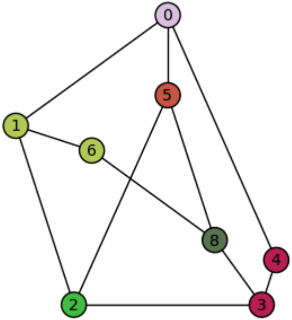

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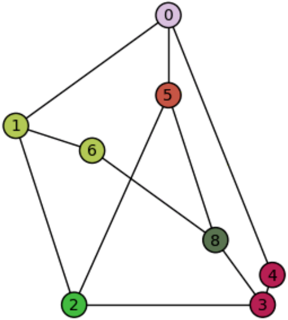

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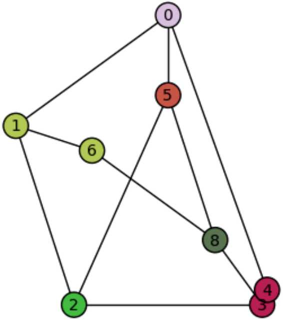

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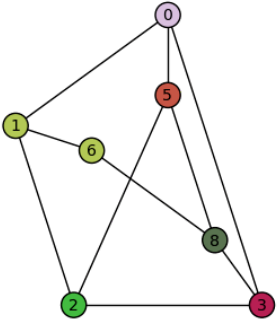

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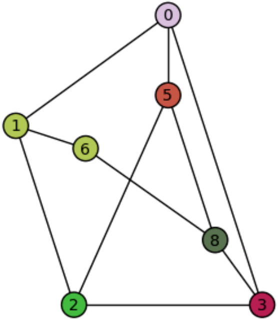

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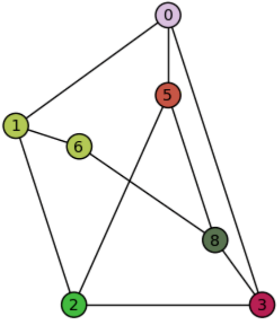

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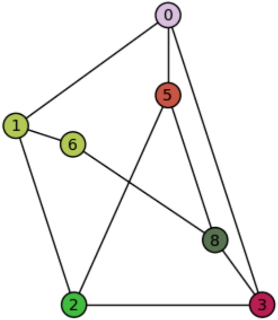

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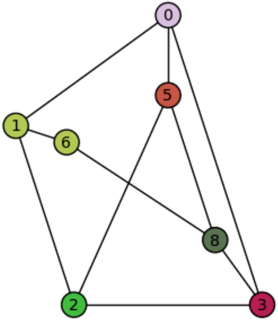

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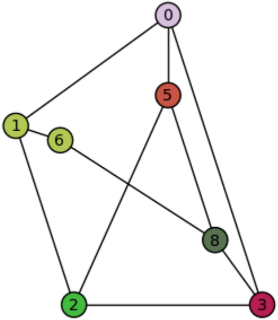

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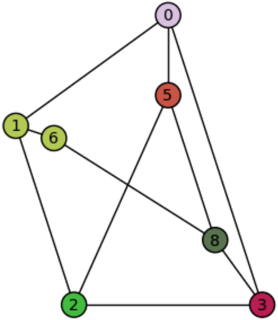

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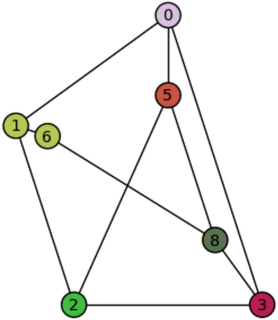

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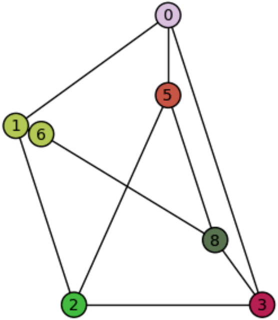

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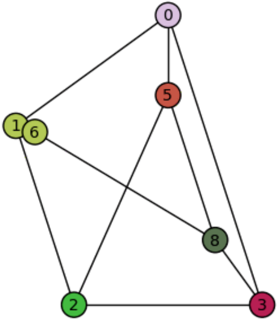

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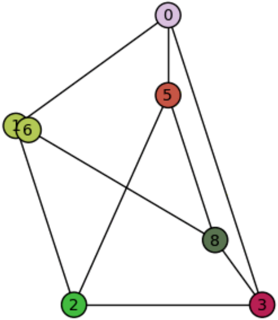

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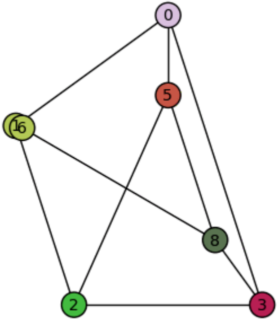

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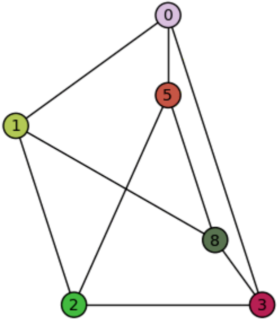

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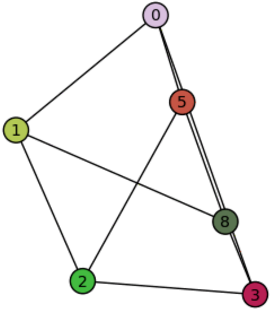

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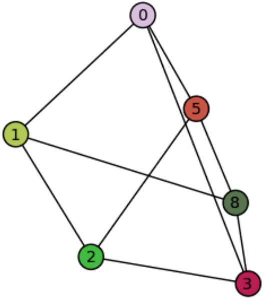

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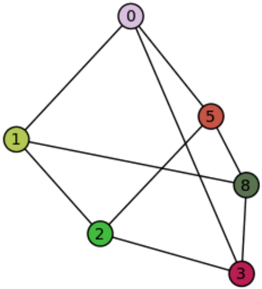

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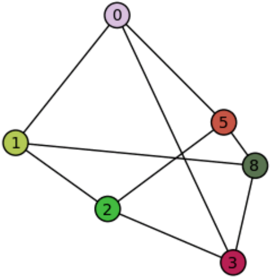

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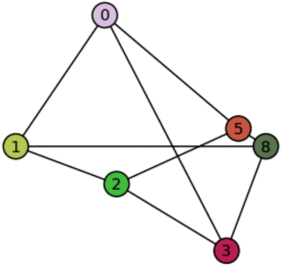

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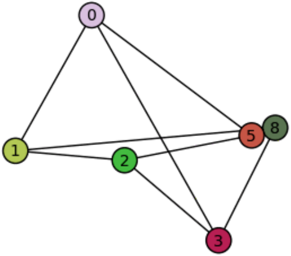

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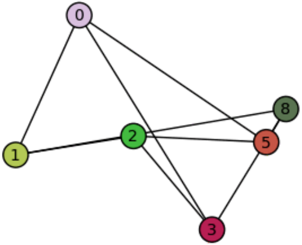

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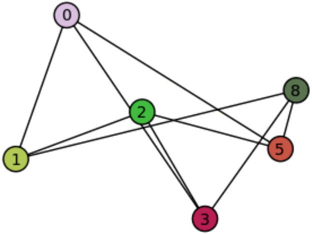

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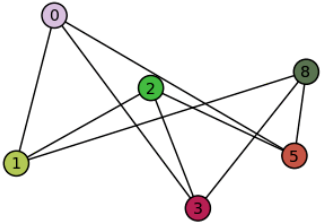

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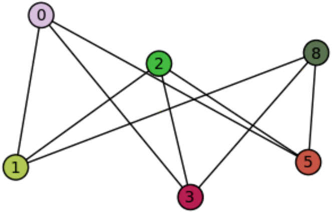

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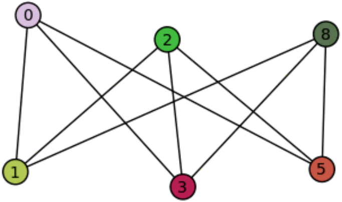

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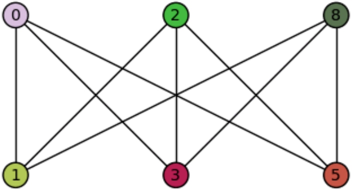

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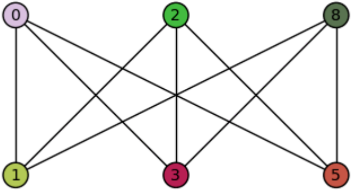

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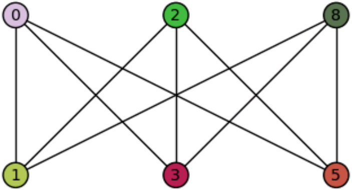

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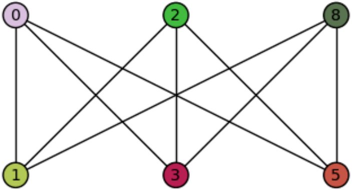

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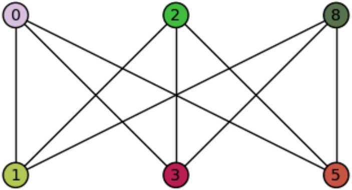

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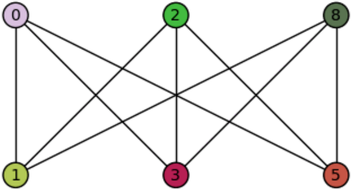

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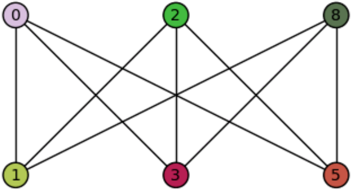

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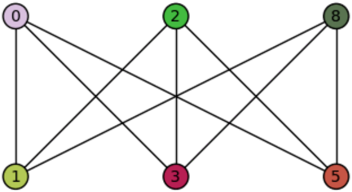

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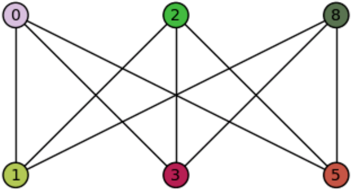

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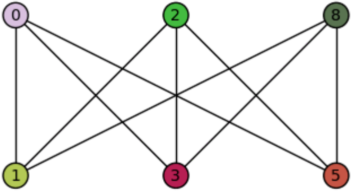

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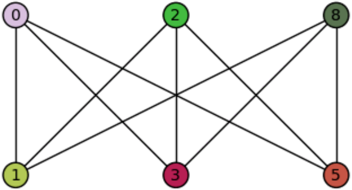

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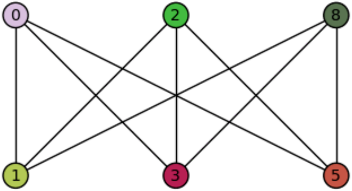
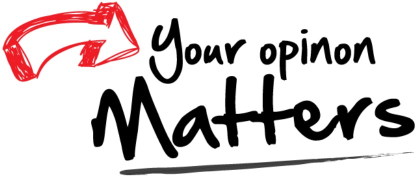



Thank You!



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