(十) 图论: 树 (Trees)

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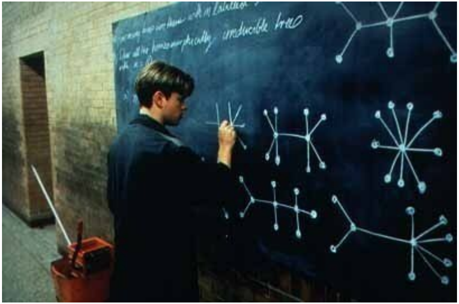
Rooted Trees in Computer Science

Definition (Rooted Trees (有根树))

bfs

dfs: in-order, pre-order, post-order

search trees



Theorem (Cayley's Formula)

The number T_n of labeled trees on $n \geq 2$ vertices is n^{n-2}.

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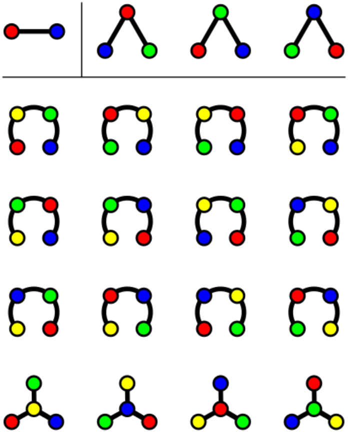
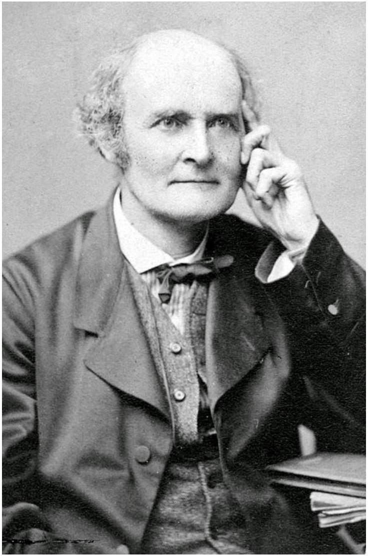
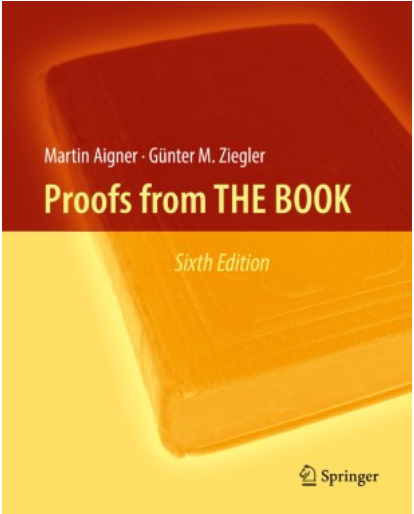




Arthur Cayley (1821 $\sim$ 1895)



Chapter 33: Cayley's formula for the number of trees

By Double Counting.

— Jim Pitman

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https://en.wikipedia.org/wiki/Double_counting_(proof_technique)#Counting_trees

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How many ways are there of forming a rooted tree from an empty graph by adding directed edges one by one?

Choose one of the T_n labeled trees on n vertices.

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Choose one of its n vertices as root.

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Choose one of the $(n-1)!$ possible sequences
in which to add its $n-1$ directed edges.

$$T_n n(n-1)! = T_n n!$$

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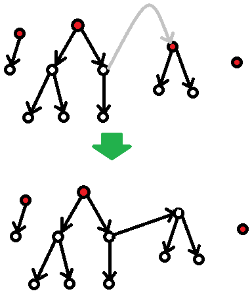

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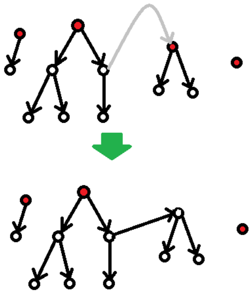

$$\prod_{k=2}^{n} n(k-1) = n^{n-1}(n-1)! = n^{n-2}n!$$

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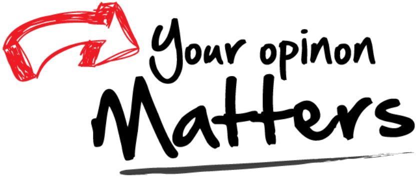
Thank
You!



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