(九) 图论: 路径与圈 (Paths and Cycles)

魏恒峰

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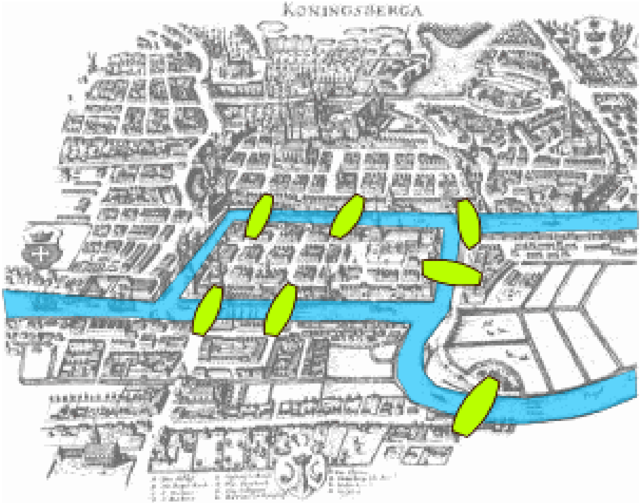
2021 年 05 月 06 日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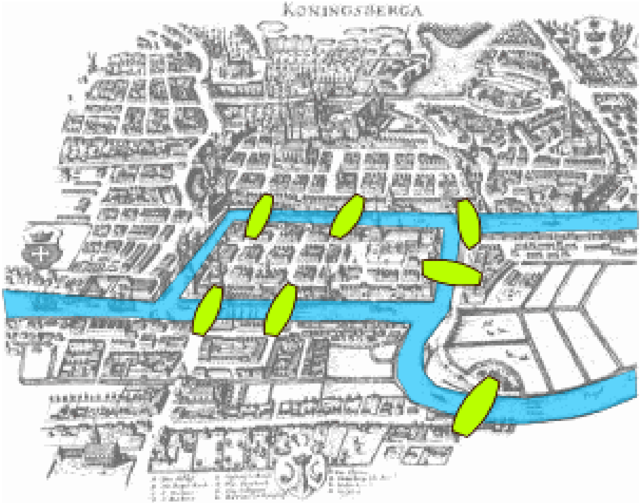
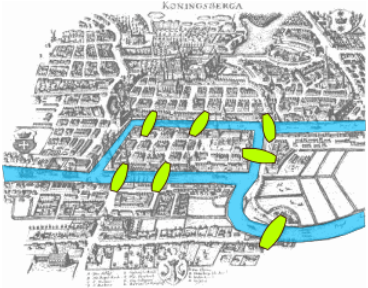


*"to devise a walk through the city
that would cross each of those bridges once and only once"*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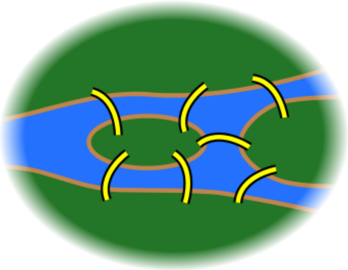

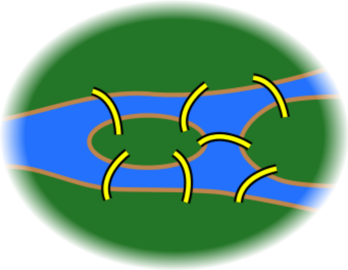

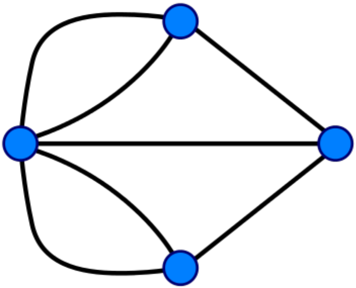

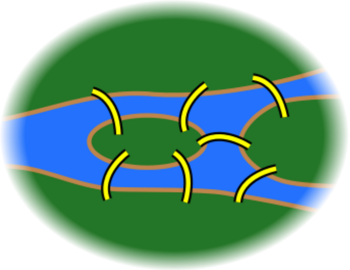 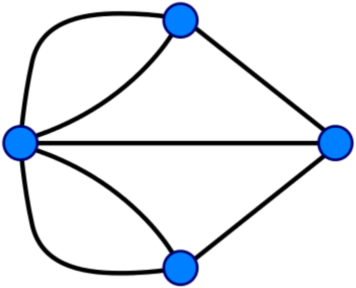

"to devise a walk through the graph
that would cross each of those edges once and only once"

Definition (Graph (图))

An (undirected simple) graph is a pair $G = (V, E)$ where

- V is a set of vertices (顶点);
- $E \subseteq \{\{x, y\} \mid x, y \in V \land x \neq y\}$ is a set of edges

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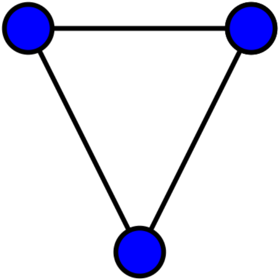
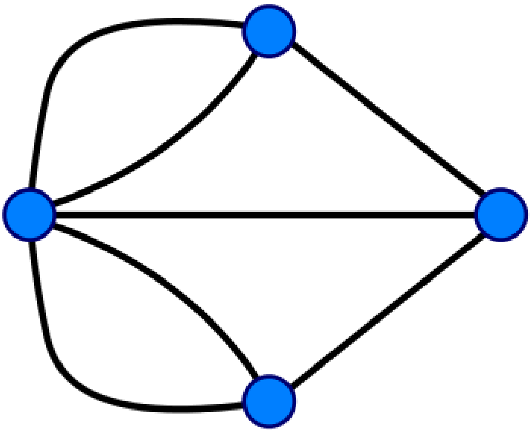
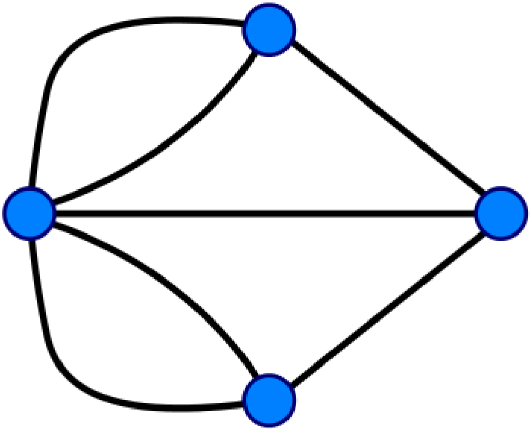
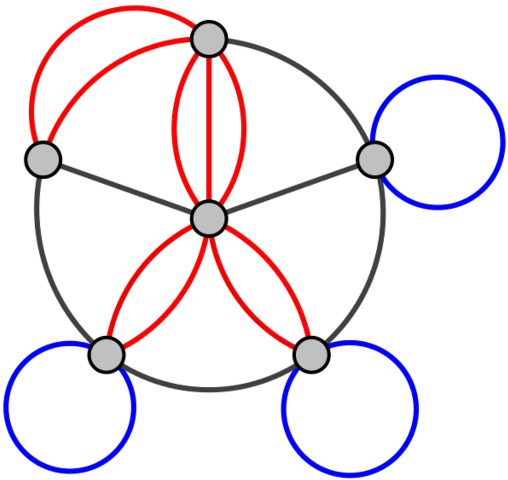
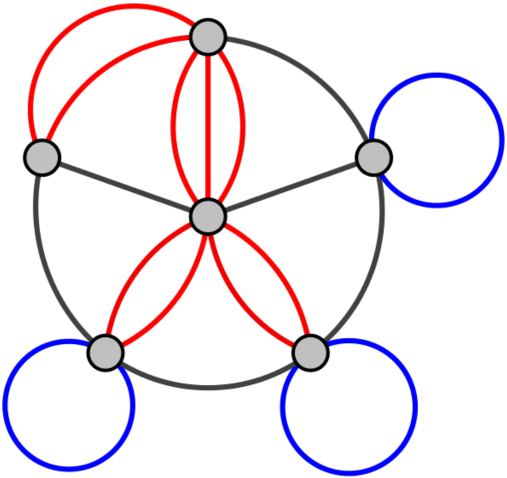






Undirected Multigraph





Undirected Multigraph Permitting Loops

Definition (Walk (道路))

Given a graph G, a (finite) walk in G is a sequence of edges of the form

$$\{v_0, v_1\}, \{v_1, v_2\}, \ldots, \{v_{m-1}, v_m\}.$$

$$(v_0 \to v_1 \to v_2 \to \cdots \to v_m)$$

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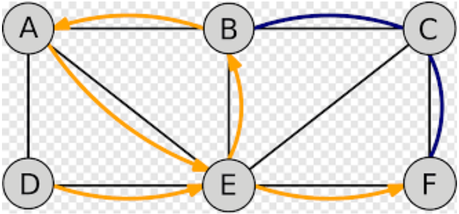
$$D \to E \to B \to A \to E \to F$$



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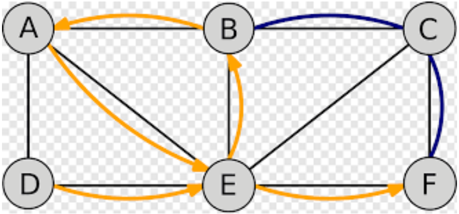

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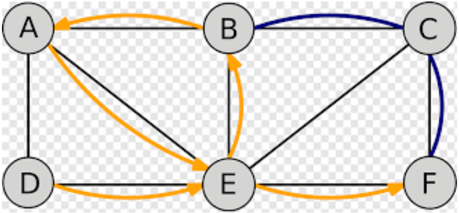

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A path is a trial in which all vertices are distinct, except possibly $v_0 = v_m$.

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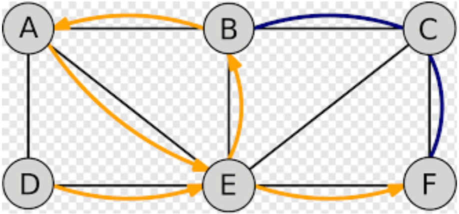

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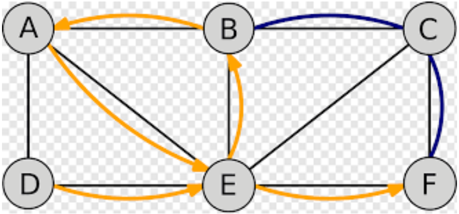

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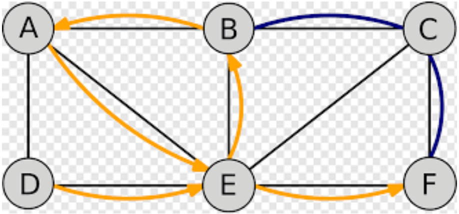
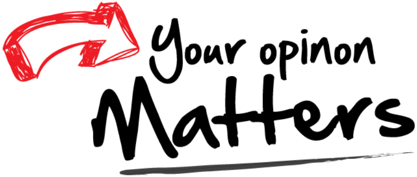


Definition (Cycle)

A cycle is a closed path with at least one edge.


Thank
You!



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