

离散数学概述

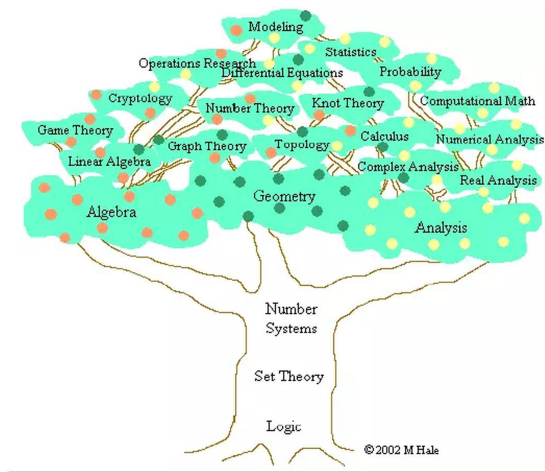
魏恒峰

hfwei@nju.edu.cn

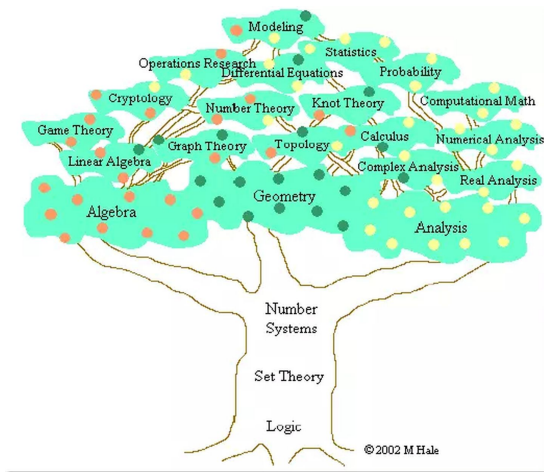
2021 年 03 月 04 日



“离散数学”在哪里？



“离散数学”在哪里？



离散数学是个大杂烩，啥都学点儿，啥都没学好

分班教学

(与计算机系赵建华老师)

授课内容与作业可能有出入, 不影响考试与成绩分配

平时作业 *vs.* 期中测试 *vs.* 期末测试

3 : 3 : 4

4 : 3 : 3

弹性制

每周四晚上发布作业 下周四 23:55 前提交作业

每次作业按 10 分计算

迟交: 周四**前**向助教登记, 可延长两天, 总分 8 分

(作业助教: 裴一凡、戴若石、肖江)

“教学立方” 课程邀请码: PLD8QKTZ



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TEX



[https://github.com/courses-at-nju-by-hfwei/
discrete-math-problem-sets](https://github.com/courses-at-nju-by-hfwei/discrete-math-problem-sets)

约法三章

非必要, 不点名

非必要, 不迟到

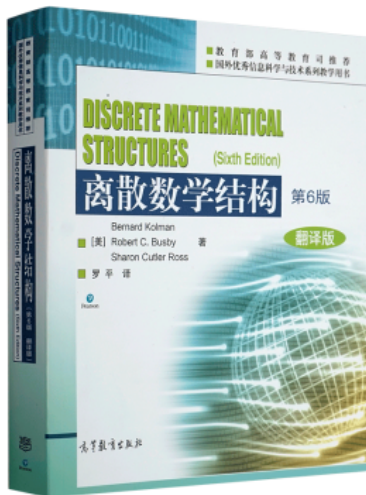
非必要，不迟到

尽量吃早餐，但不可以在教室吃早餐

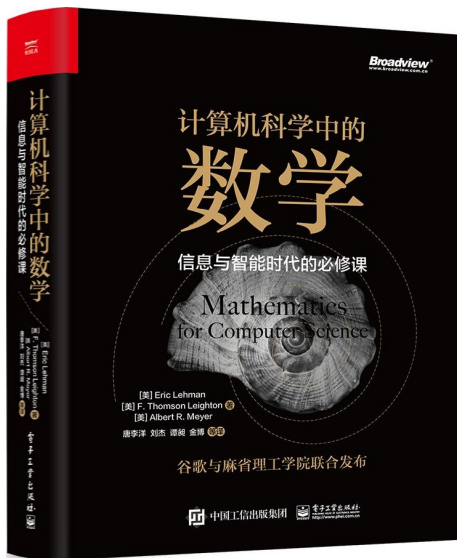
~~非/必/要/~~，不抄袭；一经发现，后果严重

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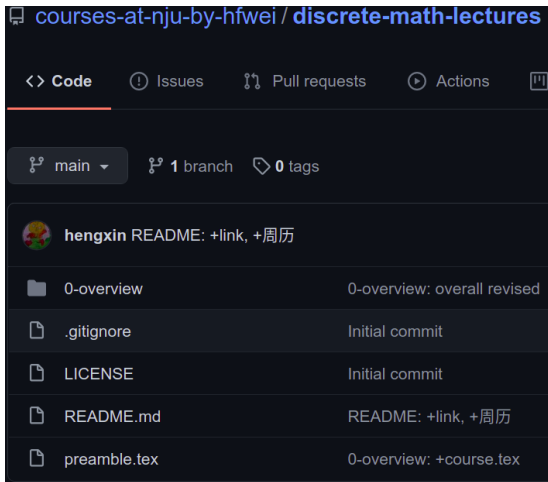
当次作业计 0 分；总评扣 10 分



教材不重要, 听讲更重要



其它参考书随课程进度安排



<https://github.com/courses-at-nju-by-hfwei/discrete-math-lectures>

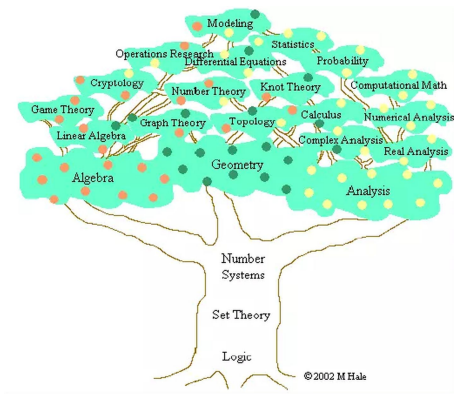


离散数学

离散数学

研究离散对象的结构、性质、操作等的数学分支 (大杂烩)

四大主题：逻辑、集合论、图论、抽象代数（群论）



支流遍布：组合与计数、数论、(离散) 概率

关于离散数学, 学长纷纷表示:

我太难了

啥用没有

真得有那么难吗?

真得有那么难吗?

确实蛮难的: 知识点多而分散、概念抽象

真得没啥用吗?

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力



Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

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Q : 这是构造性证明吗?

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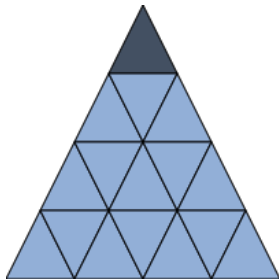
$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$$

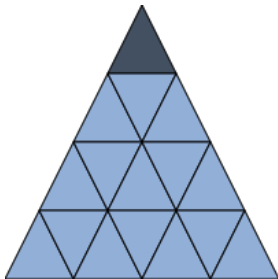
Q : 这是构造性证明吗? 这是反证法吗?

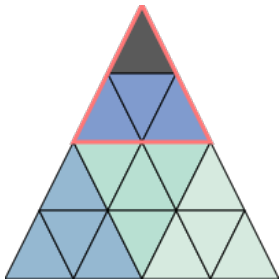
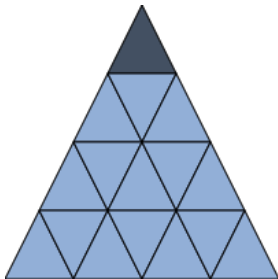
Tiling Puzzle

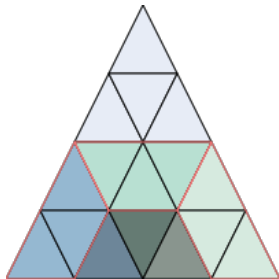
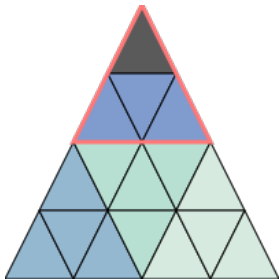
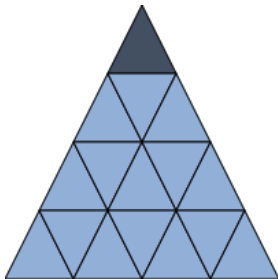
Suppose n is a positive integer. An equilateral triangle is cut into 4^n congruent equilateral triangles, and one corner is removed.

Show that the remaining area can be covered by tiles below









Base Case:

Induction Hypothesis:

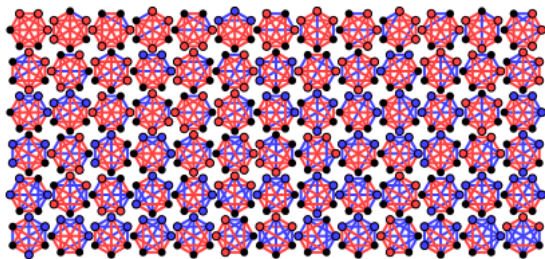
Induction Step: ... by induction hypothesis ...

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.

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In Terms of **Graph Theory**.

(Is there a **monochromatic** triangle in any 2-coloring of K_6 ?)

Theorem on Friends and Strangers

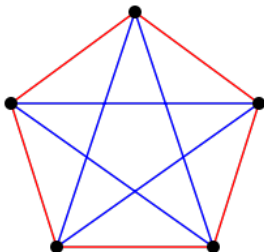
Is there a **monochromatic** triangle in any 2-coloring of K_6 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?



Ramsey theory

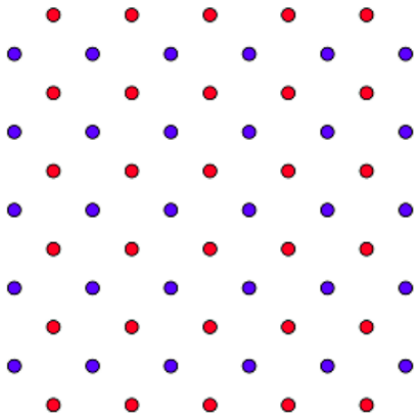
How **big** must the **structure** be
to ensure that it has a given interesting **property**?

Ramsey theory

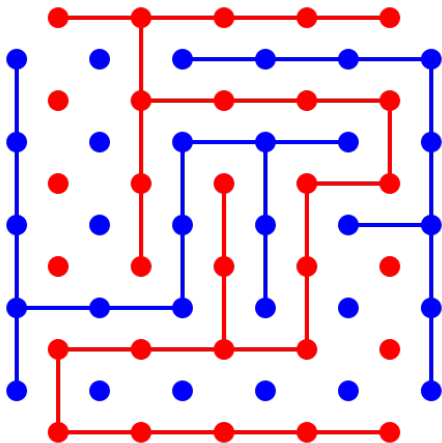
How **big** must the **structure** be
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Bridg-It Game (David Gale, 1958)



5×6 vs. 6×5



5×6 vs. 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie**?

Will Bridg-It **end in a tie**?

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

Does **Player 1** have a **winning strategy**?

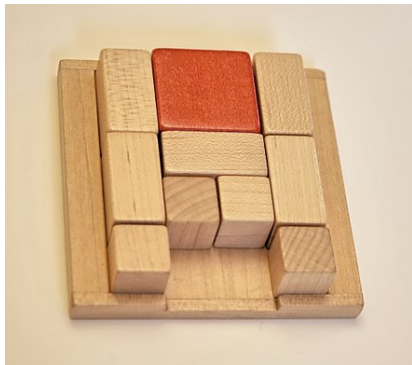
Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.



STAY TUNED

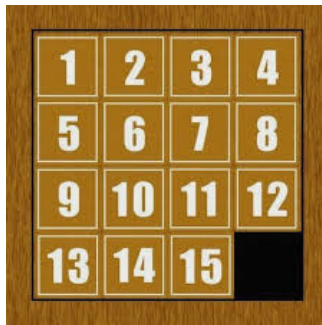
Klotski Puzzle (华容道)



Klotski Puzzle (华容道; 中国版本)



15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

How to solve it?

It uses **permutation groups** in **group theory**.



STAY TUNED

The Stable Marriage Problem (SMP)

Given n men and n women, where each person has a preference list, to establish a stable marriage.

Men $\{x, y, z, w\}$	Women $\{a, b, c, d\}$
$x : a > b > c > d$	$a : z > x > y > w$
$y : a > c > b > d$	$b : y > w > x > z$
$z : c > d > a > b$	$c : w > x > y > z$
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$$\{xb, yc, zd, wa\}$$

(x, a) is an unstable pair

$$\{xa, yb, zd, wc\}$$

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Theorem (The Gale-Shapley Algorithm (1962))

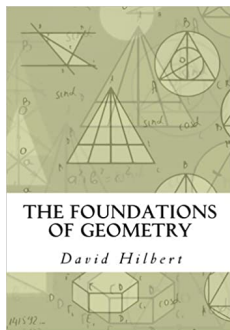
It is always possible to solve SMP.

Self-contained (自包含; 自给自足)

Axiomatic Systems

Syntax *vs.* Semantics (语法与语义对立统一)

三个公理系统: 逻辑、集合论、图论、抽象代数 (群论)



- (1) To draw a straight **line** from any **point** to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) **The parallel postulate.**

Axiomatic System for a Four-point Geometry

Undefined terms: point, line, is on

Axioms:

- (1) There are exactly four points.
- (2) It is impossible for three points to be on the same line.
- (3) For every pair of distinct points x and y , there is a unique line l such that x is on l and y is on l .
- (4) Given a line l and a point x that is not on l , there is a unique line m such that x is on m and no point on l is also on m .

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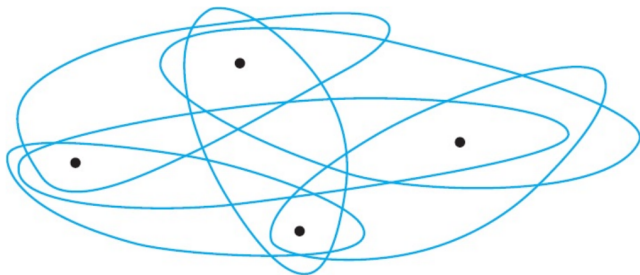
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Theorem

There are at least two distinct lines.

Syntax *vs.* Semantics

Syntax *vs.* Semantics



point : \cdot line : \bigcirc is on : $\bigcirc \cdot$

什么样的推理是正确的？

**Logic is the foundation of
the certainty of all the
knowledge we acquire.**

WhatsMyQuote.com



Leonhard Euler



Thank
You!



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