2-9 Sorting and Selection

Hengfeng Wei

hfwei@nju.edu.cn

May 28, 2018

How to Argue?

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Show that $\cdots$

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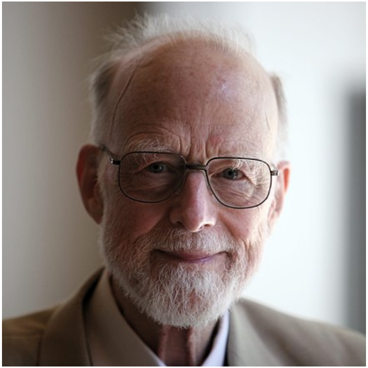
Show that $\cdots$

Argue that $\cdots$

$=$ Prove that $\cdots$

Quicksort Invented by Tony Hoare in 1959/1960



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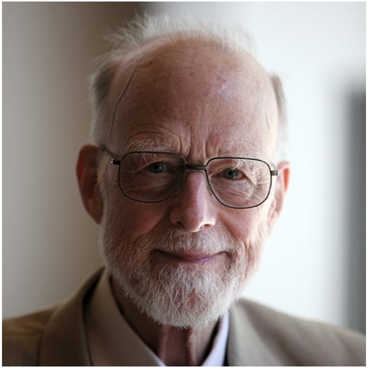

`null pointer`

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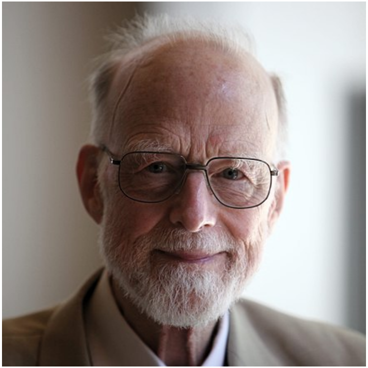

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"I call it my billion-dollar mistake."

Best-Case Complexity of QUICKSORT $(7.4-2)$

Show that QUICKSORT's best-case running time is $\Omega(n \log n)$.

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By substitution.

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Argue that in the $\Omega(n \log n)$ running time of QUICKSORT, the *median-of-3* method affects only the constant factor.

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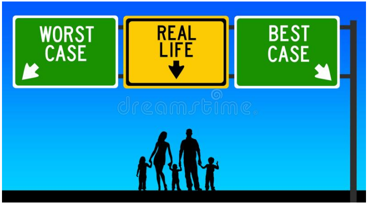

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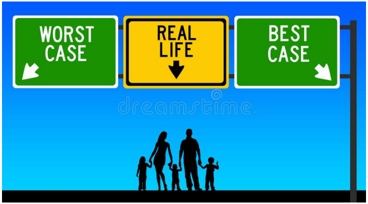

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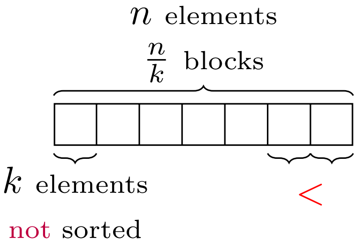
The $\frac{n}{k}$-sorted Problem (Problem $8.1 - 4$)

Sorts an already $\frac{n}{k}$-sorted array

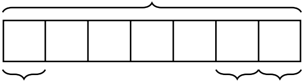


The $\frac{n}{k}$-sorted Problem (Problem $8.1-4$)

Sorts an already $\frac{n}{k}$-sorted array

n elements

$\frac{n}{k}$ blocks



k elements

$<$

not sorted

$$\Omega(n \log k)$$

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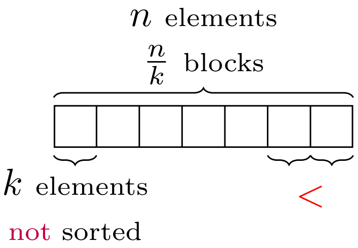

$\Omega(n \log k)$ $\qquad$ $O(n \log k)$

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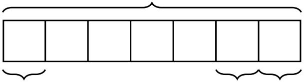

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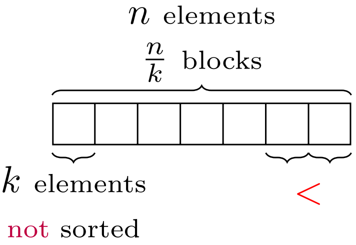
$\Omega(n \log k) \qquad O(n \log k)$

$$(k!)^{\frac{n}{k}} \le L \le 2^H$$

$\frac{n}{k}$-sorts an arbitrary array



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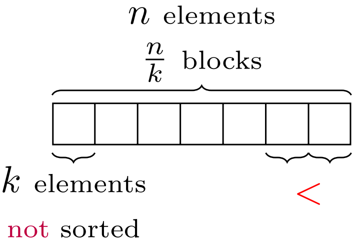
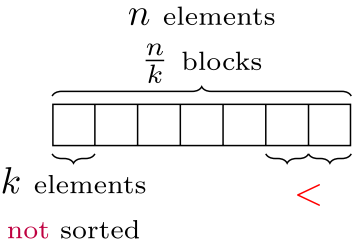


$O(?)$

$\frac{n}{k}$-sorts an arbitrary array



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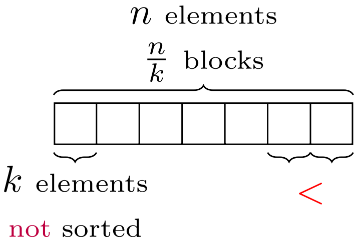

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$$L \geq \binom{n}{\underbrace{k, \ldots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}}$$

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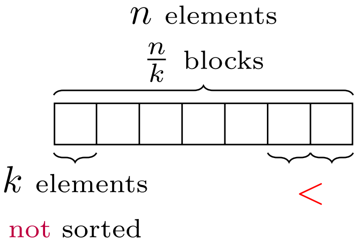

$O(?)$ $\Omega(?)$

$$L \geq \binom{n}{\underbrace{k, \ldots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}} \implies \Omega(n \log(n/k))$$

Sorting $[0, n^3 - 1]$ (Problem 8.3-4)

Sort n integers in $[0, n^3 - 1]$ in $O(n)$ time.

Sorting in Place in Linear Time (Problem $8-2$ (e))

Suppose that the n records have keys in the range $[0, k]$.
Modify COUNTING-SORT to sort them in place $(O(k))$ in $O(n+k)$ time.

Sorting in Place in Linear Time (Problem $8-2$ (e))

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	1	2	3	4	5	6	7	8
A :	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C :	2	0	2	3	0	1
C :	2	2	4	7	7	8

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B :	0	0	2	2	3	3	3	5

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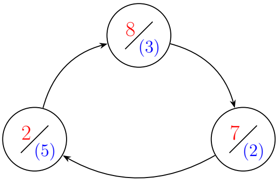

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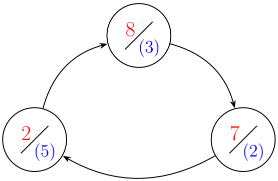
`While` $(i \geq 1)$:



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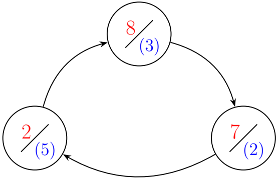
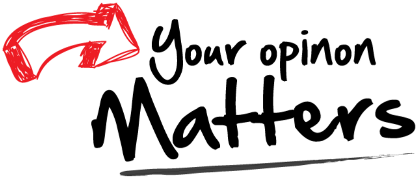


Code here


Thank
You!



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