3-10 Traversability

(Part II: Hamiltonian Graphs)

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December 10, 2018

$5.10 \quad 5.34 \quad 5.22 \quad 5.26$

Theorem (Necessary Condition; Theorem 6.5)

If G is a Hamiltonian graph, then for each nonempty set $S \subset V(G)$,

$$k(G - S) \leq |S|.$$

Theorem (Ore's Theorem, 1960; Theorem 6.6)

Let G be a graph of order $n \geq 3$. If

$$deg(u) + deg(v) \geq n$$

for each pair u, v of nonadjacent vertices of G, then G is Hamiltonian.

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Proof.

By Extremality and Contradiction.

$\square$

Theorem (Dirac's Theorem, 1952; Corollary 6.7)

Let G be a graph of order $n \geq 3$. If

$$\forall v \in V(G) : deg(v) \geq n/2,$$

then G is Hamiltonian.

Theorem (Ore's Theorem, 1960; Theorem 6.8)

Let u and v be nonadjacent vertices in a graph G of order n such that

$$deg(u) + deg(v) \geq n.$$

Then $G + uv$ is Hamiltonian $\iff$ G is Hamiltonian.

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Definition (Closure $C(G)$)

The closure $C(G)$ of a graph G is the graph obtained from G by iteratively adding edges joining pairs of nonadjacent vertices u and v such that $\deg(u) + \deg(v) \geq n$, until no such pair remains.

Theorem (Bondy-Chavatal Theorem, 1976; Theorem 6.9)

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If G is a graph of order $n \geq 3$ such that $C(G) = K_n$, then G is Hamiltonian.

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Theorem (Lajos Posa)

Let G be a graph of order $n \geq 3$. If for each integer j with $1 \leq j \leq \frac{n}{2}$, the number of vertices of G with degree at most j is less than j, then G is Hamiltonian.

Theorem

$C(G)$ is well-defined.

(Problem 6.20)

Let G be a graph of order $n \geq 3$ having the property that for each $v \in V(G)$, there is a Hamiltonian path with initial vertex v. Show that G is 2-connected but not necessarily Hamiltonian.

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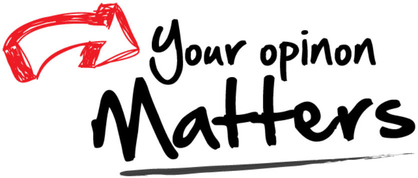
THANK YOU



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