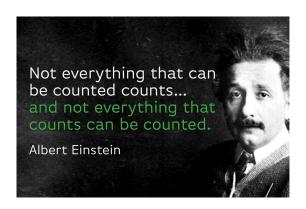
## 2-3 Counting

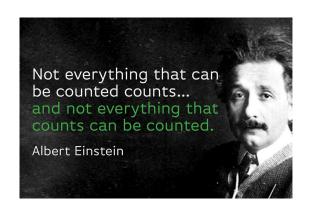
### 魏恒峰

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2018年04月11日







所以, 学好 "2-3 组合与计数" 是多么重要!

## Computing $\binom{n}{k}$ (CS 1.5:14)

1: **procedure** BINOM(n,k)

 $\triangleright \ \mathsf{Required} \colon \ n \geq k \geq 0$ 

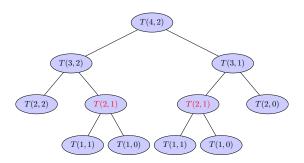
- 2: **if**  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)

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4 / 6

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$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

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(ii) # of recursive calls of BINOM:

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$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

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  - (i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

$$T(n,k) = T(n-1,k) + T(n-1,k-1) + c$$



$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix} & \begin{pmatrix} 2 \\ 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix} & \begin{pmatrix} 3 \\ 1 \end{pmatrix} & \begin{pmatrix} 3 \\ 2 \end{pmatrix} & \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix} & \begin{pmatrix} 4 \\ 1 \end{pmatrix} & \begin{pmatrix} 4 \\ 2 \end{pmatrix} & \begin{pmatrix} 4 \\ 3 \end{pmatrix} & \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 0 \end{pmatrix} & \begin{pmatrix} 5 \\ 1 \end{pmatrix} & \begin{pmatrix} 5 \\ 2 \end{pmatrix} & \begin{pmatrix} 5 \\ 3 \end{pmatrix} & \begin{pmatrix} 5 \\ 4 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

```
1: procedure BINOM(n,k)
```

 $\triangleright \ \mathsf{Required} \colon \ n \geq k \geq 0$ 

- 2: **for**  $i \leftarrow 0$  **to** n **do**
- 3:  $B[i][0] \leftarrow 1$
- 4:  $B[i][i] \leftarrow 1$
- 5: for  $i \leftarrow 2$  to n do
- 6: for  $j \leftarrow 1$  to k do
- 7:  $B[n][k] \leftarrow B[n-1][k] + B[n-1][k-1]$
- 8: return B[n][k]

# Thank You!