1-4 Algorithms

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IS THIS A
SIMULATION?

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

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```
for (int i = 0; i < N; ++i)
  statement
```

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(a) "`for-do`" by "`while-do`"

```
for (int i = 0; i < N; ++i)
  statement
```

```
int i = 0;
while (i < N)
  statement
  ++i
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

```
for (int i = 0; i < N; ++i)
  statement
```

```
int i = 0;
while (i < N)
  statement
  ++i
```

Not General

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

```
for (init; cond; inc)
  statement
```

```
init;
while (cond)
  statement
  inc
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

```
while (A)
  B
¬ A
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

```
while (A)
  B
¬ A // Wrong: side effects?
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

```
while (A)
  B
  ¬ A // Wrong: side effects?
```

```
flag = 1
while (A && flag)
  B
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B
  flag_if = 0
flag_else = 1
while (¬ A && flag_else)
  C
  flag_else = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B  // Wrong: side effects?
  flag_if = 0
flag_else = 1
while (¬ A && flag_else)
  C
  flag_else = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B  // Wrong: side effects?
  flag_if = 0
flag_else = 1
while (¬ A && flag_else)
  C
  flag_else = 0
```

```
flag = 1
while (A && flag)
  B
  flag = 0

while (¬ A && flag)
  C
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B  // Wrong: side effects?
  flag_if = 0
flag_else = 1
while (¬ A && flag_else)
  C
  flag_else = 0
```

```
flag = 1
while (A && flag)
  B
  flag = 0
// ¬A not necessary
while (¬ A && flag)
  C
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) "`while-do`" by "`if-then` & `goto`"

(d) "`while-do`" by "`repeat-until` & `if-then`"

```
while (A)
  B
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) "`while-do`" by "`if-then` & `goto`"

(d) "`while-do`" by "`repeat-until` & `if-then`"

```
while (A)
  B
```

```
L: if (A)
     B
     goto L
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) "`while-do`" by "`if-then` & `goto`"

(d) "`while-do`" by "`repeat-until` & `if-then`"

```
while (A)
  B
```

```
L: if (A)
     B
     goto L
```

```
if (A)
  repeat
    B
  until (¬ A)
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) "`while-do`" by "`if-then` & `goto`"

(d) "`while-do`" by "`repeat-until` & `if-then`"

```
while (A)
  B
```

```
L: if (A)
     B
     goto L
```

```
if (A)   // ''if'' in ''repeat''?
  repeat
    B
  until (¬ A)
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else` & `recursive`".

```
while (A)
  B
```

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Simulate "`while-do`" by "`if-then-else` & `recursive`".

```
while (A)
  B
```

```
simulateWhile() {
  if (A)
    B
    simulateWhile();
}
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else` & `recursive`".

```
while (A)
  B
```

```
simulateWhile() { // define function
  if (A)
    B
    simulateWhile();
}
```

(1) `A;B`

(2) `if-then`

(3) `if-then-else`

(4) `for-do`

(5) `while-do`

(6) `repeat-until`

(1) A;B

(2) if-then

(3) if-then-else

(4) for-do

(5) while-do

(6) repeat-until

```
repeat
  B
until (¬ A)
```

(1) A;B

(2) if-then

(3) if-then-else

(4) for-do

(5) while-do

(6) repeat-until

```
repeat
  B
until (¬ A)
```

```
B
while (A)
  B
```

(1) A;B

(2) if-then

(3) if-then-else

(4) for-do

(5) while-do

(6) repeat-until

```
repeat
  B
until (¬ A)
```

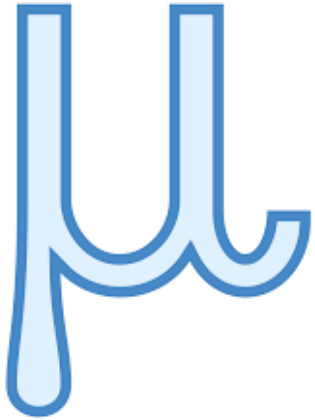
```
B
while (A)
  B
```

Theorem ("On Folk Theorems" (David Harel, 1980))

Any computable function can be computed by a "while-do" (and ";") program (with additional Boolean variables).

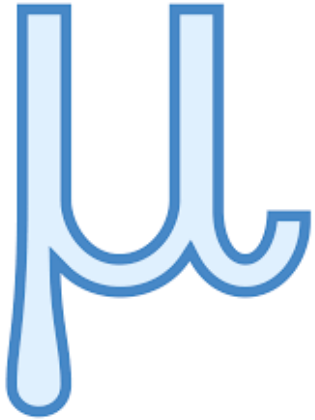
i
minimalism

What are "Computable Functions"?



μ-Recursive Functions
(1931-1934)

Kurt Gödel (1906-1978)



μ-Recursive Functions
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Lambda Calculus
(1933-1935)

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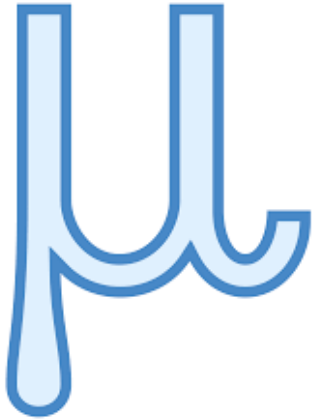
Alonzo Church (1903-1995)



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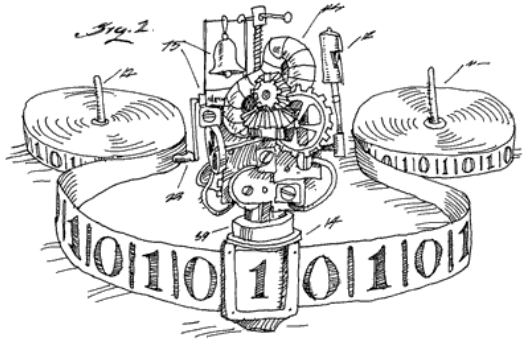


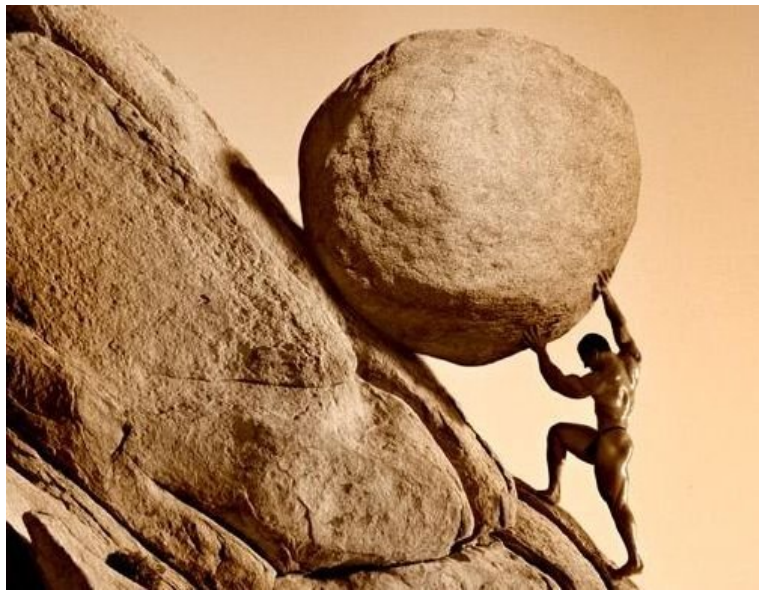
Turing Machines
(1936-1937)

Kurt Gödel (1906-1978)

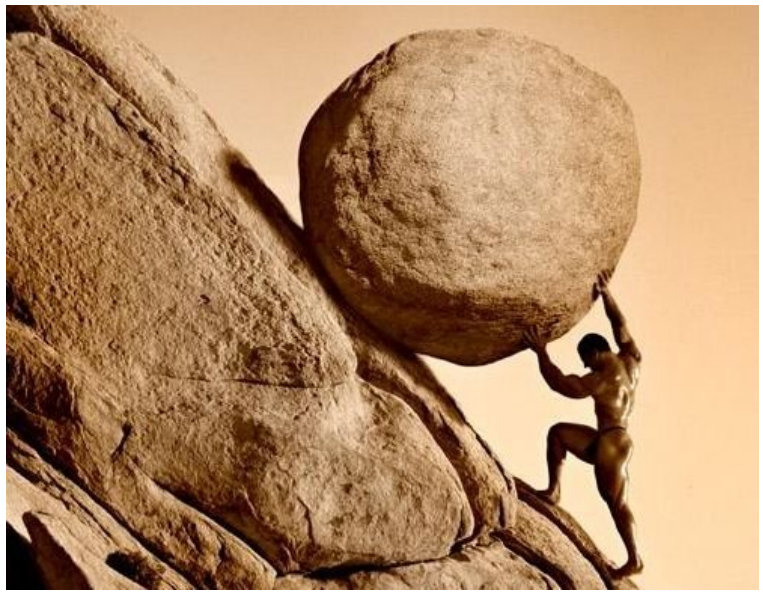
Alonzo Church (1903-1995)

Alan Turing (1912-1954)

Bounded Iterations *vs.* Unbounded Iterations



Bounded Iterations *vs.* Unbounded Iterations



Q : Why unbounded iterations?

Definition (Primitive Recursive)

$$\begin{cases} h(0) & = k, \\ h(t+1) & = g(t, h(t)) \end{cases}$$

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$$\begin{cases} h(x_1, \cdots, x_n, 0) & = f(x_1, \cdots, x_n), \\ h(x_1, \cdots, x_n, t+1) & = g(t, h(x_1, \cdots, x_n, t), x_1, \cdots, x_n) \end{cases}$$

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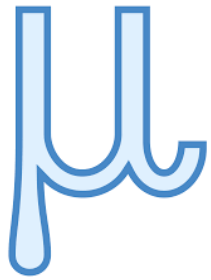
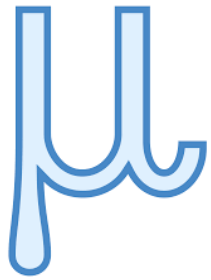
$$\forall / \exists\ t \leq y : P(t, x_1, \cdots, x_n)$$



Theorem (Ackermann Function)

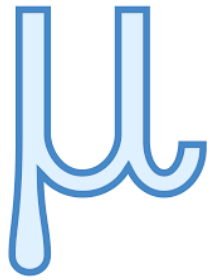
The Ackermann function is μ-recursive but not primitive recursive.



Theorem (Ackermann Function)

The Ackermann function is μ-recursive but not primitive recursive.

$$A(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1,1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m-1, A(m,n-1)) & \text{if } m > 0 \text{ and } n > 0. \end{cases}$$



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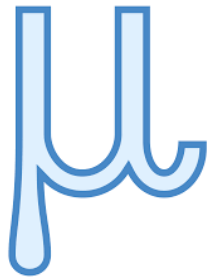
$$A(4,2) = ?$$



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$$A(4,2) = ?$$

$$g(x) \triangleq \mu y(g(x,y)) \triangleq \left(\operatorname*{argmin}_y g(x,y) = 0 \right)$$

Thank
You!