1-2 Reasoning

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2019 年 10 月 24 日

Gottfried Wilhelm Leibniz (莱布尼茨 1646 – 1716)

建立一个能够涵盖人类思维活动的"通用符号演算系统"，
让人们的思维方式变得像数学运算那样清晰。

一旦有争论，不管是科学上的还是哲学上的，人们只要坐下
来算一算，就可以毫不费力地辨明谁是对的。

Let us calculate [calculemus].

- ▶ Each inhabitant is either a truth-teller or a liar (not both).
- ▶ A truth-teller always tells the truth and a liar always lies.
- ▶ Arnie and Barnie live on the island.

(a) Arnie: "If I am a truth-teller, then each person living on this island is either a truth-teller or a liar."

(b) Arnie: "If I am a truth-teller, then so is Barnie."

(a) Is Arnie a truth-teller or a liar?

(b) Can you tell what Arnie and Barnie are?

更重要的是, 你能"算"出来吗?

(b) $A \leftrightarrow (A \to B)$

UD 3.10: Breakfast

Matilda always eats at least one of the following for breakfast:

1. cereal, bread, or yogurt.

On Monday, she is especially picky.

2. If she eats cereal and bread, she also eats yogurt.
3. If she eats bread or yogurt, she also eats cereal.
4. She never eats both cereal and yogurt.
5. She always eats bread or cereal.

Can you say what Matilda eats on Monday? If so, what does she eat?

引入命题符号：你觉得这有什么问题吗？

A : Cereal P : Cereal

B : Bread Q : Bread

C : Yogurt R : Yogurt

Look at the chart and say the <u>COLOUR</u> not the word

YELLOW BLUE ORANGE
BLACK RED GREEN
PURPLE YELLOW RED
ORANGE GREEN BLACK
BLUE RED PURPLE
GREEN BLUE ORANGE

Left – Right Conflict
Your right brain tries to say the colour but
your left brain insists on reading the word.

$$C: \text{ Cereal} \qquad B: \text{ Bread} \qquad Y: \text{ Yogurt}$$

$$C \lor B \lor Y \qquad (1)$$
$$(C \land B) \to Y \qquad (2)$$
$$(B \lor Y) \to C \qquad (3)$$
$$\neg(C \land Y) \qquad (4)$$
$$B \lor C \qquad (5)$$

Let us calculate [calculemus].

$$P \rightarrow \neg(Q \land \neg P)$$

从语法的角度讲, 这仅仅是一个字符串。

UD 2.5: 命题逻辑公式的语义

$$P \to \neg(Q \land \neg P)$$

真值表 (truth table)

命题逻辑公式的语义就是它的真值表，与原子命题的真假有关。

"真"这个概念是属于"元语言"的。

命题逻辑中的重言式

$$\alpha \to (\beta \to \alpha)$$

$$(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$$

$$(\neg\beta \to \neg\alpha) \to ((\neg\beta \to \alpha) \to \beta)$$

一阶谓词逻辑公式的语义

$$L = \{<\}$$

$$\psi : \forall x \exists y \ (y < x)$$

$Q : \psi$ 是真是假?

$$\mathcal{U} = \mathbb{N}$$

$$\mathcal{U} = \mathbb{Z}$$

一阶谓词逻辑公式的语义与它的**结构 (Structure)**有关。

"真" 这个概念是属于 "元语言" 的。

一阶谓词逻辑中的重言式

$$\Big(\forall y \neg P(y) \rightarrow \neg P(x)\Big) \rightarrow \Big(P(x) \rightarrow \exists y P(y)\Big)$$

$$\Big(\forall x(\alpha \rightarrow \beta)\Big) \rightarrow (\forall x \alpha \rightarrow \forall x \beta)$$
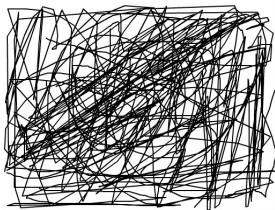
Decide whether (3) is true if (1) and (2) are both true.

Q: 该如何理解这道题? 依据什么 "decide" 真假?

逻辑知识

$$(1) \land (2) \to (3)$$

数学知识 "True" 是语义概念

▶ 与选定的 "结构" 中的知识有关

UD 4.20 (a): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

(a) (1) Everyone who loves Bill loves Sam.
      (2) I don't love Sam.
      (3) I don't love Bill.

Q: 如何在一阶谓词逻辑框架中 "算出来"?

Decide whether (3) is true if (1) and (2) are both true.

(b) (1) If Susie goes to the ball in the red dress, I will stay home.
    (2) Susie went to the ball in the green dress.
    (3) I did not stay home.

Q: 这是真的吗?

到底是真是假?

▶ (3) is true:
Whether I stay at home or
not, (3) is always true.

▶ (3) is false:
No matter what I do, the
implication is always true.

实际上，仅根据 (1)、(2), 我们无法判断 (3) 的真假。

UD 4.20 (c): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

- (c) (1) If l is a positive real number, then there exists a real number m such that $m > l$.
  - (2) Every real number m is less than t.
  - (3) The real number t is not positive.

Q : 如何符号化 (1)、(2)、(3)?

(1) $\forall l$ 还是仅是 l?   (1) $\forall l$

(2) t 究竟是不是实数?   (2) $R(t)$

(3) $R(t) \wedge \neg P(t)$ 还是 $R(t) \to \neg P(t)$?   (3) $R(t) \to \neg P(t)$

现在，让我们来 "算" 一下吧。

UD 4.20 (d): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

(d) (1) Every little breeze seems to whisper Louise or my name is Igor.
(2) My name is Stewart.
(3) Every little breeze seems to whisper Louise.

Q : 命题逻辑公式还是一阶谓词逻辑公式?

$$\{p \vee q, \neg q\} \vdash p$$

$$(p \vee q) \leftrightarrow (\neg q \rightarrow p)$$

$$\{p \lor q, \neg q\} \vdash p$$

$$\neg q \to (q \to \neg q) \qquad (1)$$

$$q \to \neg q \qquad (2)$$

$$q \to q \qquad (3)$$

$$q \to p \qquad (4)$$

$$p \to p \qquad (5)$$

$$(p \lor q) \to p \qquad (6)$$

$$p \qquad (7)$$

Decide whether (3) is true if (1) and (2) are both true.

(e) (1) There is a house on every street such that if that house is blue, the one next to it is black.
(2) There is no blue house on my street.
(3) There is no black house on my street.

(1) 在说什么? 翻译成汉语是什么意思?

$$\forall s \in S \, \exists h \in H \big(\text{On}(h, s) \wedge (\text{Blue}(h) \rightarrow \text{Black}(\text{next-to}(h))) \big)$$

UD 4.20 (f): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

(f) Let x and y be real numbers.
(1) If $x > 5$, then $y < 1/5$.
(2) We know $y = 1$.
(3) So $x \leq 5$.

先 "算一算"

Q: 在推理过程中, 我们用到了哪些数学知识 (元语言知识)?

UD 4.20 (g): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

(g) Let M and n be real numbers.

    (1) If $n > M$, then $n^2 > M^2$.

    (2) We know $n < M$.

    (3) So $n^2 \leq M^2$.

▶ (3) is false:

$$n = -2, \ M = -1$$

▶ (3) is true:

  (1) $n > 0$

  (2) $0 < n < M$

▶ 无法判断

  $(1) \wedge (2) \to (3)$

https://math.stackexchange.com/q/2471687/51434

UD 4.20 (h): 一阶谓词逻辑的推理规则与语义

Decide whether (3) is true if (1) and (2) are both true.

(h) Let x, y, and z be real numbers.
   (1) If $y > x$ and $y > 0$, then $y > z$.
   (2) We know that $y \leq z$.
   (3) Then $y \leq x$ or $y \leq 0$.

先 "算一算"

Q: 在推理过程中, 我们用到了哪些数学知识 (元语言知识)?

鄢振宇 2019/9/4 6:38:54

魏老师您还记得您第一学期的时候，降到一个逻辑相关的东西，然后您去问数学系的老师，他回答"真和假是无法定义的"这件事么？我有点好奇当时是在讲什么内容
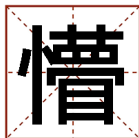
Definition (命题 (Statement/Proposition))

A **statement** is a sentence that is either true or false (but not both).

Exercise 2.1: 以下哪些是命题?

1. $X + 6 = 0$
2. $X = X$
3. 哥德巴赫猜想。
4. 今天是雨天。
5. 明天是晴天。
6. 明天是周二。
7. 这句话是假话。

来自一位数理逻辑学家的意见与建议

- (1)、(2) 不是句子 (sentence), 所以也不是命题。

- (4)、(5)、(6) 在数学中没有意义。

- "真 (Truth)" 是不能定义的。所以, (7) 不是命题。

懵

"真 (truth)" 在日常语言 (或算术) 中不可定义。
— Alfred Tarski

"我觉得你还是找一本正经的数理逻辑教材看看"

关于"命题", 我们现在知道些什么?

▶ 命题是一个语句 (sentence), 不能含有变量。

▶ 目前不知其真假, 但本身必可分辨真假的语句也是命题。

▶ 悖论不是命题。

暂时忘掉"命题"与"悖论"吧

命题逻辑与一阶谓词逻辑：

▶ 引入命题符号：将命题视为原子
▶ 关注复合命题：研究命题之间的关系

$$\wedge \qquad \vee \qquad \neg \qquad \rightarrow \qquad \leftrightarrow$$