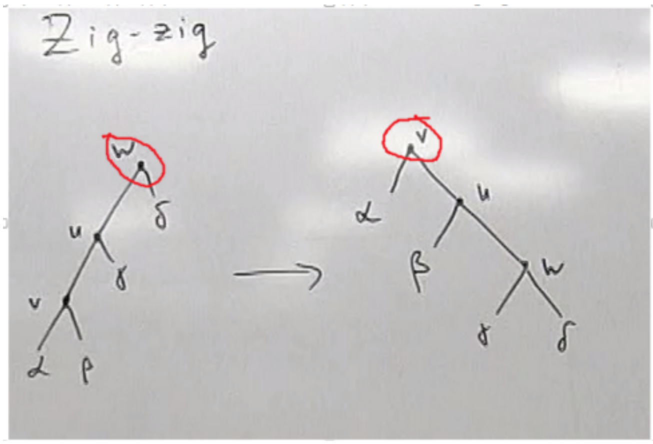
Proof of Zig-Zig step

There was a question connected with one of the video lecture lessons that I'm currently watching.

Let two trees be given - the original and the tree after the zig-zig step:



Calculate the cost of this operation.

$$r'(u)+r'(v)+r'(w)-r(u)-r(v)-r(w)$$

Where it is: the sum of the ranks of the new tree (after the zig-zig step) minus the sum of the ranks of the source tree. Now pay attention to the root tops of the trees. - because their rank is equal, then we can delete them from equation

$$r'(u)+\cancel{r'(v)}+r'(w)-r(u)-r(v)-\cancel{r(w)}$$

Now, we carry out the upper bound. r '(u) and r' (w) - they can be estimated as r '(v) in the initial tree (up to the zig-zig step) in the upper estimate - the vertices u and w are above the vertex v - therefore, when estimating from above with respect to the vertex v in the source tree - we simply consider the potential of the vertices r (u) and r (w) as r (v) with the opposite sign. As a result, we get the expression:

$$r'(u)+\cancel{r'(v)}+r'(w)-r(u)-r(v)-\cancel{r(w)} \leqslant$$
$$\leqslant r'(v)+r'(v)-r(v)-r(v)=$$
$$=2(r'(v)-r(v))$$

Now the question is: *why does index 2 change to 3*?

$$r'(u)+\cancel{r'(v)}+r'(w)-r(u)-r(v)-\cancel{r(w)} \leqslant$$
$$\leqslant r'(v)+r'(v)-r(v)-r(v)=$$
$$=2(r'(v)-r(v)) \leqslant \underline{3(r'(v)-r(v))}$$

At first I thought that it was +1 as an accounting cost for the actual action, but it turned out that this +1 should be performed for each vertex - that breaks all the evidence - there it is explained later in the lecture and also how to avoid it, But now this is not about it , but why, if this is not +1 for the actual action, then where did the index 3 come from?

P.s : Further in the lecture, attention is not focused on this - therefore I ask.

`algorithms` `trees` `splay-trees`

asked Dec 12 '17 at 19:27
BadCatss
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1 They probably only need to show the upper bound $3(r'(v) - r(v))$. It's certainly the case that $2(r'(v) - r(v)) \leq 3(r'(v) - r(v))$, assuming that $r'(v) \geq r(v)$. – Yuval Filmus Dec 12 '17 at 21:27

Could you explain more? You are welcome? – BadCatss Dec 13 '17 at 13:01

Having not seen the video, unfortunately I cannot say anything more. – Yuval Filmus Dec 13 '17 at 13:04

Excuse me, if you can - a few more questions? As I understand it, index 3 is used here as a tool for estimating the upper bound of the expression 2 (r '(v) -r (v)), but in essence the principal role as a number figure does not? those if I understood you correctly - I want to say that in general this expression: 2 (r '(v) -r (v)) ≤3 (r' (v) -r (v)) - as : (r '(v) -r (v)) ≤n + k (r' (v) -r (v)) -where k is the upper bound of the expression: n (r '(v) -r (v)) In this case, k = 1. Did I understand you correctly? Well and yes, we take into account that assuming that r '(v) ≥r (v). – BadCatss Dec 13 '17 at 14:05

1 The rest of the argument presumably only needs the upper bound $3(r'(v) - r(v))$. The argument gives the stronger upper bound $2(r'(v) - r(v))$, from which we can derive the weaker upper bound $3(r'(v) - r(v))$. For anything more, you will need to explain the rest of the argument. – Yuval Filmus Dec 13 '17 at 14:36