Home About **Problems by Topics** Linear Algebra Gauss-Jordan Elimination **Inverse Matrix** Linear Transformation **Vector Space** Eigen Value Cayley-Hamilton Theorem Diagonalization **Exam Problems** By Topics **Group Theory** Abelian Group **Group Homomorphism** Sylow's Theorem Field Theory Module Theory Ring Theory LaTex/MathJax Login/Join us Records Solve later Problems My Solved Problems

GROUP THEORY

Math 2568 Linear Algebra Spring 2018

Teaching

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I'm Yu Tsumura.

Example of an Infinite Group Whose Elements Have Finite Orders

BY YU · 10/27/2017





Problem 594

Is it possible that each element of an infinite group has a finite order?

If so, give an example. Otherwise, prove the non-existence of such a group.





Solution.

We give an example of a group of infinite order each of whose elements has a finite order. Consider the group of rational numbers $\mathbb Q$ and its subgroup $\mathbb Z$. The quotient group $\mathbb Q/\mathbb Z$ will serve as an example as we verify below.

LINEAR ALGEBRA PROBLEMS BY TOPICS

The list of linear algebra problems is available here.



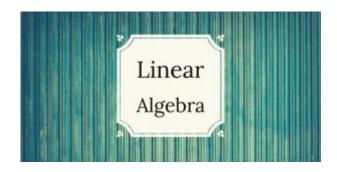
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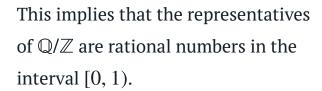




Note that each element of \mathbb{Q}/\mathbb{Z} is of the form

$$\frac{m}{n} + \mathbb{Z},$$

where m and n are integers.



There are infinitely many rational numbers in [0, 1), and hence the order of the group \mathbb{Q}/\mathbb{Z} is infinite.

On the other hand, as each element of \mathbb{Q}/\mathbb{Z} is of the form $\frac{m}{n} + \mathbb{Z}$ for $m, n \in \mathbb{Z}$, we have

$$n \cdot \left(\frac{m}{n} + \mathbb{Z}\right) = m + \mathbb{Z} = 0 + \mathbb{Z}$$

because $m \in \mathbb{Z}$.

Thus the order of the element $\frac{m}{n} + \mathbb{Z}$ is at most n.

Hence the order of each element of \mathbb{Q}/\mathbb{Z} is finite.

Therefore, \mathbb{Q}/\mathbb{Z} is an infinite group whose elements have finite orders.







CATEGORIES

- Elementary Number Theory (1)
- Field Theory (26)
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- Group Theory (126)
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Click here if solved ☆ 65

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Group of Order 18 is Solvable
Let *G* be a finite group of order 18. Show that the group *G* is solvable.

Definition
Recall that a group G is said to be solvable if G has a subnormal

$$\{e\} = G_0 \triangleleft G_1$$

such [...]

series

- Ring theory (64)
- ► Welcome (1)

MATHJAX

Mathematical equations are created by MathJax. See How to use MathJax in WordPress if you want to write a mathematical blog.



RECENT POSTS

- If the Nullity of a Linear Transformation is Zero, then Linearly Independent Vectors are Mapped to Linearly Independent Vectors
- Find All Values of *x* such that the Matrix is Invertible
- Find All Eigenvalues and Corresponding Eigenvectors for the 3×3 matrix
- Find All Values of *a* which Will Guarantee that *A* Has Eigenvalues 0, 3, and -3.
- Compute the Determinant of a Magic Square



The

Additive Group of Rational Numbers and The Multiplicative Group of Positive Rational Numbers are Not Isomorphic Let $(\mathbb{Q}, +)$ be the additive group of rational numbers and let $(\mathbb{Q}_{>0}, \times)$ be the multiplicative group of positive rational numbers. Prove that $(\mathbb{Q}, +)$ and $(\mathbb{Q}_{>0}, \times)$ are not isomorphic as groups. Proof. Suppose,

towards a [...]



The

Group of Rational

Numbers is Not

Finitely

Generated (a)

Prove that the additive group

 $\mathbb{Q} = (\mathbb{Q}, +)$ of

rational

numbers is not

finitely

generated. (b)

Prove that the multiplicative

group

 $\mathbb{Q}^* = (\mathbb{Q} \setminus \{0\},$

of nonzero

rational

numbers is not

finitely

generated.

Proof. (a) Prove

that the

additive [...]

GROUP THEORY

If the Order of a Group is Even, then the Number of Elements of Order 2 is Odd

LINEAR ALGEBRA

Diagonalize the 3 by 3 Matrix if it is Diagonalizable

GROUP THEORY

If Generators x, y Satisfy the Relation $xy^2 = y^3x$, $yx^2 = x^3y$, then the Group is Trivial

LINEAR ALGEBRA

How to Find Eigenvalues of a Specific Matrix.

LINEAR ALGEBRA

Characteristic Polynomial, Eigenvalues, Diagonalization Problem (Princeton University Exam)

TOP POSTS

- How to Diagonalize a Matrix. Step by Step Explanation.
- The Matrix for the Linear Transformation of the Reflection Across a Line in the Plane
- The Determinant of a Skew-Symmetric Matrix is Zero
- Eigenvalues of Real Skew-Symmetric
- 4 Matrix are Zero or Purely Imaginary and the Rank is Even
- How to Find a Basis for the Nullspace, Row Space, and Range of a Matrix
- The Intersection of Two Subspaces is also a Subspace



Every

Finitely Generated Subgroup of **Additive Group** Q of Rational Numbers is Cyclic Let $\mathbb{Q} = (\mathbb{Q}, +)$ be the additive group of rational numbers. (a) Prove that every finitely generated subgroup of $(\mathbb{Q}, +)$ is cyclic. (b) Prove that \mathbb{Q} and $\mathbb{Q} \times \mathbb{Q}$ are not isomorphic as groups. Proof. (a) Prove

that every

generated [...]

finitely



Commutator

Subgroup and

Abelian

Quotient Group

Let G be a

group and let

D(G) = [G, G]

be the

commutator

subgroup of *G*.

Let N be a

subgroup of *G*.

Prove that the

subgroup N is

normal in G

and G/N is an

abelian group if

and only if

 $N \supset D(G)$.

Definitions.

Recall that for

any $a, b \in G$,

the [...]

- 7 Determine Whether Each Set is a Basis for \mathbb{R}^3
- Find all Values of x such that the Given Matrix is Invertible
- Express a Vector as a Linear Combination of Other Vectors
- Positive definite Real Symmetric Matrix and its Eigenvalues

SITE MAP & INDEX

Site Map

Index



Normal

Subgroups,

Isomorphic

Quotients, But

Not Isomorphic

Let G be a

group. Suppose

that

 H_1, H_2, N_1, N_2

are all normal

subgroup of G,

 $H_1 \triangleleft N_2$, and

 $H_2 \triangleleft N_2$.

Suppose also

that N_1/H_1 is

isomorphic to

 N_2/H_2 . Then

prove or

disprove that N_1

is isomorphic to

 N_2 . Proof. We

give a [...]

Abelian Groups

Torsion

Subgroup of an

Abelian Group,

Quotient is a

Torsion-Free

Abelian Group

Let A be an

abelian group

and let T(A)

denote the set

of elements of A

that have finite

order. (a) Prove

that T(A) is a

subgroup of A.

(The subgroup

T(A) is called

the torsion

subgroup of the

abelian group A

and elements of

T(A) are called

torsion [...]



Group of *p*Power Roots of
1 is Isomorphic
to a Proper
Quotient of
Itself Let *p* be a
prime number.
Let

$$G = \{ z \in \mathbb{C} \mid z \}$$

be the group of p-power roots of 1 in \mathbb{C} . Show that the map $\Psi: G \to G$ mapping z to z^p is a surjective homomorphism. Also deduce from this that G is isomorphic to a proper quotient of G [...]

Tags: finite group group of rational numbers group theory infinite group quotient group

PREVIOUS NEXT STORY STORY The Eigenvalues Intersection of Two and Eigenvectors Subspaces is of The Cross also a Product Subspace Linear Transformation

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Group The Infinite Additive Homomor Cyclic Group of phism, Groups Preimage, Rational Do Not Numbers Have and and The Product of Compositi Multiplica Groups on Series tive 12/03/2016 09/27/2016 Group of

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Positive
Rational
Numbers
are Not
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07/15/2017

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2 RESPONSES

Q Comments 2





Lalitha © 12/15/2017 at 1:40 AM

Please explain what are representatives in Q/Z ? You mean elements of Q/Z?

"This implies that the representatives of Q/Z are rational numbers in the interval [0,1)."

From this sentence please explain the remaining part once again: "On the other hand, as each element of Q/Z is of the form mn+Z for m,n \in Z, we have..."

Why m=0? and why is the order of any element in Q/Z is atmost n?

Thanks

Reply



Yu ① 12/15/2017 at 2:04 AM

Note that each element in the group \mathbb{Q}/\mathbb{Z} is of the form $q+\mathbb{Z}$, where $q\in\mathbb{Q}$

. Wesaythatq

 $is a representative for the element \verb|q+\Z| \$.$

For the second part, m is not necessarily 0. But $m + \mathbb{Z}$ is equal to $0 + \mathbb{Z}$.

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