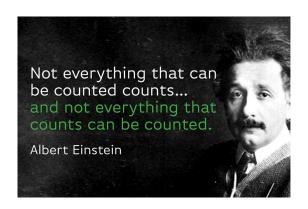
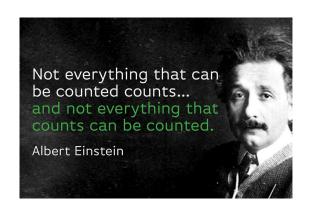
# 2-3 Counting

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所以, 学好 "2-3 组合与计数" 是多么重要!

Computing  $\binom{n}{k}$  (CS 1.5:14)

## **Algorithm 1** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n, k)

ightharpoonup Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)

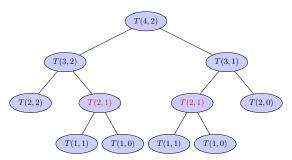
# Computing $\binom{n}{k}$ (CS 1.5:14)

#### **Algorithm 2** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n, k)

 ${\bf \triangleright} \ \mathsf{Required} \colon \ n \geq k \geq 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)



## **Algorithm 3** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n, k)

 ${\tt P Required:}\ n \geq k \geq 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)

## **Algorithm 4** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n, k)

ightharpoonup Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)
- (i) # of "+":

## **Algorithm 5** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n,k)

ightharpoonup Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)
  - (i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

#### **Algorithm 6** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n, k)

 $\triangleright$  Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)
  - (i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

#### **Algorithm 7** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n,k)

 $\triangleright$  Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)
  - (i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

#### **Algorithm 8** Computing $\binom{n}{k}$ .

1: **procedure** BINOM(n,k)

ightharpoonup Required:  $n \ge k \ge 0$ 

- 2: if  $k = 0 \lor n = k$  then
- 3: **return** 1
- 4: **return** BINOM(n-1,k) + BINOM(n-1,k-1)
  - (i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

$$T(n,k) = T(n-1,k) + T(n-1,k-1) + c$$



$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix} & \begin{pmatrix} 2 \\ 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix} & \begin{pmatrix} 3 \\ 1 \end{pmatrix} & \begin{pmatrix} 3 \\ 2 \end{pmatrix} & \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix} & \begin{pmatrix} 4 \\ 1 \end{pmatrix} & \begin{pmatrix} 4 \\ 2 \end{pmatrix} & \begin{pmatrix} 4 \\ 3 \end{pmatrix} & \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 0 \end{pmatrix} & \begin{pmatrix} 5 \\ 1 \end{pmatrix} & \begin{pmatrix} 5 \\ 2 \end{pmatrix} & \begin{pmatrix} 5 \\ 3 \end{pmatrix} & \begin{pmatrix} 5 \\ 4 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

#### **Algorithm 9** Computing $\binom{n}{k}$ .

```
\triangleright Required: n > k > 0
1: procedure BINOM(n,k)
        for i \leftarrow 0 to n do
2:
            B[i][0] \leftarrow 1
3:
            B[i][i] \leftarrow 1
4:
        for i \leftarrow 2 to n do
5:
            for j \leftarrow 1 to k do
6:
                 B[n][k] \leftarrow B[n-1][k] + B[n-1][k-1]
7:
        return B[n][k]
8:
```

# Thank You!