1-2 Reasoning

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Gottfried Wilhelm Leibniz (莱布尼茨 1646 - 1716)

"我有一个梦想 ..."

建立一个能够涵盖人类思维活动的"通用符号演算系统",让人们的思维方式变得像数学运算那样清晰。

一旦有争论,不管是科学上的还是哲学上的,人们只要坐下来算一算,就可以毫不费力地辨明谁是对的。

Let us calculate [calculemus].



- ► Each inhabitant is either a truth-teller or a liar (not both).
- ► A truth-teller always tells the truth and a liar always lies.
- ► Arnie and Barnie live on the island.
- (a) Arnie: "If I am a truth-teller, then each person living on this island is either a truth-teller or a liar."
- (b) Arnie: "If I am a truth-teller, then so is Barnie."

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- (a) Is Arnie a truth-teller or a liar?
- (b) Can you tell what Arnie and Barnie are?

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$$(b) A \leftrightarrow (A \rightarrow B)$$



UD 3.10: Breakfast

Matilda always eats at least one of the following for breakfast:

1. cereal, bread, or yogurt.

On Monday, she is especially picky.

- 2. If she eats cereal and bread, she also eats yogurt.
- 3. If she eats bread or yogurt, she also eats cereal.
- 4. She never eats both cereal and yogurt.
- 5. She always eats bread or cereal.

Can you say what Matilda eats on Monday? If so, what does she eat?

引入命题符号: 你觉得这有什么问题吗?

A: Cereal P: Cereal

B: Bread Q: Bread

C: Yogurt R: Yogurt

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Look at the chart and say the <u>COLOUR</u> not the word

YELLOW BLUE ORANGE
BLACK RED GREEN
PURPLE YELLOW RED
ORANGE GREEN BLACK
BLUE RED PURPLE
GREEN BLUE ORANGE

Left – Right Conflict

Your right brain tries to say the colour but your left brain insists on reading the word.

C: Cereal B: Bread Y: Yogurt

8/32

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$$C \vee B \vee Y \tag{1}$$

$$(C \land B) \to Y$$
 (2)

$$(B \vee Y) \to C \tag{3}$$

$$\neg (C \land Y) \tag{4}$$

$$B \vee C$$
 (5)

8/32

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$$(B \vee Y) \to C \tag{3}$$

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$$B \vee C$$
 (5)

Let us calculate [calculemus].





Syntax Semantics

UD 2.5

$$P \to \neg (Q \land \neg P)$$

从语法的角度讲,这仅仅是一个字符串。



UD 2.5: 命题逻辑公式的语义

$$P \to \neg (Q \land \neg P)$$

真值表 (truth table)

命题逻辑公式的语义就是它的真值表,与原子命题的真假有关。

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"真"这个概念是属于"元语言"的。

命题逻辑中的重言式

$$\alpha \to (\beta \to \alpha)$$

$$(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$$

$$(\neg \beta \to \neg \alpha) \to ((\neg \beta \to \alpha) \to \beta)$$

$$L = \{<\}$$

$$\psi : \forall x \exists y \ (y < x)$$

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 $Q:\psi$ 是真是假?

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一阶谓词逻辑公式的语义与它的结构 (Structure)有关。

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一阶谓词逻辑中的重言式

$$\left(\forall y \neg P(y) \rightarrow \neg P(x) \right) \rightarrow \left(P(x) \rightarrow \exists y P(y) \right)$$
$$\left(\forall x (\alpha \rightarrow \beta) \right) \rightarrow (\forall x \alpha \rightarrow \forall x \beta)$$



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数学知识"True"是语义概念

▶ 与选定的"结构"中的知识有关

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Decide whether (3) is true if (1) and (2) are both true.

- (a) (1) Everyone who loves Bill loves Sam.
 - (2) I don't love Sam.
 - (3) I don't love Bill.

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Q: 如何在一阶谓词逻辑框架中"算出来"?

Decide whether (3) is true if (1) and (2) are both true.

- (b) (1) If Susie goes to the ball in the red dress, I will stay home.
 - (2) Susie went to the ball in the green dress.
 - (3) I did not stay home.

Q: 这是真的吗?

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▶ (3) is true: Whether I stay at home or not, (3) is always true. ➤ (3) is false: No matter what I do, the implication is always true.

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实际上, 仅根据(1)、(2), 我们无法判断(3)的真假。

- (c) (1) If l is a positive real number, then there exists a real number m such that m > l.
 - (2) Every real number m is less than t.
 - (3) The real number t is not positive.

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现在,让我们来"算"一下吧。

- (d) (1) Every little breeze seems to whisper Louise or my name is Igor.
 - (2) My name is Stewart.
 - (3) Every little breeze seems to whisper Louise.

Decide whether (3) is true if (1) and (2) are both true.

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$$\{p \vee q, \neg q\} \vdash p$$

$$(p \lor q) \leftrightarrow (\neg q \to p)$$

$$\{p \vee q, \neg q\} \vdash p$$

(7)

Decide whether (3) is true if (1) and (2) are both true.

- (e) (1) There is a house on every street such that if that house is blue, the one next to it is black.
 - (2) There is no blue house on my street.
 - (3) There is no black house on my street.

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$$(1) \wedge (3) \rightarrow (2)$$
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- (f) Let x and y be real numbers.
 - (1) If x > 5, then y < 1/5.
 - (2) We know y = 1.
 - (3) So $x \le 5$.

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Q: 在推理过程中, 我们用到了哪些数学知识 (元语言知识)?

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- ▶ (3) is true:
 - (1) n > 0
 - (2) 0 < n < M

- ▶ 无法判断
 - $(1) \land (2) \rightarrow (3)$

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https://math.stackexchange.com/q/2471687/51434

- (h) Let x, y, and z be real numbers.
 - (1) If y > x and y > 0, then y > z.
 - (2) We know that $y \leq z$.
 - (3) Then $y \le x$ or $y \le 0$.

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鄢振宇 2019/9/4 6:38:54

魏老师您还记得您第一学期的时候,降到一个逻辑相关的东西,然后您去问数学系的老师,他回答"真和假是无法定义的"这件事么?我有点好奇当时是在讲什么内容

Definition (命题 (Statement/Proposition))

A statement is a sentence that is either true or false (but not both).

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Exercise 2.1: 以下哪些是命题?

- 1. X + 6 = 0
- 2. X = X
- 3. 哥德巴赫猜想。
- 4. 今天是雨天。
- 5. 明天是晴天。
- 6. 明天是周二。
- 7. 这句话是假话。

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"真 (truth)" 在日常语言 (或算术) 中不可定义。

— Alfred Tarski

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"我觉得你还是找一本正经的数理逻辑教材看看"

关于"命题", 我们现在知道些什么?

- ▶ 命题是一个语句 (sentence),不能含有变量。
- ▶ 目前不知其真假,但本身必可分辨真假的语句也是命题。
- 悖论不是命题。

暂时忘掉"命题"与"悖论"吧

命题逻辑与一阶谓词逻辑:

- ▶ 引入命题符号:将命题视为原子
- ▶ 关注复合命题:研究命题之间的关系



Thank You!