2-9 Sorting and Selection

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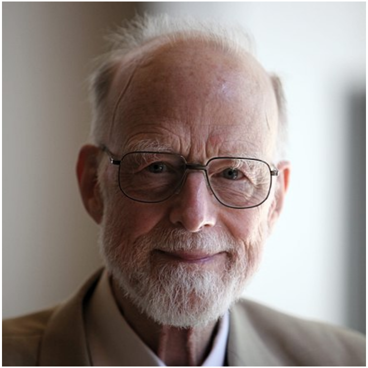
May 28, 2018

How to Argue?

Am I Alone?

QUICKSORT Invented by Tony Hoare in 1959/1960



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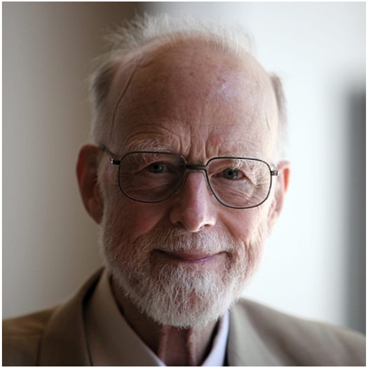

`null pointer`

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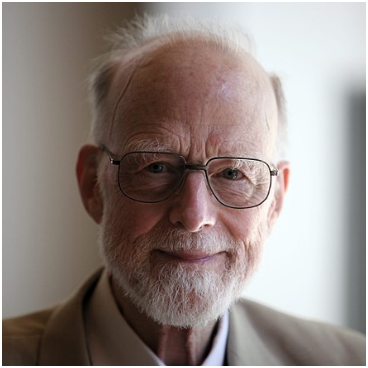

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"I call it my billion-dollar mistake."

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Show that QUICKSORT's best-case running time is $\Omega(n \log n)$.

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By substitution.

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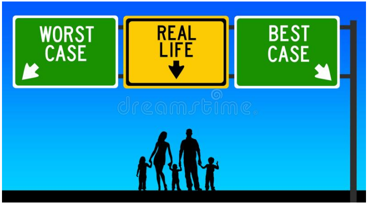

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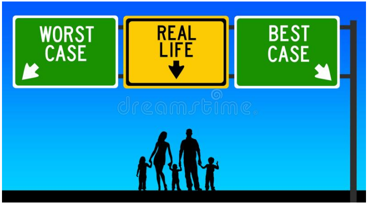

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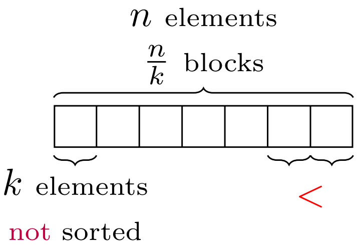
The $\frac{n}{k}$-sorted Problem (Problem $8.1 - 4$)

Sorts an already $\frac{n}{k}$-sorted array



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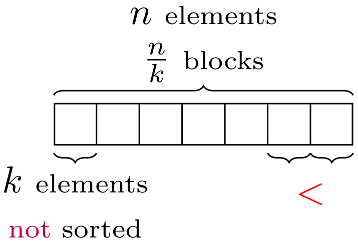

$\Omega(n \log k)$

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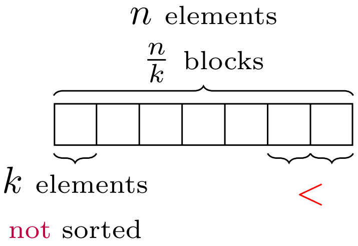

$\Omega(n \log k)$ $\qquad$ $O(n \log k)$

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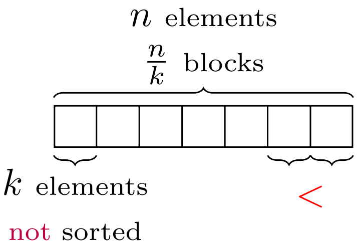
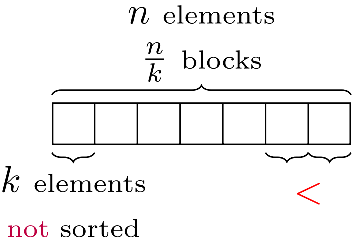


$\Omega(n \log k) \qquad O(n \log k)$

$$(k!)^{\frac{n}{k}} \leq L \leq 2^H$$

$\frac{n}{k}$-sorts an arbitrary array



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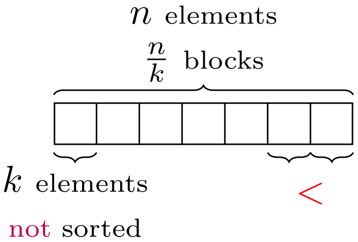

$O(?)$

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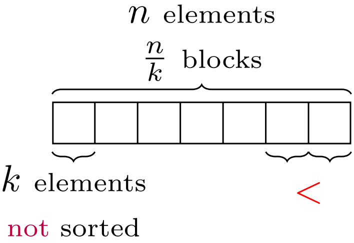

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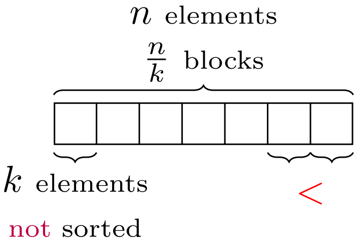

$$O(?) \qquad \Omega(?)$$

$$L \geq \binom{n}{\underbrace{k, \dots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}}$$

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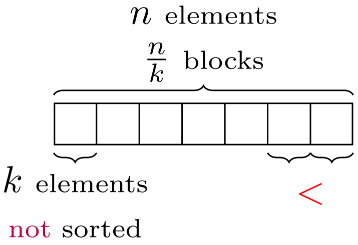

$$O(?) \qquad \Omega(?)$$

$$L \geq \binom{n}{\underbrace{k, \ldots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}} \implies \Omega(n \log(n/k))$$

Sorting $[0, n^3 - 1]$ (Problem $8.3 - 4$)

Sort n integers in $[0, n^3 - 1]$ in $O(n)$ time.

Sorting in Place in Linear Time (Problem $8-2$ (e))

Suppose that the n records have keys in the range $[0, k]$.
Modify COUNTING-SORT to sort them in place $(O(k))$ in $O(n+k)$ time.

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	1	2	3	4	5	6	7	8
A :	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C :	2	0	2	3	0	1
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B :	0	0	2	2	3	3	3	5

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B:

1	2	3	4	5	6	7	8
0	0	2	2	3	3	3	5

A:

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C:

0	1	2	3	4	5
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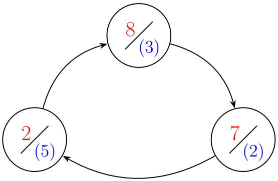

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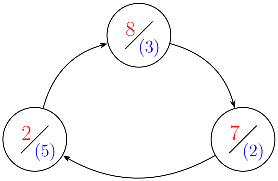
`While (`$i \geq 1$`):`



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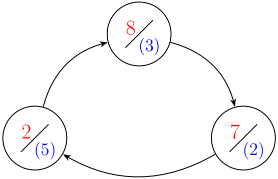

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Finding the 2nd Smallest Element (Problem $9.1-1$)

Show that the 2nd smallest of n elements can be found with $n + \lceil \log n \rceil - 2$ comparisons in the worst case.

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Show that the 2nd smallest of n elements can be found with $n + \lceil \log n \rceil - 2$ comparisons in the worst case.

$$(n-1) + (n-1-1) = 2n-3$$

$$n + \lceil \log n \rceil - 2 = (n - 1) + (\lceil \log n \rceil - 1)$$

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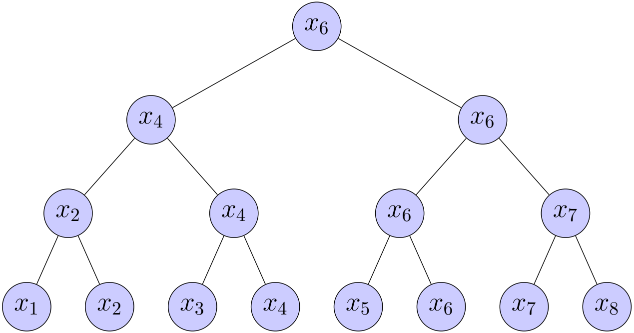

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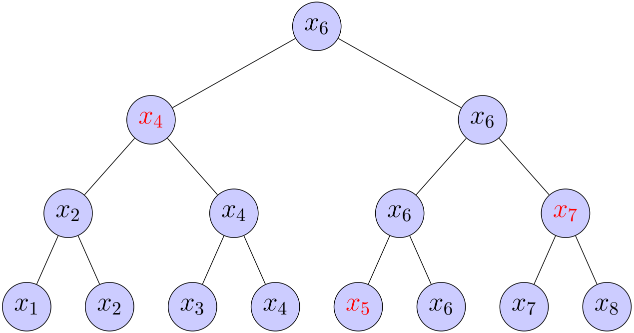

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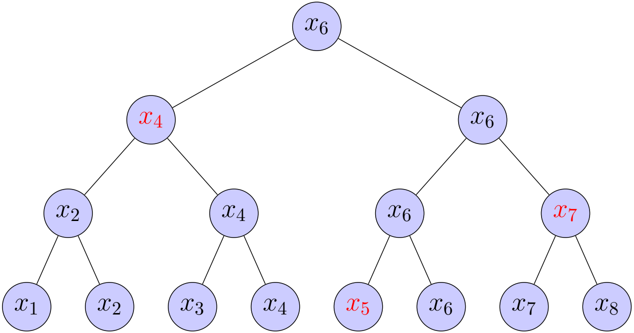

#Potential 2nd smallest elements $\leq \lceil \log n \rceil$

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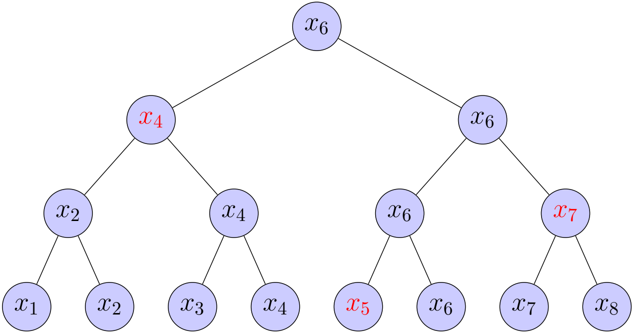

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Q : Can we do even better?

$$\Omega = n + \lceil \log n \rceil - 2$$

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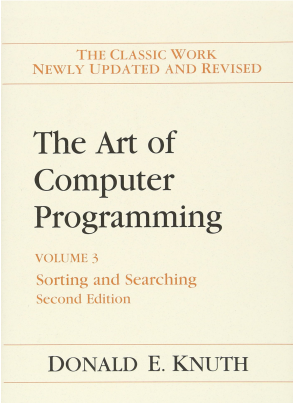

TAOCP Vol 3 (Page 209, Section 5.3.3)

k Numbers Closest to the Median (Problem $9.3 - 7$)

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$$S = \{800, 6, 900, 50, 7\}, \quad k = 2 \implies \{6, 7\}$$

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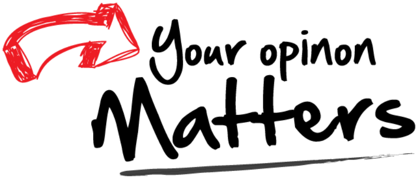
median + subtraction + $(k+1)$-th smallest + partition + add


Thank You!



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