1-5 数据与数据结构 (II)

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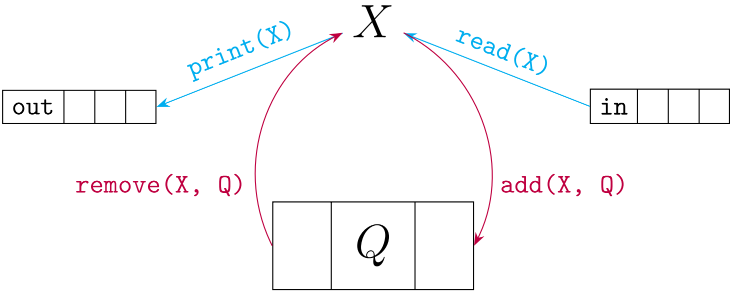
温故而知新 — 孔子

Stackable/Queueable Permutations

Treesort Algorithm

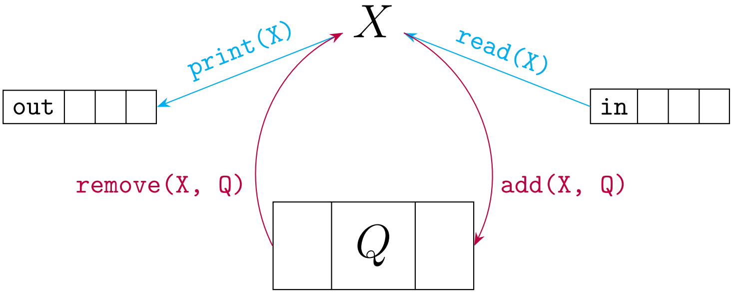
Queueable Permutations

DH 2.14: Queueable Permutations



DH 2.14: Queueable Permutations

$$\boxed{\texttt{out} = (a_1, \cdots, a_n) \xleftarrow[X=0]{Q=\emptyset} \texttt{in} = (1, \cdots, n)}$$



DH 2.14: Queueable Permutations

(a) Show that the permutations given in Excecise 2.12(b) are queueable.

   (i) $(3, 1, 2)$
   (ii) $(4, 5, 3, 7, 2, 1, 6)$

DH 2.14: Queueable Permutations

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DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

```
X = 0      Q = ∅      in != EOF
```

```
foreach 'a' ∈ out:
  if ('a' == in)
    read(X)
    print(X)
  else if ('a' > in)
    add-Q-till('a')
  else // ('a' < in)
    cycle-Q-till('a')
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```
add-Q-till('a'):
  while (('x' ∈ in) != 'a')
    add(X, Q)
  read(X)
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```
cycle-Q-till('a'):
  while (('x' ∈ Q) != 'a')
    remove(X, Q)
    add(X, Q)
  remove(X, Q)
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DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

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```
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    add(X, Q)
  remove(X, Q)
  print(X)
```

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

Proof.

```
foreach 'a' ∈ out:
  if ('a' >= in)
    add-Q-till('a')
  else // ('a' < in)
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```

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

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□

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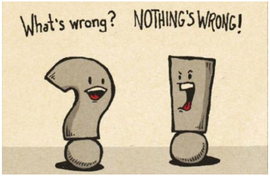

□

Pseudocode

Pseudocode


Make everything as
SIMPLE
as possible,
but not simpler.
- ALBERT EINSTEIN (1879-1955)
QUOTESEVERLASTING.COM

Pseudocode

"Executable" at an abstract level.

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

An "AHA!" Proof.

```
foreach 'a' ∈ in:
  add(X, Q)

foreach 'a' ∈ out:
  cycle-Q-till('a')
```

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

An "AHA!" Proof.

```
foreach 'a' ∈ in:
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DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.

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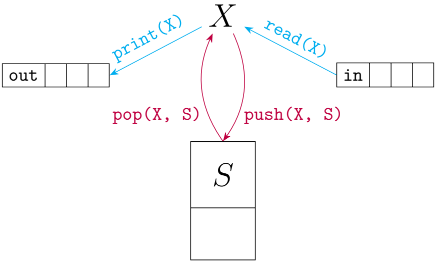

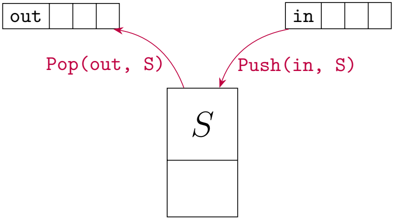

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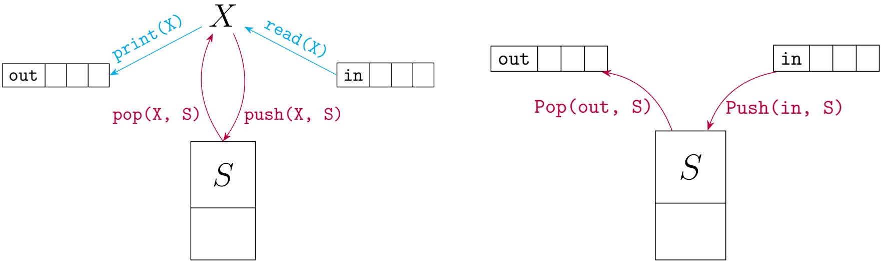
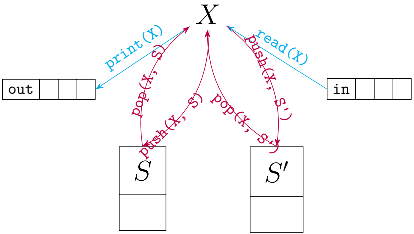


We can similarly speak of a permutation obtained by two stacks, if we permit the push and pop operations on two stacks S and S'.

DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.



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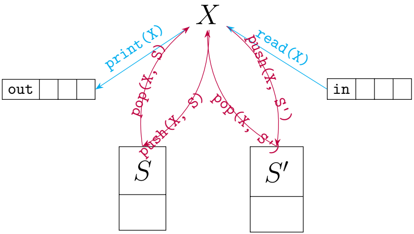


```
foreach 'a' ∈ in:
  read(X)
  push(X, S')

foreach 'a' ∈ out:
  if ('a' <= top(S')) // ∈ S'
    transfer-till(S', S, 'a')
  else // ∈ S
    transfer-till(S, S', 'a')
```

DH 2.15: Algorithm for Queueable Permutations

Extend the algorithm you were asked to design in Exercise 2.13, so that **if** the given permutation cannot be obtained by a stack, the algorithm will print the series of operations on two stacks that will generate it.

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```
two-stackable-perm(in, X, S, S′)
```

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```
two-stackable-perm(in, X, S, S′)
```

```
if (! stackable-perm(in, X, S))
  two-stackable-perm(in, X, S, S′)
```

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```
two-stackable-perm(in, X, S, S′)
```

```
if (! stackable-perm(in, X, S))
  two-stackable-perm(in, X, S, S′)
```

Embedding "`transfer`" into "`stackable-perm`".

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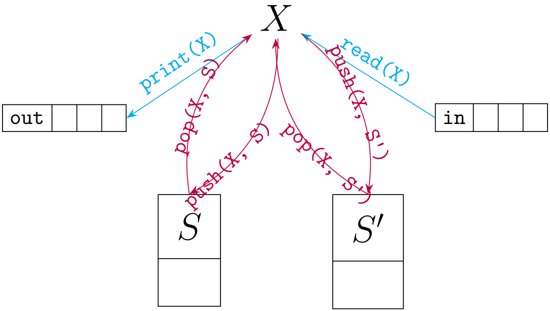

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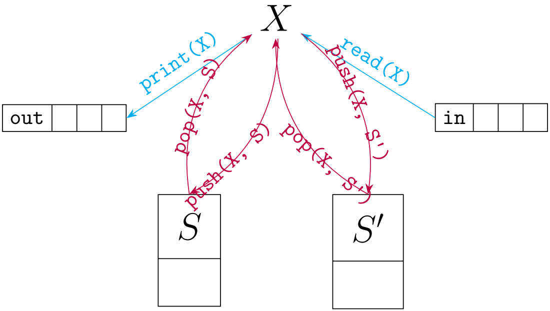

`transfer-till(S, S', top(S) == 'a')`

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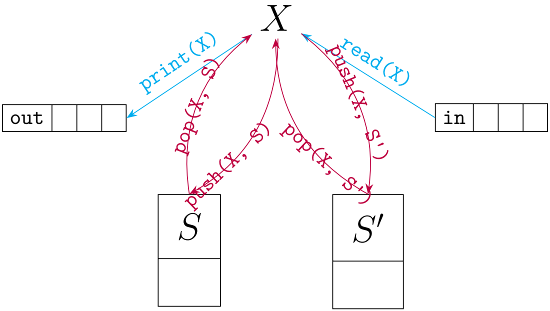
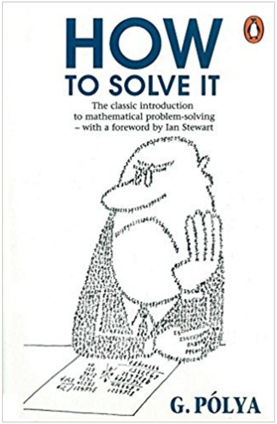
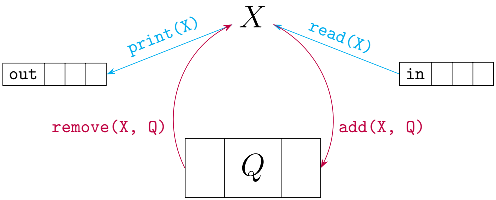
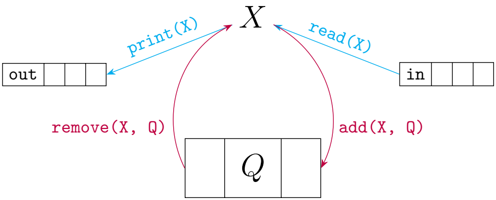



```
transfer-till(S, S', top(S) == 'a')
    transfer-till(S', S, S' == ∅)
```



Step 4: Looking Back!


X
print(X)
read(X)
out
in
remove(X, Q)
add(X, Q)
Q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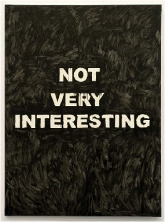
NOT
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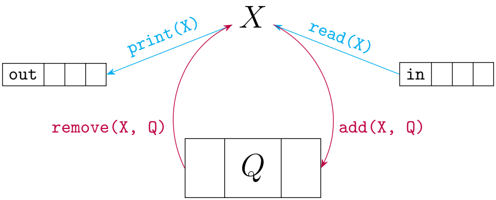
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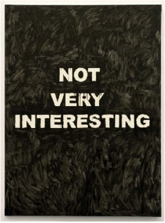
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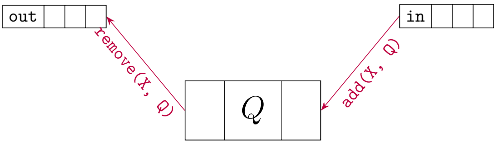
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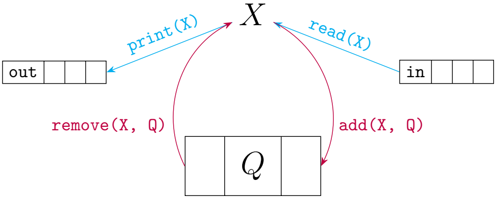
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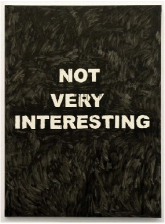
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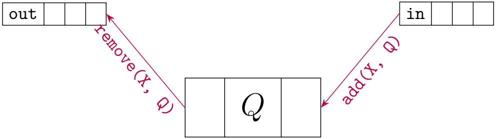
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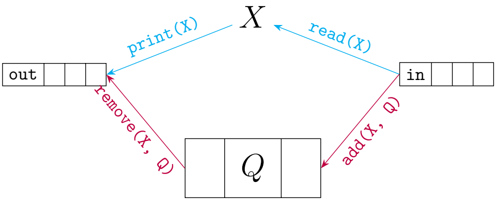
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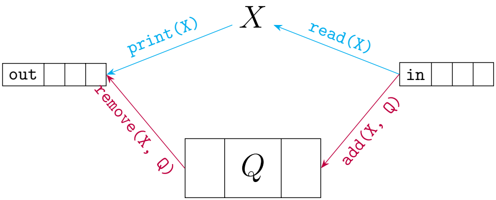
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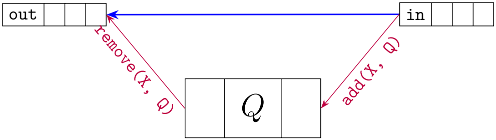
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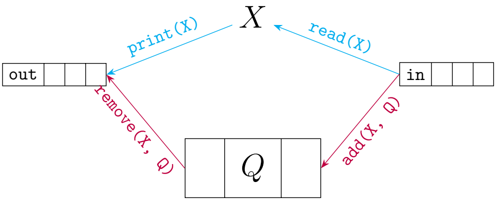
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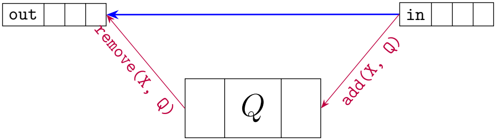
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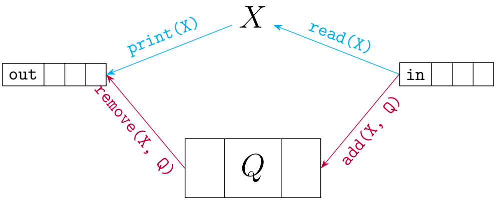
~~3 2 1~~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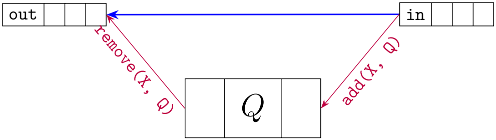
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哎呦，
有点意思

~~3 2 1~~

Theorem (Queueable Permutations)

A permutation $(a_1, \cdots, a_n)$ *is queueable* $\iff$ *it is not the case that*

$$321\text{-}\textit{Pattern}: \boxed{\textit{out} = \cdots a_i \cdots a_j \cdots a_k : i < j < k \land a_i > a_j > a_k}$$

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Proof.

Left as an exercise.

□

Theorem (# of Queueable Permutations)

The number of queueable permutations of $[1 \cdots n]$ *is* $\binom{2n}{n} - \binom{2n}{n-1}$.

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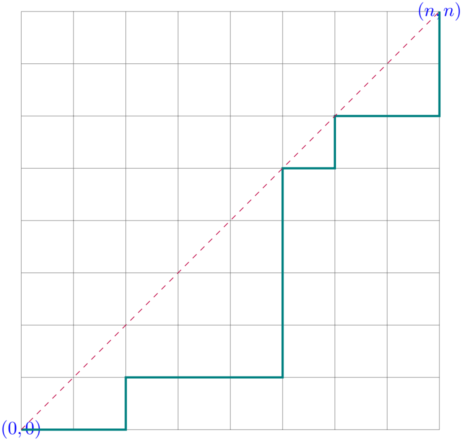

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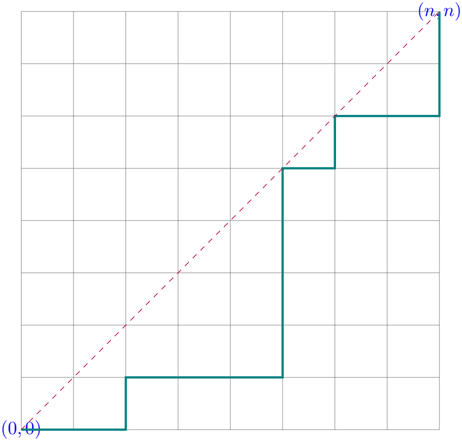

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Left for your research.

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$\square$

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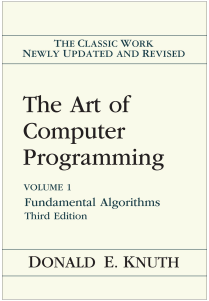
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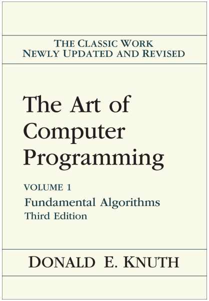
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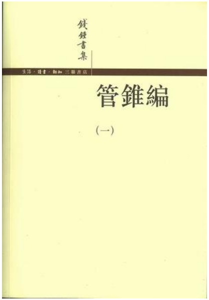
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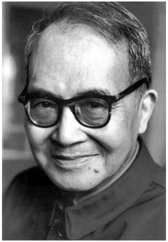
管錐編
(一)


Don Knuth




Thank You!