COMPLEXITY OF APPROXIMATION ALGORITHMS FOR COMBINATORIAL PROBLEMS: A SURVEY *

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Abstract. In this survey we examine recent results on the complexity of approximation algorithms which have appeared in Russian.

1. Introduction.

The recent works [13, 14, 18 and 44] very well reveal the current state of approximation algorithms for hard combinatorial problems. However, it is disappointing that the surveys do not include results that have been discovered in the USSR. The purpose of this survey is to outline briefly the relevant results, which have appeared in Russian.

In the paper we shall concentrate upon the worst-case behavior of the algorithms. For certain problems, such as traveling salesman, much probabilistic analysis of average and "almost everywhere" behavior has been done. However, these questions lie out of the scope of our survey, and those interested may refer to the exhaustive surveys [9, 17, 29].

We shall be making use of concepts from the theory of computational complexity [5, 22, 23, 43]; terms like classes P and NP, NP - complete problem, NP - hard problem, are considered to be known and will be used without further explanations.

2. Computational Complexity of Combinatorial Problems.

It is well known that the main credit for the interest to the P = NP? problem must go to Cook [5] and Karp [22], who first

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explored the relation between the classes P and NP of recognition combinatorial problems. At the same time it should be noted that Levin [30] independently has introduced a concept of "universal" problem, which is very close to"NP - complete problem" of Cook and Karp, and stated a theorem which is actually equivalent to that of Cook. Also, several universal problems, among them set cover, satisfiability and graph homomorphism, have been exhibited in [30] .

Frumkin and Hmelevskii have studied optimization combinatorial problems [11] . Their main result was to introduce formally a class $N\Pi$ of optimization combinatorial problems and to show that an optimization analogue of the well-known satisfiability problem is "hardest" in the class $N\Pi$, the latter fact being an optimization analogue of Cook's theorem.

We can suggest the terminology "$N\Pi$ - complete" for a problem Q such that: (i) $Q \in N\Pi$, and (ii) if $P \in N\Pi$ then P is Turing reducible to Q. Thus, the $N\Pi$ - complete problems are the most difficult problems in $N\Pi$. It is easy to show that optimization prototypes of $N\Pi$ - complete problems, considered by Karp in [22], are all belong to the class $N\Pi$, and, besides, are $N\Pi$ - complete.

With respect to reducibility among combinatorial problems, it is worth mentioning that Livshic [35] and Livshic and Rublineckii [36] have formulated a notion of reducibility, very close to that of Karp, and have proved that the partition (or, synonymously, the subset-sum problem) is reducible to the following scheduling problems:

(a) $n /1/ r_i \geq 0/T_{max}$;
(b) $n /2/ 1/T_{max}$;
(c) $n /1/ r_i \geq 0/ \sum w_i C_i$;
(d) $n /2/ I/ \sum w_i C_i$;
(e) $n /1/ \sum w_i U_i$;
(f) $n /1/ \sum w_i I_i$;
(g) $n /3/ F /F_{max}$;
(h) $n /2/ F, r_i \geq 0/F_{max}$;
(i) $n /2/$ F, start lags and stop lags / F_{max} ;
(j) $n /2/ G, n_i \leq 3/F_{max}$.

(We refer to [18] for the notation of the scheduling problems).

For many combinatorial problems it has been shown that if $P \neq NP$, not only can we not find optimal solutions in polynomial time, we cannot even guarantee coming close, various ways of measuring the "closeness" being discussed in [16, 34, 38, 43].

Livshic in the early paper [34] has showed that for certain network scheduling problems to construct an approximate solution, whose absolute difference from optimal is at most h (h being given, $h > 0$ and integer), is "just as hard" as to obtain the optimal solution. Nigmatullin [38, 39] has obtained similar results for a variety of combinatorial optimization problems, such as minimization of disjunctive normal form, maximal clique, graph coloring, node cover, clique cover, maximal cut, set packing, set cover, Hamilton circuit, traveling salesman. The main idea in obtaining the results has been to consider $h + 1$ interconnected copies of a graph given or to multiply an objective function by $h + 1$. It is clear that we may include in the list the integer linear program, Steiner tree, Johnson's scheduling problem for arbitrary number of processors and many other combinatorial problems.

It is well known that analogous fact related to the error ratio has been revealed by Sahni and Gonzales [43] , who have considered the following evaluation function for evaluating, how good an approximation is:

$$r(I_p) = |f^*(I_p) - f(I_p)|/f^*(I_p) ,$$

where I_p is an instance of a given problem P; $f^*(I_p)$ is the optimal solution value, $f(I_p)$ is the approximate solution obtained. Sahni and Gonzales have presented a list of several problems such that to find a solution no worse than k times optimal (k being given) is just as hard as to find an optimal solution. In the paper [32] the authors add two more problems to the list. These problems are: (i) the allocation problem, for its formulation readers are referred, for example to [6] , and (ii) the partition (subset-sum) problem, formulated in terms of the absolute value of a difference between two disjoint sets:

$$\left| \sum_{i=1}^{n} p_i x_i - \sum_{i=1}^{n} p_i (1 - x_i) \right| \to \min, \; x_i = 0 \text{ or } 1.$$

The evaluation function $r(I_p)$ has the disadvantage that it is not invariant for linear transformations of the objective function f. To avoid that, Yudin and Nemirovskii [48] (and independently, Cornuejols, Fisher and Nemhauser [6]) have suggested the following normalized measure:

$$r'(I_p) = \left| f^*(I_p) - f(I_p) \right| / \left| f^*(I_p) - \text{worst } f(I_p) \right|.$$

However, in this case, also, many combinatorial problems seem to have no good approximation algorithms. In particular, the authors [32] have studied " ϑ - approximative" algorithms, that is ones, which for every $\vartheta > 0$ and every input I_p of the given problem P, provide an approximate solution $f(I_p)$ such that $r'(I_p) \leq \vartheta$. We have considered so - called "fast" (or, synonymously, "fully polynomial") algorithms, i.e. those polynomial both in the problem size and $1/\vartheta$, where ϑ denotes the desired accuracy in the sense of Nemirovskii and Yudin. It has been shown that unless P = NP, no fast ϑ - approximative algorithm can exist for the traveling salesman, even if distances in the latter obey the "triangle inequality". The main idea to prove this has been in constructing the following traveling salesman problem TSP corresponding to the Hamilton circuit (in graph G) problem: a distance d_{ij} between two cities, i and j, in TSP is taken equal to 1 if there is an arc in graph G incident with nodes i and j, and $d_{ij} = 2$, otherwise. It is easy to see that $\left| f^*(I_p) - \text{worst } f(I_p) \right| \leq \sum_{i,j} d_{ij} < n(n-1)$, and a ϑ - approximative solution, with $\vartheta = 1/_{n(n-1)}$, of TSP gives an exact solution of Hamilton circuit problem.

The same fact may be established for cover, colouring and many other combinatorial problems.

Another way to measure the accuracy, which makes use of the problem discreteness, is to consider in addition to the optimal solution, the k-th best solutions and k-solutions.

Let P be a combinatorial problem, say, a maximization problem with f(x) being an objective function. Let all the feasible solutions, x , be ranked in nonincreasing order of cost f(x), the k-th solution being called the k-th best one. All the x's with the same f(x) value are assumed to be of the same group. Thus we get an ordered sequence of groups G_1, G_2, G_3,..., G_k... such that

$$f(x \mid x \in G_1) > f(x \mid x \in G_2) > f(x \mid x \in G_3) > \dots \qquad (1)$$

An approximate solution will be called a k-solution if it belongs to any of the first k groups in the sequence (1).

For example, if an objective function f(x) is integer - valued, an approximate solution, whose absolute difference from the optimal value f(x) is at most 1, will be a 2 - solution. It is clear that if x is the k-th best solution, it is also, a k - solution and in this sense to find a k - solution is "easier" than to find the k-th best solution.

For some NP - hard combinatorial problems to find k - solutions (for some k) is a tractable problems; (such a problem is considered by Dinic [7] and Dinic and Karzanov [8] , it will be discussed below). On the other hand, for some other problems (e.g. traveling salesman, knapsack, partition) the authors [16] have shown that for any k , bounded by a polynomial function of the problem size, to find k-solutions and, also, the k-th best solutions is as difficult as to find optimal solutions.

3. Performance Guarantees for Combinatorial Algorithms.

Most of the work on combinatorial algorithms outside the area of solving tractable problems has been worst-case analysis of heuristics for "intractable" problems.

One of the earliest papers on performance guarantees has been one by Korobkov and Krichevskii [26] which has analyzed heuristics for the traveling salesman problem with distances obeying the "triangle inequality"; it has been shown that there exist $O(n^2)$ heuristics that produce a solution never longer than twice optimal (n being a number of cities). Analogous results have subsequently been independently discovered and appeared in English in [40]. Leont'ev [28], Rublineckii [41] and Vizing [47] have shown that a number of $O(n^2)$ heuristics for the traveling salesman problem produce a solution never longer than an "average route", the latter being defined as a sum of all the route lengths divided by the number of routes.

Nigmatullin [37] and, independently, Johnson [21] have studied a "greedy" heuristic for the minimum set cover problem whose worst-case error ratio can be bounded in terms of the size of the largest set, and these bounds grow only with the logarithm of the largest set size.

Livshic in the paper [34] (and, independently, Kaufman [24]) has considered the problem of minimizing finishing time when scheduling n tasks on systems of m identical processors ($m < n$), the ordering relation between tasks being a tree and task lengths being bounded by k. In this case an $O(n \log n)$ algorithm is delivered which yields schedules whose maximum finishing time never differs (additively) from optimal by more than $k - k/m$.

Belov and Stolin [4] have examined the classic Johnson flowshop scheduling problem $n/m/F/C_{max}$, task lengths being bounded by k. An interesting $O(n^{m-1})$ algorithm is suggested which produce an approximate solution never differing (additively) from optimal by more than $k(m-1)(C_m(m-1)^{1/2}+1)$, C_m depending only on m. The algorithm makes use of the following fact: if in m - dimensional Euclidean space a closed broken line has n seqments, each of them of a length no more than 1, then they may be transposed in such a way that all of them belong to a sphere with a radius of C_m, where C_m depends on m and does not depends on n. It is known, that $1/2\, m^{1/2} \leq C_m \leq ((4^m-1)/3)^{1/2}$. Analogous algorithm with better bounds has been independently obtained by Sevost'yanov [45, 46] .

Dinic [7] (and Dinic and Karzanov [8]) has considered a special case of multi-dimensional knapsack problem:

Minimize $\sum_{i=1}^{n} x_i$

subject to $\sum_{i=1}^{n} a_i x_i \geq A$, $\sum_{i=1}^{n} b_i x_i \geq B$, $x_i = 0$ or 1, $a_i \geq 0, b_i \geq 0$.

The problem is shown to be NP - hard, the partition problem being reducible to it. An $O(n^2 \log n)$ algorithm is suggested, giving an approximate solution whose absolute difference from optimal is at most 1. The algorithm is combinatorial and makes use of the duality techniques. It may be generalized to the case of m constraints, $m > 2$.

We now know very little about lower bounds for combinatorial algorithms and about the relationship between time and space as measures of complexity of algorithms. An important example of research in this area is work by Grigor'ev [19] , which shows that when two nxn Boolean matrices are multiplied on a class of Boolean circuits, a lower bound for the product of running time and space is $1/8\ n^3$.

In the recent years attention has turned to the construction of "fast" or fully polynomial approximation schemes, i.e. those capable of guaranteeng performance, arbitrarily close to optimal, and having time and space polynomial in both problem size and $1/\epsilon$ (ϵ being the accuracy desired). First algorithms of this type have been discovered independently at the same period by Babat [1, 2] , Ibarra and Kim [20] , and Sahni [42] .

Babat has presented an elegant algorithm for the 0-1 min - knapsack problem. It is based on ideas of dynamic programming and on the following two facts:

Theorem 1. Let $f(x) = \sum_{i=1}^{n} C_i x_i$, $x_i \in (0,1)$.
If the C_i, ..., C_n are positive, then for any $\epsilon > 0$ all the f(x) values may be separated into ϵ - groups, the number of ϵ- groups being less or equal to $n(1+1/\log(1+\epsilon))$ (where two values, a and b, are said to be of the same ϵ- group ($\epsilon > 0$), iff $|a-b|/\min(a,b) \lesssim$ $\lesssim \epsilon$).

Theorem 2. Let $f(x) = \sum_{i=1}^{n} C_i x_i$, $x_i \in (0,1)$, $C_i > 0$ ($i = 1, \ldots, n$). For any $\epsilon > 0$, in $O(n^2/\epsilon)$ time it is possible to obtain a set of Boolean vectors $N = \{x^1, \ldots, x^t\}$, where $t \leq 2n(1 + 1/\log(1+\epsilon))$, such that for any Boolean $x = (x_1, \ldots, x_n)$ $(x \neq 0)$ there exist x^i **and** x^j **from N, satisfying** the **following** conditions: $f(x^i) \leq f(x) \leq f(x^j)$ and $f(x^j) / f(x^i) \leq 1 + \epsilon$.

One may find a more readable presentation of Babat's approach in the **surveys** given by Fridman [12] and by the authors [32].

The worst-case behaviour of the Babat algorithm, $O(n^3/\epsilon)$, is worse than that of the algorithms in [20, 42], but the former has some special good properties. For example, it is capable of solving both maximization and minimization knapsack problems, while if we want to solve the min-knapsack problem by the Ibarra-Kim or Sahni algorithms, we should modify them, and in particular, we should know a special bound $\hat{f}$, (e.g. $\hat{f} \leq f^* \leq 2\hat{f}$) on the optimum f^* of the problem considered. Besides, as the authors have mentioned in [32], the Babat's algorithm gives, in $O(n^3/\epsilon)$ time and space, ϵ - approximate solutions parametrically for all possible righthand sides of the knapsack constraint

$\sum_{i=1}^{n} a_i x_i \gtrless b$.

In [31, 32] the authors have presented an $O(n^3/\epsilon)$ algorithm for the min-job-sequencing - with-deadlines problem, which is a modification of the Babat algorithm for min-knapsack, and, thus, preserves all good properties of the latter algorithm. Another algorithm for the said sequencing problem has been later derived by the authors [16], its time being bounded by $O(n^2 \log n + n^2/\epsilon)$ and space $O(n^2/\epsilon)$; the algorithm is not capable of solving the problem parametrically.

More careful analysis of the Ibarra-Kim and Sahni algorithms, carried out by the authors [15, 16, 32, 33], made it possible to obtain bounds of $O(n/\epsilon)$ on time and space (and $O(n + 1/\epsilon^2)$ on time and space) for the subset-sum problem, instead of $O(n/\epsilon^2)$

bound in the original Ibarra-Kim algorithm. Another version of this algorithm [16], which makes use of the trade-off between time and space, has $O(n/\epsilon + 1/\epsilon^2)$ time, but only $O(n + 1/\epsilon)$ space.

Table 1.

Complexity of Approximation Algorithms.

Problem	Time Complexity	Space Complexity	Reference
1. (0-1) - min-knapsack	$O(n^3/\epsilon)$	$O(n^3/\epsilon)$	[1,2]
	$O(n^2 \log n + n^2/\epsilon)$	$O(n^2/\epsilon)$	[16]
(0-1) - max-knapsack	$O(n \log 1/\epsilon + 1/\epsilon^4)$	$O(n+1/\epsilon^3)$	[27]
2. max-multiple-choice-knapsack	$O(nm/\epsilon)$	$O(n+m^2/\epsilon)$	[16,27,32]
min-multiple-choice-knapsack	$O(n \log n + mn \log m + mn/\epsilon)$	$O(n + m^2/\epsilon)$	[16,33]
3. fixed-charge-knapsack	$O(n^3/\epsilon^2)$	$O(n^2/\epsilon)$	[16, 32, 33]
4. max-arborescent-knapsack	$O(n^3/\epsilon^2)$	$O(n^2/\epsilon)$	[33]
5. nonlinear knapsack	$O(n^4/\epsilon^2)$	$O(n^3/\epsilon)$	[32]
6. min-continuous-fixed-charge knapsack	$O(n^4/\epsilon^3)$	$O(n^3/\epsilon^2)$	[3]

Formulations of the problems considered may be found, for example, in [16, 27, 32, 33], n is the number of items, m is the number of equivalence classes in multiple-choice knapsack.

This Table 1 summarizes the complexity of fast algorithms that have been obtained for various generalizations of the knapsack problem. Two special methods, "decomposition" and "binary search", have been developed for constructing the bounds $\hat{f}$ on the optimum f^* (e.g. $\hat{f} \leq f^* \leq 8\hat{f}$) in [16, 32, 33].

Analysis of the fully polynomial approximation schemes shows that their applications for solving integer programming problems are rather narrow. We can show that to obtain a fully polynomial approximation scheme even for two-dimensional knapsack is impossible, unless P = NP. Really, let us consider a special case of that problem, with an objective function $f(x) = \sum_{i=1}^{n} x_i$. If we could find its ϵ - approximate solution, f°, we would take $\epsilon = 1/n+1$, and then, since $f(x)$ is integer, and $f^* - f^\circ \leq f^*\epsilon = f^*/n+1 \leq n/n+1 < 1$, the obtained approximate solution f° should be equal to the optimum f^*. But as it is mentioned above, to solve that problem exactly is NP - hard problem, the fact due to Dinic [7]. So, if $P \neq NP$ then it is impossible to obtain a polynomial (in problem size n) ϵ - approximation algorithm with $\epsilon = 1/n+1$ and, therefore, it is impossible to have an ϵ - approximation algorithm, polynomial both in problem size and $1/\epsilon$.

Quite recently, the question of constructing a polynomial ("non-fast") algorithm for the m-dimensional knapsack problem has become solved. This result is due, on the one hand, to a great advance made by Khachiyan [25] who has developed a special "ellipsoid" method of linear programming with a polynomial (of the eighth power) worst-case time bound, and, on the other hand, to Finkel'stein [10] who has presented an ϵ - approximation version of branch-and bound algorithm, with a linear programming problem being solved in each vertice of the branching tree and the number of vertices having polynomial (in n) growth (for every fixed m).

In conclusion, we would like to propose three open questions which have been presented in [32] and are unsolved as yet.

1. For some combinatorial problems it is unknown, whether they are NP - complete or polynomially solvable. We present one more problem of this type:

"Given natural **n**, C_i, a_i $(i = 1, \ldots, n)$, b, y.

Is there a n - dimensional 0-1 vector $x = (x_1, \ldots, x_n)$ such that

$$\sum_{i=1}^{n} C_i x_i = y, \quad \sum_{i=1}^{n} a_i x_i \leq b,$$

$$\sum_{i=1}^{n} x_i \leq k(n), \quad x_i \in \{0,1\} \text{ ?"}$$

In fact, if $k(n) = C$, C being a constant, we have a problem from the class P; if $k(n) = n$, we have the NP - complete knapsack problem. We may suggest that when $k(n) < n$, e.g. $k(n) = \log_2 n$, the problem belongs (if $P \neq NP$) to the class which do not coincide with the class of NP - complete problems.

2. Little is known about whether or not the existing fast algorithms are improvable. For example, can the worst-case time for the min-multiple-choice-knapsack, namely, $O(n \log n + nm \log m + nm/\epsilon)$, ($m$ = number of equivalence classes, see [16]) be improved to become such as that for the max-multiple-choice-knapsack, $O(nm/\epsilon)$? More generally, what can be said about lower bounds on time, space, or their product?

3. More accurate analysis of evaluation functions for measuring the accuracy of an approximation may be the subject of ongoing research. For example, the question is open, whether or not a polynomial (non-fast) ϑ - approximation algorithm exist for the traveling salesman problem (ϑ being an accuracy desired in the sense of Nemirovskii and Yudin, $o < \vartheta < 1$).

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R E F E R E N C E S

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