2-9 Sorting and Selection

Hengfeng Wei

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May 28, 2018

How to Argue?

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Show that $\cdots$

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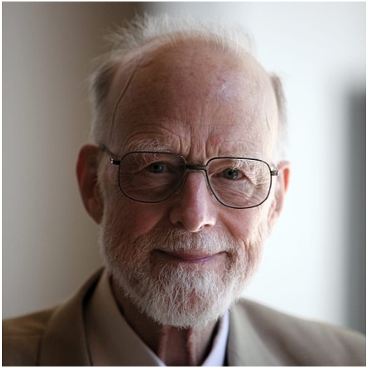
Show that $\cdots$

Argue that $\cdots$

$=$ Prove that $\cdots$

QUICKSORT Invented by Tony Hoare in 1959/1960



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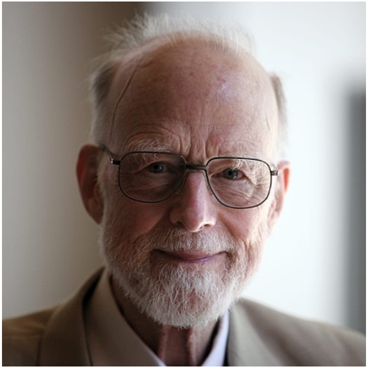

`null pointer`

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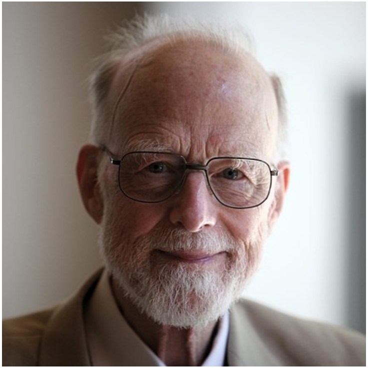

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"I call it my billion-dollar mistake."

Best-Case Complexity of QUICKSORT $(7.4-2)$

Show that QUICKSORT's best-case running time is $\Omega(n \log n)$.

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By substitution.

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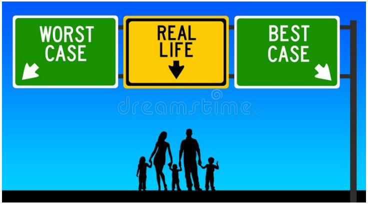

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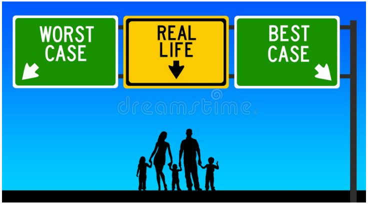

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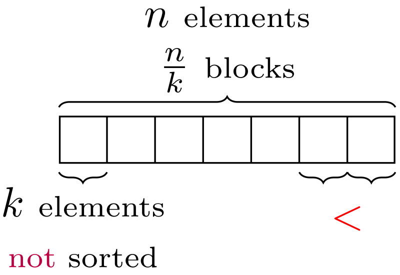
The $\frac{n}{k}$-sorted Problem (Problem $8.1-4$)

Sorts an already $\frac{n}{k}$-sorted array

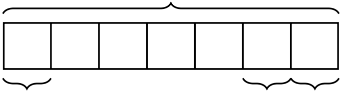


The $\frac{n}{k}$-sorted Problem (Problem $8.1-4$)

Sorts an already $\frac{n}{k}$-sorted array

n elements

$\frac{n}{k}$ blocks



k elements

$<$

not sorted

$\Omega(n \log k)$

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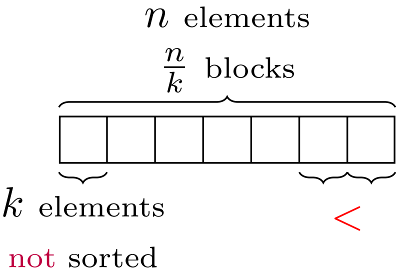

$\Omega(n \log k) \qquad O(n \log k)$

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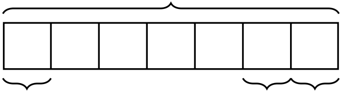

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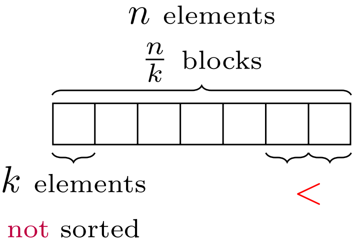
$\Omega(n \log k) \qquad O(n \log k)$

$$(k!)^{\frac{n}{k}} \le L \le 2^H$$

$\frac{n}{k}$-sorts an arbitrary array



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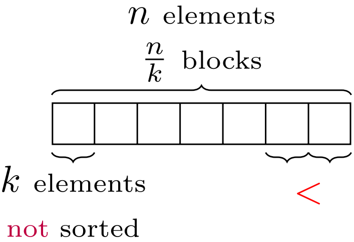

$O(?)$

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n elements

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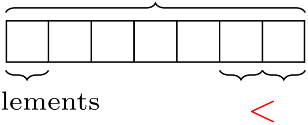
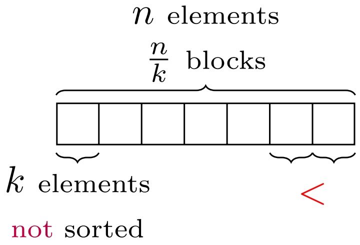


$O(?)$ $\Omega(?)$

$\frac{n}{k}$-sorts an arbitrary array



$O(?) \qquad \Omega(?)$

$$L \geq \binom{n}{\underbrace{k, \dots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}}$$

$\frac{n}{k}$-sorts an arbitrary array

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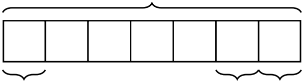


k elements

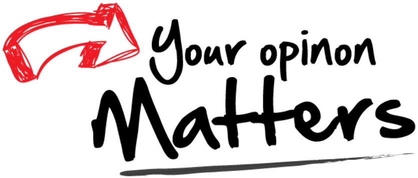
$<$

not sorted

$O(?) \qquad \Omega(?)$

$$L \geq \binom{n}{\underbrace{k, \ldots, k}_{\frac{n}{k}}} = \frac{n!}{(k!)^{\frac{n}{k}}} \implies \Omega(n \log(n/k))$$


Thank
You!



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