Homework

魏恒峰

hfwei@nju.edu.cn

2018 年 02 月 28 日

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Definition (Well-ordered Set (SM Definition 14.1))

An ordered set S is said to be *well-ordered* if every non-empty subset of S has a first element.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Definition (Well-ordered Set (SM Definition 14.1))

An ordered set S is said to be *well-ordered* if every non-empty subset of S has a first element.

Definition (Isomorphic)

Two ordered sets A and B are said to be *isomorphic*, written $A \simeq B$, if $\exists f : A \xleftrightarrow[onto]{1-1} B$ which preserves the order relations

$$\forall a, a' \in A : a \prec a' \iff f(a) \prec f(a')$$

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Remark: What if "similarity mapping"?

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Remark: What if "similarity mapping"?

Definition (Similarity Mapping)

A function $f : A \xleftrightarrow{1-1} B$ is called a *similarity mapping* from A to B if f preserves the order relations

$$\forall a, a' \in A : a \prec a' \iff f(a) \prec f(a')$$

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Remark: What if "similarity mapping"?

Definition (Similarity Mapping)

A function $f : A \xleftrightarrow{1-1} B$ is called a *similarity mapping* from A to B if f preserves the order relations

$$\forall a, a' \in A : a \prec a' \iff f(a) \prec f(a')$$

Counterexample for "similarity mapping":

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Remark: What if "similarity mapping"?

Definition (Similarity Mapping)

A function $f : A \xleftrightarrow{1-1} B$ is called a *similarity mapping* from A to B if f preserves the order relations

$$\forall a, a' \in A : a \prec a' \iff f(a) \prec f(a')$$

Counterexample for "similarity mapping":

$$A = B = \mathbb{N} \qquad f : a \mapsto a \qquad f' : a \mapsto a + 1$$

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Lemma

*X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.*

SM Problem 14.62

Suppose A and B are *well-ordered isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.

Proof.

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping. Then $\forall x \in X : f(x) \geq x$.

Proof.

By contradiction.

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.

Proof.

By contradiction.

$$Y = \{x \in X \mid f(x) < x\} \neq \emptyset$$

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.

Proof.

By contradiction.

$$Y = \{x \in X \mid f(x) < x\} \neq \emptyset$$

Well-ordered: $x = \min Y \implies f(x) < x$

Lemma

X is a well-ordered set. $f : X \to X$ is a similarity mapping.
Then $\forall x \in X : f(x) \geq x$.

Proof.

By contradiction.

$$Y = \{x \in X \mid f(x) < x\} \neq \emptyset$$

Well-ordered: $x = \min Y \implies f(x) < x$

Similarity: $f(f(x)) < f(a) \implies f(a) \in X$

$\square$

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Suppose $f : X \to X$ is an automorphism.

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Suppose $f : X \to X$ is an automorphism.

f is a similarity from X to X $\implies \forall x \in X : f(x) \geq x$

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Suppose $f : X \to X$ is an automorphism.

$$f \text{ is a similarity from } X \text{ to } X \implies \forall x \in X : f(x) \geq x$$

$$f^{-1} \text{ is a similarity from } X \text{ to } X \implies \forall x \in X : f^{-1}(x) \geq x$$

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Suppose $f : X \to X$ is an automorphism.

f is a similarity from X to $X \implies \forall x \in X : f(x) \geq x$

f^{-1} is a similarity from X to $X \implies \forall x \in X : f^{-1}(x) \geq x$

f is a similarity from X to $X \implies x = f(f^{-1}(x)) \geq f(x)$

Lemma

X is a well-ordered set.
Then X has only one automorphism, i.e., id_X.

Proof.

Suppose $f : X \to X$ is an automorphism.

f is a similarity from X to $X \implies \forall x \in X : f(x) \geq x$

f^{-1} is a similarity from X to $X \implies \forall x \in X : f^{-1}(x) \geq x$

f is a similarity from X to $X \implies x = f(f^{-1}(x)) \geq f(x)$

$$id_X : f(x) = x$$

□

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Proof.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Proof.

By contradiction.

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Proof.

By contradiction.

$\exists$ isomorphic mappings $f, g : A \to B$

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Proof.

By contradiction.

$$\exists \text{ isomorphic mappings } f, g : A \to B$$

$$h = g^{-1} \circ f \text{ is an automorphism on } A \implies h = id_A$$

SM Problem 14.62

Suppose A and B are *well-ordered* *isomorphic* sets.
Show that there is only one *isomorphic mapping* $f : A \to B$.

Proof.

By contradiction.

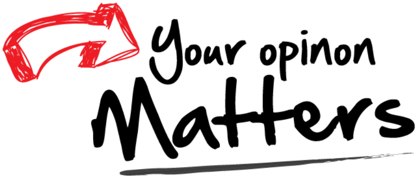
$$\exists \text{ isomorphic mappings } f, g : A \to B$$

$$h = g^{-1} \circ f \text{ is an automorphism on } A \implies h = id_A$$

$$f = g \circ h = g \circ id_A = g$$

□

Thank You!



Office 302

Mailbox: H016

hfwei@nju.edu.cn