2-4 Recurrences

魏恒峰

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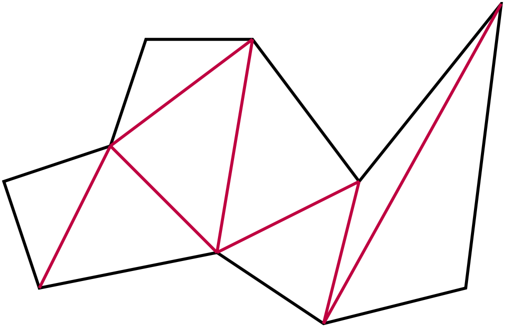
Triangulating Polygons



The Art Gallery Problem

Q : How many "BIG BROs" to hire?

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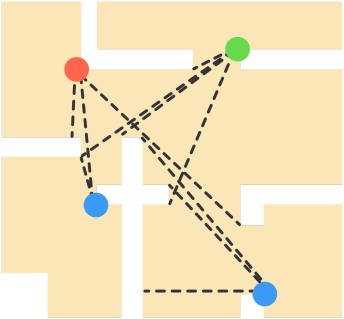

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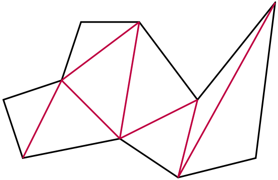
Another Version of the Ear Lemma (Problem $4.1 - 16$)

A triangulated polygon is either a triangle with three ears or has at least two ears.

of triangles (Problem $4.1 - 17$)



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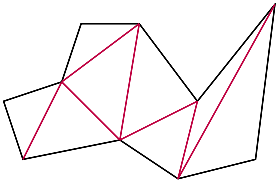

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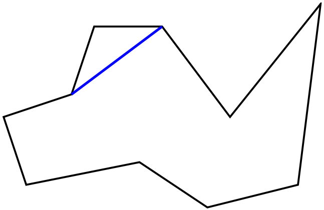

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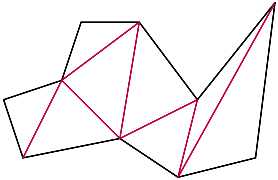

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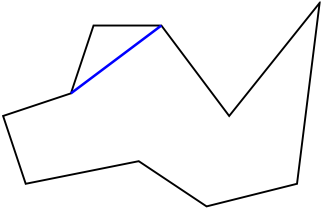

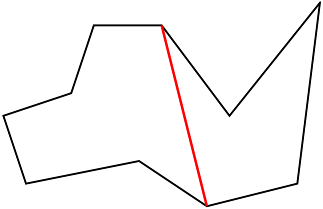

Lemma (Ear Lemma)

A triangle has 3 ears, and a larger triangulated polygon has at least 2 non-adjacent ears.

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Q : Can every polygon be triangulated?

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

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> *They break the interior of the polygon into a number of triangles, because any larger polygon can be split by adding a diagonal."*

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Every polygon with $n > 3$ has a diagonal.

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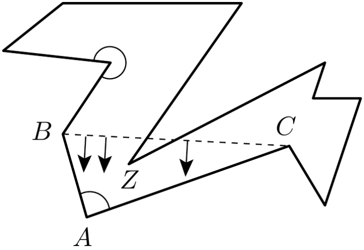
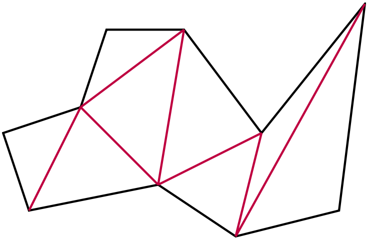


Theorem (Coloring)

Any triangulated polygon polygon is 3-colorable.



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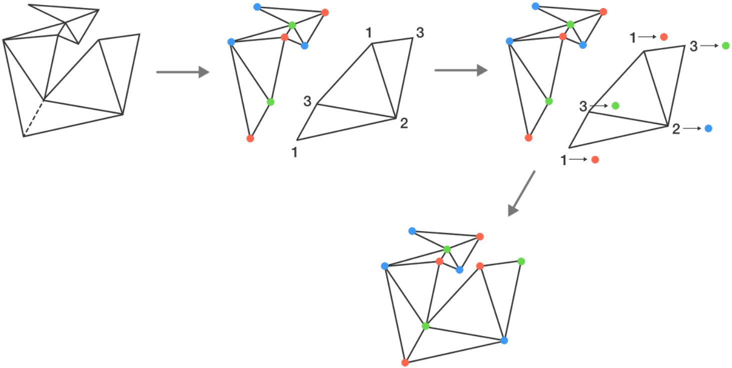

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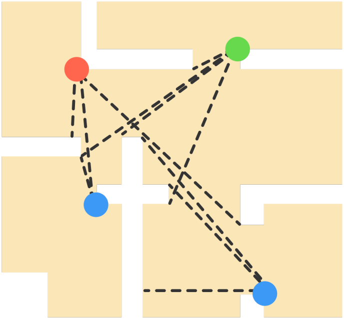

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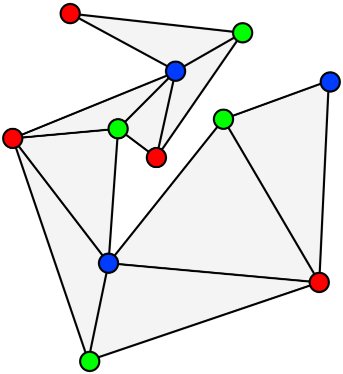

Theorem (The Art Gallery Theorem (O))

For any art gallery with n walls, $\lfloor \frac{n}{3} \rfloor$ "BIG BROs" suffice.

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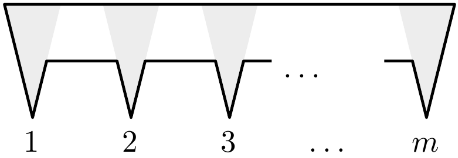
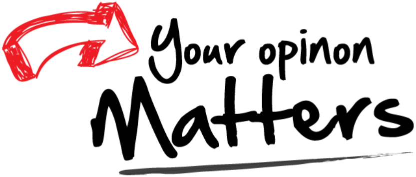


$$n = 3m$$


Thank You!



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