3-10 Traversability

(Part I: Eulerian Graphs)

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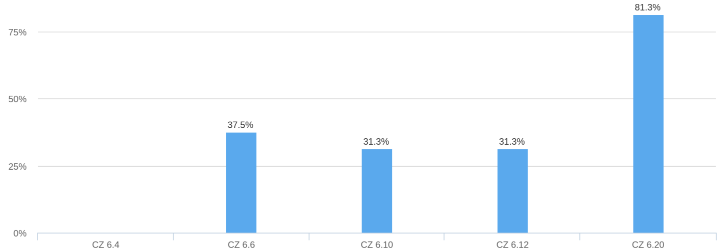


CZ 6.20 (Next Class)

这次习题相对简单【因为是必选的所以随机选了一个】，希望老师可以多回顾一下课本内容，比如哈密尔顿图的各种充分条件和证明，对哈密尔顿图和欧拉图的用法做一些拓展

欧拉图和汉密尔顿图的联系 就是在建模时如何确定图的节点和边

g-cage 对于不同大小而言都是唯一的吗？（书上只给到n=8） Kirkman triple system 和 factorization 的等价推导 Theorem 8.10的证明

6.3，不想看书，看自闭了，希望老师可以直接讲一下

如何打印欧拉回路/欧拉迹/哈密顿回路

定理6.5

无

暂无

无

陶老师上课讲的中国邮递员问题没明白

暂无，我就来抽个奖

none

定理6.12的证明

喵(^･ｪ･^)

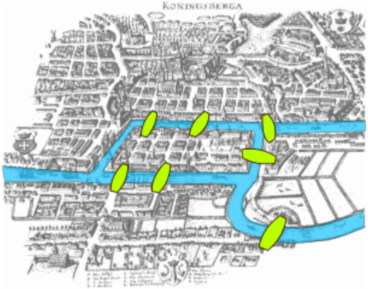
希望能讲一下fluery算法

可以总结一下证明的方法，其实每次都可以这样，不一定要课上讲，可以整理之后做成讲义课后发，比如怎么证明有欧拉回路等等

FLEURY (HIERHOLZER)

Chinese Postman Problem (Next Class)

6.3 Exploration & 6.4 Excursion (Not Required)



Leonhard Euler (1707 – 1783)

Graph Theory Topology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Theorem (Leonhard Euler 1735)

A connected graph G is Eulerian if and only if the degree of each vertex of G is even.

"$\Longleftarrow$" (Carl Hierholzer 1873); (Proofs on PPT from Tao & on CZ).

By strong mathematical induction on the number m of edges of G.

$$\text{Inductive Step}: m = k + 1$$

$$\text{Let } C = u \sim v \text{ be any trail in } G \text{ of the maximum length.}$$

$$v = u \implies C = u \sim u$$

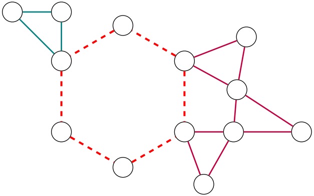
$$H = G - E(C) = \bigcup H_i$$

$\square$

$$H = G - E(C) = \bigcup H_i$$

(I) $\forall v \in H : \deg(v)$ is even

(II) $\forall i : \Big| E(H_i) \Big| < m$

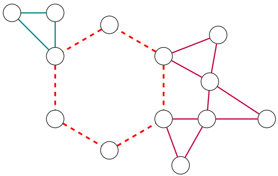


By I.H., each H_i has an Eulerian circuit C_i.

$$\forall i : V(H_i) \cap V(C) \neq \emptyset$$

Glue together each C_i with C to get an Eulerian circuit of G.

```
procedure HIERHOLZER(G)
    u ∈ V(G)
    C ← any circuit u ∼ u in G
    while ∃v ∈ C : deg(v) > 0 do
        H ← G − E(C)
        v ← any vertex in V(C) such that deg(v) > 0
        C′ ← any circuit v ∼ v in H
        C ← C ⊗ C′                    ▷ Glue C′ = v ∼ v with C via v
    return C
```



Q : Time Complexity?

Q : Data Structures?

$O(m)$: Using doubly linked list

Fleury's Algorithm (1883)

(I) $v_0 \in V(G)$; $\quad C_0 = v_0$

(II) Suppose $C_i = v_0 e_1 v_1 e_2 \cdots e_i v_i$.

Choose e_{i+1} from $E(G) - \{e_1, e_2, \cdots, e_i\}$ as follows:

(a) e_{i+1} is incident with v_i

(b) Unless there is no alternative, e_{i+1} is not a bridge of $G - \{e_1, e_2, \cdots, e_i\}$

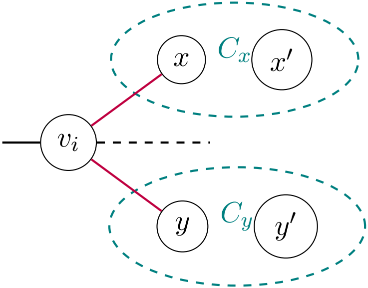
(III) Stop when step (II) can no longer be implemented

Theorem (Bridges in Fleury's Algorithm)

At any stage, v_i is incident with ≤ 1 bridge in $E(G) - \{e_1, e_2, \cdots, e_i\}$.

By Contradiction.

Suppose that v_i is incident with ≥ 2 bridges in $E(G) - \{e_1, e_2, \cdots, e_i\}$.



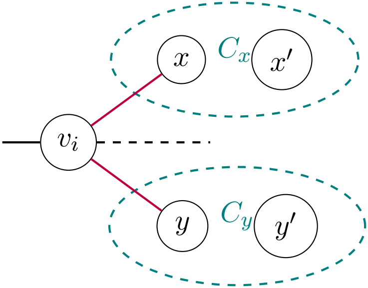
$\exists x' \in C_x : \deg(x)$ is odd

$\exists y' \in C_y : \deg(y)$ is odd

We have found 2 odd vertices.

Q : What is the contradiction?

Is $\deg(v_i)$ odd or even?



We have found 2 odd vertices.

Is $\deg(v_i)$ odd or even?

CASE I: $\deg(v_i)$ is odd.

Contradiction:

Only v_0 and v_i can have odd degrees!

CASE II: $\deg(v_i)$ is even.

$$v_i = v_0$$

Contradiction: No odd vertices!

procedure FLEURY(G)

  $v_0 \in V(G)$ ▷ Choose any starting vertex

  $C \leftarrow v_0$ ▷ Keep track of the circuit

  $i \leftarrow 0,\ V_0 \leftarrow V(G),\ E_0 \leftarrow E(G)$

  while $\deg(v_i) > 0$ in E_i **do** ▷ Stop otherwise

    if $\deg(v_i) = 1$ in E_i **then** ▷ No alternative: go the bridge

      $e_{i+1} \triangleq v_i v_{i+1}$

      $V_{i+1} \leftarrow V_i - \{v_i\}$ ▷ Delete the isolated vertex v_i

    else ▷ Have alternatives: don't go the bridge

      Choose $e_{i+1} \triangleq v_i v_{i+1}$ that is not a bridge of (V_i, E_i)

      $V_{i+1} \leftarrow V_i$ ▷ No isolated vertex produced

    $C \leftarrow C e_{i+1} v_{i+1}$

    $E_{i+1} \leftarrow E_i - \{e_{i+1}\}$

    $i \leftarrow i + 1$

  return C

PROOF

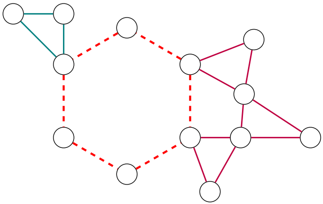
We need to prove that C returned by FLEURY is an Eulerian circuit.

Definition (Eulerian Circuit)

A connected graph is Eulerian if there exists a closed trail that includes every edge of G.

A trail is a walk in which all the edges are distinct.

Closed	Trail	Include every edge of G
$\because$ even degrees	$\because$ used edges are deleted	By Contradiction.



Include every edge of G

By Contradiction.

We know that $C : v_0 \sim v_0$ $\qquad E' \triangleq E(G) - E(C) \neq \emptyset$

$\deg(v_0) = 0$ (Otherwise, FLEURY is not terminated.)

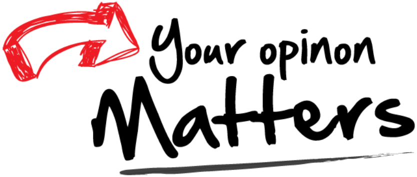
$G|_{E'}$ is disconnected from v_0

Impossible:

(I) Don't go the bridge unless there is no alternative.

(II) Delete the isolated vertex left by going the bridge.


THANK YOU



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