3-1 Dynamic Programming

(Part I: Examples)

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Taolu

Steps for Applying DP:

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(I) Define subproblems

(II) Set the goal

(III) Identify the recurrence

- larger subproblem $\leftarrow$ # smaller subproblems
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1D Subproblems

Input: $x_1, x_2, \ldots, x_n$ (array, sequence, string)

Subproblems: $x_1, x_2, \ldots, x_i$ (prefix/suffix)

#: $\Theta(n)$

Examples:

- Rod cutting
- Maximum-sum subarray
- Longest increasing subsequence
- Text justification (LaTeX)

2D Subproblems

(I) Input: $x_1, x_2, \ldots, x_m$; $y_1, y_2, \ldots, y_n$

Subproblems: $x_1, x_2, \ldots, x_i$; $y_1, y_2, \ldots, y_j$

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Examples: Edit distance, Longest common subsequence

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Examples: Edit distance, Longest common subsequence

(II) Input: $x_1, x_2, \ldots, x_n$

Subproblems: $x_i, \ldots, x_j$

#: $\Theta(n^2)$

Examples: Matrix chain multiplication, Optimal BST

3D Subproblems

- Floyd-Warshall algorithm

$$\mathsf{d}(i,j,k) = \min\Big(\mathsf{d}(i,j,k-1), \mathsf{d}(i,k,k-1) + \mathsf{d}(k,j,k-1)\Big)$$

DP on Graphs

(I) On rooted tree

Subproblems: rooted subtrees

(II) On DAG

Subproblems: nodes after/before in the topo. order

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Knapsack Problem

Subset sum problem, Change-making problem

And Others . . .

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How to identify the recurrence?

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GUESS

Make Choices by asking yourself the right question

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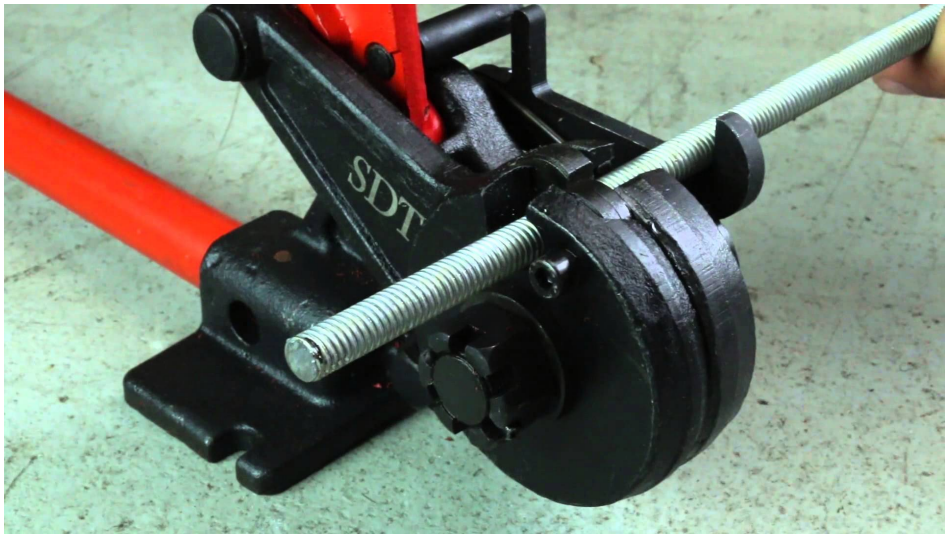
(I) Binary choice
- whether ...

(II) Multi-way choices
- where to ...
- which one ...

Rod Cutting



Rod Cutting Problem

Rod of length n

length i	1	2	3	4	5	$\cdots$
price p_i	1	5	8	9	10	$\cdots$

$$n = i_1 + i_2 + \cdots + i_k$$

$$r_n = p_{i_1} + p_{i_2} + \cdots + p_{i_k}$$

$R(i)$: max revenue obtained from *cutting a rod of length i*

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Rod Cutting Problem (Problem 15.1-3)

Each cut incurs a fixed cost of c.

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Printing Neatly (Problem 15-4)

A sequence of n words of lengths $l_1, l_2, \cdots, l_n$

Line width M

$$\mathit{extra}[i, j] = M - (j - i) - \sum_{k=i}^{j} l_k$$

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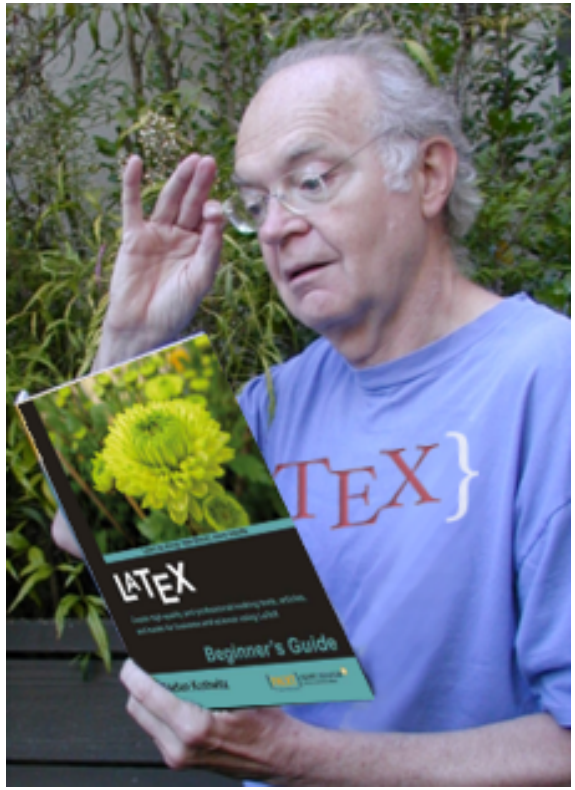
To minimize the sum, over all lines *except the last*, of the cubes of the numbers of extra space characters at the ends of lines.

$$c(n) = \min_{L} \sum_{l_{[i,j]} \in L \wedge j \neq n} \Big(\mathit{extra}[i,j]\Big)^3$$


TEX}
LATEX
Beginner's Guide

A sequence of n words of lengths $l_1, l_2, \cdots, l_n$

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$C(i)$: min cost of neatly printing the *first* i words

$C(i)$: min cost of neatly printing the *last* words i through n

$C(i, j)$: min cost of neatly printing *words i through j*

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$$O(nW)$$

1: **procedure** PRINTING-NEATLY(n)
2: **for** $i \leftarrow n \downarrow 1$ **do**
3: **if** $\mathit{extra}[i, n] \geq 0$ **then** ▷ put w_i through w_n on a line
4: $C[i] \leftarrow 0$
6: **else**
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$$B[1], \quad B[B[1]+1], \quad \cdots$$

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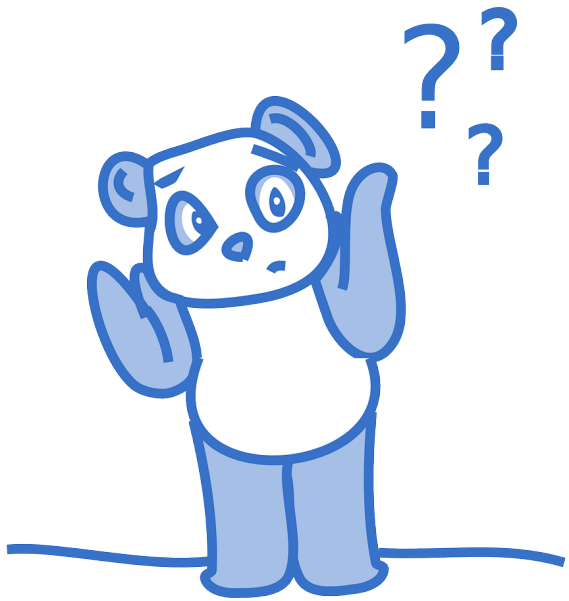

Longest Increasing Subsequence (Problem 15.4-5)

$$A[1 \ldots n]$$

$$5,\ 2,\ 8,\ 6,\ 3,\ 6,\ 9,\ 7$$

Find (the length of) a longest increasing (non-decreasing) subseqence.

$$5,\ 2,\ 8,\ 6,\ 3,\ 6,\ 9,\ 7$$

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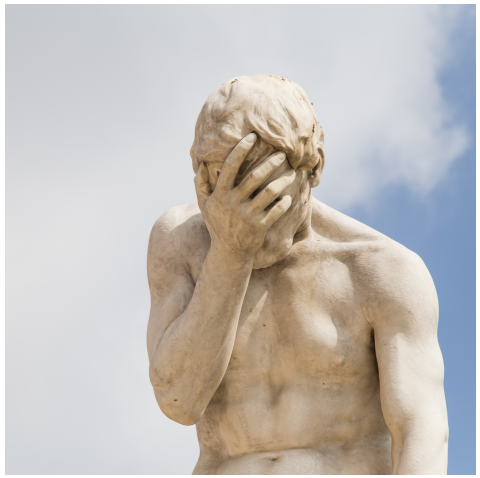

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$$O(n^2) = O(n \log n) + O(n^2)$$

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Answer by Eugene Yarovoi @ Quora

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$$\forall i < j : \textit{the ending number of } E[i] < \textit{the ending number of } E[j]$$

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Matrix-chain Multiplication

$m[i, j]$: min cost to compute the matrix $A_i \cdots A_j$

$$m[1, n]$$

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Subproblem Graph for Matrix-chain Multiplication (Problem 15.2-4)

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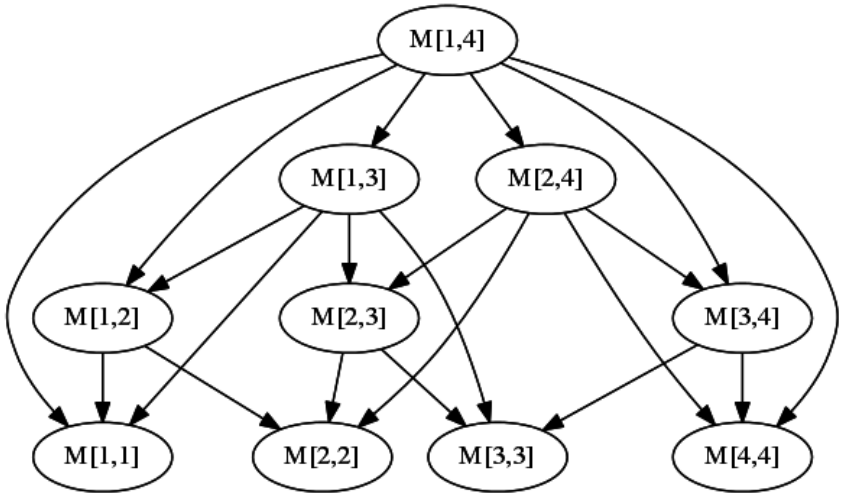

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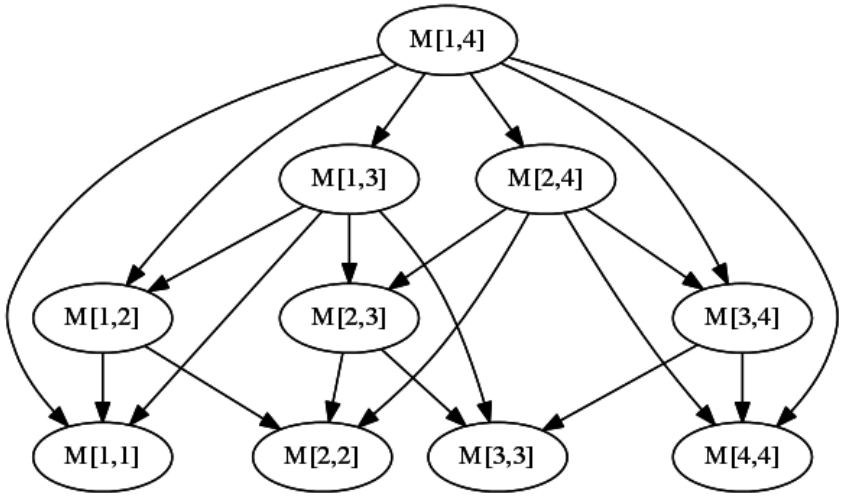

$$\left| \{(i,j) | 1 \le i \le j \le n\} \right| = \frac{n(n+1)}{2}$$

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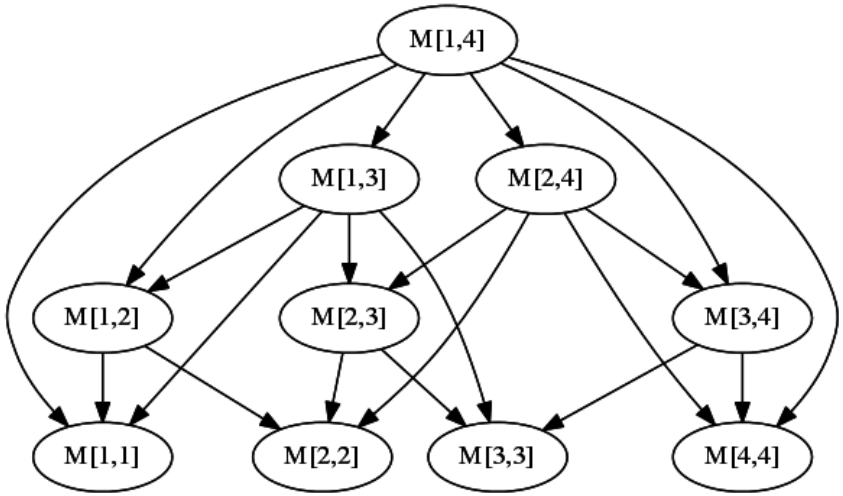

$$\Big| \{(i,j) | 1 \le i \le j \le n\} \Big| = \frac{n(n+1)}{2}$$

$$\sum_{1 \le i \le j \le n} 2(j-i) \overset{k \triangleq j-i}{=} \sum_{0 \le k \le n-1} 2k(n-k)$$

Construct-Optimal-BST(root) (Problem 15.5-1)

$$e[i,j] = \begin{cases} q_{i-1} & \text{if } j = i-1, \\ \min\limits_{i \le r \le j} \{e[i,r-1] + e[r+1,j] + w(i,j)\} & \text{if } i \le j. \end{cases}$$

CONSTRUCT-OPTIMAL-BST(root) (Problem 15.5-1)

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1: **procedure** CONSTRUCT-OPTIMAL-BST($root$, i, j)
2: **if** $j = i-1$ **then**
3: **return** node with d_{i-1}
4: $r \leftarrow$ node with key $k_{root[i,j]}$
5: $r.left \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, i, root[i,j]-1)$
6: $r.right \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, root[i,j]+1, j)$
7: **return** r

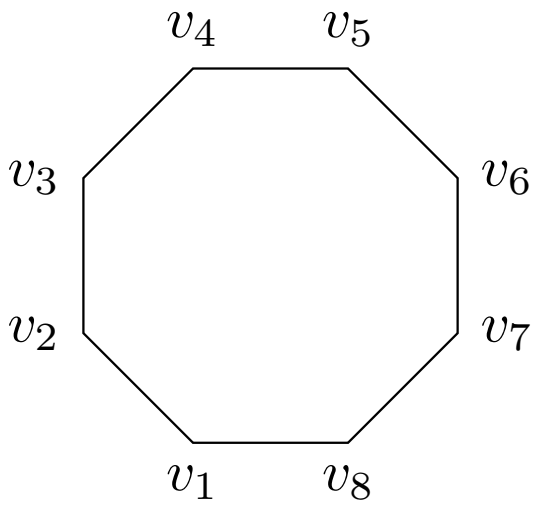
CONSTRUCT-OPTIMAL-BST(root) (Problem 15.5-1)

$$e[i,j] = \begin{cases} q_{i-1} & \text{if } j = i-1, \\ \min\limits_{i \le r \le j} \{e[i,r-1] + e[r+1,j] + w(i,j)\} & \text{if } i \le j. \end{cases}$$

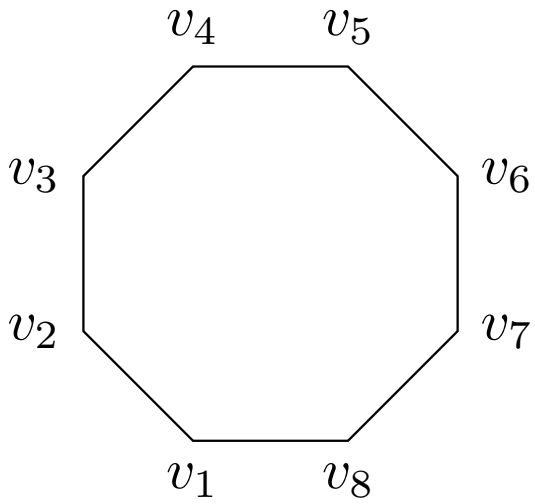
1: **procedure** CONSTRUCT-OPTIMAL-BST($root$, i, j)
2: **if** $j = i-1$ **then**
3: **return** node with d_{i-1}
4: $r \leftarrow$ node with key $k_{root[i,j]}$
5: $r.left \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, i, root[i,j]-1)$
6: $r.right \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, root[i,j]+1, j)$
7: **return** r

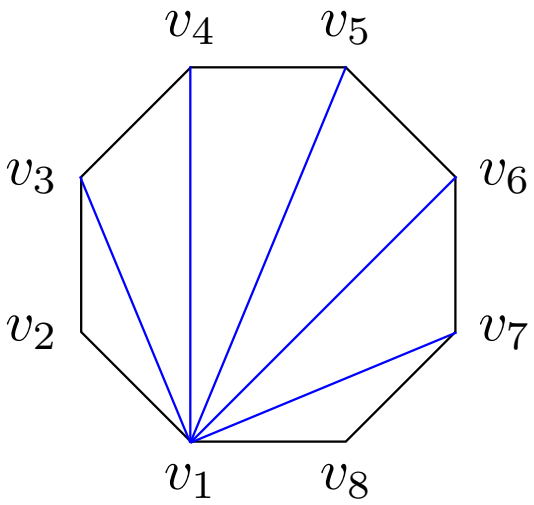
CONSTRUCT-OPTIMAL-BST$(root, 1, n)$

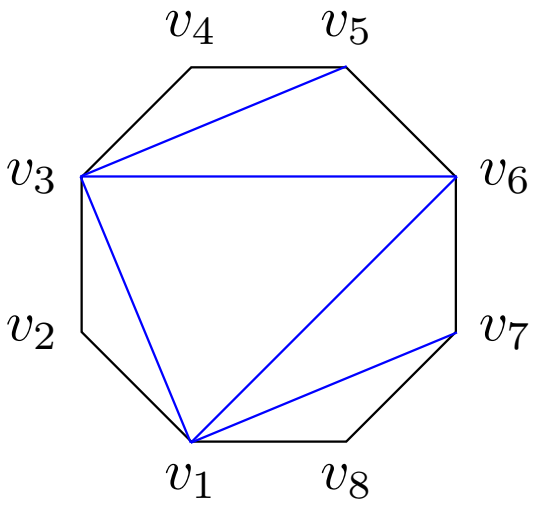
Minimum Weight Triangulation

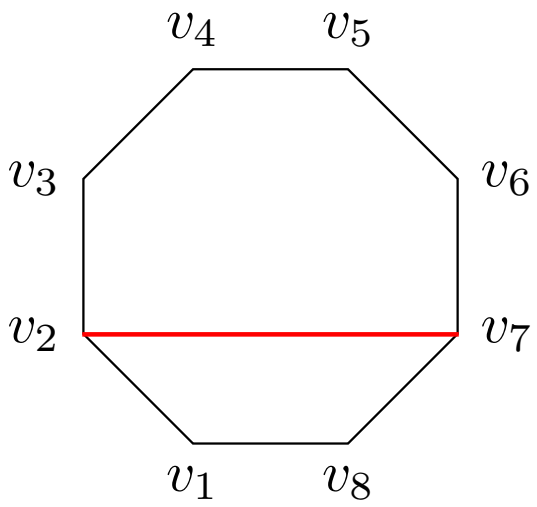


Minimum Weight Triangulation

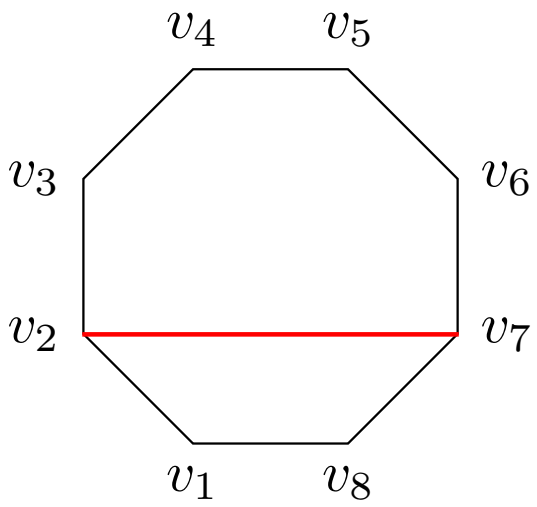








$T(i,j)$: min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), *clockwise*



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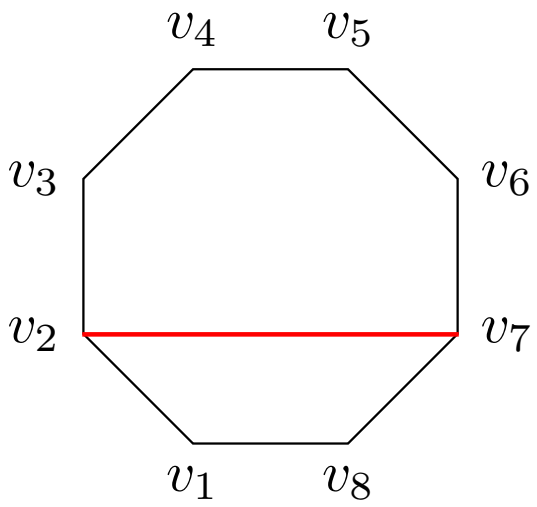
$$T(i,j) = \min_{\substack{i \le k < l-1 \le j \\ (k,l) \neq (i,j)}} \Big(T[k,l] + T[?,?] + d_{ij} \Big)$$



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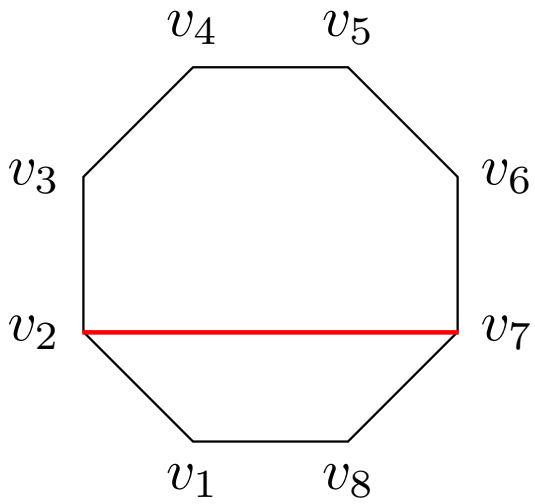
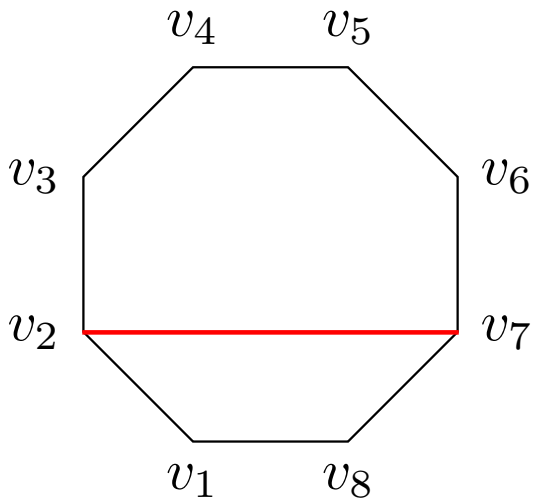
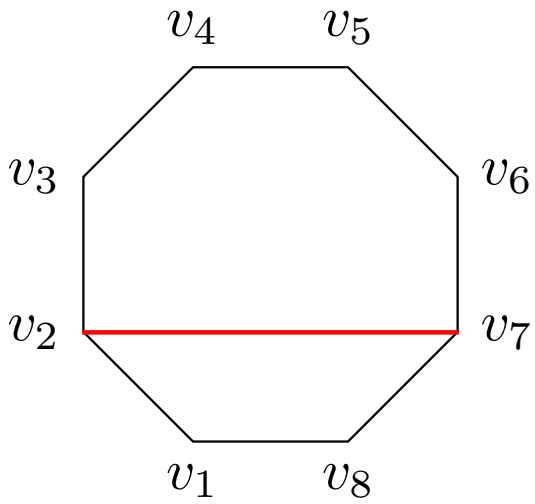
$$O(n^4) = O(n^2) \cdot O(n^2)$$


v_4
v_5
v_3
v_6
v_2
v_7
v_1
v_8

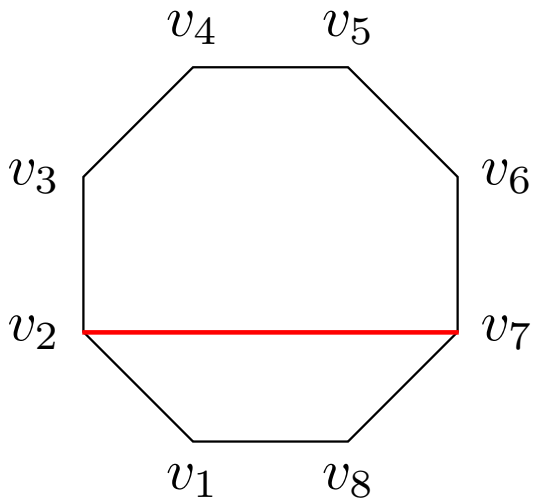


Which vertex k to pair with (i, j)?



Which vertex k to pair with (i, j)?

$T(i, j)$: min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), $i < j$



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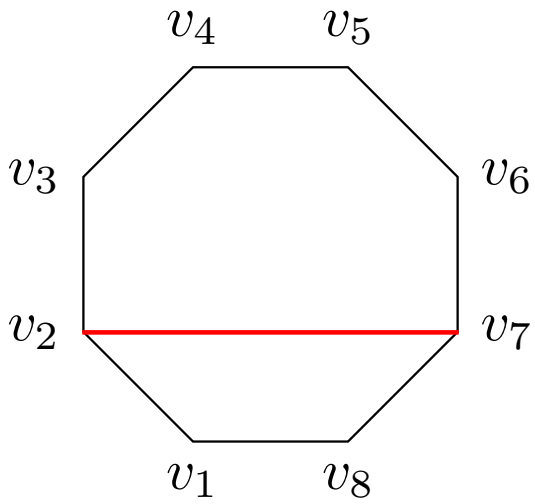

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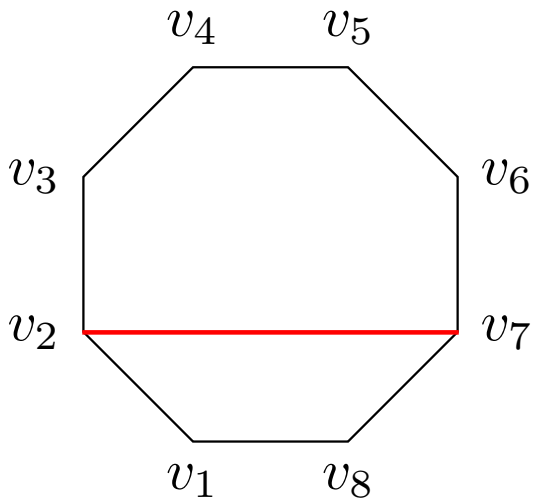
$$T[i, i+1] = 0, \quad 1 \le i \le n-1$$



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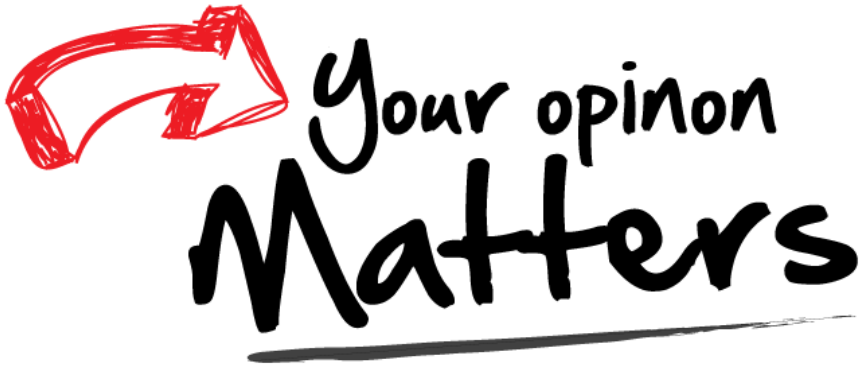
$$T[i, i + 1] = 0, \quad 1 \le i \le n - 1$$

$$O(n^3) = O(n^2) \cdot O(n)$$


THANK YOU



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