Notation and the name choice for meet and join (in order theory)

I have two simple questions:

1. From where do the names meet and join come from? I don't see any intuition between those names in context of order theory.
2. From where does the notation come? I have to admit, I always had an impression that notation we use is somewhat backwards... just take the look at the picture at the wikipedia!

I used this notation a lot of times and I have to admit that often I have to stop for a minute and convince myself that I'm indeed using it right. I'm hoping that learning a bit about origin and context of name choice and notation will make it easier.

(notation) (math-history)

asked Dec 28 '13 at 10:31

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2 I like to think of them as pointing to their counterparts, so if you're not sure what $\vee$ means, for example, then just ask, "where could I find the other one (if it existed)?" - and it'll say, "down there!" – Shaun Dec 28 '13 at 12:52

It's not the symbols that are the wrong way up, it's Hasse diagrams. If the diagrams were drawn with joins pointing downward (and if the names of 'top' and 'bottom' were reversed or changed entirely), it would all make sense :-) – Daira Hopwood Apr 25 '15 at 8:19

2 Answers

I believe the terms *meet* and *join* come from lattice theory. A lattice, after all, can be defined as a partially ordered set in which any two elements have a meet and a join. In practice, a lattice typically arises as a collection of "closed" sets (with respect to some kind of algebraic closure) ordered by set inclusion; typical examples would be the lattice of all subgroups of a group, or the lattice of all subspaces of a vector space.

Consider the lattice of subspaces of a vector space. The *meet* of two subspaces is their set-theoretic intersection; e.g., for two 2-dimensional subspaces of $\mathbb{R}^3$, their *meet* is the line where the two planes **meet**. The *join* of two subspaces is what we get when the two subspaces **join** together to make a bigger subspace; in general it's not just the set-theoretic union, but the linear span of the union.

You also wanted to know where the symbols $\vee$ and $\wedge$ come from. I don't know but I'd guess they are derived from the symbols $\cup$ and $\cap$ for union and intersection, the lattice operations in the lattice of all subsets of a set. As for the symbols $\cup$ and $\cap$, my wild guess is that they are stylized versions of the letters u (for union) and n (for intersection). And if that's not the true history, it's good enough for a mnemonic, isn't it?

edited Dec 28 '13 at 13:03

answered Dec 28 '13 at 12:15

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2 +1, reading this has finally made it possible for me to keep the words "meet" and "join" straight. I think before I was thinking of them both as nouns, but really you want to think of "join" as a verb (join the two sets, e.g., together into a bigger one) and "meet" as a noun (the place where the two sets meet). Thinking of "join" as a noun (this is the spot where the two sets join) had me endlessly confusing the two. – Mike F Apr 28 '15 at 6:33

For me, there are more pairs of similar operations: *intersection* and *union* for sets, *and* and *or* in logic. *meet* and *join* are abstractions of these pairs and so have abstract names. In context of algebraic lattices these are the primitive operations, in equivalent context of order theory these are names for inf and sup of two elements. Also note the similarity in symbols for operations $\wedge, \vee, \cap, \cup$ and in connection symbols for order $<, >, \subset, \supset$.

edited Dec 28 '13 at 12:14

answered Dec 28 '13 at 10:49

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