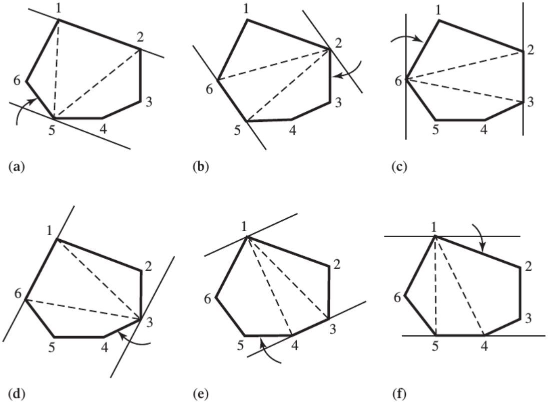
2-7 The Algorithmic Methods

魏恒峰

hfwei@nju.edu.cn

2018 年 05 月 07 日

Convex Polygon Diameter



Convex Polygon Diameter (DH 6.8)

Show that the "Convex Polygon Diameter" algorithm is of **linear-time** complexity.

Q : Linear-time of **WHAT**?

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$$\Theta(c \cdot n) = \Theta(n)$$

Correctness

Theorem

For a convex polygon, a pair of vertices determine the diameter.

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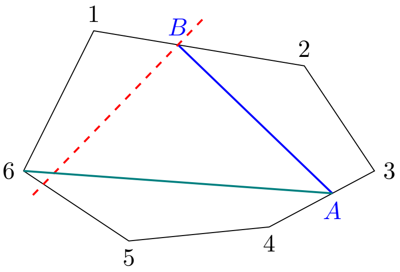

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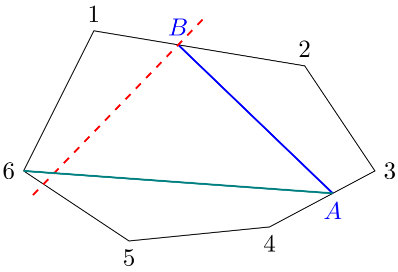

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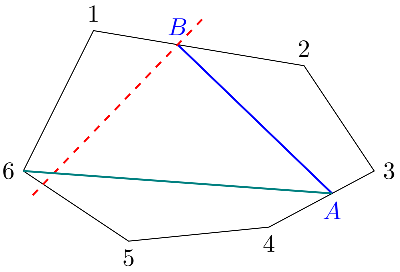

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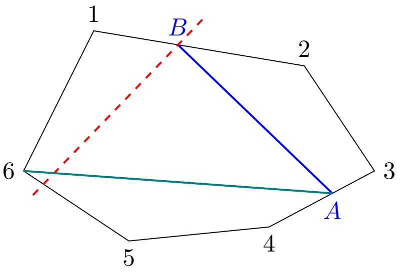

BUT, we have *not* enumerated *all* pairs of vertices.

We have enumerated *all* pairs of vertices
that *admits parallel supporting lines*.

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A line L is a *line of support* of a convex polygon P if

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We have enumerated *all* antipodals.

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If AB is a diameter of a convex polygon P, then AB is an antipodal.

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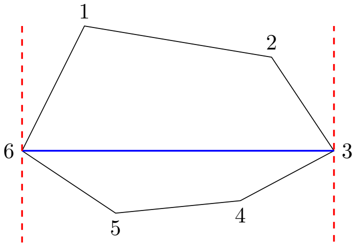
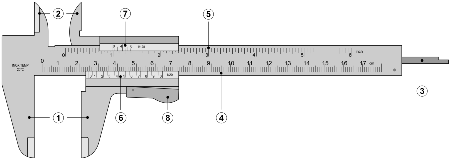


□

Rotating Caliper



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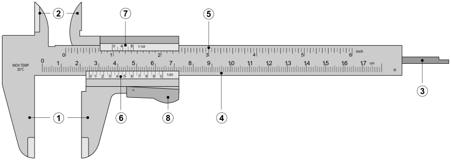

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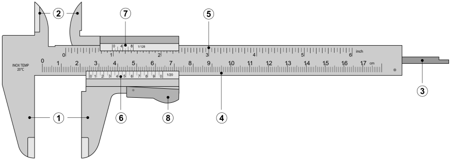

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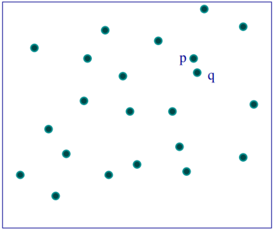
"Solving Geometric Problems with the Rotating Calipers", 1983

Finding the Closest Pair of Points



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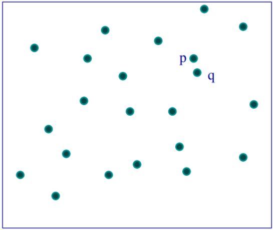
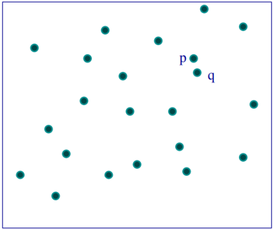


A Classic and Beautiful Divide-Conquer Algorithm:

Finding the Closest Pair of Points



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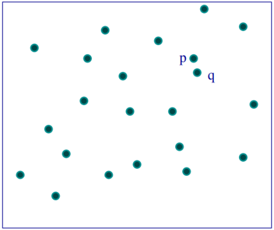

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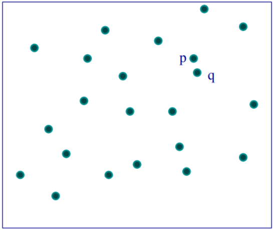
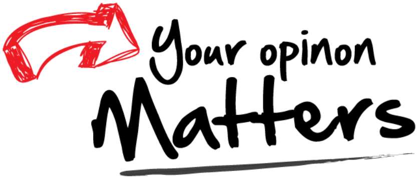


A Classic and Beautiful Divide-Conquer-Combine Algorithm:

Section 33.4, CLRS

Thank You!



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