Group Theory and the Rubik's cube

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Like many people before me, I became obsessed with trying to solve the Rubik's cube. This obsession grew into one of my large hobbies, and I now have a collection of well over fifty Rubik's cube and twisty puzzles.

If you look hard enough, you can find maths anywhere, and the Rubik's cube is no exception. It turns out that the set of configurations of the Rubik's cube can be interpreted as a mathematical object, called a group. Group theory is a rich area of study, and specific proofs, ideas and concepts central to group theory can reveal many interesting properties of the Rubik's cube. The group structure of the Rubik's cube tells us a lot about the puzzle.

Perhaps the most helpful property the group structure reveals is the number of different configurations of the Rubik's cube. There are over 43 quintillion possible ways to scramble a cube. If you have a Rubik's cube handy, give it a good scramble. Odds are that you are the first person to ever see a Rubik's cube in that configuration. *"Ideal Toy Company"* stated on the packaging of the original Rubik's cube that there are more than three billion configurations of the cube. This is such an understatement that it would be equivalent to McDonalds announcing that they've sold over 100 hamburgers.

Some people cheat when solving the Rubik's cube by disassembling the cube and putting it back together "solved". One result from group theory tells us that, if you were to randomly reassemble the cube, there is an eleven in twelve chance that the cube is unsolvable. That is, you can never return it to the solved state without disassembling it again.

One question that has been asked since the beginning of Rubik's cube mania is how many moves you need to solve any scrambling of the cube. In 1995 it was shown that one configuration needed at most 20 moves to solve, and it wasn't until 2010 that is was shown that every configuration of the cube could be solved in 20 moves or less. The reason this question was so hard is that there is an astronomical number of configurations, and a computer can't crunch them all. Luckily, some group theory trickery allowed the problem to be broken up into smaller chunks, which a computer could handle in reasonable time.

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