1-5 数据与数据结构 (II)

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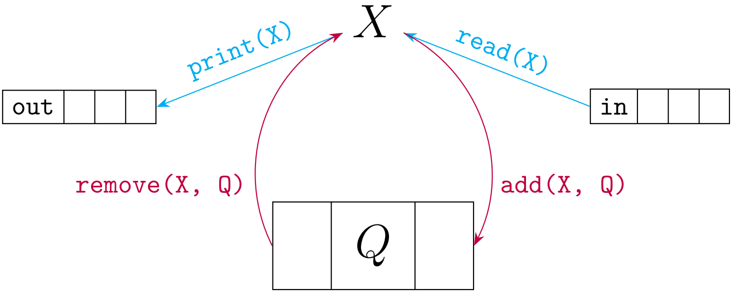
温故而知新

Stackable/Queueable Permutations

Treesort Algorithm on BST

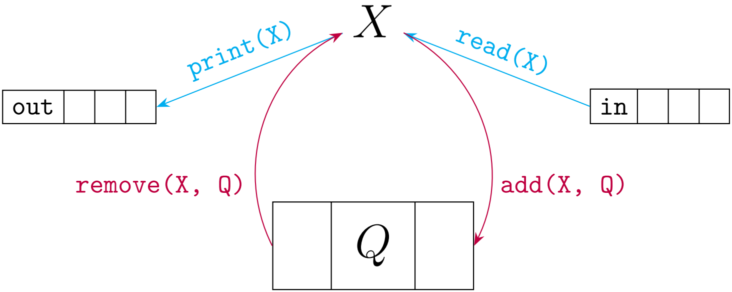
Queueable Permutations

DH 2.14: Queueable Permutations



DH 2.14: Queueable Permutations

$$\boxed{\texttt{out} = (a_1, \cdots, a_n) \xleftarrow[X=0]{Q=\emptyset} \texttt{in} = (1, \cdots, n)}$$



DH 2.14: Queueable Permutations

(a) Show that the permutations given in Excecise 2.12(b) are queueable.

   (i) $(3, 1, 2) \Longrightarrow (3, 2, 1)$

   (ii) $(4, 5, 3, 7, 2, 1, 6)$

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DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

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```
X = 0   Q = ∅ in != EOF
```

```
foreach 'a' ∈ out:
```

DH 2.14: Queueable Permutations

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foreach 'a' ∈ out:
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    read(X)
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```

```
add-Q-till('a'):
  while (('x' ∈ in) != 'a')
    read(X)
    add(X, Q)
  read(X)
  print(X)
```

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  print(X)
```

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

Proof.

```
foreach 'a' ∈ out:
  if ('a' >= in)
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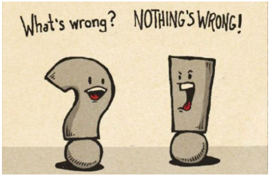

□

Pseudocode

Pseudocode


Make everything as
SIMPLE
as possible,
but not simpler.
- ALBERT EINSTEIN (1879-1955)
QUOTESEVERLASTING.COM

Pseudocode

"Executable" at an abstract level.

DH 2.14: Queueable Permutations

(b) Prove that every permutation are queueable.

An "AHA!" Proof from 杜星亮.

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DH 2.14: Queueable Permutations

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(c) Prove that every permutation can be obtained by two stacks.

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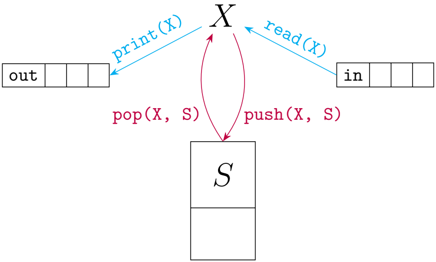

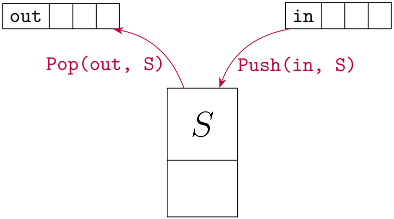

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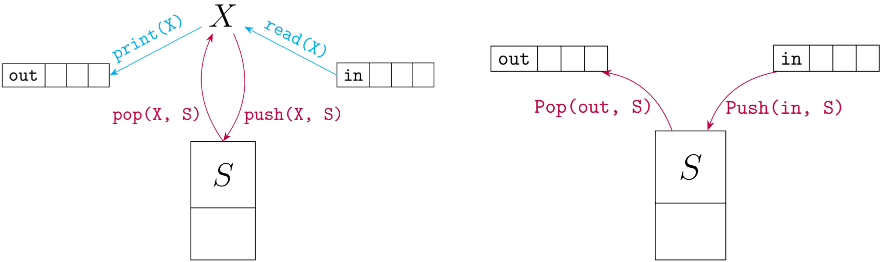
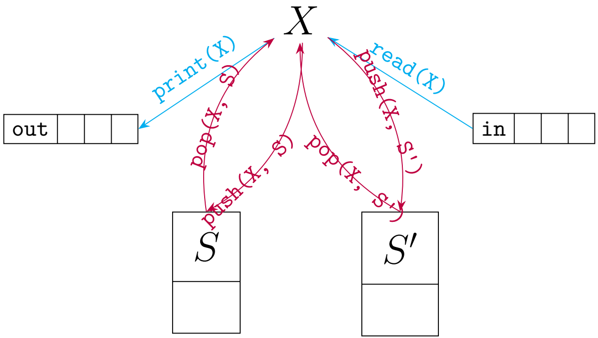


We can similarly speak of a permutation obtained by two stacks, if we permit the push and pop operations on two stacks S and S'. — *DH*

DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.



DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.

```
foreach 'a' ∈ in:
  read(X)
  push(X, S)

foreach 'a' ∈ out:
```

DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.

```
foreach 'a' ∈ in:
  read(X)
  push(X, S)

foreach 'a' ∈ out:
  transfer-till(S, S′, top(S) == 'a')
```

DH 2.14: Queueable Permutations

(c) Prove that every permutation can be obtained by two stacks.

```
foreach 'a' ∈ in:
  read(X)
  push(X, S)

foreach 'a' ∈ out:
  transfer-till(S, S′, top(S) == 'a')
  transfer-till(S′, S, S′ == ∅)
```

DH 2.15: Algorithm for Queueable Permutations

Extend the algorithm you were asked to design in Exercise 2.13, so that **if** the given permutation cannot be obtained by a stack, the algorithm will print the series of operations on two stacks that will generate it.

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```
two-stacks-perm(in, X, S, S′)
```

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```
two-stacks-perm(in, X, S, S')
```

```
if (! one-stack-perm(in, X, S))
  two-stacks-perm(in, X, S, S')
```

DH 2.15: Algorithm for Queueable Permutations

Extend the algorithm you were asked to design in Exercise 2.13, so that **if** the given permutation cannot be obtained by a stack, the algorithm will print the series of operations on two stacks that will generate it.

```
two-stacks-perm(in, X, S, S′)
```

```
if (! one-stack-perm(in, X, S))
  two-stacks-perm(in, X, S, S′)
```

Embedding "transfer" into "one-stack-perm".

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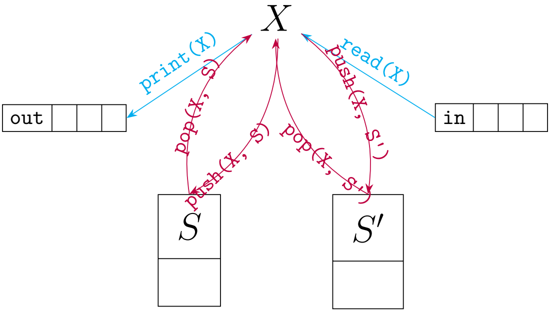

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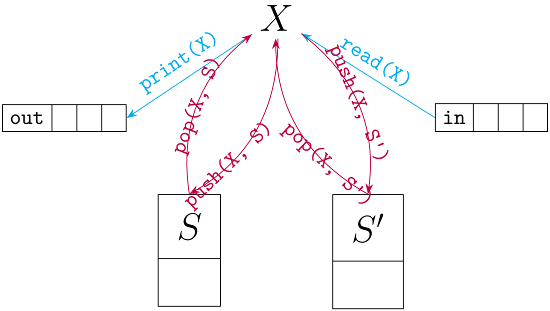

`transfer-till(S, S', top(S) == 'a')`

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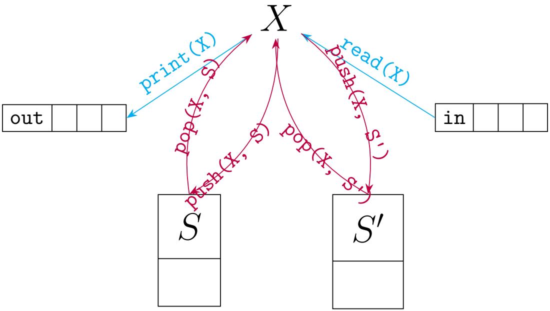
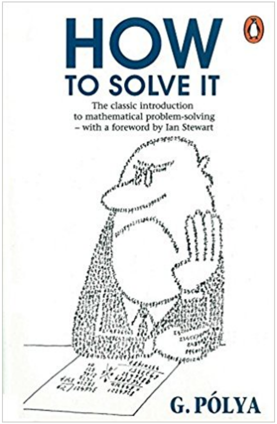
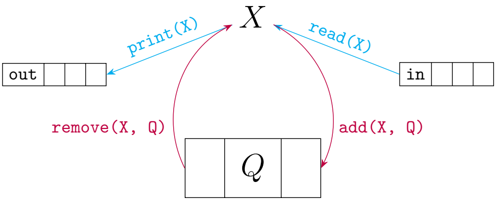
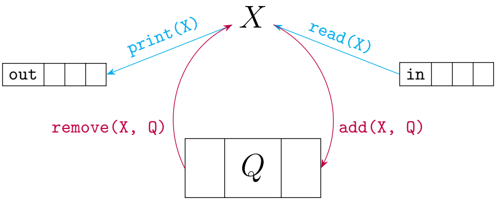



```
transfer-till(S, S', top(S) == 'a')
transfer-till(S', S, S' == ∅)
```



Step 4: Looking Back!


X
print(X)
read(X)
out
in
remove(X, Q)
add(X, Q)
Q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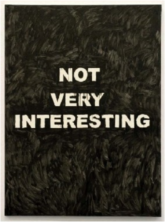
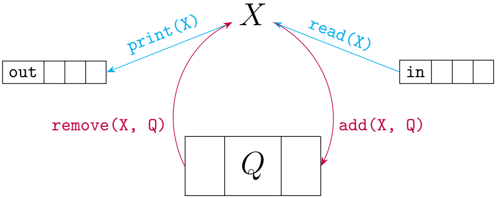

NOT
VERY
INTERESTING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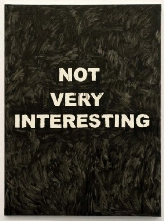
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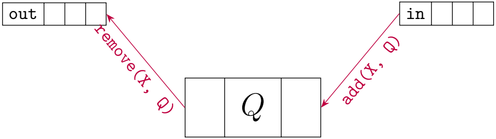
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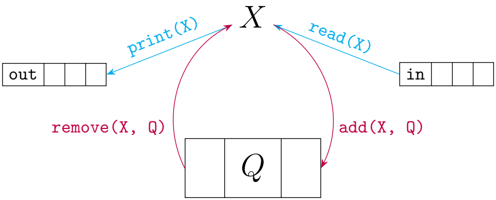
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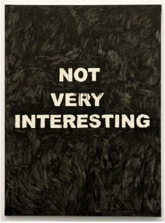
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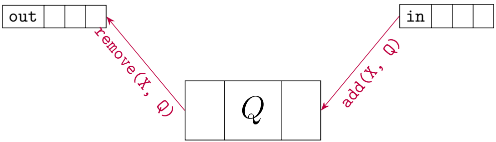
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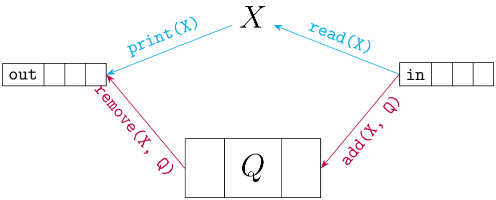
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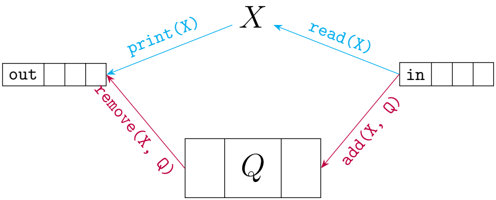
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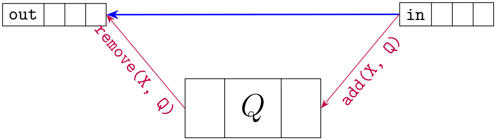
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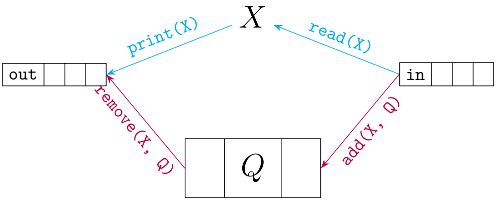
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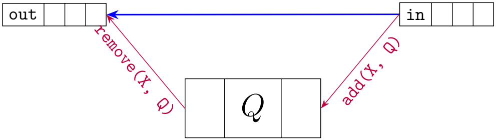
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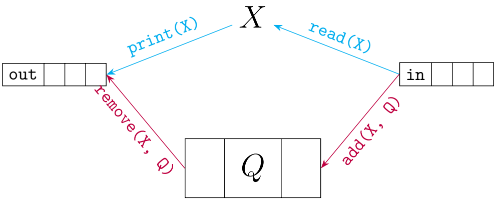
~~3 2 1~~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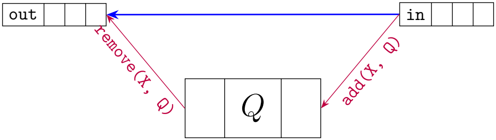
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哎哟，
有点意思

~~3 2 1~~

Theorem (Queueable Permutations)

A permutation $(a_1, \cdots, a_n)$ *is queueable* $\iff$ *it is not the case that*

$$321\text{-}\mathit{Pattern}: \boxed{\mathit{out} = \cdots a_i \cdots a_j \cdots a_k : i < j < k \land a_i > a_j > a_k}$$

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Proof.

Left as an exercise. □

Theorem (# of Queueable Permutations)

The number of queueable permutations of $[1 \cdots n]$ *is* $\binom{2n}{n} - \binom{2n}{n-1}$.

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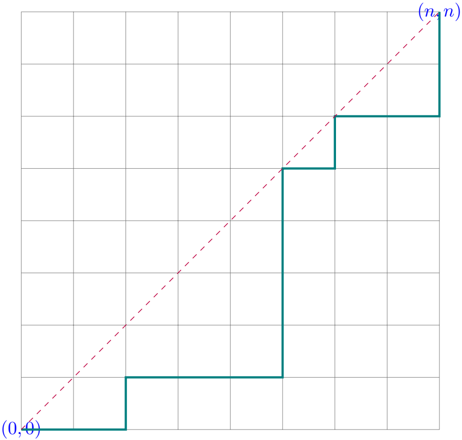

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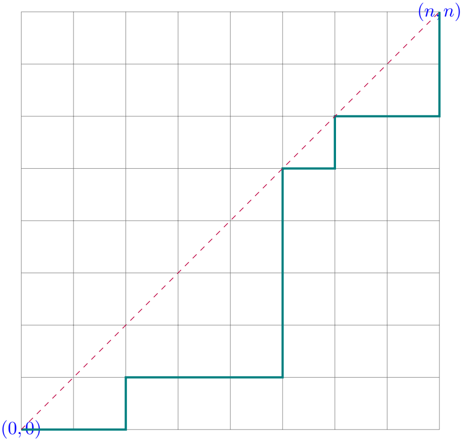

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Left for your research.

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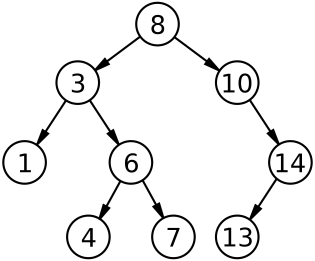
$\square$

Treesort Algorithm on BST



DH 2.16: Treesort

(i) Construct an algorithm that transforms a given list of integers into a binary search tree.

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```
Node:
  int val = NIL,
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```
buildBST(int eles[]):
  Node root(eles[0])

  foreach e ∈ eles[1..]:
    insert(root, e)
```

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```

```
insert(Node T, int e):
  if (e < T.val)
    if (T.left == NULL)
      T.left = new Node(e)
    else
      insert(T.left, e)
```

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```
Node:
  int val = NIL,
  Node left = NULL,
  Node right = NULL
```

```
buildBST(int eles[]):
  Node root(eles[0])

  foreach e ∈ eles[1..]:
    insert(root, e)
```

```
insert(Node T, int e):
  if (e < T.val)
    if (T.left == NULL)
      T.left = new Node(e)
    else
      insert(T.left, e)
  else  // e >= T.val
    if (T.right == NULL)
      T.right = new Node(e)
    else
      insert(T.right, e)
```

```
procedure put x into a BST t:
  ... call put x into t's left subtree
  ... call put x into t's right subtree
end procedure
```

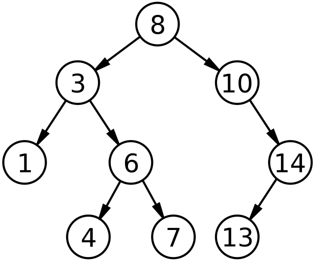
```
procedure put x into a BST t:
   ... call put x into t's left subtree
   ... call put x into t's right subtree
end procedure
```

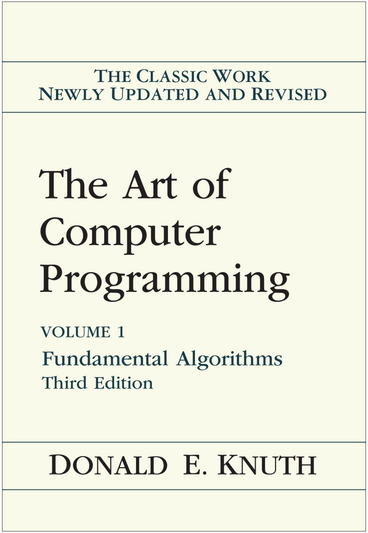
should be:

```
procedure put-x-into-BST (t):
   ... call put-x-into-BST (t's left subtree)
   ... call put-x-into-BST (t's right subtree)
end procedure
```

DH 2.16: Treesort

(ii) right; val; left

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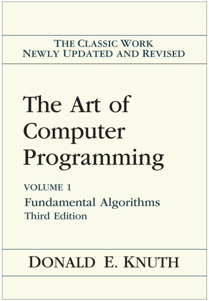
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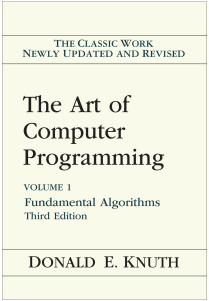
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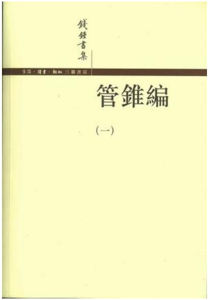
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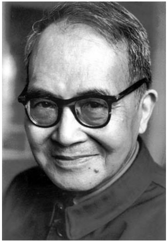


Thank You!