4-5 Polyhedral Groups

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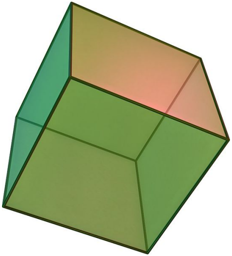
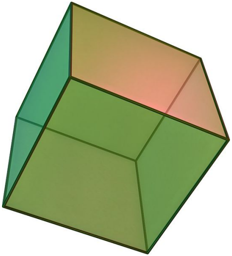
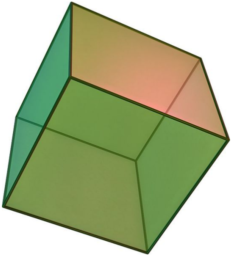
flag永不倒!



$$Sym(C) \cong$$

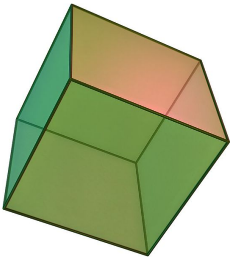


$$Sym(C) \cong S_4$$



$$Sym(C) \cong S_4$$

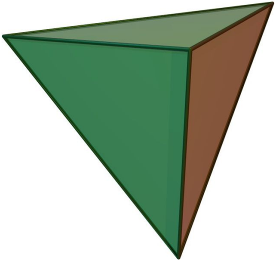
$$\left|\{H : H \leq Sym(C)\}\right| =$$



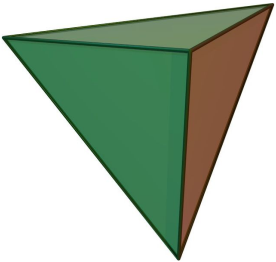
$$Sym(C) \cong S_4$$

$$\left| \left\{ H : H \leq Sym(C) \right\} \right| = 30$$

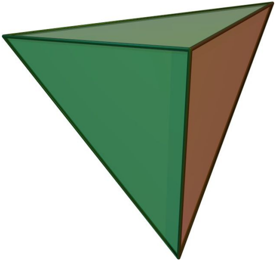

先定一个能达到的小目标



$$Sym(T) \cong$$

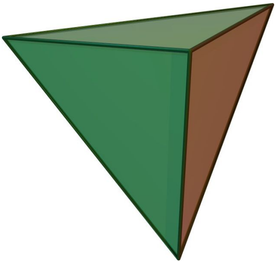


$$Sym(T) \cong A_4$$



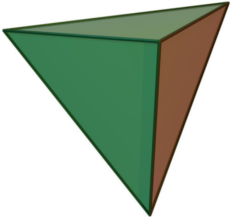
$$Sym(T) \cong A_4$$

$$\Big|\{H : H \leq Sym(T)\}\Big| =$$

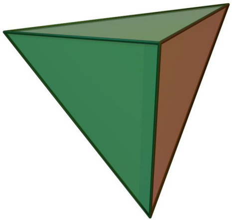


$$Sym(T) \cong A_4$$

$$\Big|\{H : H \leq Sym(T)\}\Big| = 10$$



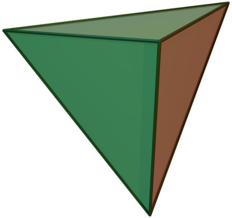
$$Sym(T) \cong A_4$$



$$Sym(T) \cong A_4$$

Proof.

(1) To find all even perms. in S_4

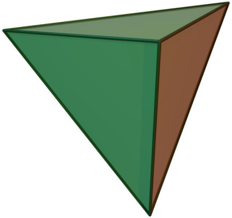


$$Sym(T) \cong A_4$$

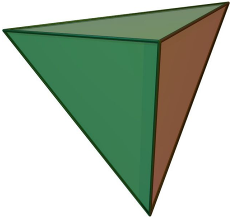
Proof.

(1) To find all even perms. in S_4

(2) To show that $\left|Sym(T)\right| < \left|S_4\right|$



$$\left|Sym(T)\right| < \left|S_4\right|$$



$$\left|Sym(T)\right| < \left|S_4\right|$$

$$\because (1\ 2) \notin Sym(T)$$

Rotate through vertices:

$$\text{Fixing } 1: \rho_1 = (2\ 3\ 4) \quad \rho_1^2 = (2\ 4\ 3) \quad \rho_1^3 = 1$$

$$\text{Fixing } 2: \rho_2 = (1\ 3\ 4) \quad \rho_2^2 = (1\ 4\ 3) \quad \rho_2^3 = 1$$

$$\text{Fixing } 3: \rho_3 = (1\ 2\ 4) \quad \rho_3^2 = (1\ 4\ 2) \quad \rho_3^3 = 1$$

$$\text{Fixing } 4: \rho_4 = (1\ 2\ 3) \quad \rho_4^2 = (1\ 3\ 2) \quad \rho_4^3 = 1$$

Rotate through vertices:

$$\text{Fixing } 1: \rho_1 = (2\ 3\ 4) \quad \rho_1^2 = (2\ 4\ 3) \quad \rho_1^3 = 1$$

$$\text{Fixing } 2: \rho_2 = (1\ 3\ 4) \quad \rho_2^2 = (1\ 4\ 3) \quad \rho_2^3 = 1$$

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$$\text{Fixing } 4: \rho_4 = (1\ 2\ 3) \quad \rho_4^2 = (1\ 3\ 2) \quad \rho_4^3 = 1$$

$$\# = 8 + 1 = 9$$

Rotate through edge-edge:

$$r_1 = (1\ 4)(2\ 3)$$

$$r_2 = (1\ 2)(3\ 4)$$

$$r_3 = (1\ 3)(2\ 4)$$

Rotate through edge-edge:

$$r_1 = (1\ 4)(2\ 3)$$

$$r_2 = (1\ 2)(3\ 4)$$

$$r_3 = (1\ 3)(2\ 4)$$

$$\# = 3$$

$$\rho_1 = (2\ 3\ 4) \quad \rho_1^2 = (2\ 4\ 3)$$
$$\rho_2 = (1\ 3\ 4) \quad \rho_2^2 = (1\ 4\ 3)$$
$$\rho_3 = (1\ 2\ 4) \quad \rho_3^2 = (1\ 4\ 2)$$
$$\rho_4 = (1\ 2\ 3) \quad \rho_4^2 = (1\ 3\ 2)$$

$$r_1 = (1\ 4)(2\ 3)$$
$$r_2 = (1\ 2)(3\ 4)$$
$$r_3 = (1\ 3)(2\ 4)$$

$$\boxed{Sym(T) \cong A_4 = \left\{ \text{id}, \quad \underbrace{\text{3-cycle},}_{\#=8} \quad \underbrace{\text{2-2-cycle}}_{\#=3} \right\}}$$

$$\boxed{\Big|\{H : H \le Sym(T)\}\Big| = 10}$$

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$$H \le A_4 \Longrightarrow \big|H\big| = 1, 2, 3, 4, 6, 12$$

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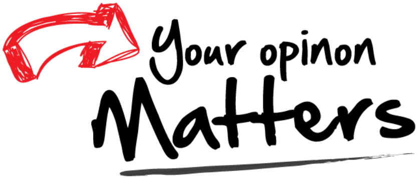
$$H \le A_4 \Longrightarrow |H| = 1, 2, 3, 4, 6, 12$$

$$|H| = \begin{cases} 1: & \text{id} \quad (\# = 1) \\ 2: & \langle r_1 \rangle, \langle r_2 \rangle, \langle r_3 \rangle \quad (\# = 3) \\ 3: & \langle \rho_1 \rangle, \langle \rho_2 \rangle, \langle \rho_3 \rangle, \langle \rho_4 \rangle \quad (\# = 4) \\ 4: & \{1, r_1, r_2, r_3\} \cong K_4 \quad (\# = 1) \\ 6: & (\# = 0) \\ 12: & A_4 \quad (\# = 1) \end{cases}$$


THANK YOU



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