COMMENTARIES

Commentaries are informative essays dealing with viewpoints of statistical practice, statistical education, and other topics considered to be of general interest to the broad readership of *The American Statistician*. Commentaries are similar in spirit to Letters to the Editor, but they involve longer discussions of background, issues, and perspectives. All commentaries will be refereed for their merit and compatibility with these criteria.

Siobhan's Problem: The Coupon Collector Revisited

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This article considers from an empirical point of view the convergence of the distribution function for the waiting time in the classical coupon collector's problem. In addition, the application of the distribution in testing for "randomness" in the decimal expansions of π and e is considered. Some remarks are also made on the inverse problems: namely, given i distinct objects obtained in sampling at random with replacement m times from some population, how many distinct objects are there in the population?

KEY WORDS: Empirical asymptotic convergence; Estimating population size; Decimal expansion of π and e; Testing randomness.

1. INTRODUCTION

While on leave from my home institution, Victoria University of Wellington, and during my visit to the University of Waterloo in 1989, my Canadian niece Siobhan (pronounced "jevon"—*je* as in French, accent on the last syllable), an ardent collector of hockey cards, asked me a question roughly equivalent to the following: How many packets of bubble gum, each having five cards, do I have to buy in order to get all 300 in the set that season? My response as a statistician was all too typical: "I can't answer that problem (at least not very easily), but I can solve this other problem." At which point I had lost my customer.

I became interested in the problem, however, which is quite well known and appears in such classic references as Feller (1968) and Mosteller (1987) and can be stated generally as follows: There are N distinct objects, and a series of independent draws is made from these, randomly selecting n at each draw, and replacing the drawn objects after each trial. Let W denote the number of draws necessary for all N objects to have been drawn at least once.

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The classical case, with $n = 1$ and equal probabilities of selection, is known as the coupon collector's problem and can be related to testing for random sampling of digits [see the footnote in Feller (1968, p. 61)]. See also Baum and Billingsley (1965), Rosen (1969, 1970, 1972), and Sen (1979) for further results concerning the problem. Rayner (1989) is a brief discussion of the cases for small N.

One of the objects of this article is to investigate empirically the classical case, in particular the asymptotic property as given in theorem 4 of Baum and Billingsley (1965), where it is shown that a certain function of the waiting time until all objects have been drawn at least once is asymptotically chi squared, with 2 df.

In addition, I will augment some of the results given in Greenwood (1955), where the digits in the decimal expansions of π and e were examined to see if they can be regarded as being randomly generated from a uniform distribution. Specifically, I look at the first 10,000 digits in the expansion of each and test the distribution of successive (disjoint) waiting times against the theoretical model given by the coupon collector's test. The waiting time is defined to be the number of digits necessary to obtain all 10 different digits.

I next look briefly at the inverse problem, that of estimating the number N of objects, when i different objects have been observed in sampling m times with replacement, assuming equal selection probabilities. An interesting property of the appropriate maximum likelihood estimator is noted in passing, and a possible application to the estimation of animal abundance is suggested.

Finally, I observe that the original problem is easily solvable once all the pertinent conditions are stated!

2. THEORETICAL BACKGROUND

I first note that it is quite easy to determine the theoretical distribution of the waiting time W [see Feller (1968, pp. 59–60) for this and related results].

$$\Pr(W = i) = \frac{1}{N^{i-1}} \sum_{j=0}^{j=N-1} \binom{N-1}{j} \times (N-1-j)^{i-1}(-1)^j \quad (1)$$

Clearly, W may assume any value greater than or equal to N.

Note. Evaluation of these probabilities is quite straightforward, provided that N is not too large, say less than 20 or so, but for values as large as 300, there appear to be serious difficulties, largely as a consequence of the loss of precision entailed in the successive additions and subtractions implied in the alternating signs in the foregoing equation, at least for computations carried out in standard precision on most machines. The difficulties are somewhat illusory, however, since although there are indeed difficulties in the evaluation for all possible values from $i = 300$ upward, the problem goes away eventually for sufficiently large i because the exponent $i - 1$ gets large enough to reduce the individual terms to within the ordinary precision of the calculations.

Now, as observed in Baum and Billingsley (1965), the distribution of W is the same as

$$X_1 + X_{1-\frac{1}{N}} + X_{1-\frac{2}{N}} + \cdots + X_{\frac{1}{N}},$$

where the X_p are independent and have distribution given by

$$\Pr(X_p = k) = (1 - p)^{k-1}p, \qquad k = 1, 2, \ldots .$$

This is more or less obvious if you consider that when, say, l distinct objects have been drawn, the waiting time until one of the remaining $N - l$ elements is drawn is geometric. Thus the X_p are independent geometric random variables and W has mean

$$\mu_N = N \sum_{k=1}^{k=N} \frac{1}{k}$$

and variance

$$\sigma^2 = N \sum_{k=1}^{k=N} \frac{N - k}{k^2} .$$

Note that the variables are not identically distributed, and hence the classical central limit theorem for iid variables does not apply. As pointed out by a referee, however, even the extended form of the theorem does not apply, essentially because the variances of the individual terms of the sum are badly behaved. More specifically, by adapting an argument used by Baum and Billingsley (1965) in the proof of their theorem 4, it can be shown that, in the present case, the coefficient of t^3 in the exponent of the limiting characteristic function $m(t)$ is nonzero.

I now state the relevant form of the result from Baum and Billingsley (1965) that is appropriate to our purpose.

Theorem 1. Using sampling with replacement from a population size N, each element of the population having probability $1/N$ of being drawn, then the waiting time W until all elements have been drawn at least once is such that as $N \to \infty$, $Z = \exp(-(W/N) + \log 2N)$ converges to a chi-squared distribution with 2 df, that is, to an exponential distribution with parameter .5.

Convergence here means that the distribution function of the variable Z converges weakly to the given distribution function. Hence $F_Z(t) \to F_U(t)$ weakly as $N \to \infty$, where U is chi squared with 2 df, $F_Z(t)$ denotes the distribution function of Z, and $F_U(t)$ denotes the distribution function of U. It is easily shown that asymptotically

$$F_W(t) \equiv \Pr(W \leq t) \approx \exp(-N \exp(-t/N)). \qquad (2)$$

This follows by first observing that, if $V = (W/N) - \log 2N$, then the result of Baum and Billingsley implies that

$$\Pr(V \leq t) \approx \exp(-\tfrac{1}{2} \exp(-t)).$$

The result then follows by noting that

$$F_W(t) = \Pr\left(V \leq \frac{t}{N} - \log 2N\right) .$$

As noted previously, part of my purpose is to investigate this result empirically, within the pertinent range of values.

3. RESULTS

For convenience I will refer to the limit distribution in (2) as Siobhan's distribution. Straightforward evaluation of the moment generating function $m(t)$ for such a variable gives

$$m(t) = N^{Nt}\Gamma(1 - Nt),$$

where $\Gamma(z)$ is the usual gamma function $\int_0^\infty t^{z-1} \exp(-t)dt$. It follows that the mean and variance of Siobhan's distribution are then

$$\mu = N (\log N - \Psi(1))$$

and

$$\sigma^2 = N^2 \pi^2/6,$$

where $\Psi(z)$ is the digamma function $\Gamma'(z)/\Gamma(z)$. See Abramowitz and Stegun (1970) for more details.

Table 1 shows the means and standard deviations for various values of N, both for the exact case and the Siobhan's distribution approximation. The contexts chosen can be thought of as

- $N = 2$: coin tossing until both head and tail have been thrown.
- $N = 4$: drawing cards until all four suits have been drawn.
- $N = 6$: throwing a die until all six sides have appeared.
- $N = 10$: random generation of digits 0–9 until all have occurred.
- $N = 13$: drawing cards from a pack until all 13 ranks have occurred.
- $N = 26$: generating letters of the alphabet until all have occurred.
- $N = 50$: waiting time until all 50 numbers in a lotto game have been drawn.
- $N = 300$: waiting time until all 300 hockey cards have been found, assuming 1 is selected at a time.

Two additional categories corresponding to $N = 100$ and $N = 150$ are included for comparison purposes.

Note that doubling the number of objects approximately doubles the standard deviation, at least over the range covered in the table. The mean is not quite so well behaved,

Table 1. Means and Standard Deviations of the Waiting Times W for Various Values of N

		Actual		Approximate	
Context	N	Mean	Standard deviation	Mean	Standard deviation
Coin tossing	2	3.0	1.4	2.5	2.6
Card suits	4	8.3	3.8	7.9	5.1
Sides of die	6	14.7	6.2	14.2	7.7
Digits	10	29.3	11.2	28.8	12.8
Card values	13	41.3	15.0	40.9	16.7
Alphabet	26	100.2	31.4	99.7	33.4
Lotto	50	225.0	62.0	224.4	64.1
Cards in pack	52	236.0	64.5	235.5	66.7
—	100	518.7	125.8	518.2	128.3
—	150	838.7	189.8	838.2	192.4
Hockey cards	300	1885.0	381.9	1884.3	384.8

with the relevant ratio ranging from about 2.8 for small N to 2.25 at the upper end of the table. Figure 1 plots the means and variances for a restricted range of N values. At the scale of the plot there is no appreciable difference between actual and approximated values.

Figure 2 displays the graphs of actual versus estimated distribution functions for $N = 10$, $N = 25$, $N = 50$, and $N = 100$. It is readily apparent that the approximation is quite acceptable for $N = 25$ and is not really adequate for $N = 10$, at least as regards "tail" values of the number of trials. Table 2 displays to four decimal places the actual and approximated values for selected values of $\Pr(a \le W \le b|N = 10)$. A continuity correction has been applied to compute the approximate values. Note that a table of individual values for $W = 10$ to $W = 76$ is given in Greenwood (1955).

I now consider the distribution of the waiting times to get all 10 decimal digits in the decimal expansions of π and e. As noted previously, this means that, starting with the first digit to the right of the decimal point, we count successive digits until all 10 different digits 0–9 have been used in the expansion. The waiting time is the number of digits used until the last one appears, and the count starts again at the next digit of the expansion. This is the convention as used in Greenwood (1955), but note that this introduces a slight "start-up" effect in that the first digit is thus nonzero, and hence the first run of digits is not strictly comparable with all others. I disregard this effect, however.

Figure 3 gives the stem-and-leaf display of the waiting times involved in the first 10,000 digits of π. Figure 4 gives the corresponding display for the first 10,000 digits in the expansion of e. In the case of π using categories 10–19, 20–29, 30–39, 40–49, 50–59, greater than or equal to 60 for a chi-squared test, with counts 59, 142, 78, 39, 11, and 8, a value of about 1.61 is obtained. (The corresponding probabilities are in Table 2.) For e with the same categories and counts 55, 140, 83, 36, 10, and 11, a value of 3.62 is obtained, much of it due to the upper tail with 11 observations, which contributes 2.81 to the chi-squared value.

The actual counts for each of the digits in the expansions are also of interest, and these are given in Table 3. Com-

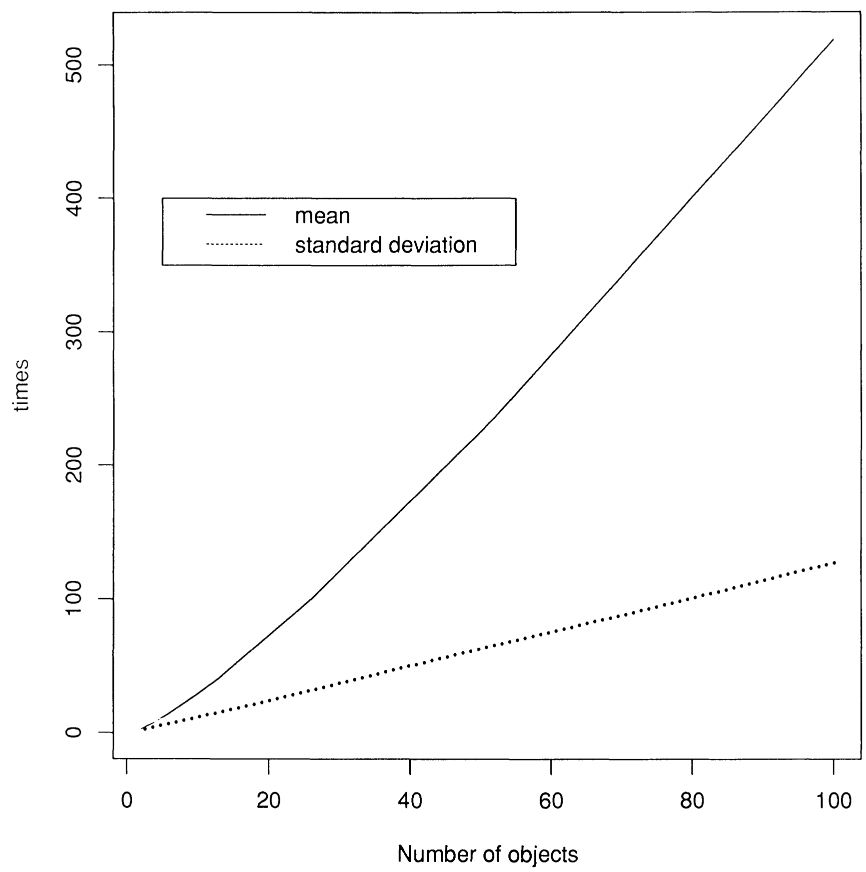

Figure 1. Means and Standard Deviations of the Waiting Time W for Various Values of the Number N of Distinct Objects. At the scale of the graph, these are essentially identical to their approximation as in Table 1.

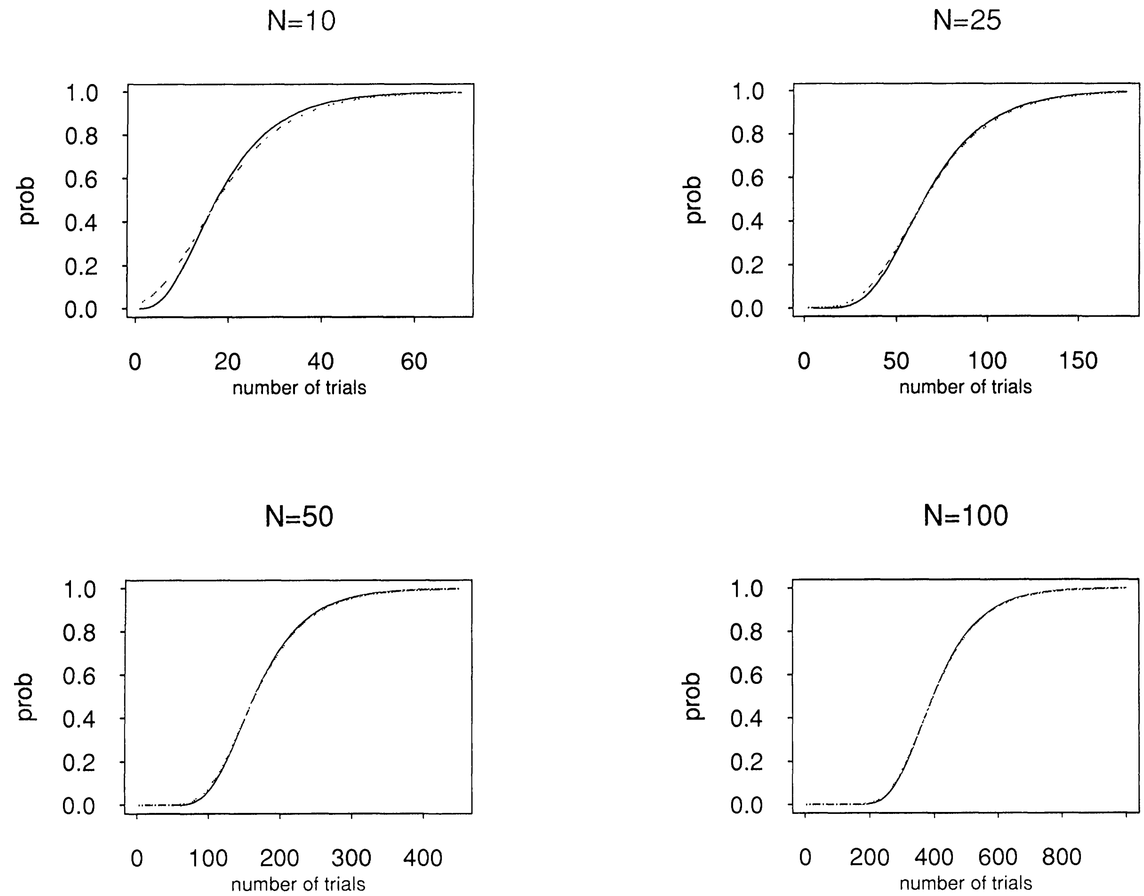

Figure 2. Actual and Approximated Distribution Function of the Waiting Time W for Various Values of N, the Number of Distinct Objects. The solid lines indicate the actual, and the dashed lines the approximated, functions. Note that strictly speaking the actual distribution function is discrete, but at the scale used there is no great disparity from that as represented.

puting chi-squared values for each of these gives 9.328 and 8.576, respectively, neither of which is particularly remarkable. Metropolis, Reitwiesner, and von Neumann (1950) reported what appeared to be excessive flatness in the distribution of the digits for the first 2,000 in the expansion of e, but this does not appear to be consistent with the foregoing results. It should be noted, however, that in the calculation of the foregoing chi-squared value, the count for the digit 6 contributed 6.241 to the value.

The expansions for π and e were obtained using the symbolic manipulation package Maple implemented on a VAX of the University of Waterloo, and computations were done using the data analysis package S implemented on a Microvax.

Table 2. The Actual and Approximate Probabilities, to Four Decimal Places, for Selected Waiting Times i When N = 10

i	*Actual*	*Approximate*
10–19	.1732	.2411
20–29	.4216	.3515
30–39	.2483	.2324
40–49	.1004	.1068
50–59	.0366	.0427
≥60	.0198	.0257

4. THE INVERSE PROBLEM

I now consider the inverse problem: that is, I suppose that I have a sample of size m, drawn with replacement from a population of N objects, each having equal chance of being drawn at a given trial. N is presumed unknown. If there are i distinct elements in the sample, I wish to obtain an estimate of N. Now it is clear that the distribution of the number X of objects in a sample of size m, when there are N objects, is given by

$$\Pr(X = i) = \frac{\binom{N}{i} Z_i(m)}{N^m},$$

where $Z_i(m)$ denotes the number of permutations of i distinct objects, taken m at a time, in which each of the i objects must appear at least once, and i ranges over 1 to $\min(m, N)$. It can be shown that

$$Z_i(m) = \sum_{j=0}^{j=i} \binom{i}{j} (i - j)^m (-1)^j.$$

[See example 12 in Riordan (1958, p. 13) or problems 5–13 in Feller (1968, pp. 59–60) for this and related results. See also Eq. (1). Rayner (1989) effectively has an informal proof of this relation.]

4	1:3334
59	1:5556666666666677777777777777788888888888888889999999999
130	2:00000000000001111111112222222222222222233333333333333334444444444444444
(71)	2:55555555555555666666666666777777777777777788888888888888999999999999999
136	3:00011111111111122223333333334444444444
98	3:5555555555556666666666667777888888888999
58	4:00000001123333333333344444
32	4:5556667788999
19	5:11222334
11	5:559
8	6:000123
High	65 77

Figure 3. Stem-and-Leaf Display for the Waiting Times for All 10 Digits Based on the First 10,000 Digits of π. The sample size is 337; the median is 27; the quartiles are 22 and 26; the decimal point is one place to the right of the colon.

The maximum likelihood principle asserts that the appropriate value to use is the value of N that maximizes the foregoing expression, for given i and m. In that case I merely need to maximize the expression

$$\frac{\binom{N}{i}}{N^m} . \tag{3}$$

Note that, if we regard N as a continuous variable, then the function has roots at $0, 1, \ldots, i - 1$, and hence there are at least $i - 1$ local extreme points between 0 and $i - 1$. It is also evident that there must be at least one maximum greater than $i - 1$, and it follows that this is then unique, since there are exactly i finite extreme values. Consideration of Example 2 below should clarify these assertions.

Example 1. Suppose that $m = 2 = i$. Then I need to maximize the expression $\binom{N}{2}/N^2 = \frac{1}{2}(N - 1)/N$. Unfortunately, this increases monotonically as N increases, implying an estimate $N = \infty$! In fact, it is easy to show that, if $m = i$, then there is no finite maximum for N. This is more convincing when m is large than when small, but in any case it is difficult to see what other "value" could possibly be supported when all trials have given rise to distinct objects.

Example 2. Consider the case for general $m > 3$ and $i = 3$. Then it is easy to show that we need to solve

$$(m - 3)N^2 - 3N(m - 2) + 2(m - 1) = 0.$$

Note that for all such m the closest integer to the smallest root of this equation is 1. For $m = 4, 5, \ldots$, the corresponding largest root is approximately 4.7, 3.3, 2.8, 2.6, 2.5, 2.5, 2.3, . . . (cf. Table 4). For the general case $i < m$, we must find the (integer) value that maximizes

$$\frac{N(N - 1)(N - 2) \cdots (N - i + 1)}{N^m} .$$

Taking logs gives

$$l = \log N + \log(N - 1) + \log(N - 2) + \cdots + \log(N - i + 1) - m \log N$$

as the expression to maximize. Taking derivatives with respect to N and equating to 0,

$$\frac{1}{N} + \frac{1}{N - 1} + \cdots + \frac{1}{N - i + 1} - \frac{m}{N} = 0.$$

Letting $S_n = \Sigma_{j=1}^{j=n} 1/j$, this can be written as $S_N - S_{N-i} = m/N$, and thus I wish to solve, subject to $N \geq i$,

$$N(S_N - S_{N-i}) - m = 0. \tag{4}$$

By the previous remarks, this exists and is unique. Of course, the integers either side of the root are then possible candidates for the actual solution. It is easily determined that there is no finite solution for $m = 2$, but that for $m = 3$ and $i = 2$ the solution is $N = 2$. Even to get an approximate general solution seems quite difficult, and some sort of iterative procedure would appear to be necessary. For small values of m, actual evaluation of (3) gives Table 4. Applying some rather crude approximations leads to $i^2/(2(m - i))$ as a rough estimate for the solution when $m - i$ is small with respect to m and i. This works quite well even in Table

9	1:233444444
55	1:5555555555566666667777777778888888888999999999
124	2:000000000001111111111122222222222222222223333333333333344444444444444
(71)	2:55555555555555566666666666677777777777777778888888888888899999999999999
140	3:00000000000000000011111222222222333444444444444
93	3:555555555566666677777777777888999999
57	4:000001111111111112222334
33	4:555677888999
21	5:0112344
14	5:567
11	6:00234
High	66 70 70 73 74 77

Figure 4. Stem-and-Leaf Display for the Waiting Times for All 10 Digits Based on the First 10,000 Digits of e. The sample size is 335; the median is 28; the quartiles are 22 and 35; the decimal point is one place to the right of the colon.

Table 3. *Counts of Digits in the First 10,000 of the Decimal Expansions of π and e, Respectively*

	0	1	2	3	4	5	6	7	8	9
π	968	1,026	1,021	975	1,012	1,046	1,021	969	948	1,014
e	974	989	1,005	1,008	982	992	1,079	1,008	994	969

4, for $m - i = 1$. For larger values of $m = 100$ and $i = 95$, say, the approximation gives about 900 as opposed to a "true" value of about 957. Much more work will be needed, however, to obtain a more satisfying approximation.

Solution of (4) can be effected using Brent's method as given in Press, Flannery, Teukolsky and Vetterling (1988, pp. 266–269). Using an implementation of this routine on a Microvax of the Department of Statistics and Actuarial Science, University of Waterloo, I obtained an estimate of 62.48 when $i = 11$ and $m = 12$.

I can apply the procedure to the data as given in Blower, Cook, and Bishop (1981, p. 83), relating to the trapping and subsequent release of the bank vole *Clethrionomys glareolus*, where 53 different animals were trapped over a six-night period. The total number of captures was 109, and the corresponding estimate is 64. Two alternative estimates given by Blower et al., based on methods detailed in Seber (1973) and Schumacher and Eschmeyer (1943), give 61 and 57, respectively.

Of course, it is almost certain that the data cannot be taken to conform strictly to the exact coupon collectors situation, but it is gratifying to see the rough agreement with the other two estimates. Following a suggestion of Blower et al. (1981, p. 82), it would seem that the estimation procedure could well be applied when birds are netted and then released immediately.

5. CONCLUSIONS AND OPEN QUESTIONS

For even modestly large values of N, it is apparent that the Baum and Billingsley approximation is quite adequate to determine the distribution of the waiting times, and hence various confidence intervals could be obtained, in the classical situation. As noted previously, it seems plausible to use the inverse problem in the estimation of such things as animal abundance. I am at present working on a comparison of this method with the two alternative methods referenced here. This requires the estimation problem to be solved at least approximately to be viable. A Monte Carlo technique can be applied, of course, but a well-determined approximate method would be preferable. I have not as yet been able to derive acceptable approximations.

As far as the "randomness" of the expansions for π and e are concerned, it appears from our results that the first 10,000 digits of each are consistent with being drawn at random, both coupon collector's test and one based on the distribution of individual digits supporting this assertion.

Exact solution of the more general coupon collector's problem seems difficult. The more general problem can be regarded as a classical problem with associated W, in which the sequence of observations is divided into blocks of n and the integral number X of blocks is observed. It seems evident that the number X of draws required behaves roughly like W/n, but attempts to obtain an exact solution seem to lead to intractable equations. Inequalities relating the moments of W to those of X are reasonably easy to determine, however, and these could conceivably be put to use to obtain approximate solutions. It is not at all evident how one might easily obtain interval estimates in the general case. The actual problem, however, as originally stated to me by Siobhan, is solvable exactly when further information on the nature of the hockey cards is made available—namely, that the producers of the cards undertake to allow exchanges of duplicates. In that case, it is evident that to obtain 300 cards it is merely necessary to purchase 60 packets of gum!

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Table 4. *Maximum Likelihood Estimator Values for the Number N of Objects, for Various Sample Sizes m and Numbers i of Distinct Objects*

	i										
m	1	2	3	4	5	6	7	8	9	10	11
1	∞	—	—	—	—	—	—	—	—	—	—
2	1	∞	—	—	—	—	—	—	—	—	—
3	1	2	∞	—	—	—	—	—	—	—	—
4	1	2	5	∞	—	—	—	—	—	—	—
5	1	2	3	8	∞	—	—	—	—	—	—
6	1	2	3	6	13	∞	—	—	—	—	—
7	1	2	3	5	8	19	∞	—	—	—	—
8	1	2	3	4	7	11	25	∞	—	—	—
9	1	2	3	4	6	9	15	33	∞	—	—
10	1	2	3	4	5	8	12	19	42	∞	—
11	1	2	3	4	5	7	10	15	24	51	∞
12	1	2	3	4	5	7	9	12	18	29	62

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CORRECTIONS

Julio L. Peixoto, "A Property of Well-Formulated Polynomial Regression Models," *The American Statistician*, 44, 26–30.

The main result (Theorem 3.1) in my article had previously been discussed (although not as formally) by Driscoll and Anderson (1980, sec. 4). If I had known of this article, I would have cited it and credited its priority. I apologize to the authors for overlooking it.

Reference

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In Table 1 on page 69, the term of presidency of Barbara A. Bailar should be 1987, and not 1897. John Chung-Chiang Liu called our attention to the error.