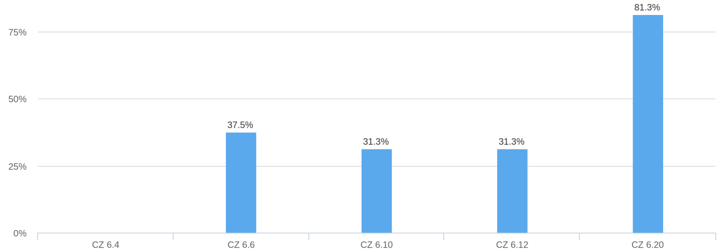
3-10 Traversability

(Part I: Eulerian Graphs)

Hengfeng Wei

hfwei@nju.edu.cn

December 03, 2018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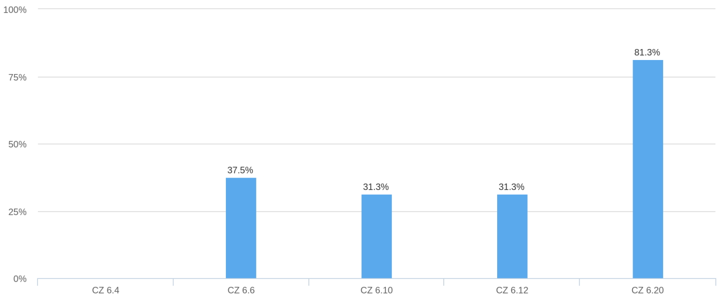
CZ 6.20



CZ 6.20 (Next Class)

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欧拉图和汉密尔顿图的联系 就是在建模时如何确定图的节点和边

g-cage 对于不同大小而言都是唯一的吗？（书上只给到n=8） Kirkman triple system 和 factorization 的等价推导 Theorem 8.10的证明

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如何打印欧拉回路/欧拉迹/哈密顿回路

定理6.5

无

暂无

无

陶老师上课讲的中国邮递员问题没明白

暂无，我就来抽个奖

none

定理6.12的证明

喵(^･ｪ･^)

希望能讲一下fluery算法

可以总结一下证明的方法，其实每次都可以这样，不一定要课上讲，可以整理之后做成讲义课后发，比如怎么证明有欧拉回路等等

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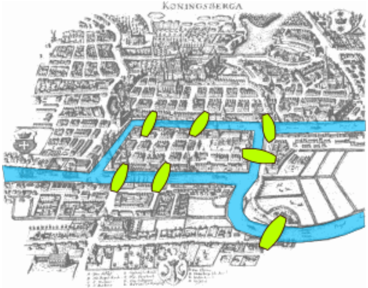
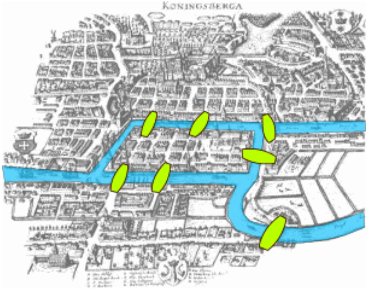
6.3 Exploration & 6.4 Excursion (Not Required)



Leonhard Euler (1707 – 1783)



Leonhard Euler (1707 – 1783)

Graph Theory Topology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$$H = G - E(C) = \bigcup H_i$$

$\square$

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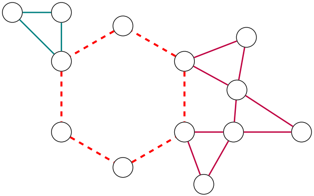
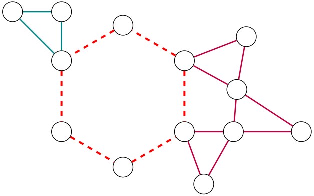


$$H = G - E(C) = \bigcup H_i$$

(I) $\forall v \in H : \deg(v)$ is even

(II) $\forall i : \left| E(H_i) \right| < m$



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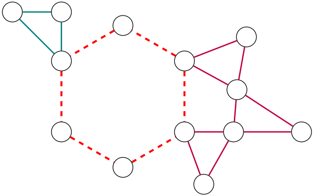

By I.H., each H_i has an Eulerian circuit C_i.

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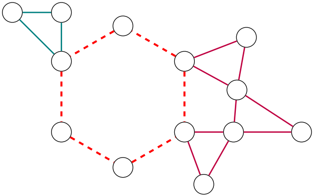

By I.H., each H_i has an Eulerian circuit C_i.

$$\forall i : V(H_i) \cap V(C) \neq \emptyset$$

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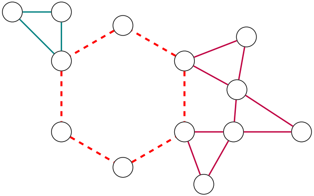

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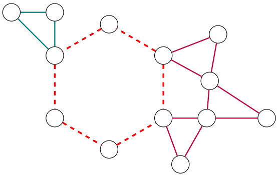
Glue together each C_i with C to get an Eulerian circuit of G.

```
procedure HIERHOLZER(G)
    u ∈ V(G)
    C ← any circuit u ~ u in G
    while ∃v ∈ C : deg(v) > 0 do
        H ← G − E(C)
        v ← any vertex in V(C) such that deg(v) > 0
        C′ ← any circuit v ~ v in H
        C ← C ⊗ C′                          ▷ Glue C′ = v ~ v with C via v
    return C
```



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    $C' \leftarrow$ any circuit $v \sim v$ in H
    $C \leftarrow C \otimes C'$ ▷ Glue $C' = v \sim v$ with C via v
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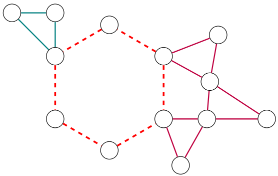

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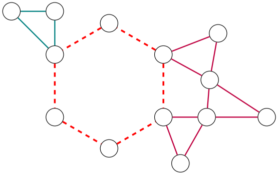

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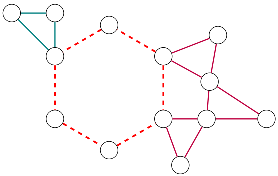

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Q : Data Structures?

$O(m)$: Using doubly linked list

Fleury's Algorithm (1883)

(I) $v_0 \in V(G); \quad C_0 = v_0$

(II) Suppose $C_i = v_0 e_1 v_1 e_2 \cdots e_i v_i$.

Choose e_{i+1} from $E(G) - \{e_1, e_2, \cdots, e_i\}$ as follows:

(a) e_{i+1} is incident with v_i

(b) Unless there is no alternative, e_{i+1} is not a bridge of $G - \{e_1, e_2, \cdots, e_i\}$

(III) Stop when step (II) can no longer be implemented

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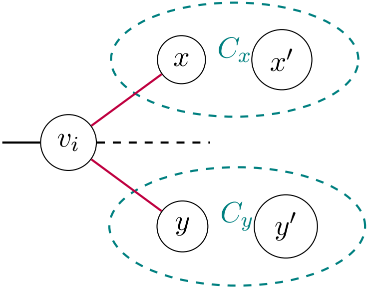

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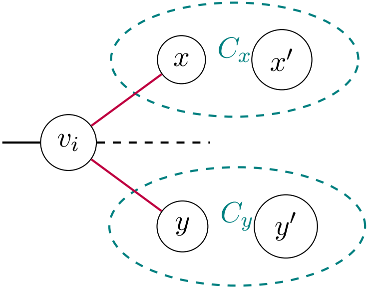

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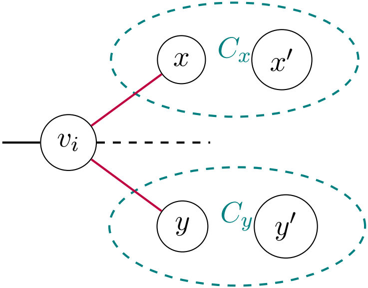

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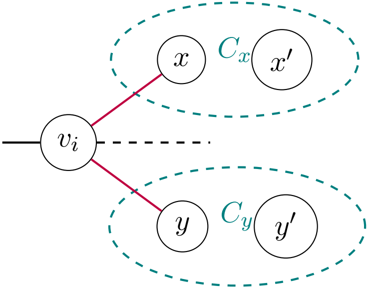

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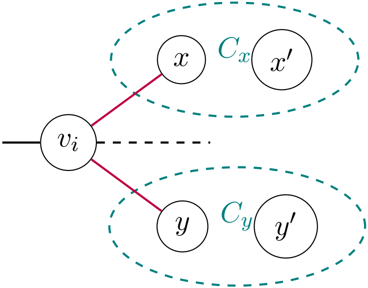

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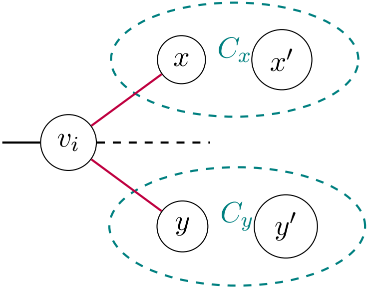

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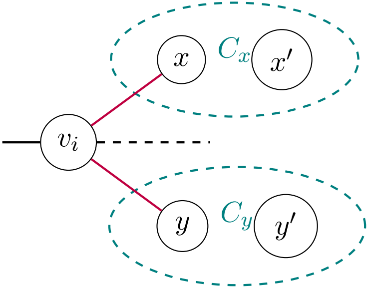
Is $\deg(v_i)$ odd or even?



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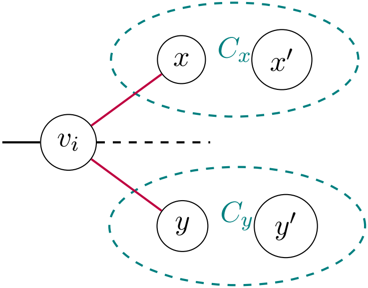

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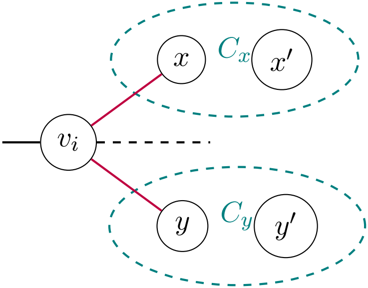

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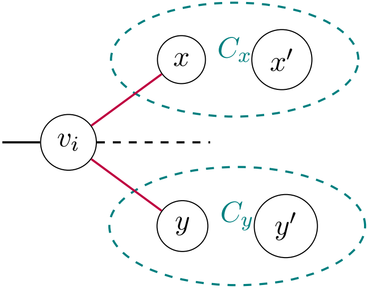

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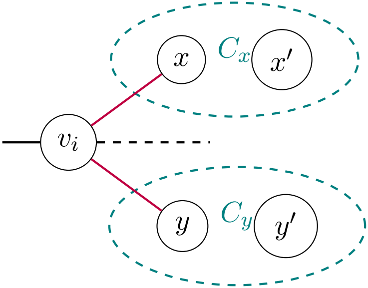

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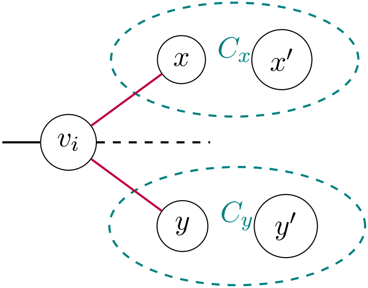

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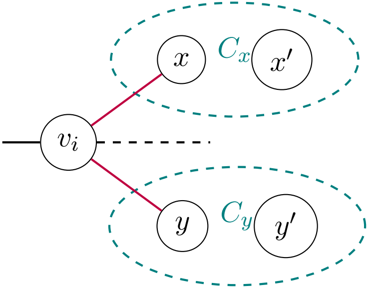

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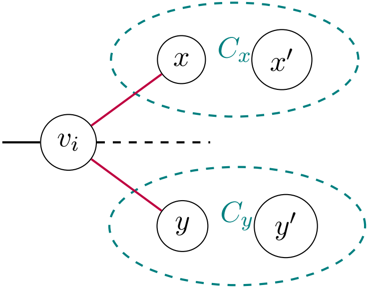

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Contradiction: No odd vertices!

```
procedure FLEURY(G)
    v_0 ∈ V(G)                              ▷ Choose any starting vertex
    C ← v_0                                 ▷ Keep track of the circuit
    i ← 0, V_0 ← V(G), E_0 ← E(G)
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    while deg(v_i) > 0 in E_i do                 ▷ Stop otherwise

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    $e_{i+1} \triangleq v_i v_{i+1}$

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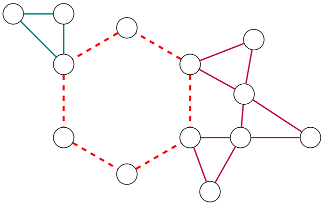
Closed	Trail	Include every edge of G
$\because$ even degrees	$\because$ used edges are deleted	By Contradiction.



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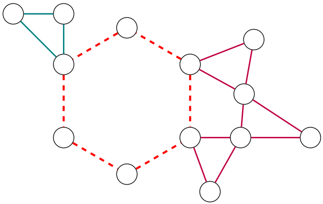

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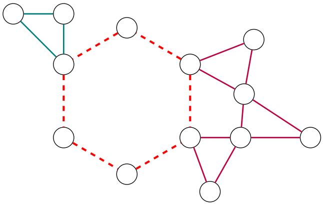

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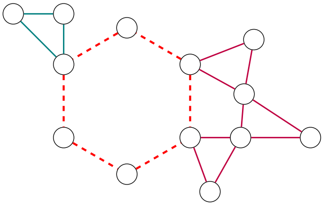
We know that $C : v_0 \sim v_0$ $\quad E' \triangleq E(G) - E(C) \neq \emptyset$



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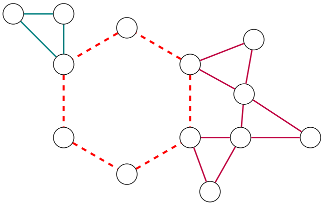
$\deg(v_0) = 0$



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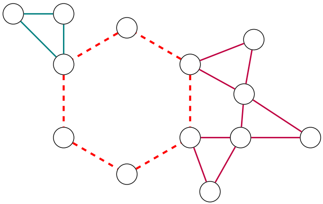

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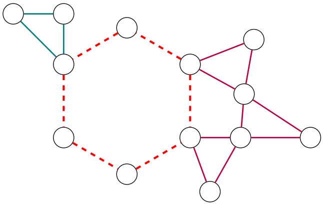

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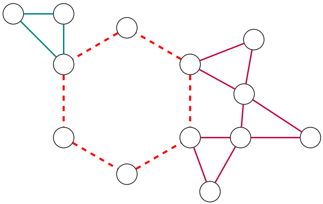

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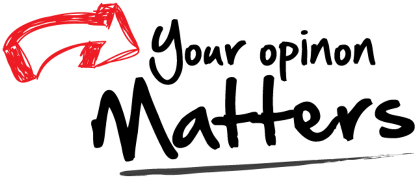
Impossible:

(I) Don't go the bridge unless there is no alternative.

(II) Delete the isolated vertex left by going the bridge.


THANK YOU



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