2-1 The Correctness of Algorithms

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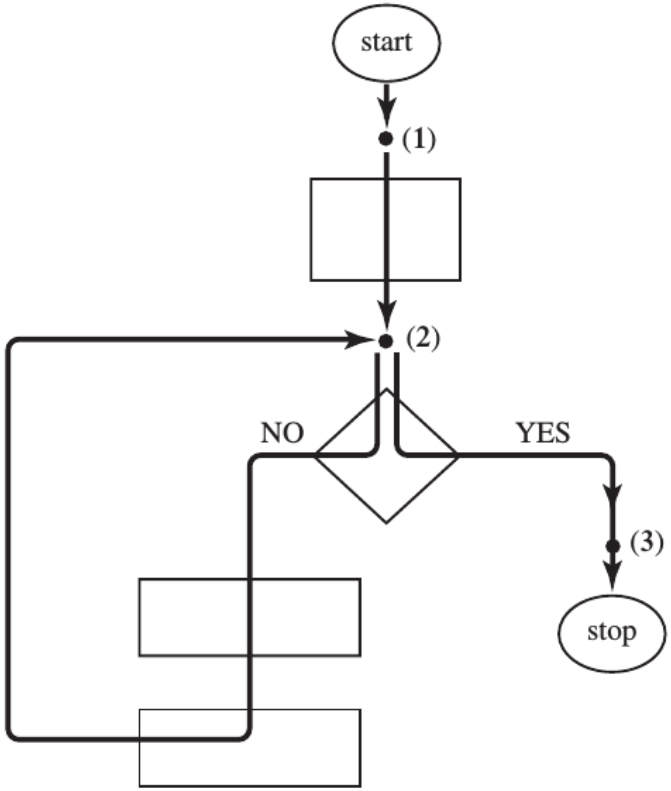
学生反馈

Q : Assertion, Invariant, Loop invariant 之间是什么关系?

Q : How to prove a loop partially correct?



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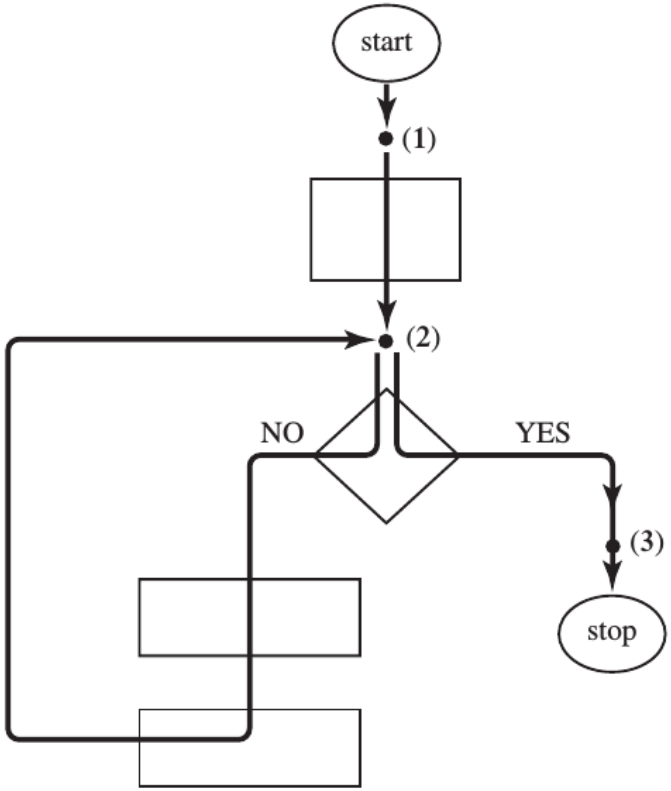

$\{P\}$ `loop` $\{Q\}$

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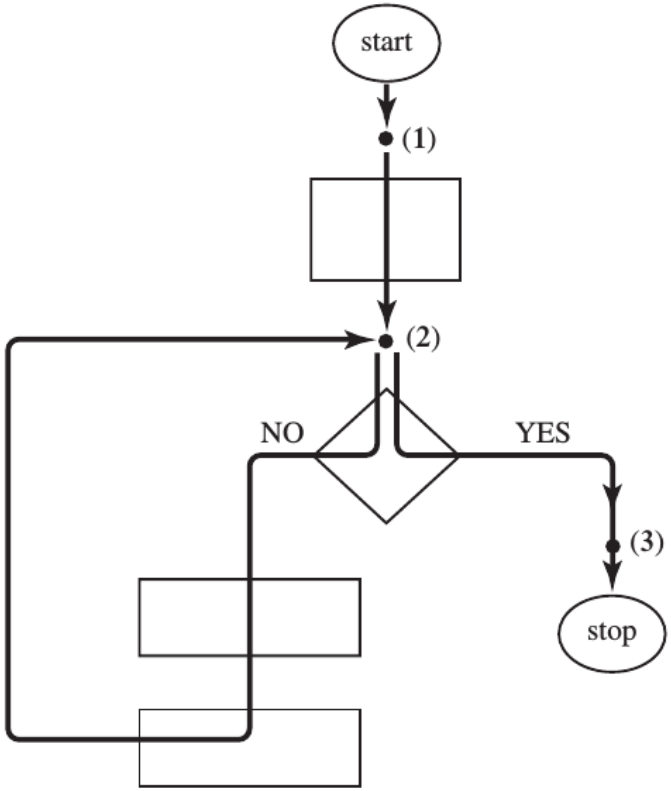


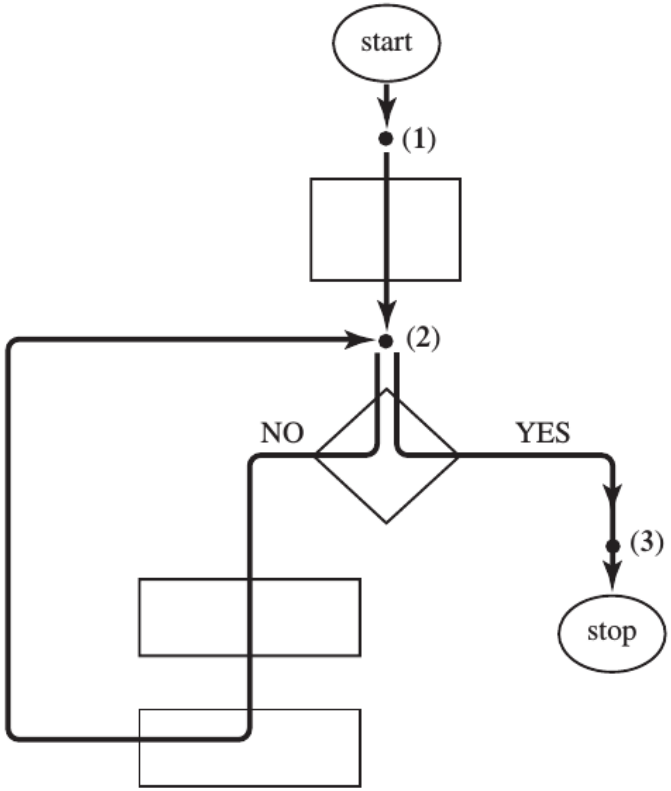
$\{P\}$ `loop` $\{Q\}$

(1) $\{P\}$ `init` $\{I\}$

(2) $\{I \wedge C\}$ `body` $\{I\}$

(3) $\{I \wedge \neg C\} \Longrightarrow \{Q\}$

Q : How to prove a loop partially correct?



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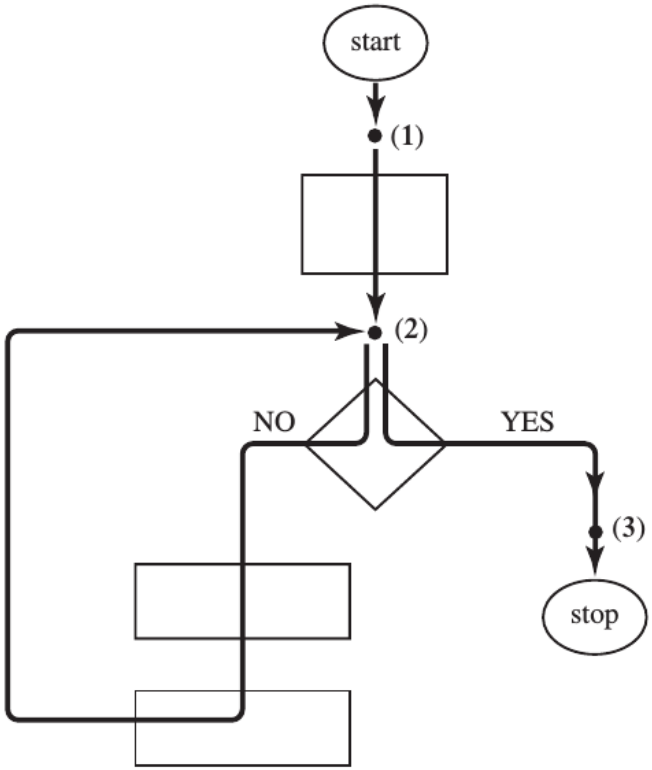
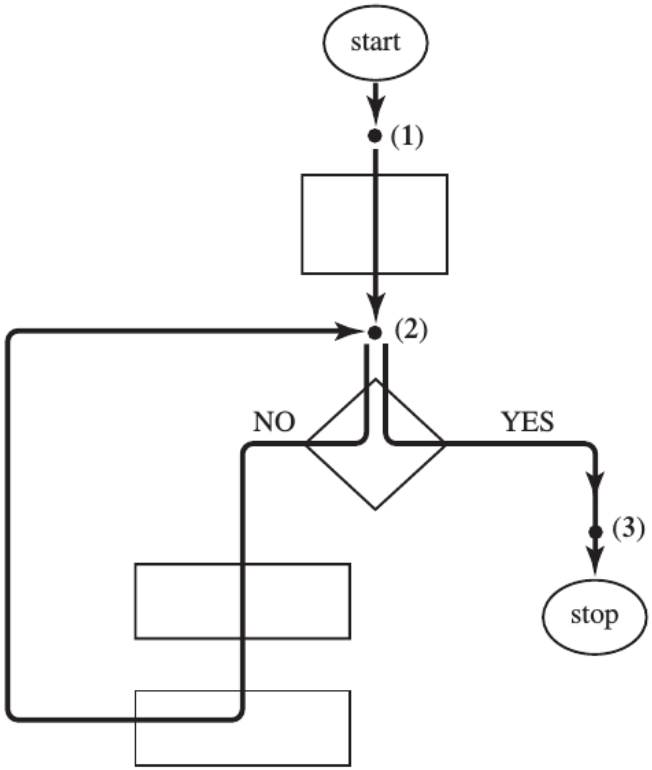
I is *before* the loop.

Q : How to prove a loop totally correct?



$D(X)$

Q : How to prove a loop totally correct?



$$D(X)$$

(1) $\{I \wedge C\}$ `body` $\{D(X') < D(X)\}$

(2) $\{I \wedge D(X) = \min\} \implies \neg C$

Q : How to develop loop invariants?

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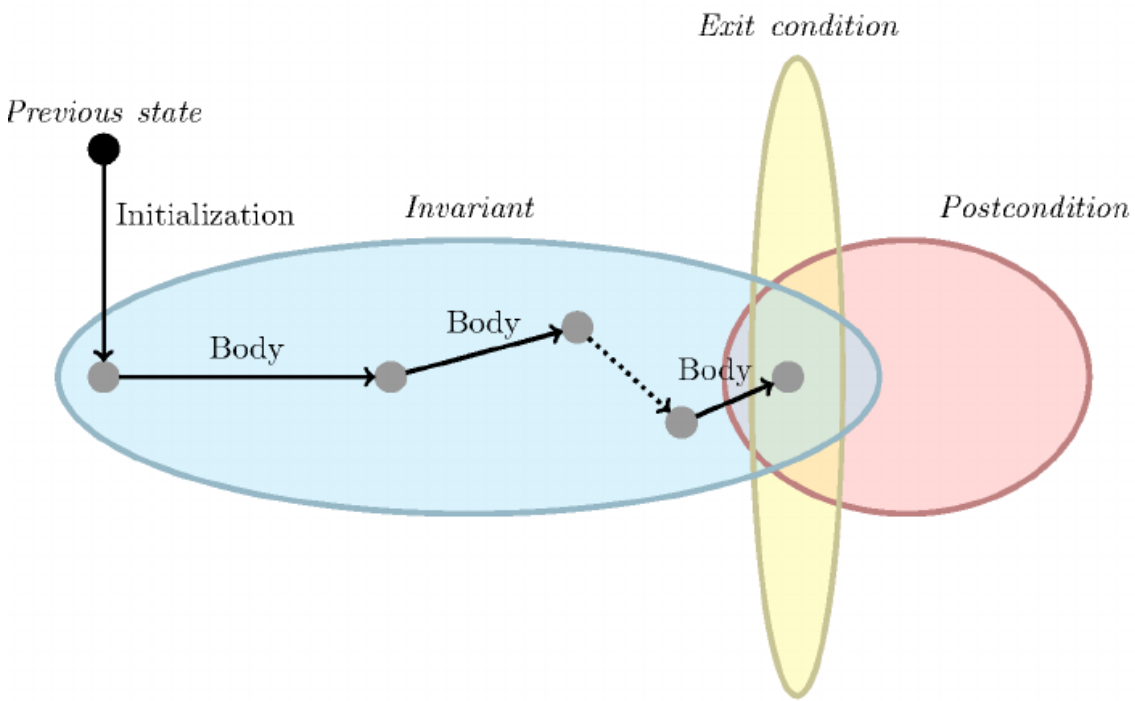
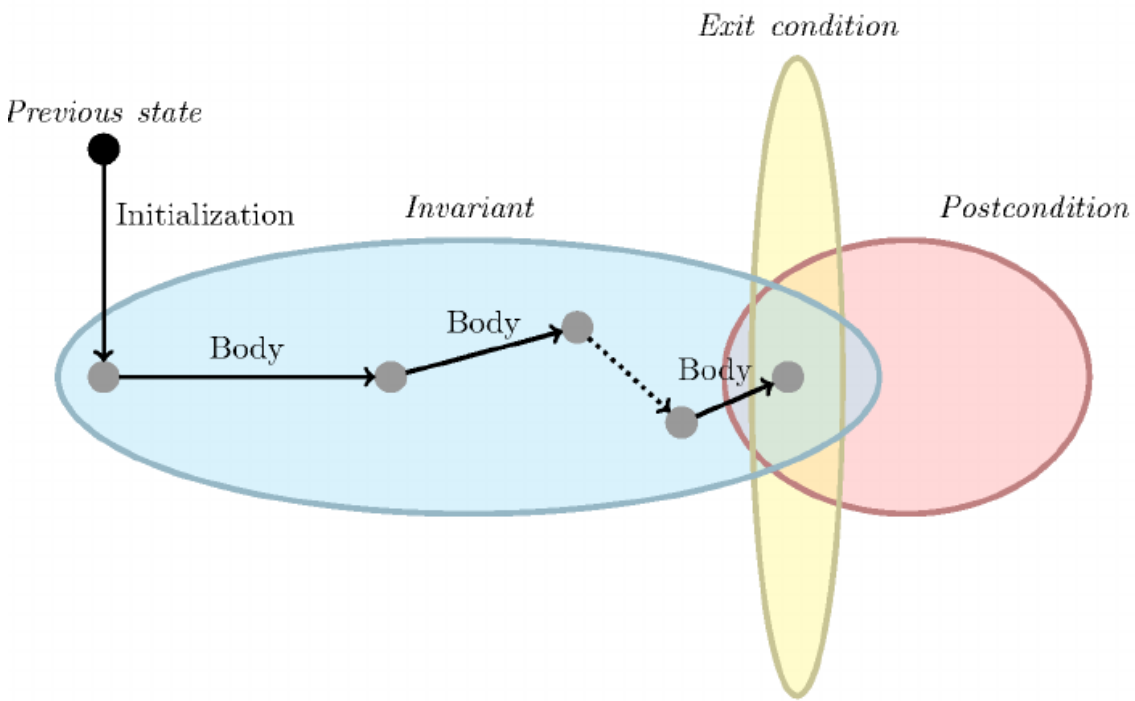
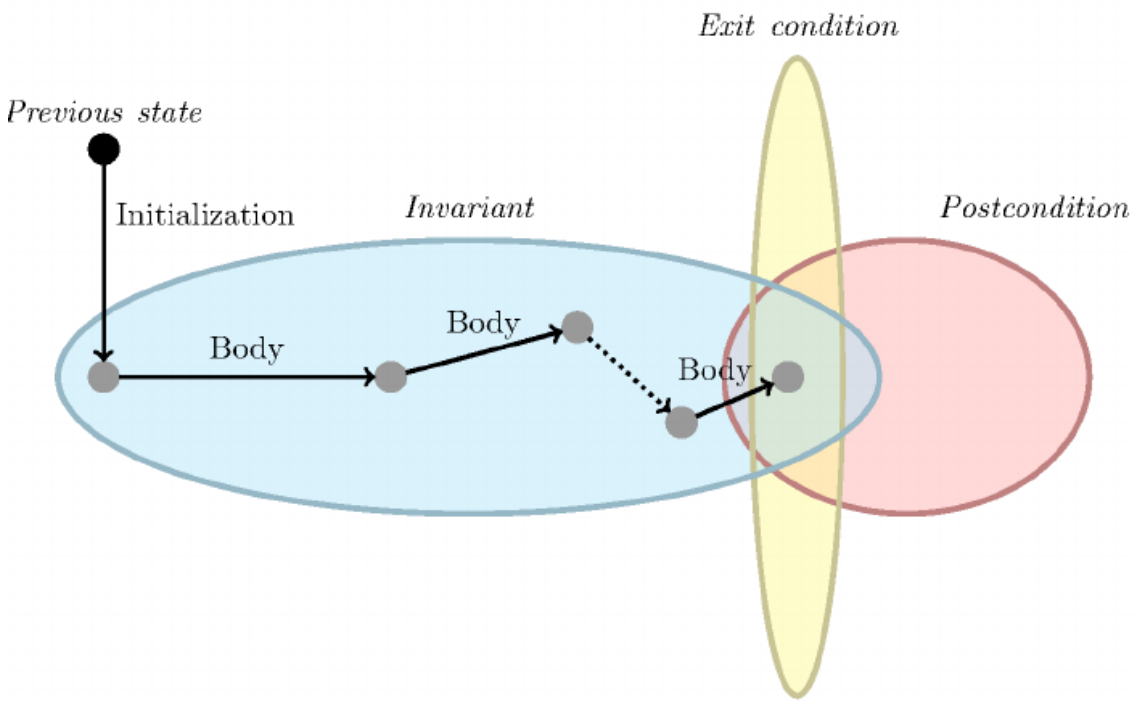


Q : How to develop loop invariants?



$$I \equiv \Big(\texttt{totalWork} = \texttt{workDone} + \texttt{workToDo}\Big)$$

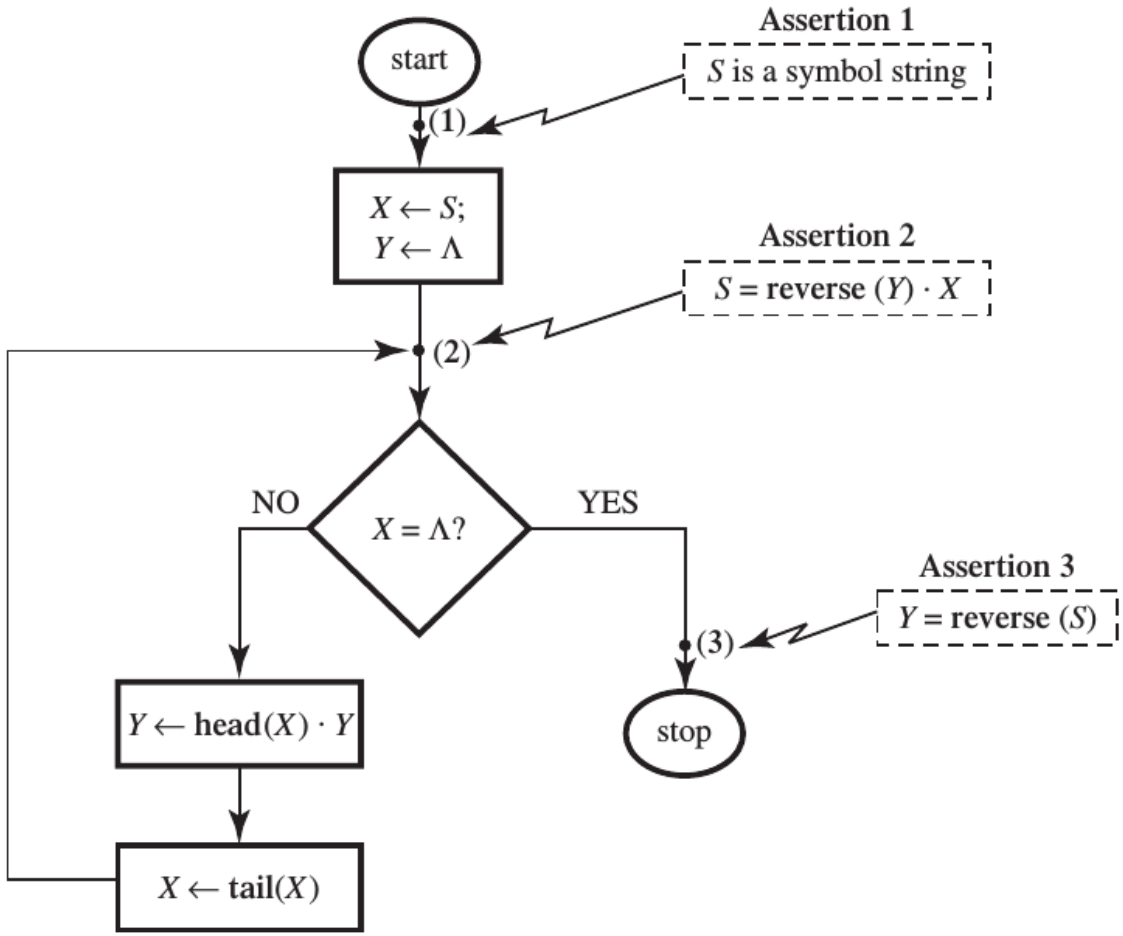
Q : How to develop loop invariants?



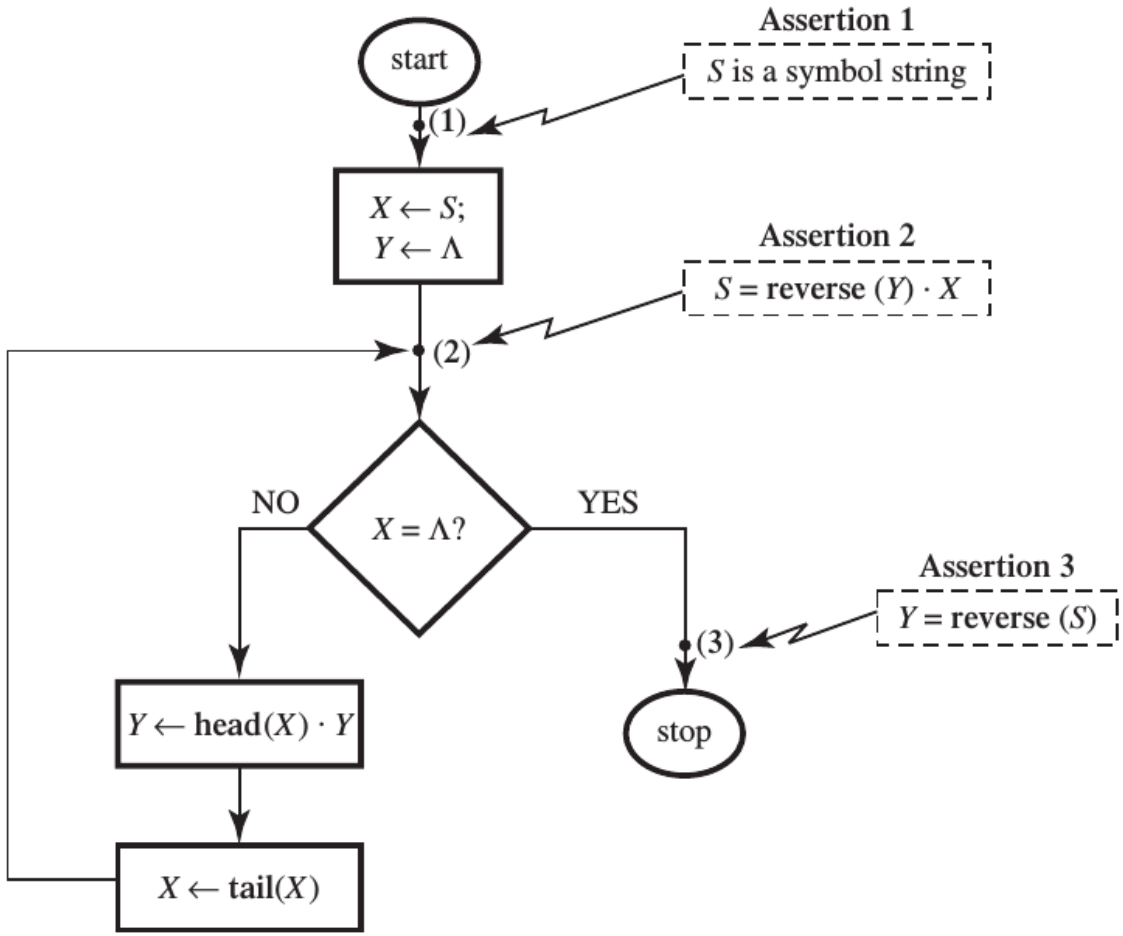
$$I \equiv \Big(\texttt{totalWork} = \texttt{workDone} + \texttt{workToDo}\Big)$$

$$\texttt{workDone} \xLeftarrow{\texttt{data}} \texttt{workToDo}$$

Reverse(S)


start
(1)
$X \leftarrow S$;
$Y \leftarrow \Lambda$
(2)
$X = \Lambda$?
NO
YES
$Y \leftarrow \text{head}(X) \cdot Y$
$X \leftarrow \text{tail}(X)$
(3)
stop
Assertion 1
S is a symbol string
Assertion 2
$S = \text{reverse}(Y) \cdot X$
Assertion 3
$Y = \text{reverse}(S)$

Reverse(S)



$$I \equiv \Big(\texttt{reverse}(S) = Y \cdot \texttt{reverse}(X)\Big)$$

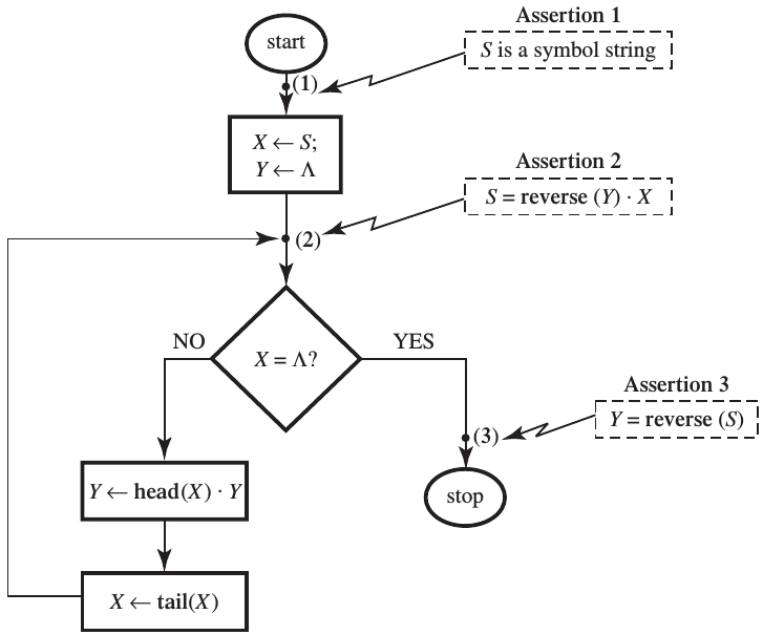
of invariants (DH Problem 5.6)

(a) Structure of `Reverse`(X)

(b) Only two invariants

(c) Sufficent #invariants for two loops

(d) Necessary #invariants for two loops



equal(X, Y) (Problem 5.9)

- head(X)
- tail(X)
- last(X)
- all-but-last(X)
- eq(s, t)

$X \leftarrow S_1$
$Y \leftarrow S_2$

$E \leftarrow \top$
while $(X \neq \Lambda \wedge E == \top)$
  if $(Y == \Lambda)$
    $E \leftarrow \bot$
  else if $\texttt{eq}(\texttt{head}(X), \texttt{head}(Y))$
      $X \leftarrow \texttt{tail}(X)$
      $Y \leftarrow \texttt{tail}(Y)$
  else
    $E \leftarrow \bot$

if $Y \neq \Lambda$
  $E \leftarrow \bot$

return E

改变判断条件


Thank
You!