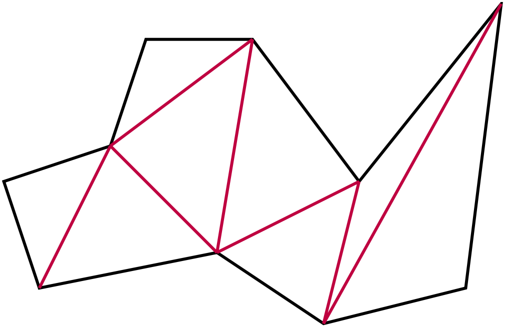
2-4 Recurrences

魏恒峰

hfwei@nju.edu.cn

2018 年 04 月 23 日

Triangulating Polygons



The Art Gallery Problem

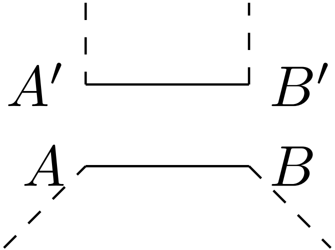
Q : How many “BIG BROs” to hire?

Another Version of the Ear Lemma (Problem $4.1-16$)

A triangulated polygon is either a triangle with three ears or has at least two ears (which are *not necessarily non-adjacent*).

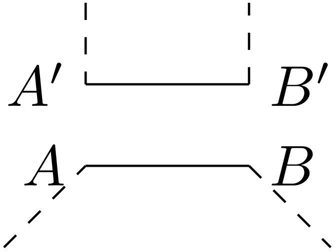
Another Version of the Ear Lemma (Problem $4.1-16$)

A triangulated polygon is either a triangle with three ears or has at least two ears (which are *not necessarily non-adjacent*).



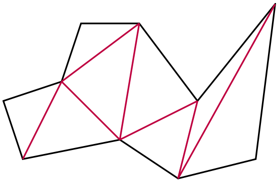
Another Version of the Ear Lemma (Problem $4.1 - 16$)

A triangulated polygon is either a triangle with three ears or has at least two ears (which are *not necessarily non-adjacent*).



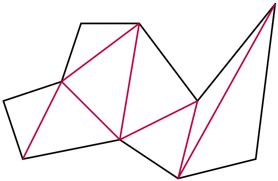
Q : *Adjacent ears?*

$\#$ of triangles (Problem $4.1 - 17$)

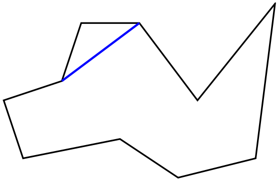


$$T(n) = n - 2$$

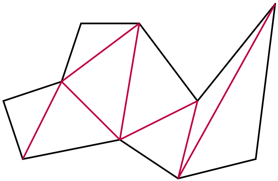
$\#$ of triangles (Problem $4.1 - 17$)



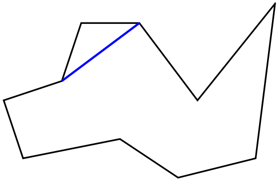
$$T(n) = n - 2$$

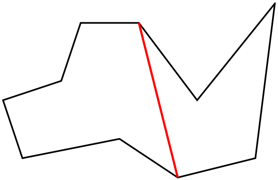


$\#$ of triangles (Problem $4.1 - 17$)

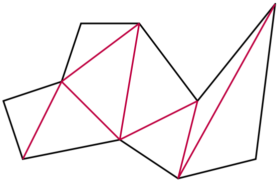


$$T(n) = n - 2$$

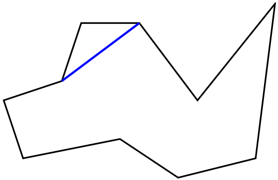


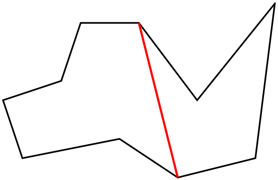


of triangles (Problem $4.1-17$)



$$T(n) = n - 2$$





Q : Existence of diagonals?

Lemma (Ear Lemma)

A triangle has 3 ears, and a larger triangulated polygon has at least 2 non-adjacent ears.

Lemma (Ear Lemma)

A triangle has 3 ears, and a larger triangulated polygon has at least 2 non-adjacent ears.

Q : *Can every polygon be triangulated?*

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Proof.

"To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added.

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Proof.

> *"To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added.*
>
> *These diagonals must lie entirely interior to the polygon and are not allowed to intersect.*

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Proof.

> *"To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added.*
>
> *These diagonals must lie entirely interior to the polygon and are not allowed to intersect.*
>
> *They break the interior of the polygon into a number of triangles, because any larger polygon can be split by adding a diagonal."*

□

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Proof.

> *"To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added.*
>
> *These diagonals must lie entirely interior to the polygon and are not allowed to intersect.*
>
> *They break the interior of the polygon into a number of triangles, because any larger polygon can be split by adding a diagonal."*

□

"(This fact is perhaps not obvious,
but we won't get sidetracked by proving it here.)"

Theorem (Existence of Triangulation)

Any polygon can be triangulated.

Proof.

> *"To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added.*
>
> *These diagonals must lie entirely interior to the polygon and are not allowed to intersect.*
>
> *They break the interior of the polygon into a number of triangles, because any larger polygon can be split by adding a diagonal."*

□

"(This fact is perhaps not obvious,
but we won't get sidetracked by proving it here.)"

Theorem (Existence of Diagonal)

Every polygon with $n > 3$ *has a diagonal.*

Theorem (Existence of Diagonal)

Every polygon with $n > 3$ has a diagonal.

Definition (Convex Vertex)

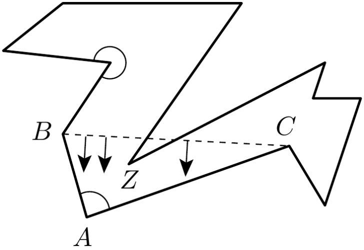
A vertex v is *convex* if the *interior* angle at v is less than 180°.

Theorem (Existence of Diagonal)

Every polygon with $n > 3$ has a diagonal.

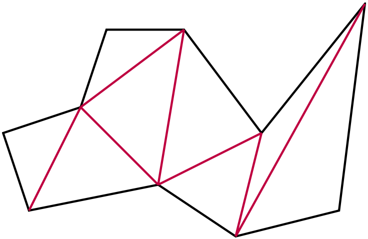
Definition (Convex Vertex)

A vertex v is *convex* if the *interior* angle at v is less than 180°.



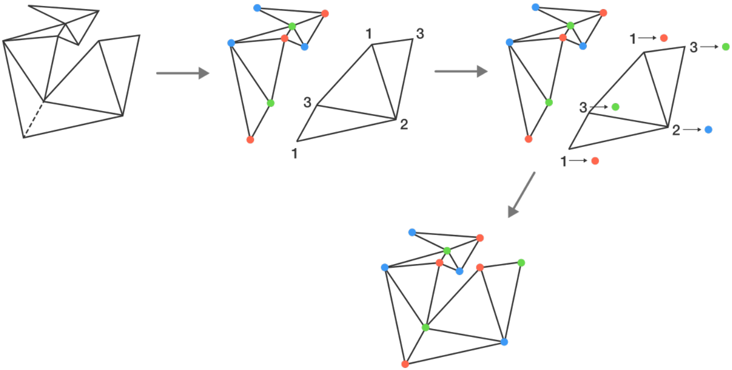
Theorem (Coloring)

Any triangulated polygon polygon is 3-colorable.



Theorem (Coloring)

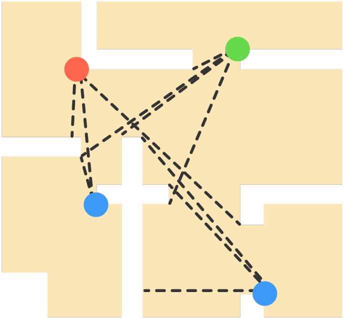
Any triangulated polygon polygon is 3-colorable.



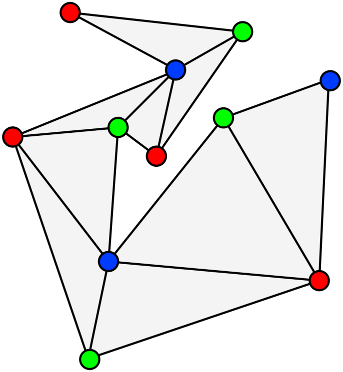
The Art Gallery Problem

Q : How many "BIG BROs" to hire?

The Art Gallery Problem



Q : How many "BIG BROs" to hire?



Theorem (The Art Gallery Theorem (O))

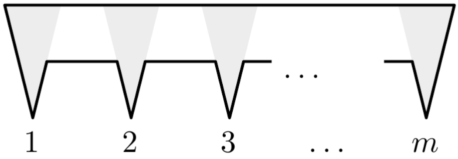
For any art gallery with n walls, $\lfloor \frac{n}{3} \rfloor$ "BIG BROs" suffice.

Theorem (The Art Gallery Theorem (Ω))

There exists an art gallery with n walls such that $\lfloor \frac{n}{3} \rfloor$ "BIG BROs" are necessary.

Theorem (The Art Gallery Theorem (Ω))

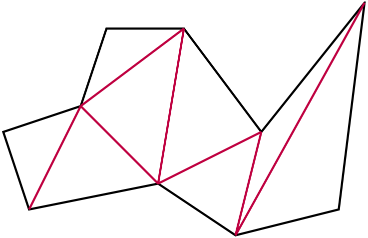
There exists an art gallery with n walls such that $\lfloor \frac{n}{3} \rfloor$ "BIG BROs" are necessary.



$$n = 3m$$

"How to Guard a Museum?"

"How to Guard a Museum?"

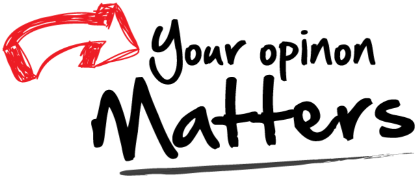


$$O(n \log n)$$

$$O(n \log \log n)$$

$$O(n)$$


Thank You!



Office 302

Mailbox: H016

hfwei@nju.edu.cn