Absorption law

In algebra, the **absorption law** or **absorption identity** is an identity linking a pair of binary operations.

Two binary operations, ¤ and ⁂, are said to be connected by the absorption law if:

$$a \mathbin{¤} (a \mathbin{⁂} b) = a \mathbin{⁂} (a \mathbin{¤} b) = a.$$

A set equipped with two commutative, associative and idempotent binary operations ∨ ("join") and ∧ ("meet") that are connected by the absorption law is called a lattice.

Examples of lattices include Boolean algebras, the set of sets with *union* and *intersection* operators, Heyting algebras, and ordered sets with *min* and *max* operations.

In classical logic, and in particular Boolean algebra, the operations OR and AND, which are also denoted by ∨ and ∧, satisfy the lattice axioms, including the absorption law. The same is true for intuitionistic logic.

The absorption law does not hold in many other algebraic structures, such as commutative rings, *e.g.* the field of real numbers, relevance logics, linear logics, and substructural logics. In the last case, there is no one-to-one correspondence between the free variables of the defining pair of identities.

See also

- Identity (mathematics)

References

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