3-2 Greedy Algorithms

(How to justify your greed?)

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Change-making (Problem 16-1 (a))

Making change for v cents using the fewest number of coins.

Change-making (Problem 16-1 (a))

25

10

5

1

Making change for v cents using the fewest number of coins.

$$\text{OPT}: v = 25 \times A^* + 10 \times B^* + 5 \times C^* + 1 \times D^*$$

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$$(A^*, B^*, C^*, D^*) \textit{ vs. } (A, B, C, D)$$

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$$\text{G}: v = 25 \times A + 10 \times B + 5 \times C + 1 \times D$$

$$(A^*, B^*, C^*, D^*) \textit{ vs. } (A, B, C, D)$$

$$A^* = A$$

$$\text{OPT}: v = 25 \times A^* + 10 \times B^* + 5 \times C^* + 1 \times D^*$$

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Lemma (Properties of OPT)

(I) $B^* \leq 2$

(II) $C^* \leq 1$

(III) $D^* \leq 4$

(IV) $\neg(B^* = 2 \wedge C^* = 1)$

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(IV) $\neg(B^* = 2 \wedge C^* = 1)$

Theorem

$$10 \times B^* + 5 \times C^* + 1 \times D^* \leq 24$$

$$5 \times C^* + 1 \times D^* \leq 9$$

$$1 \times D^* \leq 4$$

$$\text{OPT}: v = 25 \times A^* + 10 \times B^* + 5 \times C^* + 1 \times D^*$$

Lemma (Properties of OPT)

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Theorem

$$10 \times B^* + 5 \times C^* + 1 \times D^* \leq 24$$

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$$(A^*, B^*, C^*, D^*) = (A, B, C, D)$$

Change-making (Problem 16-1 (c))

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$$\{1, 3, 4\}, \qquad v = 10$$

Change-making (Problem 16-1 (c))

$$\{1, 3, 4\}, \qquad v = 10$$

G : $4, 4, 1, 1$ *vs.* OPT : $4, 3, 3$

Change-making (Problem 16-1 (c))

$$\{1, 3, 4\}, \qquad v = 10$$

G : $4, 4, 1, 1$ *vs.* OPT : $4, 3, 3$

Why does the previous proof fail?

Change-making (Problem 16-1 (b))

$$c^0, c^1, c^2, \cdots, c^k, \quad c > 1, k \geq 1$$

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Change-making (Problem 16-1 (b))

$$c^0, c^1, c^2, \cdots, c^k, \quad c > 1, k \geq 1$$

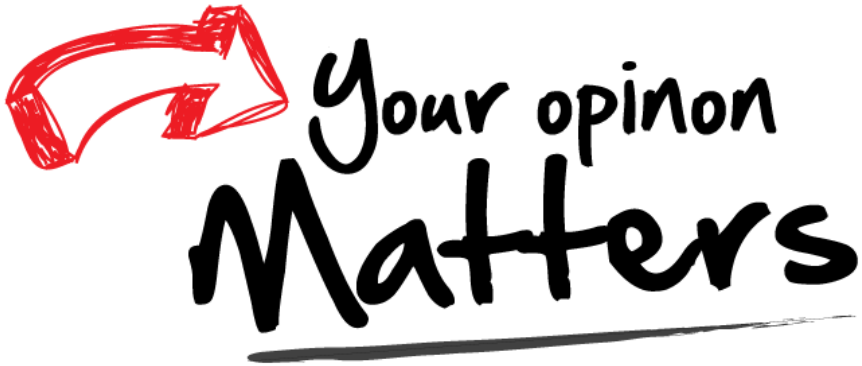
$$\text{OPT} : v = \sum_{i=0}^{i=k} c^i a_i^*$$

$$\text{G} : v = \sum_{i=0}^{i=k} c^i a_i$$

$$a_i^* = a_i$$

Canonical Coin Systems


THANK YOU



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