2-11 Heapsort

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ALGORITHM 245
TREESORT 3 [M1]
ROBERT W. FLOYD (Recd. 22 June 1964 and 17 Aug. 1964)
Computer Associates, Inc., Wakefield, Mass.

procedure *TREESORT* 3 (M, n);
value n; **array** M; **integer** n;
comment *TREESORT* 3 is a major revision of *TREESORT* [R. W. Floyd, Alg. 113, *Comm. ACM 5* (Aug. 1962), 434] suggested by *HEAPSORT* [J. W. J. Williams, Alg. 232, *Comm. ACM 7* (June 1964), 347] from which it differs in being an in-place sort. It is shorter and probably faster, requiring fewer comparisons and only one division. It sorts the array $M[1:n]$, requiring no more than $2 \times (2 \uparrow p-2) \times (p-1)$, or approximately $2 \times n \times (\log_2(n)-1)$ comparisons and half as many exchanges in the worst case to sort $n = 2 \uparrow p - 1$ items. The algorithm is most easily followed if M is thought of as a tree, with $M[j \div 2]$ the father of $M[j]$ for $1 < j \leq n$;

Heap Identity

$$\forall h \geq 1 : \lceil \log(\lfloor \frac{1}{2} h \rfloor + 1) \rceil + 1 = \lceil \log(h + 1) \rceil$$

# of Nodes at Height h (TC $6.3-3$)

There are at most $\lceil \frac{n}{2^{h+1}} \rceil$ nodes of height h in any n-element heap.

Sum of Heights of Nodes

In an n-element heap, we have

$$\sum_{\text{node } x} H(x) \leq n - 1$$

Worst-case of MAX-HEAPIFY (TC $6.2 - 6$)

Show that the worst-case running time of MAX-HEAPIFY on an n-element heap is $\Omega(\log n)$.

反馈: O, Θ, Ω 傻傻分不清。

什么时候用哪个?

这道题为什么问的是 Ω, 而不问 O 或 Θ?

	O	Ω	Θ
Best-case			
Worst-case			
Average-case			

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By Example.

Worst-case of HEAPSORT (TC $6.4-4$)

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HARD.

Best-case of HEAPSORT (TC $6.4 - 5$)

Show that when all elements are distinct, the best-case running time of HEAPSORT is $\Omega(n \log n)$.

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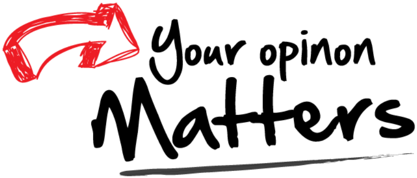
Priority Queue (TC $6.5 - 7$)

Show how to implement a FIFO queue/stack with a priority queue.


Thank
You!



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