MATH 11008: Fleury's Algorithm Section 5.6

The following table summaries the previous two theorems for a connected graph G. Remember that if a graph is disconnected, it cannot have an Euler path nor an Euler circuit.

Summary of Euler's Theorems (Assuming G is connected)

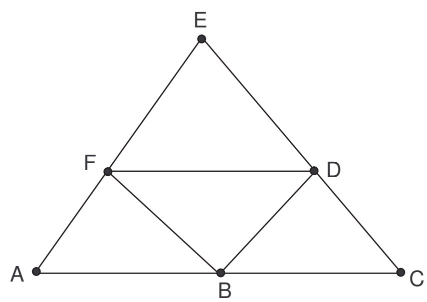
Number of odd vertices	Conclusion
0	G has an Euler circuit.
2	G has an Euler path.
$4, 6, 8 \ldots$	G has neither.
$1, 3, 5, \ldots$	This is impossible.

Fleury's Algorithm for finding an Euler Circuit (Path): While following the given steps, be sure to label the edges in the order in which you travel them.

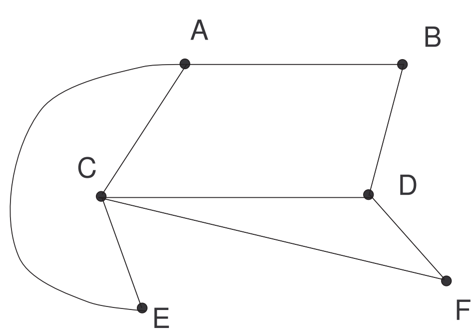
1. Make sure the graph is connected and either (1) has no odd vertices (circuit) or (2) has just two odd vertices (path).
2. Choose a starting vertex. For a circuit this can be any vertex, but for a path it must be one of the two odd vertices.
3. At each step, if you have a choice, do NOT choose a bridge of the yet-to-be-traveled part of the graph. However, if you have only one choice, take it.
4. When you can't travel any more, the circuit (or path) is complete. For a circuit you will be back at the starting vertex; and for a path you will end at the other odd vertex.

- It is critical when using Fleury's Algorithm to separate the past (the part of the graph you have already traveled) with the future (the part of the graph that still needs traveled).

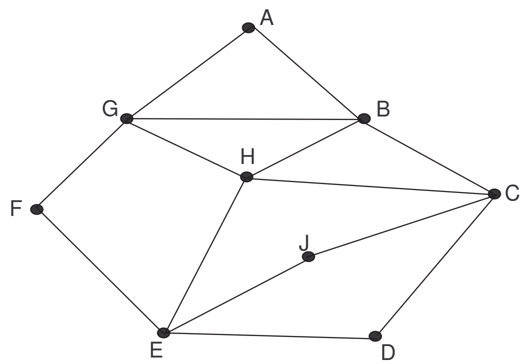
Example 1: Determine if the following graph has an Euler circuit, an Euler path, or neither. If it has an Euler circuit or Euler path, identify one.



Example 2: Determine if the following graph has an Euler circuit, an Euler path, or neither. If it has an Euler circuit or Euler path, identify one.



Example 3: Determine if the following graph has an Euler circuit, an Euler path, or neither. If it has an Euler circuit or Euler path, identify one.



Example 4: Determine if the following graph has an Euler circuit, an Euler path, or neither. If it has an Euler circuit or Euler path, identify one.