2-11 Heapsort

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ALGORITHM 245
TREESORT 3 [M1]
ROBERT W. FLOYD (Recd. 22 June 1964 and 17 Aug. 1964)
Computer Associates, Inc., Wakefield, Mass.

procedure *TREESORT* 3 (M, n);
value n; **array** M; **integer** n;
comment *TREESORT* 3 is a major revision of *TREESORT* [R. W. Floyd, Alg. 113, *Comm. ACM 5* (Aug. 1962), 434] suggested by *HEAPSORT* [J. W. J. Williams, Alg. 232, *Comm. ACM 7* (June 1964), 347] from which it differs in being an in-place sort. It is shorter and probably faster, requiring fewer comparisons and only one division. It sorts the array $M[1:n]$, requiring no more than $2 \times (2 \uparrow p-2) \times (p-1)$, or approximately $2 \times n \times (\log_2(n)-1)$ comparisons and half as many exchanges in the worst case to sort $n = 2 \uparrow p - 1$ items. The algorithm is most easily followed if M is thought of as a tree, with $M[j \div 2]$ the father of $M[j]$ for $1 < j \leq n$;

Worst-case of MAX-HEAPIFY (TC $6.2-6$)

Show that the worst-case running time of MAX-HEAPIFY on an n-element heap is $\Omega(\log n)$.

反馈: O, Θ, Ω 傻傻分不清。

什么时候用哪个?

这道题为什么问的是 Ω, 而不问 O 或 Θ?

Algorithm $\mathcal{A}$

Inputs $\mathcal{I}$ of size n

	O	Ω	Θ
Best-case			
Worst-case			
Average-case			

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Best-case			$O = \Omega$
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Best-case	by example	"weakness" of $\mathcal{A}$	$O = \Omega$
Worst-case			$O = \Omega$
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Worst-case	"power" of $\mathcal{A}$	by example	$O = \Omega$
Average-case			$O = \Omega$

Algorithm $\mathcal{A}$

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Worst-case	"power" of $\mathcal{A}$	by example	$O = \Omega$
Average-case	$\leq$	$\geq$	$O = \Omega$

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By Example.

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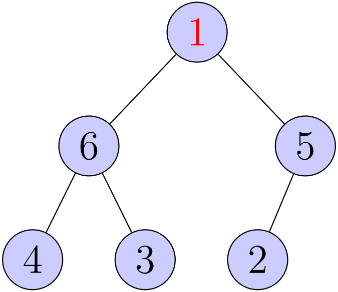

Worst-case of HEAPSORT (TC $6.4-4$)

Show that the worst-case running time of HEAPSORT is $\Omega(n \log n)$.

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Non-proof.

$$\underbrace{\Theta(n)}_{\text{EXTRACT-MAX}} \times \underbrace{\Omega(\log n)}_{\text{MAX-HEAPIFY}} = \Omega(n \log n)$$

□

Worst-case of HEAPSORT (TC $6.4 - 4$)

Show that the worst-case running time of HEAPSORT is $\Omega(n \log n)$.

Heap in decreasing order?

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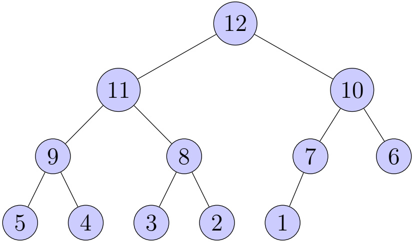
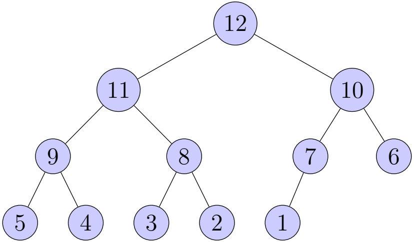
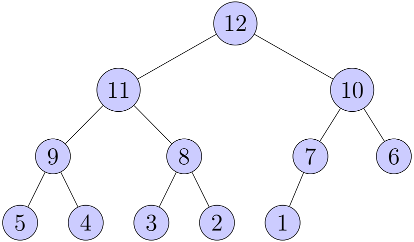


Heap in decreasing order?



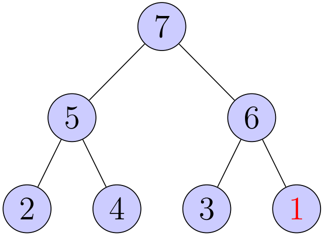
HARD.

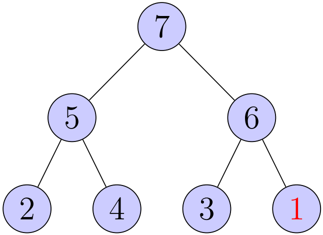
Heap in decreasing order?



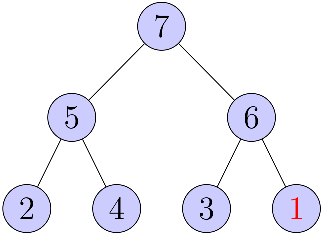
HARD.

$$T(12) = 3 + 2 + 2 + 2 + 2 + 2 + 2 + 1 + 1 + 0 + 0 + 0 = 17$$


7
5
6
2
4
3
1



(Ex. 23, Section 5.2.3, TAOCP Vol 3)



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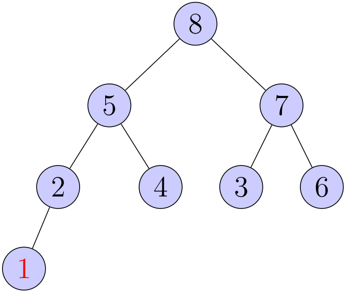
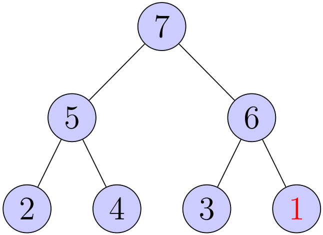
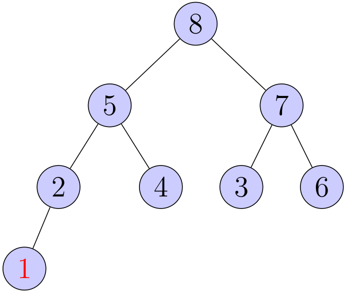
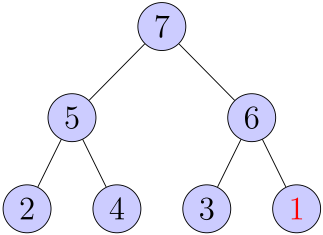


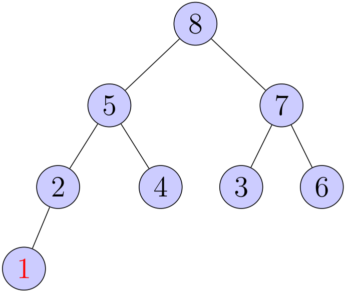




(Ex. 23, Section 5.2.3, TAOCP Vol 3)

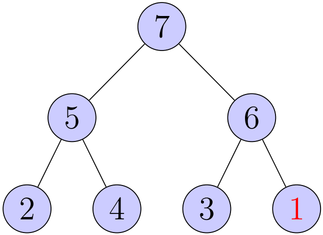
$$\sum_{r=1}^{n-1} \lfloor \log r \rfloor$$

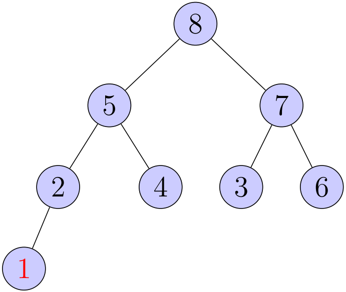




(Ex. 23, Section 5.2.3, TAOCP Vol 3)

$$\sum_{r=1}^{n-1} \lfloor \log r \rfloor = n \lfloor \log n \rfloor - 2^{\lfloor \log n \rfloor + 1} + 2$$





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$$\sum_{r=1}^{n-1} \lfloor \log r \rfloor = n \lfloor \log n \rfloor - 2^{\lfloor \log n \rfloor + 1} + 2 = \Omega n \log n$$

Worst-case of HEAPSORT (TC $6.4-4$)

Show that the worst-case running time of HEAPSORT is $O(n \log n)$.

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No Examples Needed!

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No Examples Needed!

$$\underbrace{\Theta(n)}_{\text{EXTRACT-MAX}} \times \underbrace{O(\log n)}_{\text{MAX-HEAPIFY}} = O(n \log n)$$

Therefore . . .

Worst-case of HEAPSORT (TC $6.4 - 4$)

Show that the worst-case running time of HEAPSORT is $\Theta(n \log n)$.

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Worst-case	"power" of $\mathcal{A}$	by example	$O = \Omega$

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Show that when all elements are distinct, the best-case running time of HEAPSORT is $\Omega(n \log n)$.

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Best-case of HEAPSORT (TC $6.4 - 5$)

Show that when all elements are distinct, the best-case running time of HEAPSORT is $\Theta(n \log n)$.

	O	Ω	Θ
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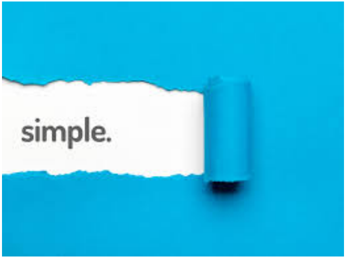

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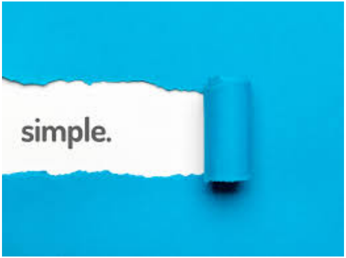

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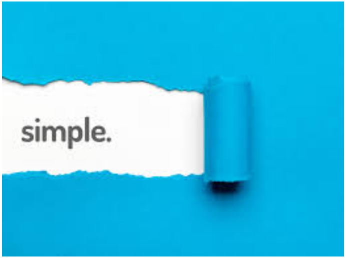

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not easy.

By an elegant counting argument.

$$\boxed{f(n) \triangleq \#\text{ of heaps of } n \text{ distinct keys}}$$

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$$f(n) = \binom{n-1}{m} f(m) f(n-1-m)$$

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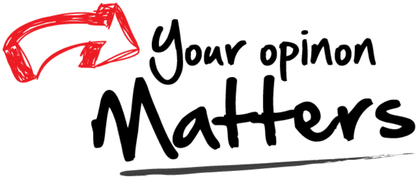
$$\boxed{f(n) = \frac{n!}{\prod_{1 \le i \le n} s_i}}$$

$s_i \triangleq$ size of the subtree rooted at i

$$f(13) = \frac{13!}{13 \cdot 7 \cdot 5 \cdot 3 \cdot 3 \cdot 3} = 506880$$


Thank
You!



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