1-10 函数

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2017 年 12 月 25 日

Well-defined Functions (UD 13.3)

(g) Define $f : \mathbb{Q} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x + 1 & \text{if } x \in 2\mathbb{Z} \\ x - 1 & \text{if } x \in 3\mathbb{Z} \\ 2 & \text{otherwise} \end{cases}$$

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$$x \in 6\mathbb{Z}$$

$$f : A \to B$$

One-to-One (Injective)

$$\forall a_1, a_2 \in A : f(a_1) = f(a_2) \implies a_1 = a_2$$

$$\forall a_1, a_2 \in A : a_1 \neq a_2 \implies f(a_1) \neq f(a_2)$$

Onto (Surjective)

$$\forall b \in B \ \exists a \in A : f(a) = b$$

Bijective

One-to-one correspondence

One-to-one and/or Onto Functions (UD 14.8)

(f) Let A and B be nonempty sets and *let $b \in B$*.

$$f : A \to A \times B$$

$$f(a) = (a, b)$$

One-to-one and/or Onto Functions (UD 14.8)

(f) Let A and B be nonempty sets and *let* $b \in B$.

$$f : A \to A \times B$$

$$f(a) = (a, b)$$

$$B = \{b\}$$

$$B \neq \{b\}$$

One-to-one and/or Onto Functions (UD 14.13)

Let $F([0,1])$ denote the set of all *real-valued* functions defined on the closed interval $[0,1]$.
Define a new function

$$\phi : F([0,1]) \to \mathbb{R}$$

by

$$\phi(f) = f(0).$$

(a) Is ϕ a function from $F([0,1])$ to $\mathbb{R}$?

(b) Is ϕ one-to-one?

(c) Is ϕ onto?

Inverse

Definition (Inverse)

Let $f : A \to B$ be a bijective function.
The inverse of f is the function $f^{-1} : B \to A$ defined by

$$f^{-1}(y) = x \iff f(x) = y.$$

(UD 15.11)

$$f : A \to B$$

$$g_1, g_2 : B \to A$$

(i)

$$f \circ g_1 = f \circ g_2, \quad f \text{ is bijective} \implies g_1 = g_2$$

(ii)

$$g_1 \circ f = g_2 \circ f, \quad f \text{ is bijective} \implies g_1 = g_2$$

Images and Inverse Images

$$f : X \to Y, \quad A \subseteq X, \quad B \subseteq Y$$

$$f(A) = \{f(a) : a \in A\}$$

$$f^{-1}(B) = \{x \in X : f(x) \in B\}$$

Partition (UD 16.19)

$$f : A \to B$$

$$f \text{ is onto}$$

To prove that

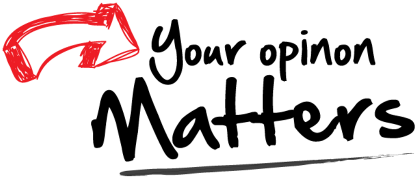
$$\{f^{-1}(\{b\}) : b \in B\}$$

is a partition of A.

Images (UD 16.20)

Inverse Images (UD 16.21)


Thank
You!



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