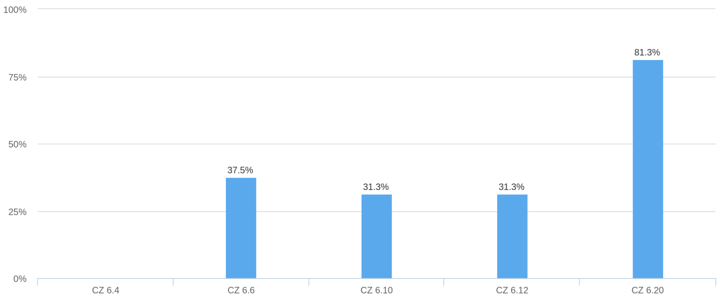
3-10 Traversability

(Part I: Eulerian Graphs)

Hengfeng Wei

hfwei@nju.edu.cn

December 03, 2018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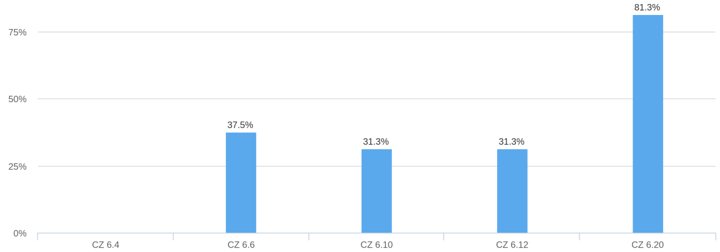

CZ 6.20 (Next Class)

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欧拉图和汉密尔顿图的联系 就是在建模时如何确定图的节点和边

g-cage 对于不同大小而言都是唯一的吗？（书上只给到n=8） Kirkman triple system 和 factorization 的等价推导 Theorem 8.10的证明

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如何打印欧拉回路/欧拉迹/哈密顿回路

定理6.5

无

暂无

无

陶老师上课讲的中国邮递员问题没明白

暂无，我就来抽个奖

none

定理6.12的证明

喵(^･ｪ･^)

希望能讲一下fluery算法

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Chinese Postman Problem (Next Class)

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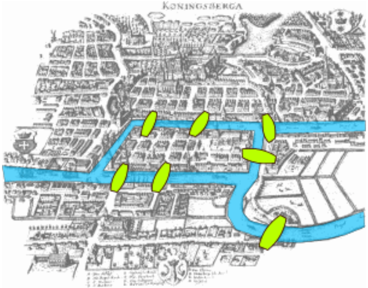
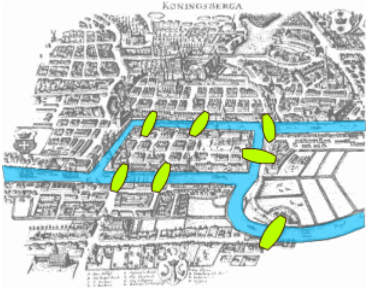
Chinese Postman Problem (Next Class)

6.3 Exploration & 6.4 Excursion (Not Required)



Leonhard Euler (1707 – 1783)



Leonhard Euler (1707 – 1783)

Graph Theory Topology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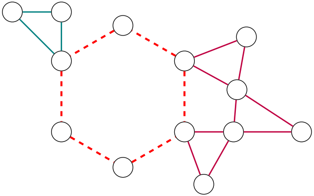
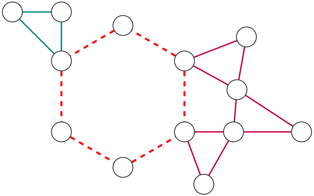


$$H = G - E(C) = \bigcup H_i$$

(I) $\forall v \in H : \deg(v)$ is even

(II) $\forall i : \left| E(H_i) \right| < m$



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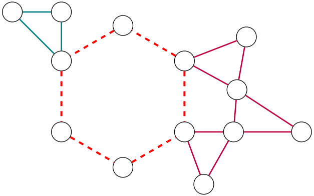

By I.H., each H_i has an Eulerian circuit C_i.

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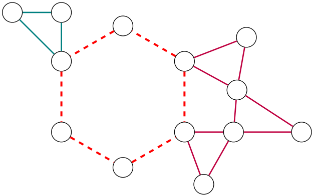

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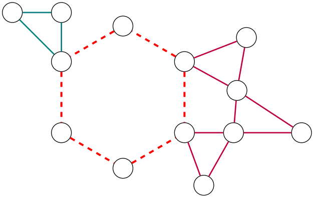

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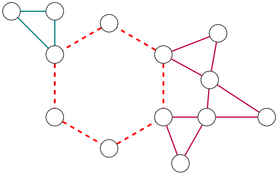
Glue together each C_i with C to get an Eulerian circuit of G.

```
procedure HIERHOLZER(G)
    u ∈ V(G)
    C ← any circuit u ∼ u in G
    while ∃v ∈ C : deg(v) > 0 do
        H ← G − E(C)
        v ← any vertex in V(C) such that deg(v) > 0
        C′ ← any circuit v ∼ v in H
        C ← C ⊗ C′                              ▷ Glue C′ = v ∼ v with C via v
    return C
```



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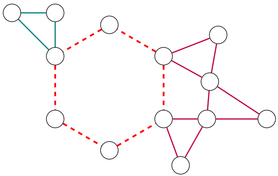

Q : Time Complexity?

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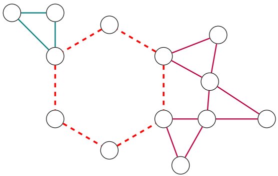

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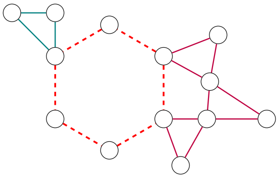

Q : Time Complexity?

Q : Data Structures?

$O(m)$: Using doubly linked list

Fleury's Algorithm (1883)

(I) $v_0 \in V(G); \quad P_0 = v_0$

(II) Suppose $C_i = v_0 e_1 v_1 e_2 \cdots e_i v_i$.

Choose e_{i+1} from $E(G) - \{e_1, e_2, \cdots, e_i\}$ as follows:

(a) e_{i+1} is incident with v_i

(b) Unless there is no alternative, e_{i+1} is not a bridge of $G - \{e_1, e_2, \cdots, e_i\}$

(III) Stop when step (II) can no longer be implemented

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Theorem (Bridges in Fleury's Algorithm)

At any stage, v_i *is incident with* ≤ 1 *bridge in* $E(G) - \{e_1, e_2, \cdots, e_i\}$.

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Suppose that v_i is incident with ≥ 2 bridges in $E(G) - \{e_1, e_2, \cdots, e_i\}$.

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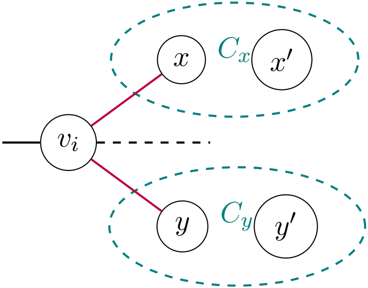

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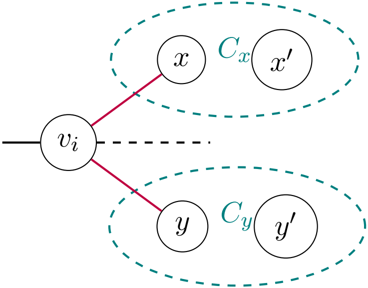

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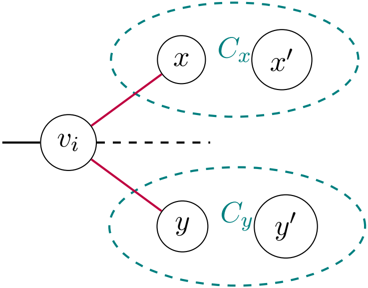

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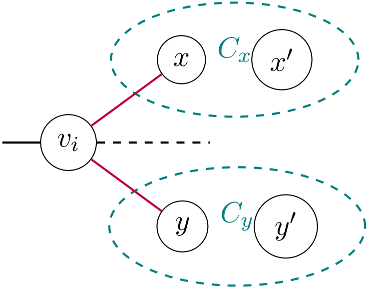

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We have found 2 odd vertices.

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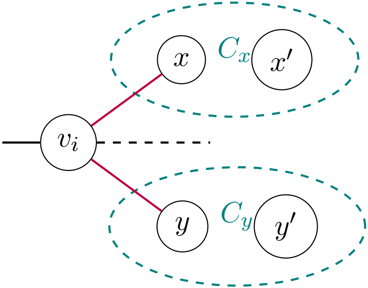

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Q : What is the contradiction?

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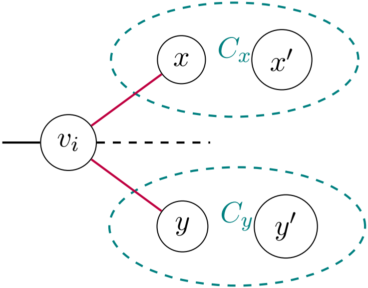

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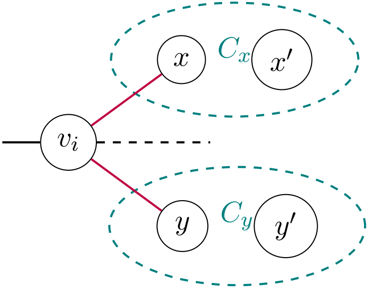
Is $\deg(v_i)$ odd or even?



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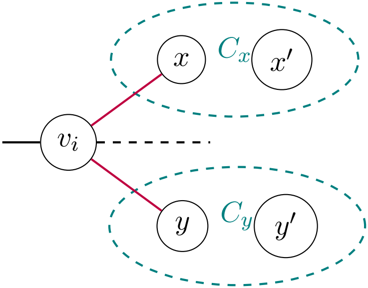

Case I: $\deg(v_i)$ is odd.

We have found 2 odd vertices.

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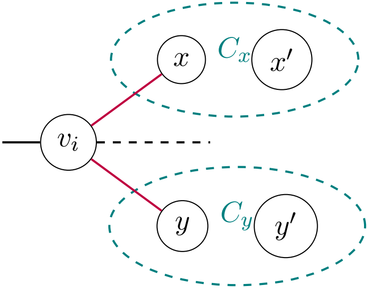

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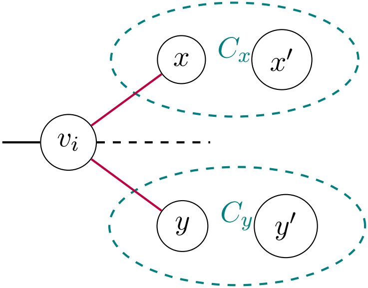
Contradiction:



We have found 2 odd vertices.

Is $\deg(v_i)$ odd or even?

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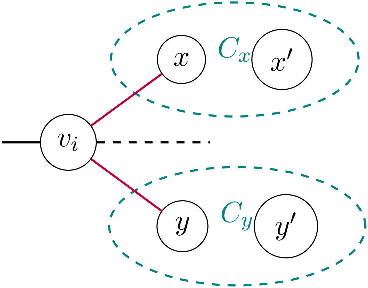

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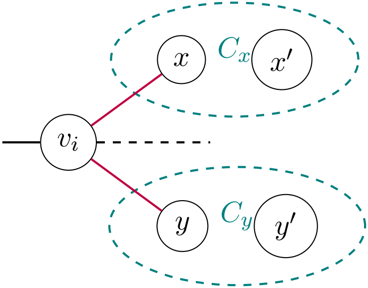

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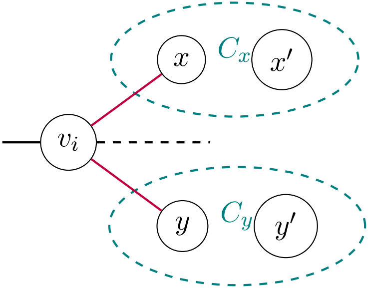

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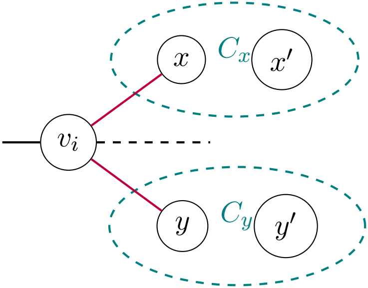

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Contradiction: No odd vertices!

procedure FLEURY(G)

$v_0 \in V(G)$ ▷ Choose any starting vertex

$C \leftarrow v_0$ ▷ Keep track of the circuit

$i \leftarrow 0,\ V_0 \leftarrow V(G),\ E_0 \leftarrow E(G)$

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A trail is a walk in which all the edges are distinct.

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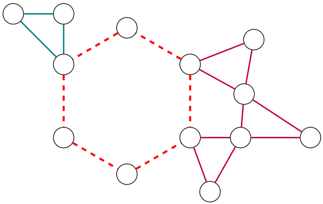
$\because$ even degrees	$\because$ used edges are deleted	By Contradiction.



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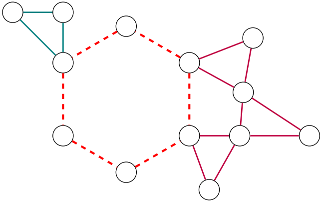

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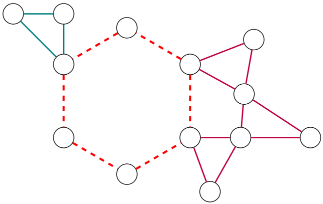

Include every edge of G

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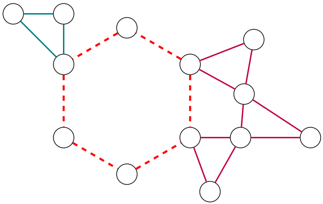
We know that $C : v_0 \sim v_0$ $\quad E' \triangleq E(G) - E(C) \neq \emptyset$



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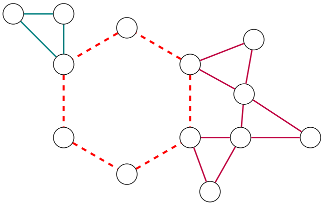

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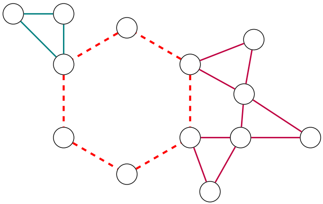

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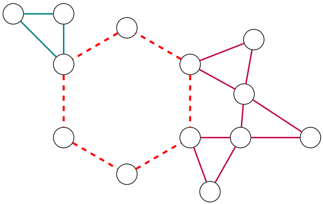

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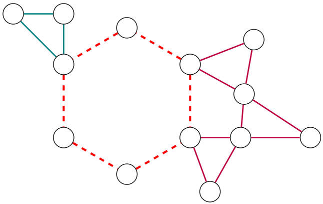

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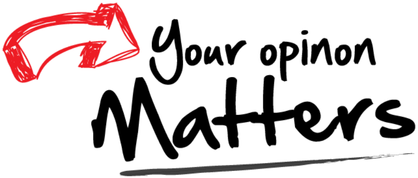
Impossible:

(I) Don't go the bridge unless there is no alternative.

(II) Delete the isolated vertex left by going the bridge.


THANK YOU



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