1-9 Set Theory (II): Relations

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数学科学文化理念传播丛书 (第一辑)

了呢？如果要详细地回答这
此只能举例一二，点到为止.

现在计算机专业的大学一、二年级学生，普遍不愿意学习逻辑演算与集合论课程，认为相关内容与计算机专业没有什么用. 那么我们的教育管理部门和相关专业人士又是如何认知的呢？据我所知，南京大学早年不仅要给计算机专业本科生开设这两门课程，而且还要开设递归论和模型论课程. 然而随着思维模式的不断转移，不仅递归论和模型论早已停开，而且逻辑演算与集合论课程的学时数也在逐步缩减. 现在国内坚持开设这两门课的高校已经很少了，大部分高校只在离散数学课程中，给学生讲很少一点逻辑演算与集合论知识. 其实，相关知识对于培养计算机专业的高科技人才来说是至关重要的，即使不谈这是最起码的专业文化素养，难道不明白我们所学之程序设计语言是靠逻辑设计出来的？而且柯特(E. P. Codd)博士创立关系数据库，以及许华兹(J. T. Schwartz)教授开发的集合论程序设计语言 SETL，可谓全都依靠数理逻辑与集合论知识的积累. 但却很少有专业教师能从历史的角度并依此为例去教育学生，甚至还有极个别的专家教授，竟然主张把“计算机科学理论”这门硕士研究生学位课取消，认为这门课相对于毕业后去公司就业的学生太空洞，这真是令人瞠目结舌. 特别是对于那些初涉高等学府的学子来说，其严重性
更在于他们的

The Relational Data Model

— 如何靠“关系”赢得图灵奖 (1981)?

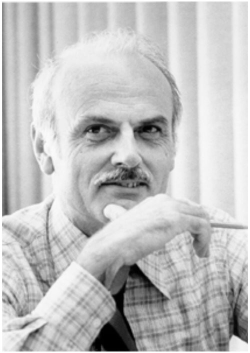
A Relational Model of Data for Large Shared Data Banks

E. F. Codd
IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

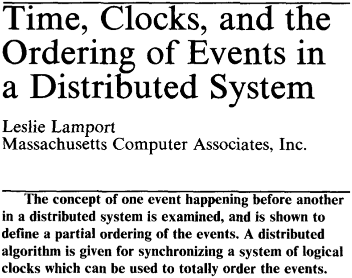
Existing noninferential, formatted data systems provide users with tree-structured files or slightly more general network models of the data. In Section 1, inadequacies of these models are discussed. A model based on *n*-ary relations, a normal form for data base relations, and the concept of a universal data sublanguage are introduced. In Section 2, certain operations on relations (other than logical inference) are discussed and applied to the problems of redundancy and consistency in the user's model.

Codd@CACM'1970



Edgar F. Codd (1923 – 2003)

Ordering of Events in Distributed Systems
— 如何靠“关系”赢得图灵奖 (2013)?

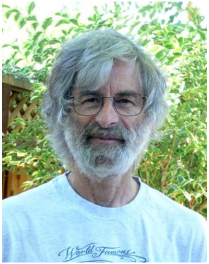


Time, Clocks, and the Ordering of Events in a Distributed System

Leslie Lamport
Massachusetts Computer Associates, Inc.

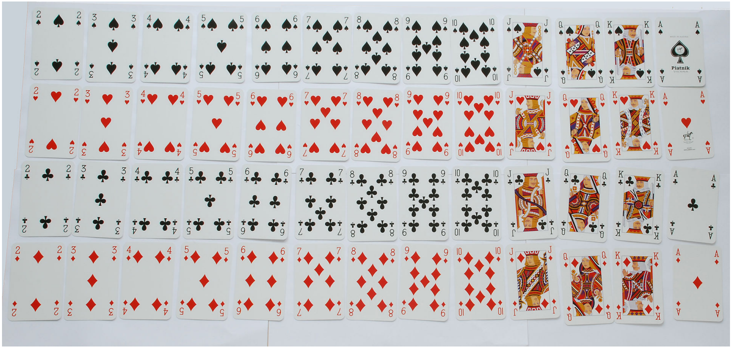
The concept of one event happening before another in a distributed system is examined, and is shown to define a partial ordering of the events. A distributed algorithm is given for synchronizing a system of logical clocks which can be used to totally order the events.

Lamport@CACM’1978



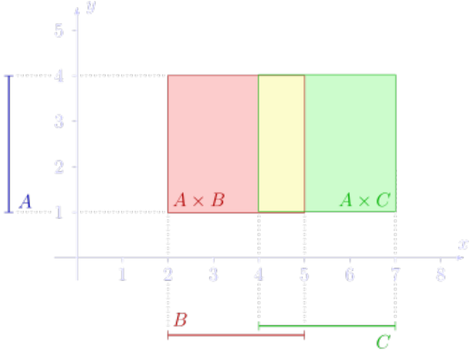
Leslie Lamport (1941 ∼)

Ordered Pair and Cartesian Product



UD Problem 9.19: Distributive Laws

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$
$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$
$$A \times (B \setminus C) = (A \times B) \setminus (A \times C)$$



UD Problem 9.18: Cartesian Product and "$\subseteq$"

$$A \times B \subseteq C \times D \iff A \subseteq C \land B \subseteq D$$

Proof.

$$(x, y) \in A \times B \implies (x, y) \in C \times D$$

$$x \in A \land y \in B \implies x \in C \land y \times D$$

$$(x \in A \implies x \in C) \land (y \in B \implies y \in D)$$

$$(A \subseteq C) \land (B \subseteq D)$$

$$A = \emptyset \lor B = \emptyset$$

□

$$A \times B \subseteq C \times D \xRightarrow{A,B \neq \emptyset} A \subseteq C \land B \subseteq D$$

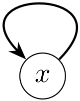
By contradiction.

Relation

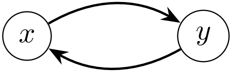
Definition (Equivalence Relation)

R is an equivalence relation on X if R is

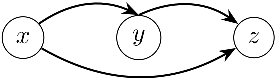
Reflexive: $\forall x \in X : xRx$



Symmetric: $\forall x, y \in X : xRy \implies yRx$



Transitive: $\forall x, y, z \in X : xRy \land yRz \implies xRz$



Definition (Equivalence Class)

Equivalance Relation $R \subseteq X \times X$

The equivalence class of a modulo R is a *set*:

$$[a]_R = \{x \in X : a \sim x\}$$

UD Problem 10.10

$$\sim \text{ is an equivalence relation on } X$$

Prove that

$$\forall x, y \in X : [x]_{\sim} = [y]_{\sim} \iff x \sim y.$$

UD Problem 10.9

$$\sim \subseteq \mathbb{R}^2 \times \mathbb{R}^2$$

$$(x_1, x_2) \sim (y_1, y_2) \iff \mathsf{Even}(x_1 - y_1) \land \mathsf{Even}(x_2 - y_2)$$

Q : Is $\sim$ an equivalence relation?

Q : What is the partition of $\mathbb{R}^2$?

UD Problem 10.13: Equivalence Relations/Classes on Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 \quad (a_j \in \mathbb{R}, n \in \mathbb{N})$$

(a)

$$p \sim q \iff p(0) = q(0)$$

$$[p(x) = x]_{\sim}$$

(b)

$$p \sim q \iff \deg(p) = \deg(q)$$

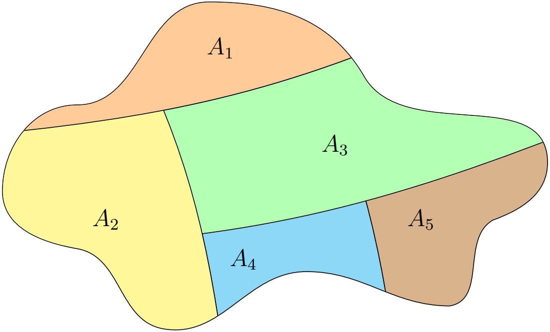
$$[p(x) = 3x + 5]_{\sim}$$

(c)

$$p \sim q \iff \deg(p) \leq \deg(q)$$

$$[p(x) = x^2]_{\sim}$$

Partition



Definition (Partition)

A family of sets $\{A_\alpha : \alpha \in I\}$ is a *partition* of X if

(i)

$$\forall \alpha \in I : A_\alpha \neq \emptyset$$

$$\forall \alpha \in I \; \exists x \in X : x \in A_\alpha$$

(ii)

$$\bigcup_{\alpha \in I} A_\alpha = X$$

$$\forall x \in X \; \exists \alpha \in I : x \in A_\alpha$$

(iii)

$$\forall \alpha, \beta \in I : A_\alpha \cap A_\beta = \emptyset \vee A_\alpha = A_\beta$$

$$\forall \alpha, \beta \in I : A_\alpha \cap A_\beta \neq \emptyset \implies A_\alpha = A_\beta$$

UD Problem 11.8: Partitions of the Set of Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 \quad (a_j \in \mathbb{R}, n \in \mathbb{N})$$

$$\deg(p = 0) = -\infty$$

(a)

$$A_m = \{p : \deg(p) = m\} \quad m \in \mathbb{N}$$

$$(p = 0) \notin \bigcup_{m \in \mathbb{N}} A_m$$

(c)

$$A_q = \{p : \exists r (p = qr)\} \quad q \in P$$

$$q \in A_q$$

$$p \in A_p$$

$$p \neq q \land r = pq \implies (r \in A_p \cap A_q) \land (A_p \neq A_q)$$

UD Problem 11.8: Partitions of the Set of Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 \quad (a_j \in \mathbb{R}, n \in \mathbb{N})$$

$$\deg(p = 0) = -\infty$$

(b)

$$A_c = \{p : p(0) = c\} \quad c \in \mathbb{R}$$

(d)

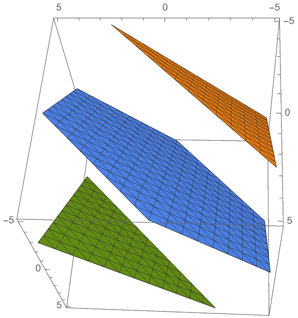
$$A_c = \{p : p(c) = 0\} \quad c \in \mathbb{R}$$

$$(p(x) = x^2 + 1) \notin \bigcup_{c \in \mathbb{R}} A_c$$

UD Problem 11.4: Partitions of $\mathbb{R}^3$

Is $\{A_r \mid r \in \mathbb{R}\}$ a partition of $\mathbb{R}^3$?

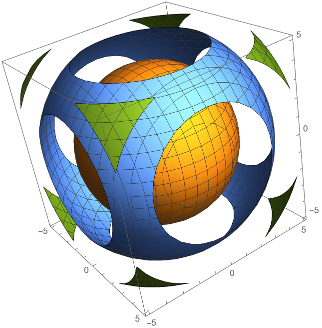
$$A_r = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = r\}$$



UD Problem 11.4: Partitions of $\mathbb{R}^3$

Is $\{A_r \mid r \in \mathbb{R}\}$ a partition of $\mathbb{R}^3$?

$$A_r = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = r^2\}$$



UD Problem 11.9: Subset and Partition

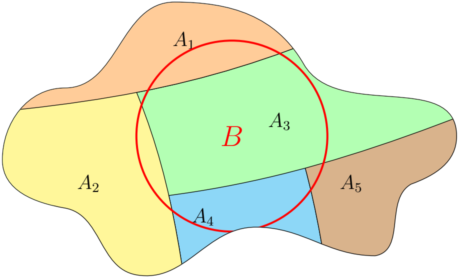
Let $\{A_\alpha : \alpha \in I\}$ be a partition of $X \neq \emptyset$.

(a)

$$B \subseteq X, \quad \forall \alpha \in I : A_\alpha \cap B \neq \emptyset$$

To prove that

$$\{A_\alpha \cap B : \alpha \in I\} \text{ } \textit{is} \text{ a partition of } B.$$



$$\bigcup_{i \in I} (A \cap X_i) = A \cap \bigcup_{i \in I} X_i$$

$$\begin{aligned}
& x \in \bigcup_{i \in I} (A \cap X_i) \\
\iff & \exists i \in I : x \in A \cap X_i \\
\iff & \exists i \in I : x \in A \land x \in X_i \\
\iff & x \in A \land \exists i \in I : x \in X_i \\
\iff & x \in A \land x \in \bigcup_{i \in I} X_i \\
\iff & x \in A \cap \bigcup_{i \in I} X_i
\end{aligned}$$

$$\bigcup_{i \in I} (A \cap X_i) = A \cap \bigcup_{i \in I} X_i$$

$$\bigcap_{i \in I} (A \cup X_i) = A \cup \bigcap_{i \in I} X_i$$

$$A \setminus \bigcup_{i \in I} X_i = \bigcap_{i \in I} (A \setminus X_i)$$

$$A \setminus \bigcap_{i \in I} X_i = \bigcup_{i \in I} (A \setminus X_i)$$

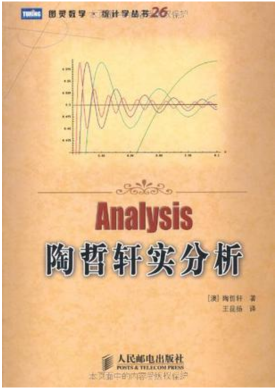
$$\bigcup_{i\in I} X_i \cap \bigcup_{i\in I} Y_i \supseteq \bigcup_{i\in I}(X_i \cap Y_i)$$

$$\bigcap_{i\in I} X_i \cup \bigcap_{i\in I} Y_i \subseteq \bigcap_{i\in I}(X_i \cup Y_i)$$

$$\begin{aligned}
& x \in \bigcap_{i\in I} X_i \cup \bigcap_{i\in I} Y_i \\
\iff & x \in \bigcap_{i\in I} X_i \lor x \in \bigcap_{i\in I} Y_i \\
\iff & \forall i \in I : x \in X_i \lor \forall i \in I : x \in Y_i \\
\implies & \forall i \in I : (x \in X_i \lor x \in Y_i) \\
\iff & x \in \bigcap_{i\in I}(X_i \cup Y_i)
\end{aligned}$$

$$X_1 = \{1\}, X_2 = \{2\}, \qquad Y_1 = \{2\}, Y_2 = \{1\}$$

Order in the Reals




Thank
You!