Show that f is one-to-one if and only if it is onto.

Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto.

How should I begin?

(functions)

edited Apr 16 '15 at 15:09
GFauxPas
4,069 1 11 27

asked Apr 16 '15 at 15:03
Alim Teacher
462 1 6 17

@JessicaK, isn't f is a function from A to B, so $b \in A$, not in B for f to be evaluated there? – Ilham Apr 16 '15 at 15:13

@Ilham Opps, I'll fix that. – JessicaK Apr 16 '15 at 15:14

Do you know what the definition of 1-1 and onto are? First suppose f is 1-1, (i.e., for $a_1, a_2 \in A$ $f(a_1) = f(a_2)$ implies $a_1 = a_2$. Note that $f(a_1), f(a_2) \in B$.) and show that f is onto using this definition and the hypothesis of the question. Then use the definition of onto and show f must be 1-1. – JessicaK Apr 16 '15 at 15:17

1 Answer

Use the pigeonhole principle, to see that you can't have the $|B|$ pigeonholes of B having only one "pigeon" of A in them without filling them all up since $|B| = |A|$. Thus injectivity implies surjectivity.

The other direction is a dual statement. Now let for each $b \in B$, let $g(b)$ be the number of distinct elements of A mapped to b by f. Since f is surjective, each $g(b)$ is at least one. Suppose for contradiction f is not injective, then at least one of these $g(b)$ is greater than 1. So their sum is greater than $|B|$, and hence greater than $|A|$. Is that possible, considering any function from $A \to B$ maps exactly one element of B to each element of A?

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answered Apr 16 '15 at 15:18
Ilham
1,343 4 13