Direct Products and Quotient Groups

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$$\mathbb{Z}_2 \times \mathbb{Z}_3 = G \cong \mathbb{Z}_6$$

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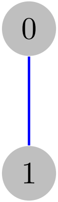
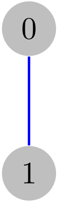


$\mathbb{Z}_2$

$$\mathbb{Z}_2 \times \mathbb{Z}_3 = G \cong \mathbb{Z}_6$$



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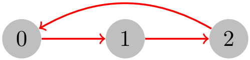

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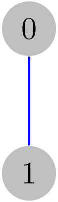

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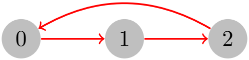

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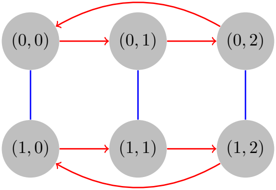

$\mathbb{Z}_2 \times \mathbb{Z}_3$

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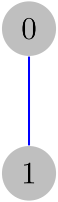

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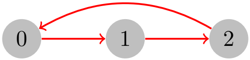

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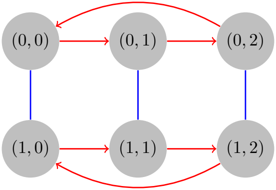

$\mathbb{Z}_2 \times \mathbb{Z}_3$

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$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$

$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$G = H \times K$$

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$$H' \cong H, \quad K' \cong K$$

$$G = H \times K$$

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$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$
$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$
$$H' \leq G, \quad K' \leq G$$

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$
$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$
$$H' \leq G, \quad K' \leq G$$

$$G = H'K'$$

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$
$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$
$$H' \le G, \quad K' \le G$$

$$G = H'K'$$
$$H' \cap K' = \{(e_H, e_K)\} = \{e\}$$

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$

$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$

$$H' \leq G, \quad K' \leq G$$

$$G = H'K'$$

$$H' \cap K' = \{(e_H, e_K)\} = \{e\}$$

H' and K' commute.

Theorem

$$\textit{If } G = H \times K,$$

$$\textit{then } \exists H' \cong H, K' \cong K,$$

$$\textit{such that } G \textit{ is the internal direct product of } H' \textit{ and } K'.$$

Theorem

$$\text{If } G = H \times K,$$
$$\text{then } \exists H' \cong H, K' \cong K,$$
$$\text{such that } G \text{ is the internal direct product of } H' \text{ and } K'.$$

Definition (Internal Direct Product)

Let G be a group with subgroups H and K satisfying

$$G = HK$$

$$H \cap K = \{e\}$$

H and K commute.

Then, G is the internal direct product of H and K.

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$
$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$
$$H' \leq G, \quad K' \leq G$$

$$G = H'K'$$
$$H' \cap K' = \{e\}$$

H' and K' commute.

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$
$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$

$$H' \triangleleft G, \quad K' \triangleleft G$$

$$G = H'K'$$

$$H' \cap K' = \{e\}$$

H' and K' commute.

$$G = H \times K$$

Definition

$$H' = H \times \{e_K\} = \{(h', e_K) : h' \in H)\}$$

$$K' = \{e_H\} \times K = \{(e_H, k') : k' \in K)\}$$

$$H' \cong H, \quad K' \cong K$$

$$H' \lhd G, \quad K' \lhd G$$

$$G = H'K'$$

$$H' \cap K' = \{e\}$$

H' and K' commute.

Definition (Internal Direct Product)

Let G be a group with subgroups H and K satisfying

$$G = HK$$

$$H \cap K = \{e\}$$

$$H \text{ and } K \text{ commute}$$

Then, G is the internal direct product of H and K.

Definition (Internal Direct Product)

Let G be a group with subgroups H and K satisfying

$$G = HK$$

$$H \cap K = \{e\}$$

$$H \text{ and } K \text{ commute}$$

Then, G is the internal direct product of H and K.

Definition (Internal Direct Product (Equivalent))

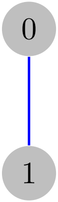
Let G be a group with normal subgroups H and K satisfying

$$G = HK$$

$$H \cap K = \{e\}$$

Then, G is the internal direct product of H and K.

$$\mathbb{Z}_2 \times \mathbb{Z}_3 = G \cong \mathbb{Z}_6$$



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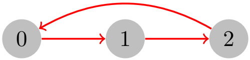

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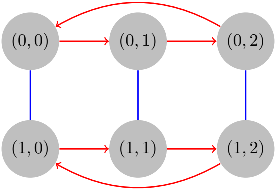
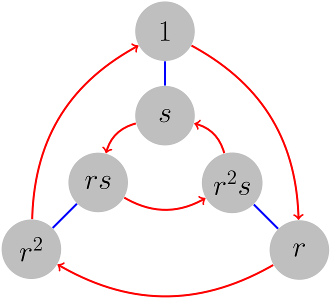
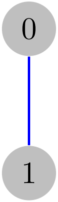


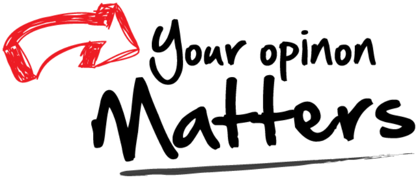
$\mathbb{Z}_2 \times \mathbb{Z}_3$

$$D_6 \cong D_3 \times \mathbb{Z}_2$$






THANK YOU



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