4-8 RSA

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May 06, 2019

这是一段关于“坚持”与“灵感”的旅程 · · ·

Programming Techniques

S.L. Graham, R.L. Rivest* Editors

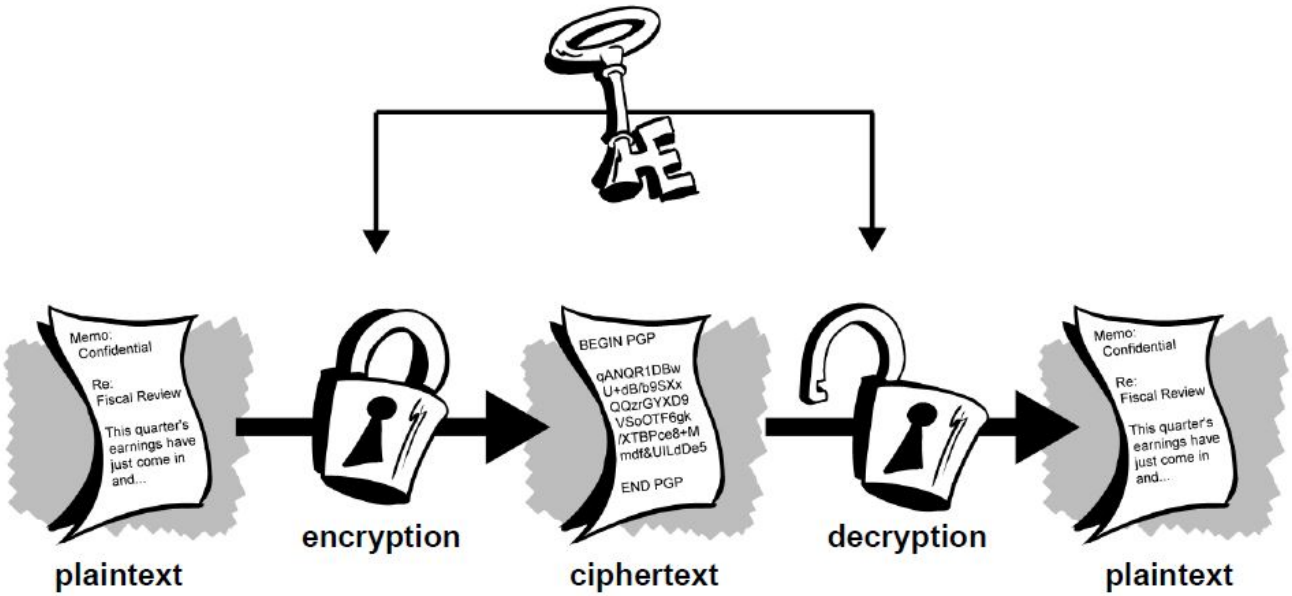
A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R. L. Rivest, A. Shamir, and L. Adleman
MIT Laboratory for Computer Science
and Department of Mathematics

CACM'1978

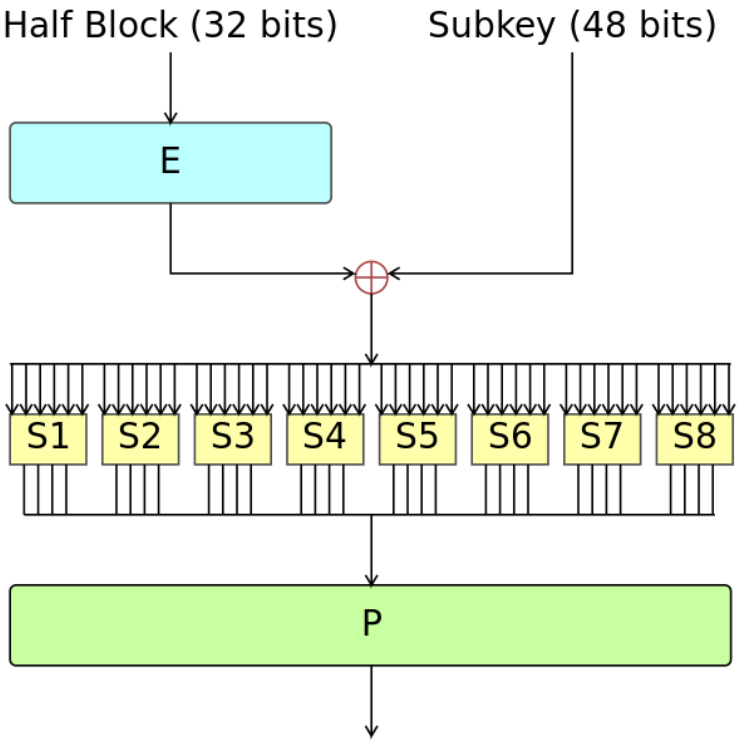

BACK
to the
Past

Symmetric-key Encryption



Q : How to share this KEY?

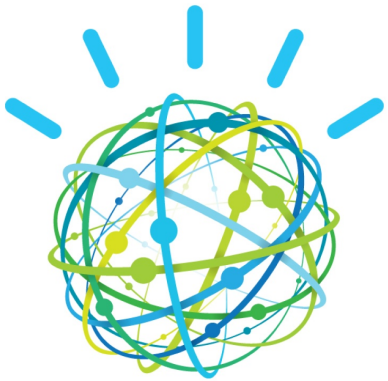
Data Encryption Standard (DES; 1976)



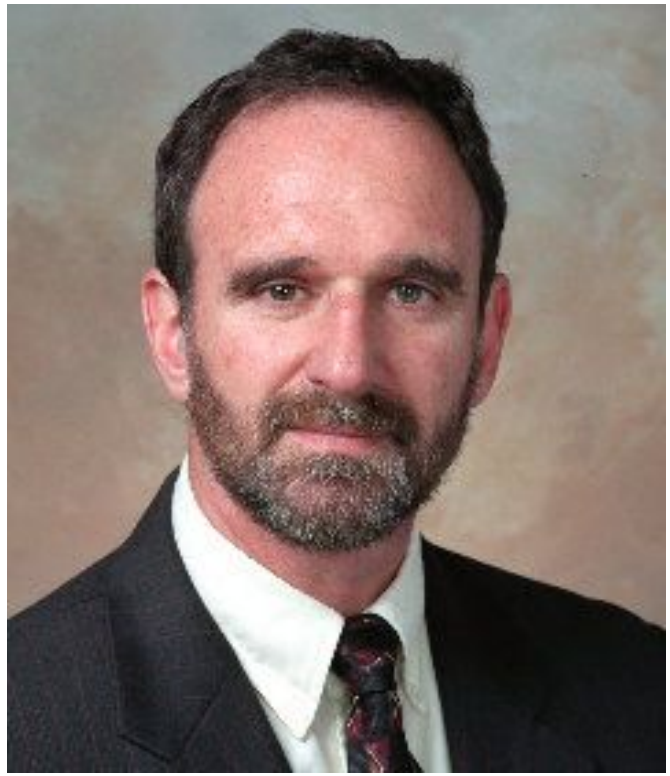
Keys Sent by People

Q : How to share this KEY?

Whitfield Diffie (1944 $\sim$)



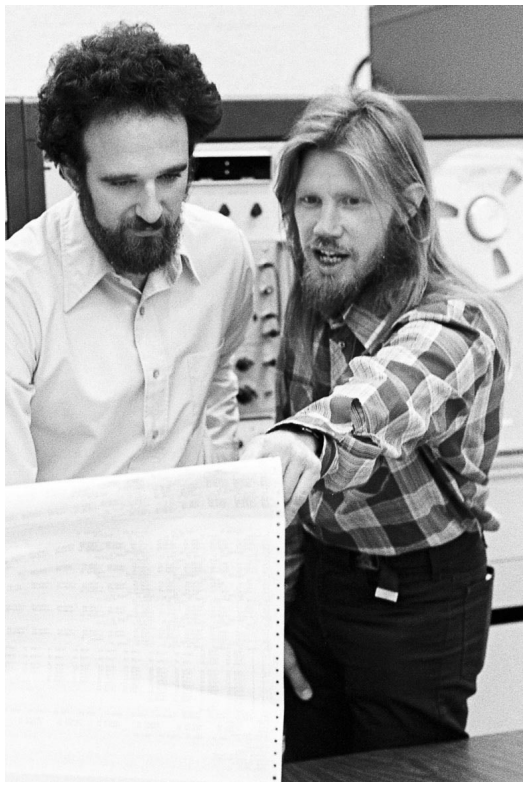
Whitfield Diffie@IBM Watson'1974



Martin Hellman (1945 $\sim$)

两千多年来的密码学 “公理”:

“不管采用什么方法, 密钥就是一定得发送,
这个动作无论如何避免不了”

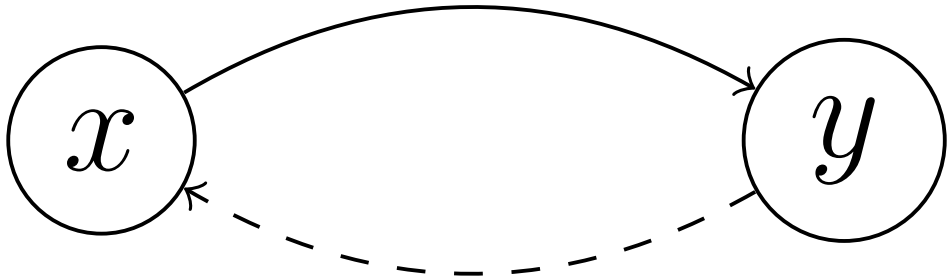


+ Ralph Merkle

Martin Hellman, Whitfield Diffie

Definition (One-way Function)

$$f : \text{Easy}$$



$$f^{-1} : \text{Hard}$$

Q : Hard in *worst case* or in *average case*?

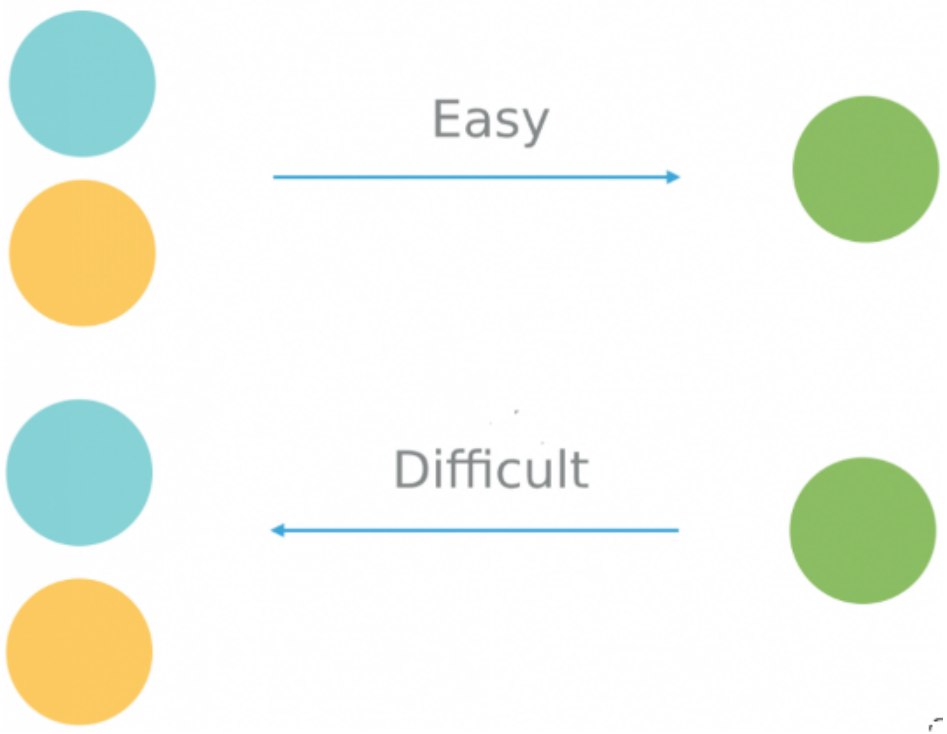
Q : Do one-way functions exist?

One-way Function (wiki)

Unsolved problem in computer science:

? *Do one-way functions exist?*

(more unsolved problems in computer science)



Q : How to share a Color?

Diffie-Hellman-Merkle Key Exchange

(Martin Hellman; Spring, 1976)

Definition (Discrete Logarithm)

$$g^x \equiv a \pmod{p}$$

p g (generator for $\mathbb{Z}_p$)

Alice	Bob
1. Randomly choose a	1. Randomly choose b
2. Compute $A = g^a \bmod p$	2. Compute $B = g^b \bmod p$
3. Send A to Bob	3. Send B to Alice
4. Compute $K = B^x \bmod p$	4. Compute $K = A^y \bmod p$

$K = g^{ab} \bmod p$

K used for DES

Man-in-the-Middle Attack

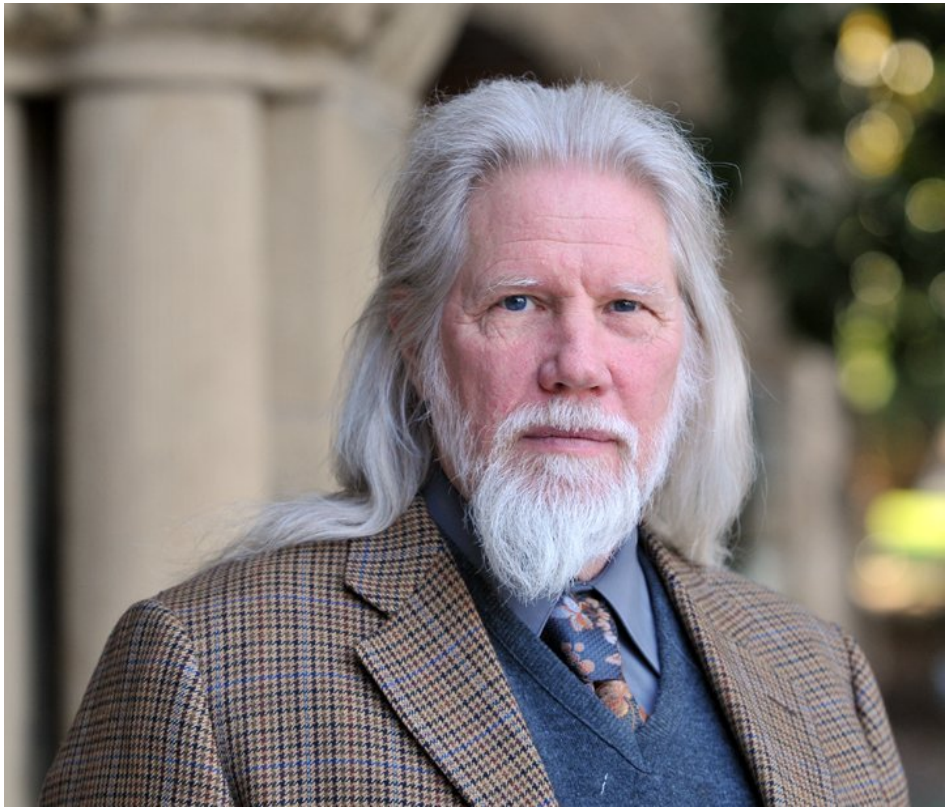
Alice and Bob need to be online simultaneously.

两千多年来的密码学 “公理”:

“不管采用什么方法, 密钥就是一定得发送,
这个动作无论如何避免不了”

Proof.

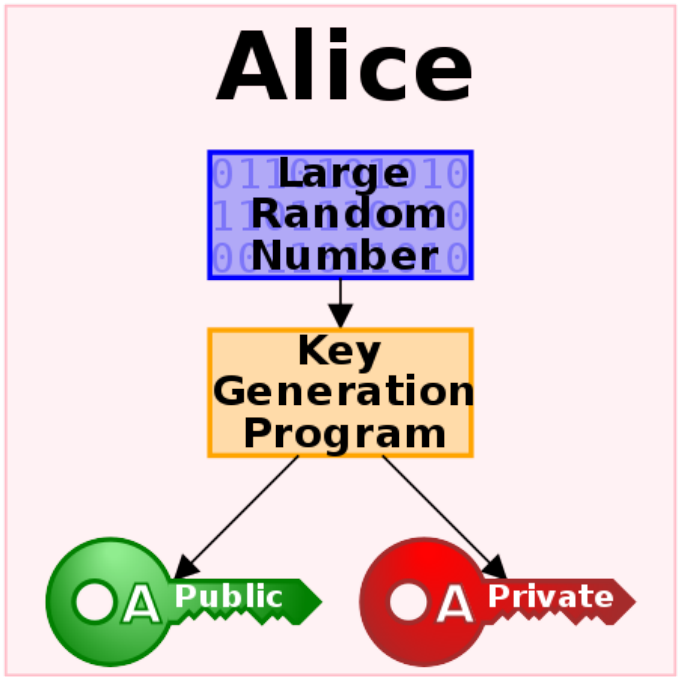
□

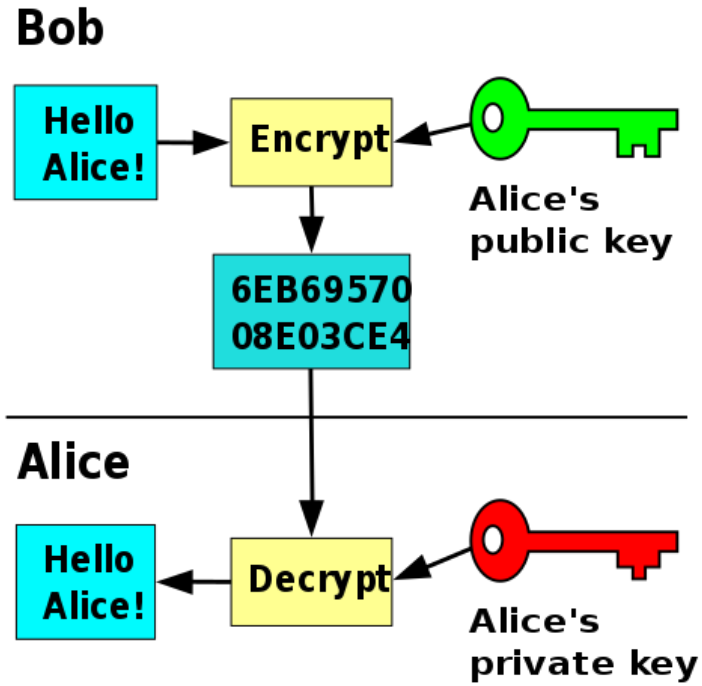


Whitfield Diffie

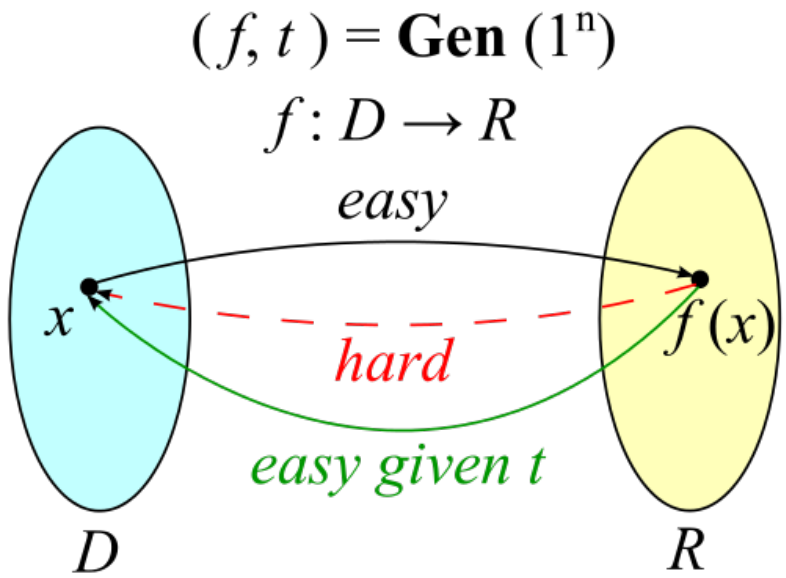
“我是个没用的科学家，一辈子将一事无成”

Asymmetric Cryptography (Public-Key Cryptography)
(Whitfield Diffie; 1975)





Definition (Trapdoor (陷门) One-Way Functions)



$$g^x \equiv a \pmod{p}$$

Q : How to send (encrypt and decrypt) a Color?

New Directions in Cryptography

Invited Paper

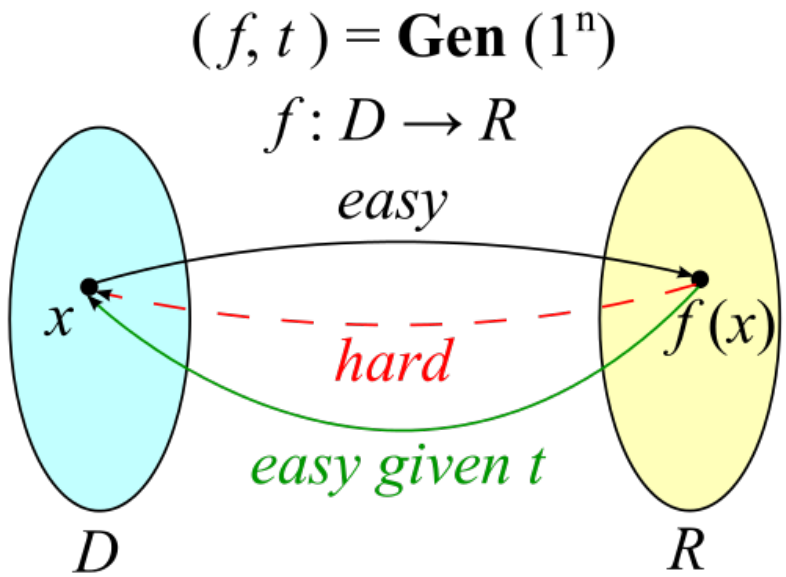
WHITFIELD DIFFIE AND MARTIN E. HELLMAN, MEMBER, IEEE

***Abstract*—Two kinds of contemporary developments in cryptography are examined. Widening applications of teleprocessing have given rise to a need for new types of cryptographic systems, which minimize the need for secure key distribution channels and supply the equivalent of a written signature. This paper suggests ways to solve these currently open problems. It also discusses how the theories of communication and computation are beginning to provide the tools to solve cryptographic problems of long standing.**

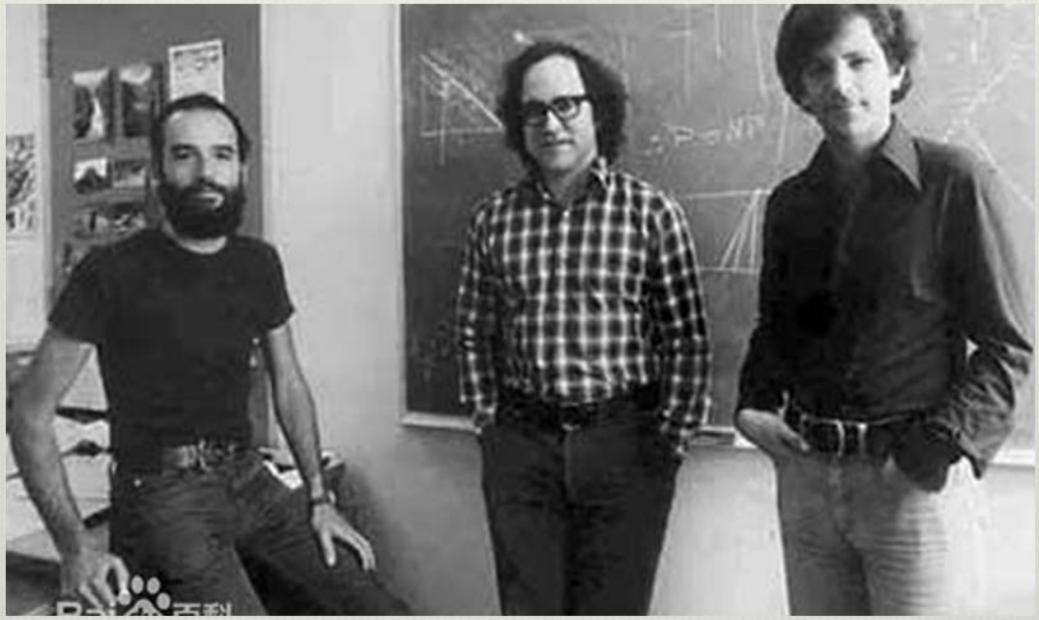
The best known cryptographic p
vacy: preventing the unauthorized
tion from communications over an
order to use cryptography to insure
currently necessary for the commun
a key which is known to no one else.
ing the key in advance over some s
private courier or registered mail. A

Whitfield Diffie, Martin Hellman (1976)

Task: Search for Trapdoor One-way Functions



Ron Rivest, Adi Shamir and Leonard Adleman



“也许、大概、或许并不存在这样的 *Trapdoor One-Way Functions*”

RSA

(Ron Rivest; April 1977)

Programming Techniques

S.L. Graham, R.L. Rivest*
Editors

A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R. L. Rivest, A. Shamir, and L. Adleman
MIT Laboratory for Computer Science
and Department of Mathematics

$$p, \quad q$$

$$n = pq$$

$$\phi(n) = (p-1)(q-1)$$

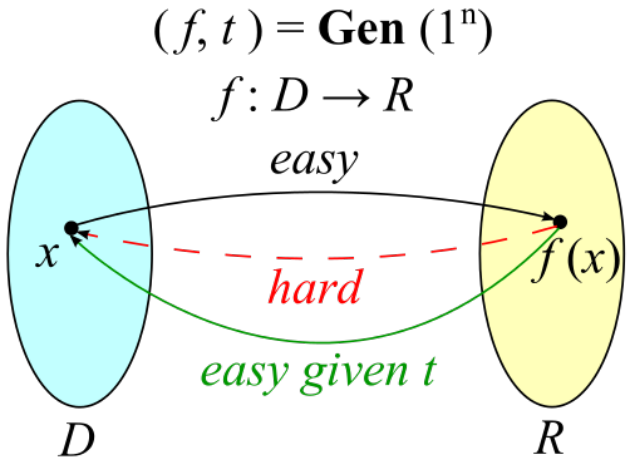
$$e : (e, \phi(n)) = 1$$

$$d : ed \equiv 1 \bmod \phi(n)$$

$$P = (e, n) \qquad S = (d, n)$$

$$P(M) = M^e \bmod n \qquad S(C) = C^d \bmod n$$

Q : What is the trapdoor one-way function in RSA?



$$f(x) = x^e \bmod n$$

d

RSA-number : $n = pq$

number	decimal digits	prize	factored (references)
RSA-100	100		Apr. 1991
RSA-110	110		Apr. 1992
RSA-120	120		Jun. 1993
RSA-129	129		Apr. 1994 (Leutwyler 1994, Cipra 1995)
RSA-130	130		Apr. 10, 1996
RSA-140	140		Feb. 2, 1999 (te Riele 1999a)
RSA-150	150		Apr. 6, 2004 (Aoki 2004)
RSA-155	155		Aug. 22, 1999 (te Riele 1999b, Peterson 1999)
RSA-160	160		Apr. 1, 2003 (Bahr et al. 2003)
RSA-200	200		May 9, 2005 (see Weisstein 2005a)
RSA-576	174	$10000	Dec. 3, 2003 (Franke 2003; see Weisstein 2003)
RSA-640	193	$20000	Nov. 4, 2005 (see Weisstein 2005b)
RSA-704	212	withdrawn	Jul. 1, 2012 (Bai et al. 2012, Bai 2012)
RSA-768	232	withdrawn	Dec. 12, 2009 (Kleinjung 2010, Kleinjung et al. 2010)
RSA-896	270	withdrawn	
RSA-1024	309	withdrawn	
RSA-1536	463	withdrawn	
RSA-2048	617	withdrawn	

"Small e Attack" (CLRS 31.7-2)

$$e = 3$$

$$0 < d < \phi(n)$$

$$n = pq$$

Why do I factor n if I have obtained d?

Common n; Different e's and d's

$$ed \equiv 1 \pmod{\phi(n)}$$

$$ed = 1 + k\phi(n) (k \in \mathbb{Z})$$

$$\boxed{ed = 1 + k\phi(n) \ (k \in \mathbb{N}, k < \min\{e, d\})}$$

$$3d = 1 + k\phi(n) \ (k \in 1, 2)$$

$$\phi(n) = (p-1)(q-1) = n - (p+q) + 1$$

$$n = pq$$

Common Modulus Attack

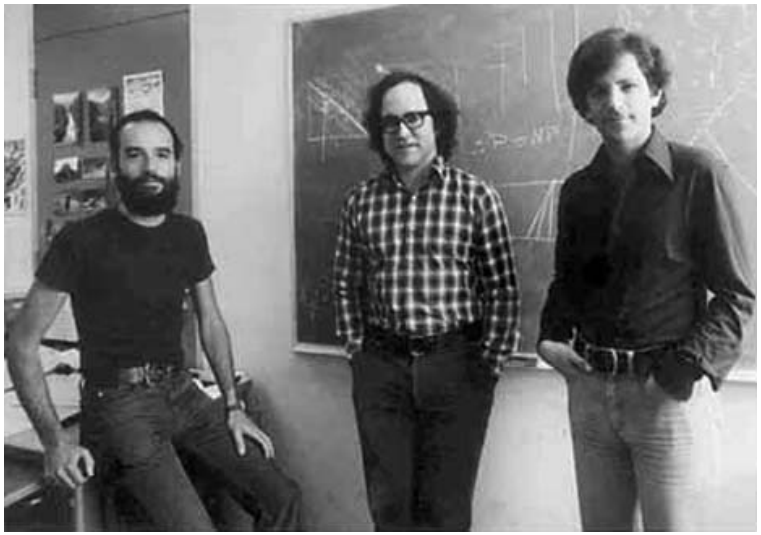
Common n; Different e's and d's

$$(e_i, e_j) = 1$$

$$E = M^{e_1} \pmod{n} \qquad F = M^{e_2} \pmod{n}$$

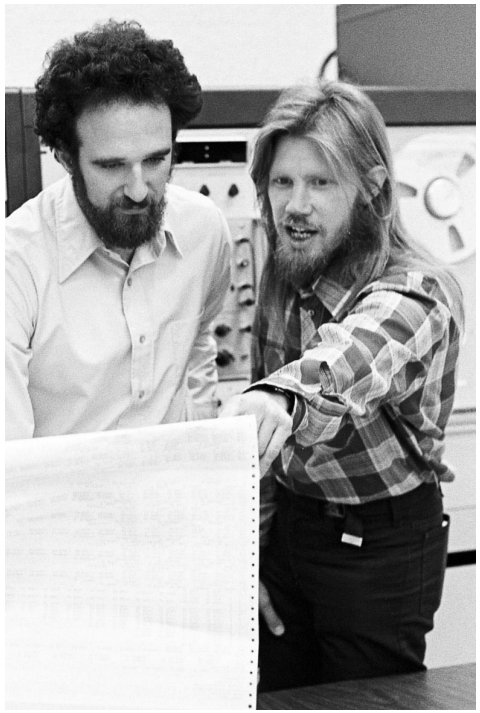
$$(e_1, e_2) = 1 \Longrightarrow \ e_1 x + e_2 y = 1$$

$$M = E^x \cdot F^y \pmod{n}$$



"For their ingenious contribution for making public-key cryptography useful in practice."

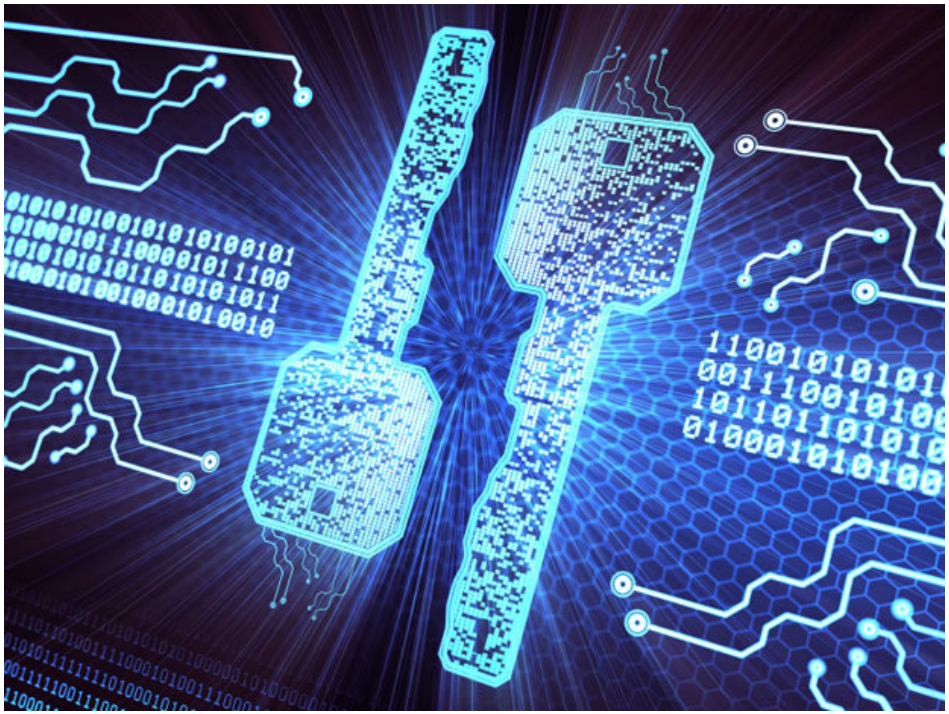
— *ACM Turing Award*, 2002



"For fundamental contributions to modern cryptography."

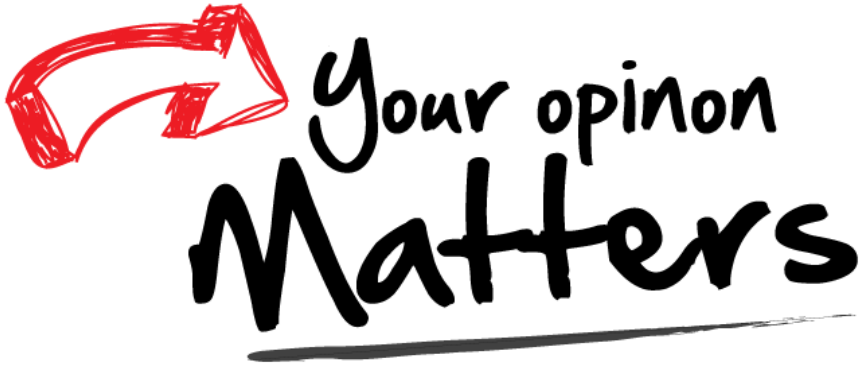
— *ACM Turing Award*, 2015


I don't know' has become 'I don't know yet'.
Bill Gates
www.thequotes.in



Will Quantum Computers Break Encryption?


THANK YOU



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