3-7 Relax! We are SSSP Algorithms.

Hengfeng Wei

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Definition (Shortest Path)

$$G = (V, E, w): \text{ weighted digraph}$$

$$\delta(u, v) = \begin{cases} \min\{w(p) : u \rightsquigarrow^p v\} & \text{if } u \rightsquigarrow v \\ \infty & \text{o.w.} \end{cases}$$

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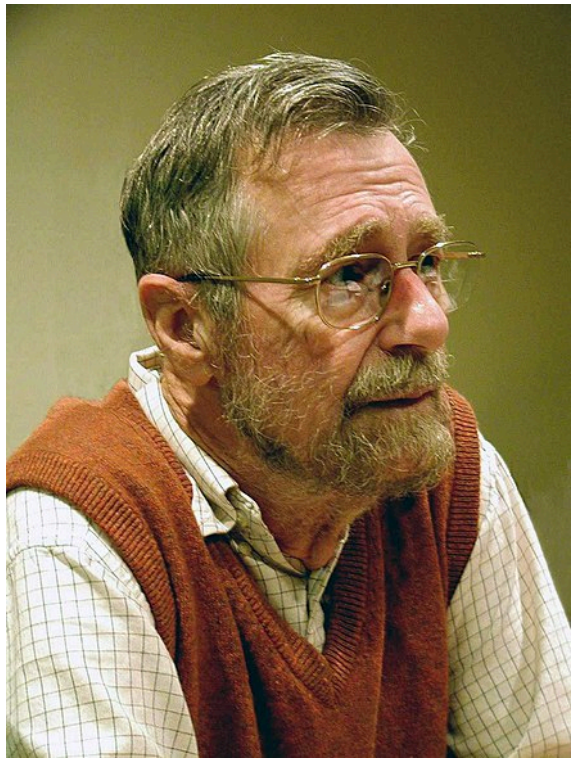
Q : What about undirected graphs?



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2:     INIT-SINGLE-SOURCE(G, s)
3:     S = ∅
4:     Q = G.V
5:     while Q ≠ ∅ do
6:         u ← EXTRACT-MIN(Q)
7:         S ← S ∪ {u}
8:         for v ∈ G.Adj[u] do
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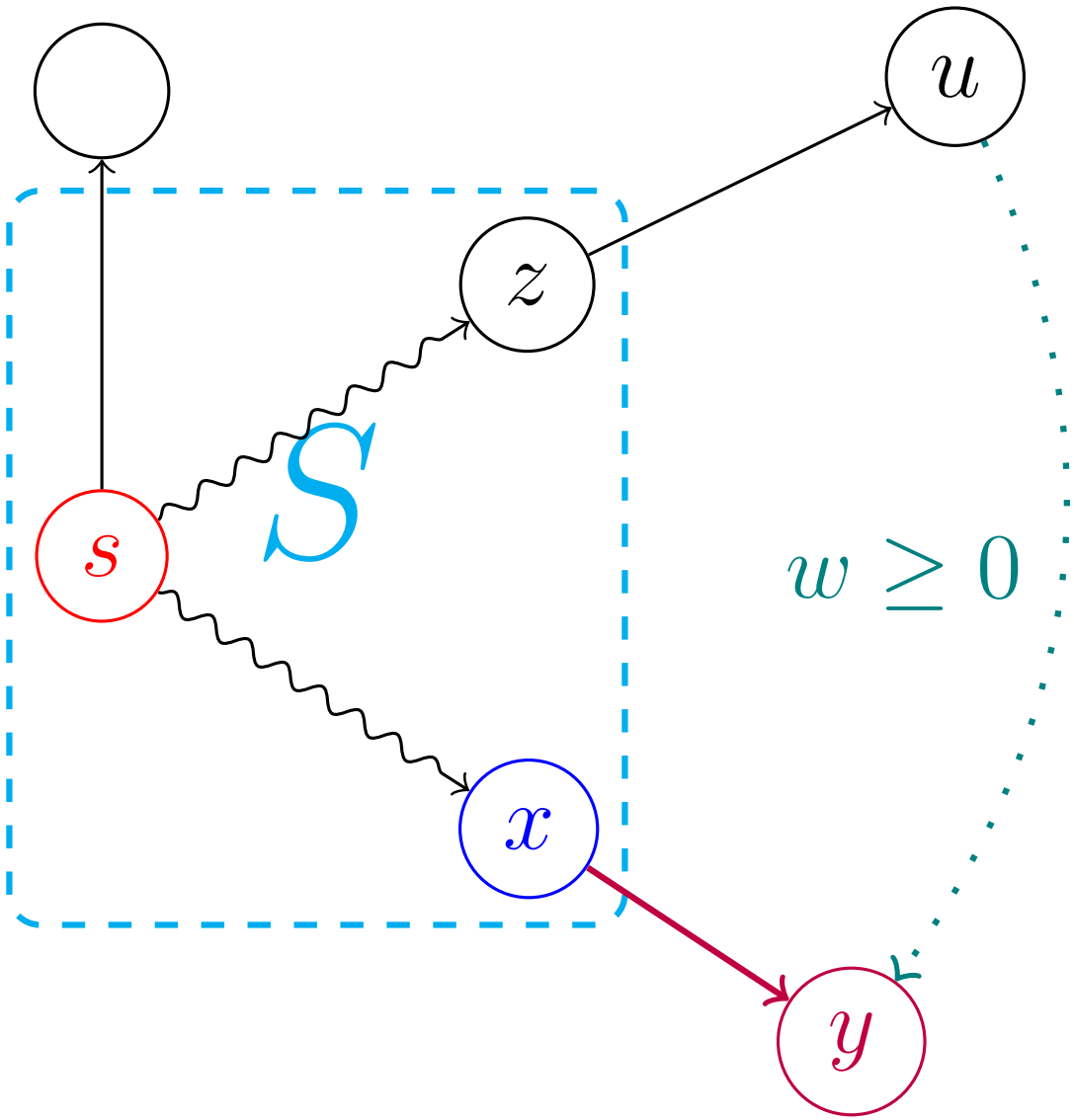
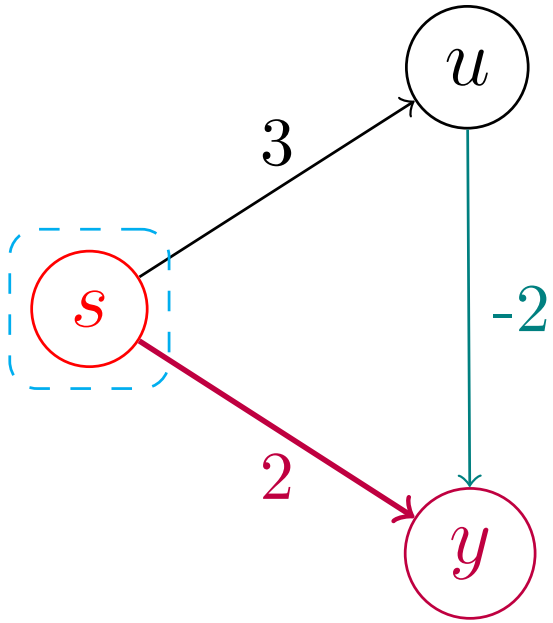

u
z
S
s
w ≥ 0
x
y

Negative-weight Edges for Dijkstra's Algorithm (Problem 24.3-2)



Negative-weight Edges for Dijkstra's Algorithm (Additional Problem 24.3-10)

- All negative-weight egdes are from s
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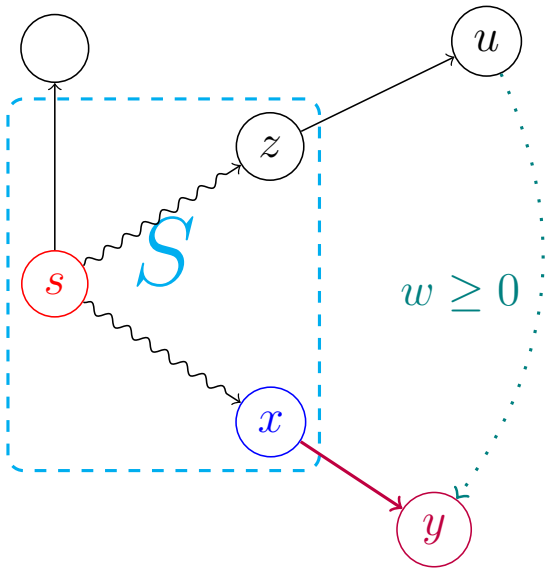


Checking Output of Dijkstra's Algorithm (Problem 24.3-4)

Dijkstra's Algorithm on Digraph with Nonnegative-weight Edges

Lawler's Algorithm on DAG

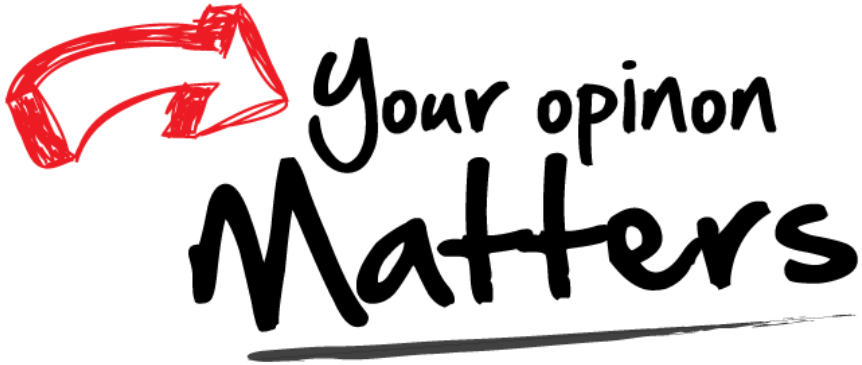
$\Longleftarrow$

Dijkstra's Algorithm on Digraph with Nonnegative-weight Edges

$\Longrightarrow$

Bellman-Ford Algorithm on Digraph with Negative-weight Edges


THANK YOU



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