2-11 Heapsort

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Organization and Maintenance of Large Ordered Indexes

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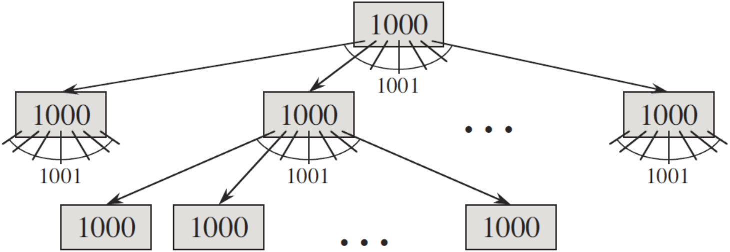
Summary. Organization and maintenance of an index for a dynamic random access file is considered. It is assumed that the index must be kept on some pseudo random access backup store like a disc or a drum. The index organization described allows retrieval, insertion, and deletion of keys in time proportional to $\log_k I$ where I is the size of the index and k is a device dependent natural number such that the performance of the scheme becomes near optimal. Storage utilization is at least 50% but generally much higher. The pages of the index are organized in a special data-structure, so-called B-trees. The scheme is analyzed, performance bounds are obtained, and a near optimal k is computed. Experiments have been performed with indexes up to 100000 keys. An index of size 15000 (100000) can be maintained with an average of 9 (at least 4) transactions per second on an IBM 360/44 with a 2311 disc.

"Bayer and McCreight introduced B-trees in 1972;

they did not explain their choice of name."



2-way *vs.* multi-way

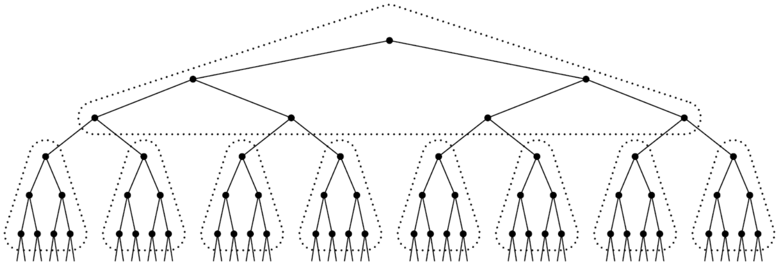
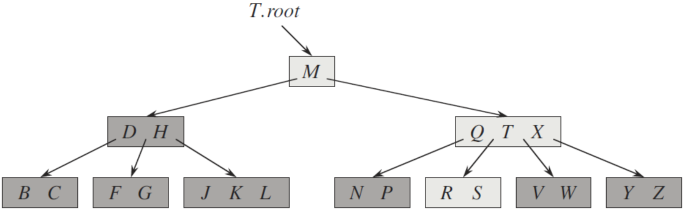


Fig. 29. A large binary search tree can be divided into "pages."

node *vs.* pages

Minimum (TC 18.2-3)

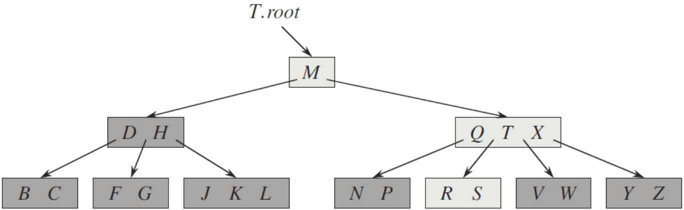
Explain how to find the minimum key stored in a B-tree.



the leftmost key in the leftmost node

Predecessor (TC 18.2-3)

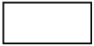
Explain how to find the predecessor of a given key stored in a B-tree.



Insertion (TC 18.2-4 *)

Suppose that we insert the keys $\{1, 2, \ldots, n\}$ in increasing order into an empty B-tree with minimum degree 2.

How many nodes, denoted X_n, does the final B-tree have?

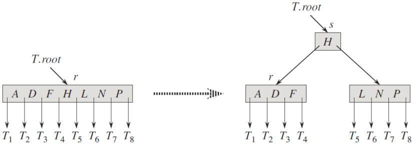


$$X_0 = 1$$

By Yangjing Dong (June 2018)

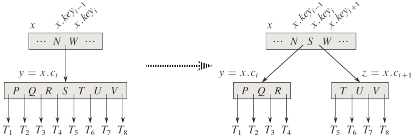
https://maxmute.com/TC18.2-4.html

Only SPLIT can create new nodes.



root SPLIT

+2



non-root SPLIT

+1

(I) Which nodes will SPLIT? S

(II) When does each node $s \in S$ SPLIT? $T_s = \{s_1, s_2, \dots\}$

(III) How does it SPLIT, as a root or a non-root? $T_s = s_R \uplus s_{NR}$

$$X_n = 1 + \sum_{s \in S} \Big(2\,|s_R| + |s_{NR}| \Big)$$

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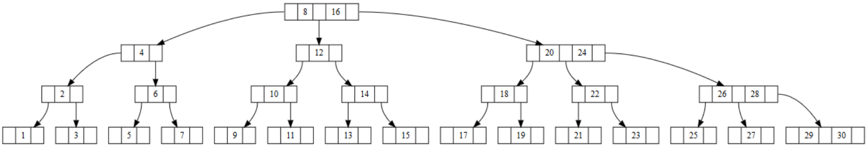
(I) Which nodes will SPLIT?

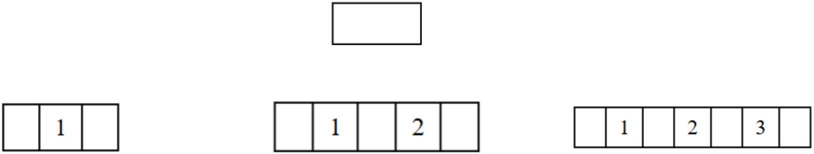


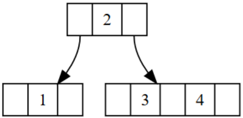
$$1, 2, \ldots, 30$$

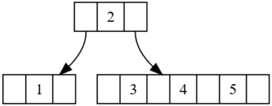
$S = \{$the nodes in the rightmost chain$\}$

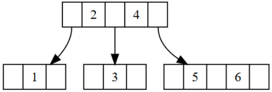
(II) When does each node SPLIT?

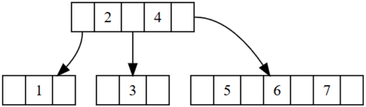
Let's focus the rightmost node first, denoted A.


1
1 2
1 2 3


2
1
3
4


2
1
3
4
5


2
4
1
3
5
6


2
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1
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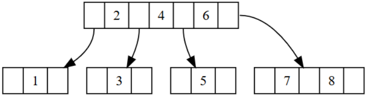
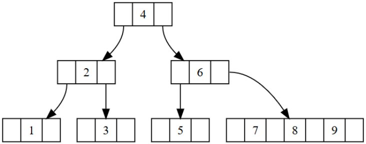
5
6
7


2
4
6
1
3
5
7
8


4
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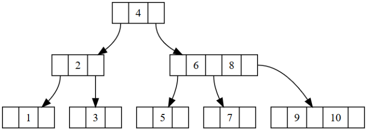
9


4
2
6
8
1
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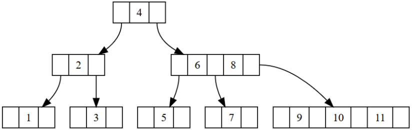
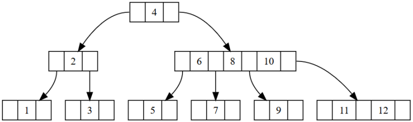
5
7
9
10



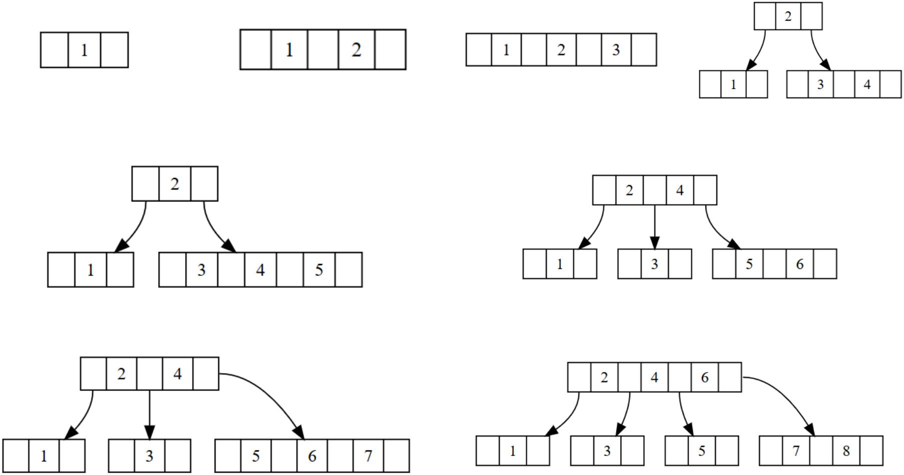


A SPLIT : 4, 6, 8, 10, 12, ...

(II) When does each node SPLIT?

Let's consider the parent of A, denoted $B \triangleq p(A)$.

Every time A splits, B obtains a new key.


1
1 2
1 2 3
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1
3 4
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1
3 4 5
2 4
1
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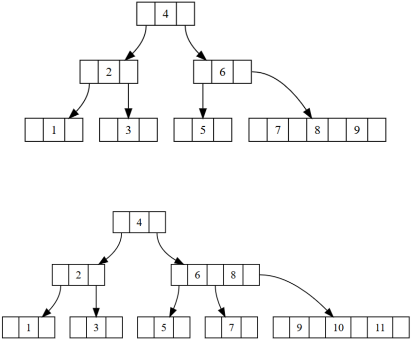
5 6
2 4
1
3
5 6 7
2 4 6
1
3
5
7 8


4
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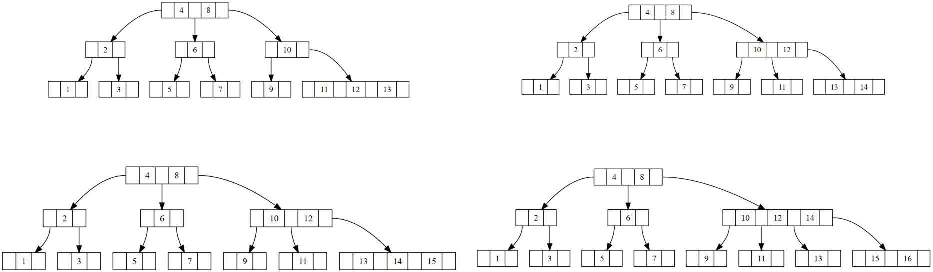
7
9
11
12



B SPLIT : $9, \quad 13, \quad 17, \quad 21, \quad 25, \quad \ldots$

A SPLIT : $4, \quad 6, \quad 8, \quad 10, \quad 12, \quad \ldots$

B SPLIT : $9, \quad 13, \quad 17, \quad 21, \quad 25, \quad \ldots$

(II) When does each node SPLIT?

Let's consider the parent of B, denoted $C = p(B)$.

$$A \text{ SPLIT}: 4, \quad 6, \quad 8, \quad 10, \quad 12, \quad \ldots$$

$$B \text{ SPLIT}: 9, \quad 13, \quad 17, \quad 21, \quad 25, \quad \ldots$$

$$C \text{ SPLIT}: 18, \quad 26, \quad 34, \quad 42, \quad 50, \quad \ldots$$

$$A : 1 \qquad B : 2 \qquad C : 3$$

$T_i :$ the first time point the i-th node splits

$$T_1 = 4$$

$$T_i = \underbrace{T_{i-1}}_{\text{its right child first split}} + \underbrace{2 \times 2^{i-1}}_{\text{its right child split twice more}} + \underbrace{1}_{\text{insert one more}}$$

$$T_i = 2^{i+1} + i - 1$$

$$X_n = 1 + \sum_{s \in S} \Big(2\ |s_R| + |s_{NR}| \Big)$$

$$(T_s = s_R \uplus s_{NR})$$

(III) How does it SPLIT, as a root or a non-root?

$$s_R = \{s_1\} \qquad s_{NR} = \{s_2, s_3, \dots\}$$

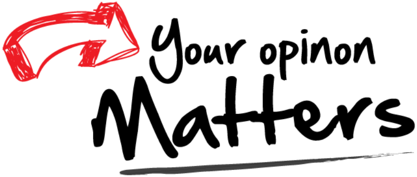
$$|s_R| = 1 \qquad |s_{NR}| = |T_s| - 1$$

$$X_n = 1 + \sum_{s \in S} \Big(2 + |T_s| - 1 \Big) = 1 + \sum_{s \in S} \Big(|T_s| + 1 \Big)$$

$$X_n = 1 + \sum_{s \in S} \Big(|T_s| + 1 \Big) \qquad T_i = 2^{i+1} + i - 1$$

$$\begin{aligned}
X_n &= 1 + \sum_{i=1}^{\infty} [T_i \le n] \Big(\Big(\Big\lfloor \frac{n - T_i}{2^i} \Big\rfloor + 1 \Big) + 1 \Big) \\
&= 1 + \sum_{i=1}^{\infty} [T_i \le n] \Big(\Big\lfloor \frac{n - T_i}{2^i} \Big\rfloor + 2 \Big) \\
&= 1 + \sum_{i=1}^{\infty} [T_i \le n] \Big(\Big\lfloor \frac{n - 2^{i+1} - i + 1}{2^i} \Big\rfloor \Big) \\
&= 1 + \sum_{i=1}^{2^{i+1}+i-1 \le n} \Big\lfloor \frac{n - i + 1}{2^i} \Big\rfloor \\
&= 1 + \sum_{i=0}^{2^{i+2}+i \le n} \Big\lfloor \frac{n - i}{2^{i+1}} \Big\rfloor
\end{aligned}$$


Thank
You!



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