3-9 Connectivity

(Part II: Menger's Theorem)

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如果两个割点相连，那么联通块怎么划分
联通快呢）

menger定理吧

暂无

好像没有……

Menger定理的证明看不懂

menger定理的证明

不。。不记得了

还好理解，只是都不怎么容易理解

menger定理的证明没太理解 老师辛苦了！

点割集，边割集

Menger's Theorem

(Theorem 5.16; Theorem 5.21)

Theorem (Menger's Theorem (Theorem 5.16))

Let u and v be nonadjacent vertices in a graph G.

The minimum number of vertices in a $u-v$ separating set equals the maximum number of internally disjoint $u-v$ paths in G.

How do Case 1, Case 2, and Case 3 cover all possibilities?

Are Case 1 and Case 2 mutually exclusive?

What is the key to use the induction hypothesis in Case 2?

Are Case 1 and Case 3 mutually exclusive?

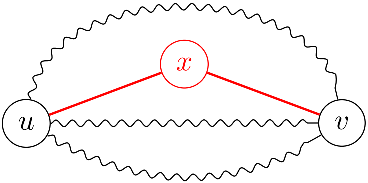
What will fail if we do not exclude Case 1 from Case 3?

Can you restate these three cases in terms of $N(u)$ and $N(v)$?

Can you rearrange these three cases to make them (hopefully) easier to understand?

By induction on the number m of edges of G.

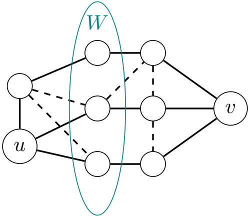
CASE I: There exists a minimum $u - v$ separating set W in G containing a vertex x that is adjacent to both u and v.

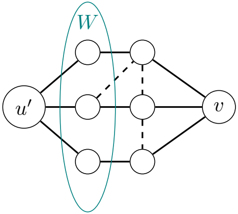


$W - \{x\}$ is a minimum $u - v$ separating set in $G - x$

By induction on the number m of edges of G.

Case II: There exists a minimum $u - v$ separating set W in G containing a vertex in W that is not adjacent to u and a vertex in W that is not adjacent to v.

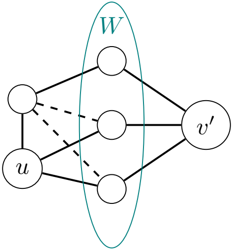




G'_v

$\boxed{m(G'_v) < m(G)}$

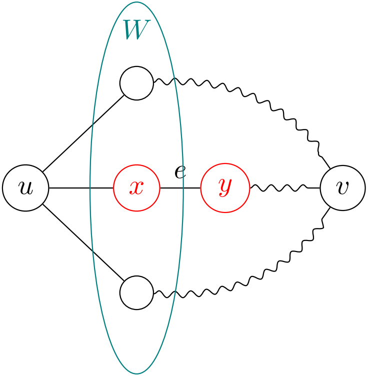
G'_u



$\boxed{m(G'_u) < m(G)}$

By induction on the number m of edges of G.

CASE III: For each minimum $u - v$ separating set W in G, either every vertex of W is adjacent to u and not adjacent to v or every vertex of W is adjacent to v and not adjacent to u.



$$\boxed{P = u, x, y, \ldots, v}$$

A $u - v$ shortest simple path in G

$$m(G - e) < m(G)$$

A minimum $u - v$ separating set in $G - e$ contains k vertices.

Case I: There exists a minimum $u - v$ separating set W in G containing a vertex x that is adjacent to both u and v.

$$\exists W : \exists x \in W : x - u \wedge x - v$$

Case II: There exists a minimum $u - v$ separating set W in G containing a vertex in W that is not adjacent to u and a vertex in W that is not adjacent to v.

$$\exists W : \exists x \in W : x \not- u$$
$$\wedge \exists y \in W : y \not- v$$

Case III: For each minimum $u - v$ separating set W in G, either every vertex of W is adjacent to u and not adjacent to v or every vertex of W is adjacent to v and not adjacent to u.

$$\forall W : \forall x \in W : x - u \wedge x \not- v$$
$$\vee \forall x \in W : x - v \wedge x \not- u$$

$$\mathrm{I} : \exists W : \exists x \in W : x - u \wedge x - v \qquad \mathrm{I}' : \forall W : \forall x \in W : x \not- u \vee x \not- v$$

$$\mathrm{II} : \exists W : \exists x \in W : x \not- u \wedge \exists y \in W : y \not- v$$

$$\mathrm{II}' : \forall W : \forall x \in W : x - u \vee \forall y \in W : y - v$$

$$\mathrm{III} : \forall W : \forall x \in W : x - u \wedge x \not- v \vee \forall x \in W : x - v \wedge x \not- u$$

$$\mathrm{III} \equiv \mathrm{II}' \wedge \mathrm{I}'$$

II

II′

I

III

II

$$
\begin{aligned}
\text{II} : \exists W : \exists x \in W : x \not- u \\
\land \exists y \in W : y \not- v
\end{aligned}
$$

II

$$
\begin{aligned}
\exists W : W \nsubseteq N(u) \\
\land W \nsubseteq N(v)
\end{aligned}
$$

II′

$$\text{I} : \exists W : \exists x \in W : x - u \land x - v$$

II′

$$\exists W : \exists x \in W : x \in N(u) \cap N(v)$$

$$
\begin{aligned}
\text{III} : \forall W : \forall x \in W : x - u \land x \not- v \\
\lor \forall x \in W : x - v \land x \not- u
\end{aligned}
$$

$$
\begin{aligned}
\forall W : W \subseteq N(u) \land W \cap N(v) = \emptyset \\
\lor W \subseteq N(v) \land W \cap N(u) = \emptyset
\end{aligned}
$$

II

$$\text{II}: \exists W : W \not\subseteq N(u) \\ \land W \not\subseteq N(v)$$

II′

$$\text{I}: \exists W : \exists x \in W : x \in N(u) \cap N(v)$$

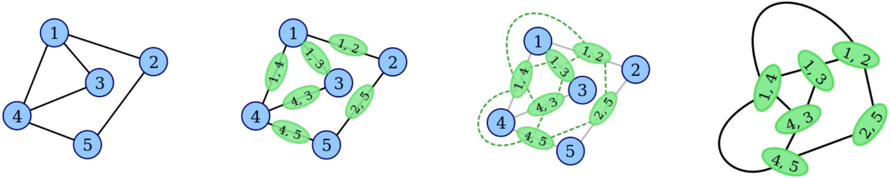
$$\text{III}: \forall W : W \subseteq N(u) \land W \cap N(v) = \emptyset \\ \lor W \subseteq N(v) \land W \cap N(u) = \emptyset$$

Theorem (Menger's Theorem for Edge-Connectivity (Theorem 5.21))

For distinct vertices u and v in a graph G,

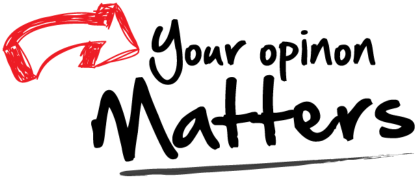
the minimum number of edges of G that separate u and v equals the maximum number of pairwise edge-disjoint $u-v$ paths in G.

Line Graph



Definition 4.2.18 & Theorem 4.2.19
of "Introduction to Graph Theory" by Douglas B. West


THANK YOU



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