2-4 Recurrences

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Binary Search (CLRS $4.5-3$)

$$T(n) = 2T(n/2) + \Theta(1)$$

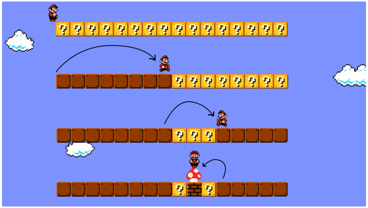
$$T(n) = \Theta(n \lg n)$$

People who analyze algorithms have double happiness.

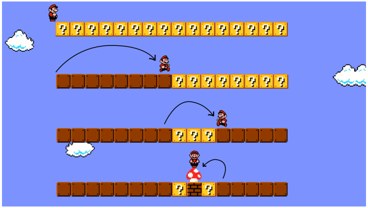
First of all they experience the sheer beauty of elegant mathematical patterns that surround elegant computational procedures.

Then they receive a practical payoff when their theories make it possible to get other jobs done more quickly and more economically.

— Donald E. Knuth (1995)



$$T(n) = \begin{cases} \max\left\{T(\lfloor \frac{n-1}{2} \rfloor), T(\lceil \frac{n-1}{2} \rceil)\right\} + 1, & n > 2 \\ 1, & n = 1 \end{cases}$$



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$$T(n) = \lfloor \lg n \rfloor + 1$$

Theorem

The worst case time complexity (# of comparisons) of BinarySearch *on an input size of* n $=$ *# of bits in the binary representation of* n.


TREASURE
HUNT
on foot


Hints
&
Tips


Help
Wanted

Analysis of Mergesort in CLRS (# of Comparisions; $a_i : \infty$ not Counted)

(a) Analyze the worst case $W(n)$ and the best case $B(n)$ time complexity of mergesort *as accurately as possible*.
Explore the relation between them and the binary representations of numbers.
Plot $W(n)$ and $B(n)$ and explain what you observe.

(b) Analyze the average case $A(n)$ time complexity of mergesort.
Plot $A(n)$ and explain what you observe.

(c) Prove that: The minimum number of comparisons needed to merge two sorted arrays of equal size m is $2m - 1$.

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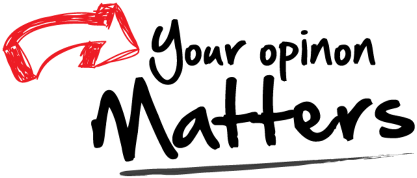
$W(n)$: Consider $W(n+1)$


Thank You!



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