PROFESSOR QUIBB

A TACTICAL GUIDE TO THE INFINITE REALM OF SCIENCE. ALTHOUGH THE WORLD OF SCIENCE WOULD TAKE ETERNITY TO EXPLORE, PROFESSOR QUIBB ATTEMPTS TO SCRAPE THE EDGE OF THIS UNIVERSE. THIS BLOG HELPS YOU TO UNDERSTAND PARTICULAR TOPICS UNDER THE MORE GENERAL CATEGORIES: COSMOLOGY, MATHEMATICS, QUANTUM PHYSICS, METEOROLOGY AND OTHERS. JOIN ME ON MY TREK ACROSS THE UNTRAVERSED LANDS OF THE UNKNOWN.

THURSDAY, FEBRUARY 2, 2012

Infinity: Operations on Cardinals

Before reading this post, make sure you have read the first three parts of the Infinity Series, the first of which is found here. For all posts, see the Infinity Series Portal.

Having found the mathematical relationship between aleph-zero and the cardinality of the continuum, one wonders if it is possible to perform other operations with infinity cardinals, and whether these equations create any numbers not yet discussed. To start, take a simple addition from cardinal arithmetic:

$$\aleph_0 + 1 = \aleph_0 \qquad (1)$$

What exactly does this equation mean? We are asked to "add" two quantities, one of which is an infinite cardinal, and one is a simple number. That it is *possible* to evaluate the sum (1) follows from the fact that aleph-zero and 1 are both cardinal numbers. Since they are of the same number system, they are "compatible" in a way, and can be combined by the use of sets. We have already proved (1) in the first post of the series, but let us recap. It has been discussed that adding the element 0 to the set of natural numbers does not change its cardinality, since the function $y=x-1$ from one set to the

HURRICANE INFORMATION

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http://www.nhc.noaa.gov or
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SPACECRAFT UPDATE

As of June 2019...

other is a bijection. In set notation, with vertical lines representing cardinality: |{0}|+|{1,2,3...}|=|{0,1,2,3...}| Note that if one replaces the values in this equation with the actual cardinalities, one obtains the equation (1) above! The equation in simple sets that has the same meaning as a cardinal equation will henceforth be known as the Corresponding Set Equality (CSE).

The generalization of (1) for any natural number *n* is

$$\aleph_0 + n = \aleph_0 \tag{2}$$

This can also be transformed into a CSE, since any subset of integers also has cardinality aleph-zero. The the set {-*n*+1,-*n*+2,...-2,-1,0} clearly has *n* elements, and can be added to the natural numbers to form the CSE, namely: |{-*n*+1,-*n*+2,...-2,-1,0}|+|{1,2,3...}|=|{-*n*+1,-*n*+2,...-2,-1,0,1,2,3...}|. By evaluating the cardinalities on both sides, one obtains equation (2). It is easy to continue on to other arithmetic operations, such as multiplication:

$$2 \bullet \aleph_0 = \aleph_0 + \aleph_0 = \aleph_0 \tag{3}$$

This particular equation also has a CSE. First consider the set of integers. It is clear that it can be split into two components, both of which have a cardinality of aleph-zero, namely the whole numbers: {0,1,2,3...} and the negative integers: {...-3,-2,-1} However, when these two sets are combined, the set of integers results, which we already know has cardinality aleph-zero as well. The CSE here is |{0,1,2,3...}|+|{-1,-2,-3...}|=|{...-3,-2,-1,0,1,2,3...}|. All three of the cardinalities are evaluated as aleph-zero, and (3) results. Now we shall prove the general multiplication result

$$n \bullet \aleph_0 = \aleph_0 \tag{4}$$

for any natural number *n*. We have already determined that the rational numbers, and any infinite subset of them have cardinality aleph-zero. Consider the set {*a*/*n*,1+(*a*/*n*),2+(*a*/*n*),...}. This is simply the set of natural numbers with the rational number *a*/*n* added to each term. For a constant *n*, one could consider creating a set for each integral value of *a* from 0, to *n*-1, inclusive. This produces *n* sets of cardinality aleph-zero.

An example will make this more clear. Consider the case with *n*=3. For *a*=0, the set produced is simply the whole numbers ({0/3,1+(0/3),2+(0/3)...}={0,1,2...}) For *a*=1, the set is {1/3,1+(1/3),2+(1/3)...} and for *a*=2, the set is {2/3,1+(2/3),2+(2/3)...}, all of which have cardinality aleph-zero. The sum of these sets is {0,1/3,2/3,1,1+(1/3),1+(2/3),2...}, or the set of all multiples of 1/3, which also has the same cardinality. We have just proved (4) for *n*=3. The more general CSE for (4) is |{0/*n*,1+(0/*n*),2+(0/*n*)...}|+|{1/*n*,1+(1/*n*),2+(1/*n*)...}|+
|{2/*n*,1+(2/*n*),2+(2/*n*)...}|+...+|{(*n*-1)/*n*,1+((*n*-1)/*n*),2+((*n*-1)/*n*)...}|=
|{0,1/*n*,2/*n*...,1,1+(1/*n*),1+(2/*n*),...}|, the final set being the set of multiples of 1/*n*.

This equation is tedious, but it simply is the division of the set of multiples of 1/*n* into *n* parts, all of which have cardinality of aleph-zero. Since the set of multiples on the right hand side of the equation as an equivalent cardinality, this proves (4). Moving on, even

New Horizons: Launched in January 2006, the probe successfully flew by Pluto on July 14, 2015. It made a flyby of Kuiper Belt object 2014 MU69, nicknamed "Ultima Thule," on Jan 1, 2019. For more information about the New Horizons mission, see the main post, New Horizons.

Juno: Launched in 2011, the Juno probe has been orbiting Jupiter since 2016. It has already provided unprecedented insights into the structure, evolution, and composition of the Solar System's largest planet. For more information, see the main post, Juno.

Mars Science Laboratory: This mission's primary payload is a rover, Curiosity, by far the largest rover to date. Since it landed on Mars in 2012, this mission has analyzed the red planet with more than 5 times the scientific equipment of any of its predecessors. The rover has discovered, among other things, the existence of liquid water on Mars and compelling evidence that Mars could have supported life in the past. For more information, see the main post, Mars Science Laboratory.

MAVEN: Launched on November 18, 2013, MAVEN is a Martian orbiter which arrived at Mars on in 2014. Its mission is to investigate the Martian atmosphere and its interaction with solar wind. These Data have revealed by what means and how quickly Mars's atmosphere has been lost over time, the nature of the planet's magnetic field, and has given clues that Mars could have supported life billions of years ago. For more information, see the main post, MAVEN.

ExoMars: ExoMars is a mission to investigate possible traces of life on the planet Mars. The mission includes two launches: one in 2016 and one in 2020, with the first delivering an orbiter and a lander to Mars and the second the ExoMars rover. The first mission's lander crashed upon entry, but the Trace Gas Orbiter has begun its science mission. For more

the multiplication of two infinite quantities is possible.

$$(\aleph_0)^2 = \aleph_0 \bullet \aleph_0 = \aleph_0 \quad (5)$$

The CSE for this equation follows from the countability of the set of ordered pairs. The set of ordered pairs (*x*,*y*) with natural numbers *x* and *y* can be split into components by setting a value for *x*, for example 1, and letting *y* vary among the natural numbers. For each constant value of *x*, a set of cardinality aleph-zero is generated, and since there are aleph-zero choices for *x*, the above result (5) follows. In CSE form, |{(1,1),(1,2),(1,3)...}|+|{(2,1),(2,2),(2,3)...}|+...=[the cardinality of the set of ordered pairs]. Each quantity in the equality has value aleph-zero when evaluated, and since there are as many members on the left side, it follows that the cardinality of the natural numbers, when multiplied by itself, yields the same quantity. This result can too be generalized to any positive integer power:

$$(\aleph_0)^n = \aleph_0 \quad (6)$$

Since the case *n*=2 makes use of ordered pairs, it is natural to assume that higher powers will involve the corresponding ordered *n*-tuplet. This is correct. There are aleph-zero possibilities for each element of an integral ordered *n*-tuplet, and each choice of element contributes an aleph-zero to the product. The end result is aleph-zero to the *n*th power, but since it has already been said that the cardinalities of the sets of ordered *n*-tuplets for finite *n* are all aleph-zero, the equality (6) is a direct result.

Summarizing the above, no additions, multiplications, or *n*th powers, when applied to aleph-zero, change its value. However, it has already been shown that two taken to the power of aleph-zero produces a different infinite cardinal, namely the cardinality of the continuum. But what about a general natural number *n* taken to the same power, or even aleph-zero taken to the power of itself?

$$(\aleph_0)^{\aleph_0} = \mathfrak{c} \quad (7)$$

Remarkably, we find that this quantity is equal to the cardinality of the continuum! This can be derived intuitively from the result (6). Since aleph-zero to the power of *n* is equal to the cardinality of the set of the ordered *n*-tuplets, one obtains (7) by letting *n* increase without bound to aleph-zero, at which point one obtains the cardinality of the set of infinite sequences, which has previously been shown to be *greater* than aleph-zero, and named the cardinality of the continuum. Any other *n* taken to the aleph-zero power is also equal to the cardinality of the continuum, as such quantities would clearly be greater than 2 to that power and less than the left side of (7). Since these both have the same value, those of the general case do as well. This is all summarized below.

$$(\aleph_0)^{\aleph_0} = (n)^{\aleph_0} = (2)^{\aleph_0} = \mathfrak{c}$$

The next post explores uncountable sets, namely stating what other sets besides the

information, see the main post, ExoMars.

OSIRIS-REx: OSIRIS-REx is a sample return mission to the asteroid 101955 Bennu. Launched on September 8, 2016, the probe arrived at its destination on December 3, 2018. It is currently surveying the asteroid for potential sample collection sites. For more information, see the main post, OSIRIS-REx.

Posts about completed missions: MESSENGER, Rosetta, Kepler, Dawn.

LATEST NEWS

March 9, 2018
This date marks one decade of Professor Quibb! Marking the occasion is the beginning of a post-series touching on mathematics and logic, see here.

January 1, 2018
The new year begins with a two-part series on neutrinos.

April 19, 2017
Another early start to the hurricane season with the formation of Tropical Storm Arlene.

January 1, 2017
Professor Quibb switches off of tracking hurricanes, publishes the first post of the year: Voronoi Diagrams and Metrics.

November 8, 2016
Professor Quibb passes 300,000 page views!

January 13, 2016
The 2016 Atlantic hurricane season begins very early with the formation of Subtropical Storm Alex.

January 1, 2016
First post of the year, Introduction to Ion Propulsion, is published.

May 8, 2015
Professor Quibb transitions to Hurricane Tracker Mode when Subtropical Storm Ana forms almost a month before the season officially begins.

January 1, 2015
Professor Quibb exits Hurricane Tracker Mode. The first post of the year, Derechos, is published.

June 12, 2014
Professor Quibb passes 150,000 page views!

For older news, see the News Archive.

real or complex numbers have a cardinality equal to, or even greater than, the cardinality of the continuum.

Sources: http://en.wikipedia.org/wiki/Power_set

POSTED BY LOUIS AT 7:52:00 AM
LABELS: MATHEMATICS

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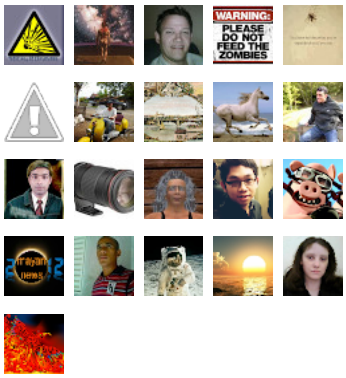

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Fates of the Universe (7)
Cell Parts (1)
Evolution (1)

ABOUT ME

LOUIS

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SITEMETER

HURRICANES AND TROPICAL STORMS

2008

TROPICAL STORM ARTHUR
HURRICANE BERTHA
TROPICAL STORM CRISTOBAL
HURRICANE DOLLY
TROPICAL STORM EDOUARD
TROPICAL STORM FAY
HURRICANE GUSTAV (NAME RETIRED)
HURRICANE HANNA
HURRICANE IKE (NAME RETIRED)
TROPICAL STORM JOSEPHINE
HURRICANE KYLE
TROPICAL STORM LAURA
TROPICAL STORM MARCO
TROPICAL STORM NANA
HURRICANE OMAR
TROPICAL DEPRESSION SIXTEEN
HURRICANE PALOMA (NAME RETIRED)

2009

TROPICAL DEPRESSION ONE
TROPICAL STORM ANA
HURRICANE BILL
TROPICAL STORM CLAUDETTE
TROPICAL STORM DANNY
TROPICAL STORM ERIKA
HURRICANE FRED
TROPICAL DEPRESSION EIGHT
TROPICAL STORM GRACE
TROPICAL STORM HENRI
HURRICANE IDA

2010

HURRICANE ALEX
TROPICAL DEPRESSION TWO

TROPICAL STORM BONNIE
TROPICAL STORM COLIN
TROPICAL DEPRESSION FIVE
HURRICANE DANIELLE
HURRICANE EARL
TROPICAL STORM FIONA
TROPICAL STORM GASTON
TROPICAL STORM HERMINE
HURRICANE IGOR (NAME RETIRED)
HURRICANE JULIA
HURRICANE KARL
HURRICANE LISA
TROPICAL STORM MATTHEW
TROPICAL STORM NICOLE
HURRICANE OTTO
HURRICANE PAULA
HURRICANE RICHARD
HURRICANE SHARY
HURRICANE TOMAS (NAME RETIRED)

2011

TROPICAL STORM ARLENE
TROPICAL STORM BRET
TROPICAL STORM CINDY
TROPICAL STORM DON
TROPICAL STORM EMILY
TROPICAL STORM FRANKLIN
TROPICAL STORM GERT
TROPICAL STORM HARVEY
HURRICANE IRENE (NAME RETIRED)
TROPICAL DEPRESSION TEN
TROPICAL STORM JOSE
HURRICANE KATIA
UNNAMED TROPICAL STORM
TROPICAL STORM LEE
HURRICANE MARIA
HURRICANE NATE
HURRICANE OPHELIA
HURRICANE PHILIPPE
HURRICANE RINA
TROPICAL STORM SEAN

2012

TROPICAL STORM ALBERTO
TROPICAL STORM BERYL
HURRICANE CHRIS
TROPICAL STORM DEBBY
HURRICANE ERNESTO
TROPICAL STORM FLORENCE
TROPICAL STORM HELENE
HURRICANE GORDON
HURRICANE ISAAC
TROPICAL STORM JOYCE
HURRICANE KIRK

HURRICANE LESLIE
HURRICANE MICHAEL
HURRICANE NADINE
TROPICAL STORM OSCAR
TROPICAL STORM PATTY
HURRICANE RAFAEL
HURRICANE SANDY (NAME RETIRED)
TROPICAL STORM TONY

2013

TROPICAL STORM ANDREA
TROPICAL STORM BARRY
TROPICAL STORM CHANTAL
TROPICAL STORM DORIAN
TROPICAL STORM ERIN
TROPICAL STORM FERNAND
TROPICAL STORM GABRIELLE
TROPICAL DEPRESSION EIGHT
HURRICANE HUMBERTO
HURRICANE INGRID (NAME RETIRED)
TROPICAL STORM JERRY
TROPICAL STORM KAREN
TROPICAL STORM LORENZO
TROPICAL STORM MELISSA
UNNAMED SUBTROPICAL STORM

2014

HURRICANE ARTHUR
TROPICAL DEPRESSION TWO
HURRICANE BERTHA
HURRICANE CRISTOBAL
TROPICAL STORM DOLLY
HURRICANE EDOUARD
HURRICANE FAY
HURRICANE GONZALO
TROPICAL STORM HANNA

2015

TROPICAL STORM ANA
TROPICAL STORM BILL
TROPICAL STORM CLAUDETTE
HURRICANE DANNY
TROPICAL STORM ERIKA (NAME RETIRED)
HURRICANE FRED
TROPICAL STORM GRACE
TROPICAL STORM HENRI
TROPICAL DEPRESSION NINE
TROPICAL STORM IDA
HURRICANE JOAQUIN (NAME RETIRED)
HURRICANE KATE

2016

HURRICANE ALEX
TROPICAL STORM BONNIE
TROPICAL STORM COLIN
TROPICAL STORM DANIELLE
HURRICANE EARL
TROPICAL STORM FIONA
HURRICANE GASTON
TROPICAL DEPRESSION EIGHT
HURRICANE HERMINE
TROPICAL STORM IAN
TROPICAL STORM JULIA
TROPICAL STORM KARL
TROPICAL STORM LISA
HURRICANE MATTHEW (NAME RETIRED)
HURRICANE NICOLE
HURRICANE OTTO (NAME RETIRED)

2017

TROPICAL STORM ARLENE
TROPICAL STORM BRET
TROPICAL STORM CINDY
TROPICAL DEPRESSION FOUR
TROPICAL STORM DON
TROPICAL STORM EMILY
HURRICANE FRANKLIN
HURRICANE GERT
HURRICANE HARVEY (NAME RETIRED)
HURRICANE IRMA (NAME RETIRED)
HURRICANE JOSE
HURRICANE KATIA
HURRICANE LEE
HURRICANE MARIA (NAME RETIRED)
HURRICANE NATE (NAME RETIRED)
TROPICAL STORM PHILIPPE
TROPICAL STORM RINA

2018

SUBTROPICAL STORM ALBERTO
HURRICANE BERYL
HURRICANE CHRIS
TROPICAL STORM DEBBY
TROPICAL STORM ERNESTO
HURRICANE FLORENCE (NAME RETIRED)
TROPICAL STORM GORDON
HURRICANE HELENE
HURRICANE ISAAC
TROPICAL STORM JOYCE
TROPICAL DEPRESSION ELEVEN
TROPICAL STORM KIRK
HURRICANE LESLIE
HURRICANE MICHAEL (NAME RETIRED)
TROPICAL STORM NADINE
HURRICANE OSCAR