Functions

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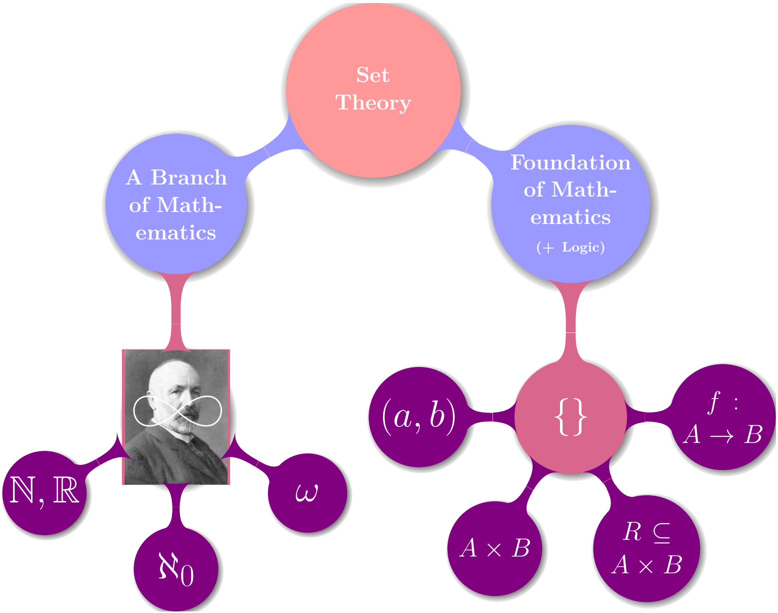
Set Theory
A Branch of Math-ematics
Foundation of Math-ematics
(+ Logic)
$\mathbb{N}, \mathbb{R}$
$\aleph_0$
ω
(a, b)
$\{\}$
$f : A \to B$
$A \times B$
$R \subseteq A \times B$

Definition of Function

Definition (Function)

Let A and B be sets.
A **function** f from A to B is a *relation* f from A to B such that

$$\forall a \in A \; \exists! b \in B \; (a, b) \in f.$$

$$\exists! : \forall b, b' \in B, (a, b) \in f \wedge (a, b') \in f \implies b = b'.$$

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$$A : \mathrm{dom}(f) \qquad B : \mathrm{cod}(f)$$

$$f : a \mapsto f(a)$$

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$$A : \mathrm{dom}(f) \qquad B : \mathrm{cod}(f)$$

$$f : a \mapsto f(a)$$

$$\mathrm{ran}(f) = \{b \in B \mid \exists a \in A : f(a) = b\} \subseteq B$$

$$f = g \iff \mathsf{dom}(f) = \mathsf{dom}(g) \land (\forall x \in \mathsf{dom}(f) : f(x) = g(x))$$

$$f : A \to B$$

$$f \subseteq A \times B$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$$(a, b) = \{\{a\}, \{a, b\}\}$$

Properties of Functions

Definition (Injective (one-to-one; 1-1))

$$\forall a_1, a_2 \in A : f(a_1) = f(a_2) \implies a_1 = a_2$$

$$\forall a_1, a_2 \in A : a_1 \neq a_2 \implies f(a_1) \neq f(a_2)$$

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$$\exists a_1, a_2 \in A : a_1 \neq a_2 \wedge f(a_1) = f(a_2)$$

Definition (Surjective (onto))

$$\forall b \in B : \exists a \in A : f(a) = b$$

$$\mathrm{ran}(f) = B$$

Definition (Surjective (onto))

$$\forall b \in B : \exists a \in A : f(a) = b$$

$$\mathsf{ran}(f) = B$$

$$\exists b \in B : \forall a \in A : f(a) \neq b$$

Definition (Bijective (one-to-one correspondence))

Bijective: injective + surjective

proof examples

Operations on Functions

Definition (Intersection, Union)

$$f_1, f_2 : A \to B$$

(1) Q : Is $f_1 \cup f_2$ a function?
(2) Q : Is $f_1 \cap f_2$ a function?

Definition (Composition)

$$f : A \to B \qquad g : C \to D$$

$$\text{ran}(f) \subseteq C$$

The composition function

$$g \circ f : A \to D$$

$$(g \circ f)(x) = g(f(x))$$

Theorem (Properties of Composition)

$$f \circ g \neq g \circ f$$

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Theorem (Properties of Composition)

$$f : A \to B \qquad g : B \to C$$

(1) *If* f, g *are injective, then* $g \circ f$ *is injective.*

(2) *If* f, g *are surjective, then* $g \circ f$ *is injective.*

(3) *If* f, g *are bijective, then* $g \circ f$ *is bijective.*

Definition (Inverse)

$$f : A \to B$$

$$f : X \to Y \quad A \subseteq X \quad B \subseteq Y$$

Definition (Image)

The image of A under f is the set

$$f(A) = \{f(a) \mid a \in A\}.$$

Definition (Inverse Image)

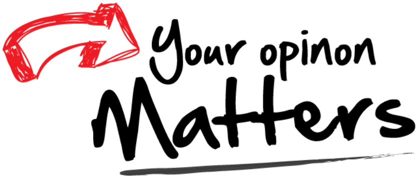
The inverse image of B under f is the set

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}.$$

$$Q_1 : A \text{ vs. } f^{-1}(f(A))$$

$$Q_2 : B \text{ vs. } f(f^{-1}(B))$$


Thank You!



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