# Jensen's Inequality

**Theorem**

For any concave function $f$,

$$E[f(X)] \leq f(E[X])$$

*Proof.* Suppose $f$ is differentiable. The function $f$ is concave if, for any $x$ and $y$,

$$f(x) \leq f(y) + (x - y)f'(y)$$

Let $x = X$ and $y = E[X]$. We can write

$$f(X) \leq f(E[X]) + (X - E[X])f'(E[X])$$

This inequality is true for all $X$, so we can take expectation on both sides to get

$$E[f(X)] \leq f(E[X]) + f'(E[X])E[(X - E[X])] = f(E[X])$$