3-1 Dynamic Programming

(Part I: Examples)

Hengfeng Wei

hfwei@nju.edu.cn

September 17, 2018

Taolu

Steps for Applying DP:

Steps for Applying DP:

(I) Define subproblems

(II) Set the goal

(III) Identify the recurrence

- larger subproblem $\leftarrow$ # smaller subproblems
- init. conditions

Steps for Applying DP:

(I) Define subproblems

(II) Set the goal

(III) Identify the recurrence
- larger subproblem $\leftarrow$ # smaller subproblems
- init. conditions

(IV) Write pseudo-code: filling in "tables" in some order

(V) Analyze the time complexity

(VI) Extract the optimal solution (optionally)

Steps for Applying DP:

(I) Define subproblems

(II) Set the goal

(III) Identify the recurrence

- larger subproblem $\leftarrow$ # smaller subproblems
- init. conditions

(IV) Write pseudo-code: filling in "tables" in some order

(V) Analyze the time complexity

(VI) Extract the optimal solution (optionally)

Steps for Applying DP:

(I) Define subproblems

(II) Set the goal

(III) Identify the recurrence
- larger subproblem $\leftarrow$ # smaller subproblems
- init. conditions

(IV) Write pseudo-code: filling in "tables" in some order

(V) Analyze the time complexity

(VI) Extract the optimal solution (optionally)

1D Subproblems

Input: $x_1, x_2, \ldots, x_n$ (array, sequence, string)

Subproblems: $x_1, x_2, \ldots, x_i$ (prefix/suffix)

#: $\Theta(n)$

Examples:
- Rod cutting
- Maximum-sum subarray
- Longest increasing subsequence
- Text justification ($\LaTeX$)

2D Subproblems

(I) Input: $x_1, x_2, \ldots, x_m; \quad y_1, y_2, \ldots, y_n$

Subproblems: $x_1, x_2, \ldots, x_i; \quad y_1, y_2, \ldots, y_j$

#: $\Theta(mn)$

Examples: Edit distance, Longest common subsequence

2D Subproblems

(I) Input: $x_1, x_2, \ldots, x_m; \quad y_1, y_2, \ldots, y_n$

Subproblems: $x_1, x_2, \ldots, x_i; \quad y_1, y_2, \ldots, y_j$

#: $\Theta(mn)$

Examples: Edit distance, Longest common subsequence

(II) Input: $x_1, x_2, \ldots, x_n$

Subproblems: $x_i, \ldots, x_j$

#: $\Theta(n^2)$

Examples: Matrix chain multiplication, Optimal BST

3D Subproblems

- Floyd-Warshall algorithm

$$\mathsf{d}(i,j,k) = \min\Big(\mathsf{d}(i,j,k-1), \mathsf{d}(i,k,k-1) + \mathsf{d}(k,j,k-1)\Big)$$

DP on Graphs

(I) On rooted tree

Subproblems: rooted subtrees

(II) On DAG

Subproblems: nodes after/before in the topo. order

DP on Graphs

(I) On rooted tree

Subproblems: rooted subtrees

(II) On DAG

Subproblems: nodes after/before in the topo. order

Knapsack Problem

Subset sum problem, Change-making problem

And Others . . .

And Others . . .

How to identify the recurrence?

How to identify the recurrence?

GUESS

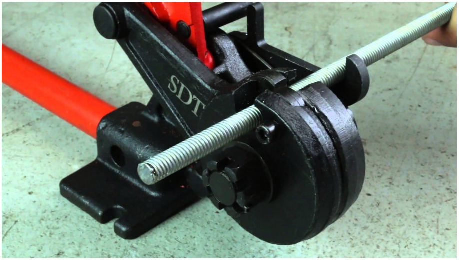
Make Choices by asking yourself the right question

Make Choices by asking yourself the right question

(I) Binary choice
- whether ...

(II) Multi-way choices
- where to ...
- which one ...

Rod Cutting



Rod Cutting Problem

Rod of length n

length i	1	2	3	4	5	$\cdots$
price p_i	1	5	8	9	10	$\cdots$

$$n = i_1 + i_2 + \cdots + i_k$$

$$r_n = p_{i_1} + p_{i_2} + \cdots + p_{i_k}$$

$R(i) : \max$ revenue obtained from *cutting a rod of length i*

$$R(n)$$

$R(i) : \max$ revenue obtained from *cutting a rod of length i*

$$R(n)$$

Where is the first cut?

$R(i)$: max revenue obtained from *cutting a rod of length i*

$$R(n)$$

Where is the first cut?

$$R(i) = \max_{1 \le j \le i} \Big(p_j + R(i-j) \Big)$$

$R(i)$: max revenue obtained from *cutting a rod of length i*

$$R(n)$$

Where is the first cut?

$$R(i) = \max_{1 \le j \le i} \Big(p_j + R(i-j) \Big)$$

$$R(0) = 0$$

$R(i) : \max$ revenue obtained from *cutting a rod of length i*

$$R(n)$$

Where is the first cut?

$$R(i) = \max_{1 \le j \le i} \Big(p_j + R(i-j)\Big)$$

$$R(0) = 0$$

$$O(n^2) = \Theta(n) \cdot O(n)$$

Rod Cutting Problem (Problem 15.1-3)

Each cut incurs a fixed cost of c.

Rod Cutting Problem (Problem 15.1-3)

Each cut incurs a fixed cost of c.

$$R(i) = \max_{1 \le j \le i} \Big(p_j - c + R(i - j) \Big)$$

Rod Cutting Problem (Problem 15.1-3)

Each cut incurs a fixed cost of c.

$$R(i) = \max_{1 \le j \le i} \Big(p_j - c + R(i-j) \Big)$$

$$R(i) = \max \Big(p_i, \max_{1 \le j < i} \left(p_j - c + R(i-j) \right) \Big)$$

Printing Neatly (Problem 15-4)

A sequence of n words of lengths $l_1, l_2, \cdots, l_n$

Line width M

$$\textit{extra}[i, j] = M - (j - i) - \sum_{k=i}^{j} l_k$$

Printing Neatly (Problem 15-4)

A sequence of n words of lengths $l_1, l_2, \cdots, l_n$

Line width M

$$\textit{extra}[i,j] = M - (j-i) - \sum_{k=i}^{j} l_k$$

To minimize the sum, over all lines *except the last*, of the cubes of the numbers of extra space characters at the ends of lines.

$$c(n) = \min_{L} \sum_{l_{[i,j]} \in L \,\wedge\, j \neq n} \Big(\textit{extra}[i,j]\Big)^3$$


LaTeX
Beginner's Guide
TEX}

$\TeX$ 3.14159265 $\sim \pi$

A $\boxed{\text{sequence}}$ of n words of lengths $l_1, l_2, \cdots, l_n$

A $\boxed{\text{sequence}}$ of n words of lengths $l_1, l_2, \cdots, l_n$

$C(i)$: $\min$ cost of neatly printing the *first* i words

$C(i)$: $\min$ cost of neatly printing the *last* words i through n

$C(i, j)$: $\min$ cost of neatly printing *words* i *through* j

$C(i)$: $\min$ cost of neatly printing the *last* words i through n

$C(i)$: $\min$ cost of neatly printing the *last* words i through n

$\boxed{\textit{How many words to place on the first line?}}$

$C(i)$: $\min$ cost of neatly printing the *last* words i through n

How many words to place on the first line?

$$C(i) = \min_{\substack{0 < k \le (n-i+1) \\ \textit{extra}[i,\, i+k-1] \ge 0}} \Big(\textit{extra}[i, i+k-1]\Big)^3 + C(i+k)$$

$C(i)$: min cost of neatly printing the *last* words i through n

How many words to place on the first line?

$$C(i) = \min_{\substack{0 < k \le (n-i+1) \\ \textit{extra}[i, i+k-1] \ge 0}} \left(\textit{extra}[i, i+k-1]\right)^3 + C(i+k)$$

$$C(i) = 0, \quad \text{if } \textit{extra}[i, n] \ge 0$$

$C(i)$: $\min$ cost of neatly printing the *last* words i through n

$\boxed{\textit{How many words to place on the first line?}}$

$$C(i) = \min_{\substack{0 < k \le (n-i+1) \\ \textit{extra}[i, i+k-1] \ge 0}} \Big(\textit{extra}[i, i+k-1]\Big)^3 + C(i+k)$$

$$C(i) = 0, \quad \text{if } \textit{extra}[i, n] \ge 0$$

$$O(nW)$$

procedure PRINTING-NEATLY(n)

  for $i \leftarrow n \downarrow 1$ **do**

    if $extra[i, n] \geq 0$ **then** $\triangleright$ put w_i through w_n on a line

      $C[i] \leftarrow 0$

    else

      $$C(i) = \min_{\substack{0 < k \leq (n-i+1) \\ extra[i, i+k-1] \geq 0}} \Big(extra[i, i+k-1]\Big)^3 + C(i+k)$$

procedure $\textsc{Printing-Neatly}(n)$
    for $i \leftarrow n \downarrow 1$ **do**
        if $extra[i, n] \geq 0$ **then** ▷ put w_i through w_n on a line
            $C[i] \leftarrow 0$
            $B[i] \leftarrow n$
        else
            $C(i) = \min\limits_{\substack{0 < k \leq (n-i+1) \\ extra[i, i+k-1] \geq 0}} \Big(extra[i, i+k-1]\Big)^3 + C(i+k)$
            $B[i] = i + k$ ▷ line break

```
procedure PRINTING-NEATLY(n)
    for i ← n ↓ 1 do
        if extra[i, n] ≥ 0 then                    ▷ put w_i through w_n on a line
            C[i] ← 0
            B[i] ← n
        else
            C(i) = min_{0 < k ≤ (n−i+1), extra[i, i+k−1] ≥ 0} (extra[i, i+k−1])^3 + C(i+k)
            B[i] = i + k                                       ▷ line break
```

$$C(i) = \min_{\substack{0 < k \le (n-i+1) \\ \textit{extra}[i, i+k-1] \ge 0}} \Big(\textit{extra}[i, i+k-1]\Big)^3 + C(i+k)$$

$B[1],$

procedure $\textsc{Printing-Neatly}(n)$

  for $i \leftarrow n \downarrow 1$ **do**

    if $extra[i, n] \geq 0$ **then** ▷ put w_i through w_n on a line

      $C[i] \leftarrow 0$

      $B[i] \leftarrow n$

    else

      $$C(i) = \min_{\substack{0 < k \leq (n-i+1) \\ extra[i, i+k-1] \geq 0}} \Big(extra[i, i+k-1] \Big)^3 + C(i+k)$$

      $B[i] = i + k$ ▷ line break

$$B[1], \quad B[B[1]+1], \quad \cdots$$

Neatness: $c[i, j] = \textit{extra}[i, j]^1$

— *TianYun Zhang*

Neatness: $c[i,j] = \textit{extra}[i,j]^1$

— *TianYun Zhang*

Neatness: $c[i,j] = \textit{extra}[i,j]^{1}$

— *TianYun Zhang*

Proof: Exchange Argument

Longest Increasing Subsequence (Problem 15.4-5)

$$A[1 \ldots n]$$

$$5,\ 2,\ 8,\ 6,\ 3,\ 6,\ 9,\ 7$$

Find (the length of) a longest increasing (non-decreasing) subseqence.

$$5,\ \color{red}{2},\ 8,\ 6,\ \color{red}{3},\ \color{red}{6},\ \color{red}{9},\ 7$$

$L(i)$: the length of an LIS *ending with* $A[i]$

$$\max_i L(i)$$

$L(i)$: $\boxed{\text{the length of an LIS } \textit{ending with } A[i]}$

$$\max_i L(i)$$

$\boxed{\textit{What is the previous element?}}$

$L(i)$: the length of an LIS *ending with* $A[i]$

$$\max_i L(i)$$

What is the previous element?

$$L(i) = 1 + \max_{j < i \,\wedge\, A[j] \le A[i]} L(j)$$

$L(i)$: the length of an LIS *ending with* $A[i]$

$$\max_i L(i)$$

What is the previous element?

$$L(i) = 1 + \max_{j<i \,\wedge\, A[j] \le A[i]} L(j)$$

$$L(1) = 1$$

$L(i)$: $\boxed{\text{the length of an LIS } \textit{ending with } A[i]}$

$$\max_i L(i)$$

$\boxed{\textit{What is the previous element?}}$

$$L(i) = 1 + \max_{j<i \,\wedge\, A[j] \le A[i]} L(j)$$

$$L(1) = 1$$

$$O(n^2) = \Theta(n) \cdot O(n)$$

$$\mathsf{LIS(A)} = \mathsf{LCS}\Big(A, \textsc{Sort}(A)\Big)$$

$$\mathsf{LIS(A)} = \mathsf{LCS}\Big(A, \textsc{Sort}(A)\Big)$$

$$O(n^2) = O(n \log n) + O(n^2)$$

Longest Increasing Subsequence (Problem 15.4-6)

$$O(n \log n)$$

Longest Increasing Subsequence (Problem 15.4-6)

$$O(n \log n)$$

Longest Increasing Subsequence (Problem 15.4-6)

$$O(n \log n)$$

Answer by Eugene Yarovoi @ Quora

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

procedure *LIS*(n)

  $E[l] \leftarrow \emptyset, \quad \forall 1 \le i \le n$

  for $i \leftarrow 1 \uparrow n$ **do**

    $\forall 1 \le l \le i : E[l] \leftarrow$

      $\{\textit{all inc. subseq. of length } l\}$

  return

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

procedure *LIS*(n)
$\quad E[l] \leftarrow \emptyset, \quad \forall 1 \le i \le n$
$\quad$**for** $i \leftarrow 1 \uparrow n$ **do**
$\quad\quad \forall 1 \le l \le i : E[l] \leftarrow$
$\quad\quad\quad \{\textit{all inc. subseq. of length } l\}$
$\quad$**return** $\max \{l \mid E[l] \neq \emptyset\}$

$$\color{red}{E[l]} = \{\textit{all increasing subsequences of length } l\}$$

---

procedure *LIS*(n)

  $E[l] \leftarrow \emptyset, \quad \forall 1 \le i \le n$

  for $i \leftarrow 1 \uparrow n$ **do**

    $\forall 1 \le l \le i : E[l] \leftarrow$ ▷ *By extending each shorter subseq.*

      $\{\textit{all inc. subseq. of length } l\}$

  return $\max \{l \mid E[l] \neq \emptyset\}$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$\langle 3 \quad 9 \quad 11 \quad 13 \rangle$$

$$\langle 4 \quad 6 \quad 10 \quad 15 \rangle$$

$$\langle 2 \quad 5 \quad 12 \quad 18 \rangle$$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$\langle 3 \quad 9 \quad 11 \quad 13 \rangle$$

$$\langle 4 \quad 6 \quad 10 \quad 15 \rangle$$

$$\langle 2 \quad 5 \quad 12 \quad 18 \rangle$$

$\Big|E[l]\Big| = 1$: *the one with the smallest ending number*

procedure *LIS*(n)

  $E[l] \leftarrow \emptyset, \quad \forall 1 \leq i \leq n$

  for $i \leftarrow 1 \uparrow n$ **do**

    $\forall 1 \leq l \leq i : E[l] \leftarrow$ ▷ *By extending each shorter subseq.*

      the inc. subseq. of length l *with the smallest ending number*

  return $\max \{l \mid E[l] \neq \emptyset\}$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$\Big|E[l]\Big| = 1 : \textit{the one with the smallest ending number}$$

$E[l] = \{$*all increasing subsequences of length* $l\}$

$\Big|E[l]\Big| = 1$: *the one with the smallest ending number*

$$\langle 2 \quad 8 \quad 15 \rangle$$

$$\langle 4 \quad 6 \quad 11 \quad 13 \rangle$$

$$\langle 3 \quad 9 \quad 11 \quad 12 \quad 13 \rangle$$

$$E[l] = \{\textit{all increasing subsequences of length } l\}$$

$$\Big|E[l]\Big| = 1 : \textit{the one with the smallest ending number}$$

$$\langle 2 \quad 8 \quad 15 \rangle$$

$$\langle 4 \quad 6 \quad 11 \quad 13 \rangle$$

$$\langle 3 \quad 9 \quad 11 \quad 12 \quad 13 \rangle$$

$$\forall i < j : \textit{the ending number of } E[i] < \textit{the ending number of } E[j]$$

procedure *LIS*(n)

$E[l] \leftarrow \emptyset, \quad \forall 1 \leq i \leq n$

for $i \leftarrow 1 \uparrow n$ **do**

$\forall 1 \leq l \leq i : E[l] \leftarrow$ ▷ *By extending each shorter subseq.*

the inc. subseq. of length l *with the smallest ending number*

return $\max \{l \mid E[l] \neq \emptyset\}$

procedure *LIS*(n)

  $E[l] \leftarrow \emptyset, \quad \forall 1 \le i \le n$

  for $i \leftarrow 1 \uparrow n$ **do**

    $\forall 1 \le l \le i : E[l] \leftarrow$ ▷ *Extending only one subseq using binary search*

      the inc. subseq. of length l *with the smallest ending number*

  return $\max \{ l \mid E[l] \neq \emptyset \}$

Extending

Extending

by looking at only the ending number of the inc. subseq

Extending

by looking at only the ending number of the inc. subseq

```
procedure LIS(n)
    E[l] ←        ∀1 ≤ i ≤ n
    for i ← 1 ↑ n do
        E[l] ←        ▷ Extending only one subseq using binary search
            the smallest ending number for the inc. subseq. of length l
    return
```

Extending

by looking at only the ending number of the inc. subseq

---

procedure *LIS*(n)

  $E[l] \leftarrow \infty \quad \forall 1 \le i \le n$

  for $i \leftarrow 1 \uparrow n$ **do**

    $E[l] \leftarrow$ ▷ *Extending only one subseq using binary search*

      the smallest ending number for the inc. subseq. of length l

  return

Extending

by looking at only the ending number of the inc. subseq

```
procedure LIS(n)
    E[l] ← ∞   ∀1 ≤ i ≤ n
    for i ← 1 ↑ n do
        E[l] ←            ▷ Extending only one subseq using binary search
            the smallest ending number for the inc. subseq. of length l
    return max {l | E[l] < ∞}
```

Matrix-chain Multiplication

$m[i, j]$: min cost to compute the matrix $A_i \cdots A_j$

$$m[1, n]$$

$m[i, j]$: $\min$ cost to $\boxed{\text{compute the matrix } A_i \cdots A_j}$

$$m[1, n]$$

Where is the last parentheses?

$m[i,j]$: min cost to $\boxed{\text{compute the matrix } A_i \cdots A_j}$

$$m[1,n]$$

Where is the last parentheses?

$$m[i,j] = \min_{i \le k < j} \Big(m[i,k] + m[k+1,j] + p_{i-1}p_k p_j \Big)$$

$m[i, j]$: min cost to compute the matrix $A_i \cdots A_j$

$$m[1, n]$$

Where is the last parentheses?

$$m[i, j] = \min_{i \le k < j} \Big(m[i, k] + m[k+1, j] + p_{i-1} p_k p_j \Big)$$

$$m[i, i] = 0$$

$m[i, j]$: min cost to $\boxed{\text{compute the matrix } A_i \cdots A_j}$

$$m[1, n]$$

Where is the last parentheses?

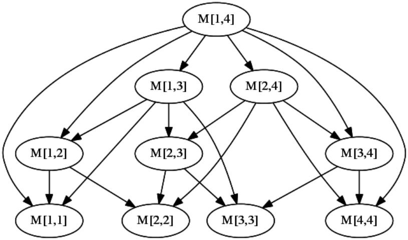
$$m[i, j] = \min_{i \le k < j} \Big(m[i, k] + m[k+1, j] + p_{i-1} p_k p_j \Big)$$

$$m[i, i] = 0$$

$$O(n^3) = \Theta(n^2) \cdot O(n)$$

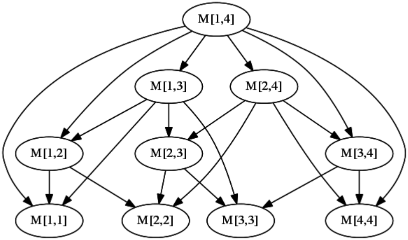
Subproblem Graph for Matrix-chain Multiplication (Problem 15.2-4)

$$m[i,j] = \min_{i \le k < j} \Big(m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \Big)$$



Subproblem Graph for Matrix-chain Multiplication (Problem 15.2-4)

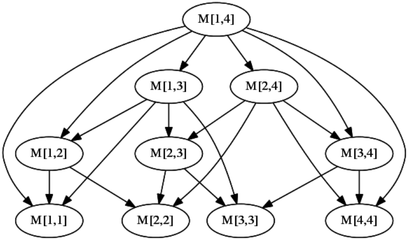
$$m[i,j] = \min_{i \le k < j} \Big(m[i,k] + m[k+1,j] + p_{i-1}p_k p_j \Big)$$



$$\Big| \{(i,j) | 1 \le i \le j \le n\} \Big| = \frac{n(n+1)}{2}$$

Subproblem Graph for Matrix-chain Multiplication (Problem 15.2-4)

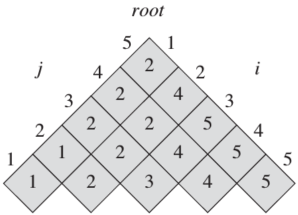
$$m[i,j] = \min_{i \le k < j} \Big(m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \Big)$$

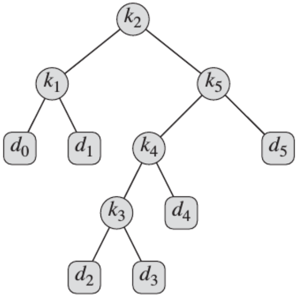


$$\Big| \{(i,j) | 1 \le i \le j \le n\} \Big| = \frac{n(n+1)}{2}$$

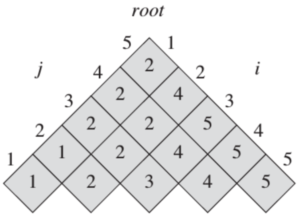
$$\sum_{1 \le i \le j \le n} 2(j-i) = \frac{n^3 - n}{3}$$

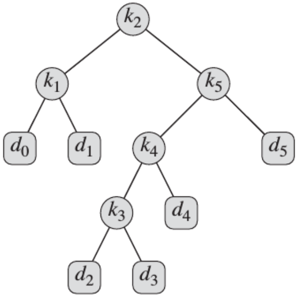
Construct-Optimal-BST(root) (Problem 15.5-1)





Construct-Optimal-BST(root) (Problem 15.5-1)





What about $d_0, d_1, \cdots, d_n$*?*

Construct-Optimal-BST(root) (Problem 15.5-1)

$$e[i, j] = \begin{cases} q_{i-1} & \text{if } j = i - 1, \\ \min\limits_{i \le r \le j} \{e[i, r-1] + e[r+1, j] + w(i, j)\} & \text{if } i \le j. \end{cases}$$

CONSTRUCT-OPTIMAL-BST(root) (Problem 15.5-1)

$$e[i,j] = \begin{cases} q_{i-1} & \text{if } j = i-1, \\ \min\limits_{i \le r \le j} \{e[i,r-1] + e[r+1,j] + w(i,j)\} & \text{if } i \le j. \end{cases}$$

```
procedure CONSTRUCT-OPTIMAL-BST(root, i, j)
    if j = i − 1 then
        return node with d_{i−1}

    rn ← node with key k_{root[i,j]}
    rn.left ← CONSTRUCT-OPTIMAL-BST(root, i, root[i,j] − 1)
    rn.right ← CONSTRUCT-OPTIMAL-BST(root, root[i,j] + 1, j)

    return rn
```

Construct-Optimal-BST(root) (Problem 15.5-1)

$$e[i,j] = \begin{cases} q_{i-1} & \text{if } j = i-1, \\ \min\limits_{i \le r \le j} \{e[i,r-1] + e[r+1,j] + w(i,j)\} & \text{if } i \le j. \end{cases}$$

```
procedure CONSTRUCT-OPTIMAL-BST(root, i, j)
    if j = i − 1 then
        return node with d_{i−1}

    rn ← node with key k_{root[i,j]}
    rn.left ← CONSTRUCT-OPTIMAL-BST(root, i, root[i,j] − 1)
    rn.right ← CONSTRUCT-OPTIMAL-BST(root, root[i,j] + 1, j)

    return rn
```

$$\textsc{Construct-Optimal-BST}(root, 1, n)$$

procedure CONSTRUCT-OPTIMAL-BST($root$, i, j, r) ▷ r : *parent*

  if $r = i$ **then**

    print "d_{i-1} is the *left* child of k_r" **return** node with d_{i-1}

  if $r = j$ **then**

    print "d_{i-1} is the *right* child of k_r" **return** node with d_{i-1}

  $r' \leftarrow root[i, j]$ $rn \leftarrow$ node with key $k_{r'}$

  if $r = 0$ **then**

    print "$k_{r'}$ is the root"

  else if $j < r$ **then**

    print "$k_{r'}$ is the *left* child of k_r"

  else if $i > r$ **then**

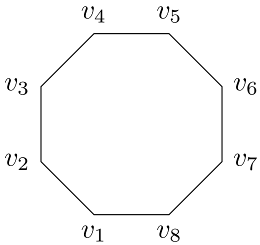
    print "$k_{r'}$ is the *right* child of k_r"

  $rn.left \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, i, r' - 1, r')$

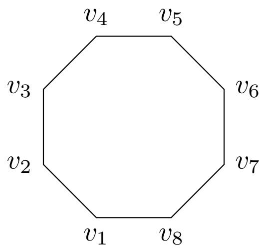
  $rn.right \leftarrow$ CONSTRUCT-OPTIMAL-BST$(root, r' + 1, j, r')$

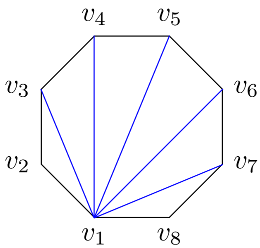
  return rn

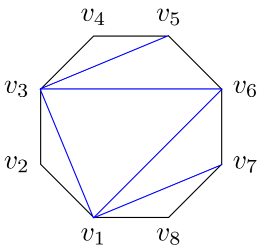
Minimum Weight Triangulation

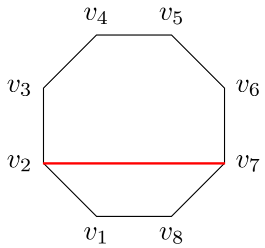


Minimum Weight Triangulation

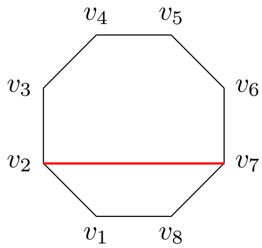






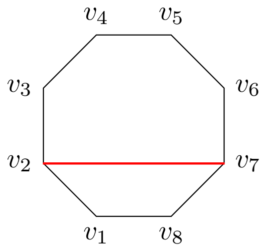


$T(i, j):$ min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), *clockwise*



$T(i,j):$ min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), *clockwise*

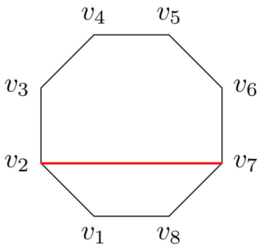
$$T(i,j) = \min_{\substack{i \le k < l-1 \le j \\ (k,l) \neq (i,j)}} \Big(T[k,l] + T[?,?] + d_{ij} \Big)$$

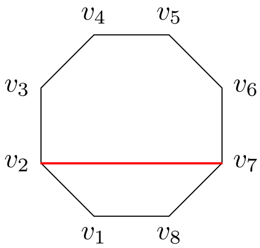


$T(i,j):$ min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), *clockwise*

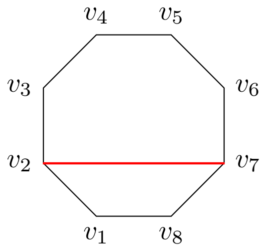
$$T(i,j) = \min_{\substack{i \le k < l-1 \le j \\ (k,l) \neq (i,j)}} \Big(T[k,l] + T[?,?] + d_{ij} \Big)$$

$$O(n^4) = O(n^2) \cdot O(n^2)$$


v_4
v_5
v_3
v_6
v_2
v_7
v_1
v_8

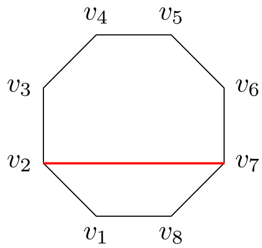


Which vertex k to pair with (i, j)?



Which vertex k to pair with (i, j)?

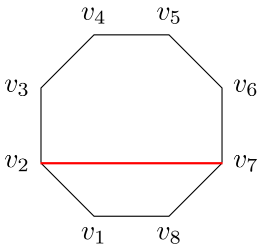
$T(i, j)$: min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), $i < j$



Which vertex k to pair with (i, j)?

$T(i, j):$ min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), $i < j$

$$T(i,j) = \min_{i<k<j} \Big(T(i,k) + T(k,j) + d_{ik} + d_{kj} \Big)$$

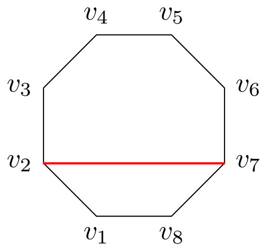


Which vertex k to pair with (i, j)?

$T(i, j):$ min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), $i < j$

$$T(i, j) = \min_{i<k<j} \Big(T(i, k) + T(k, j) + d_{ik} + d_{kj} \Big)$$

$$T[i, i+1] = 0, \quad 1 \leq i \leq n-1$$



Which vertex k to pair with (i, j)?

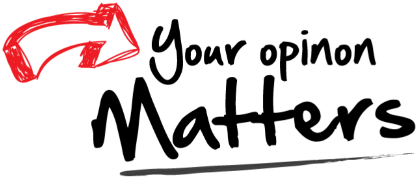
$T(i, j)$: min cost of triangulating $v_i \cdots v_j$ (with $v_j - v_i$), $i < j$

$$T(i, j) = \min_{i<k<j} \Big(T(i, k) + T(k, j) + d_{ik} + d_{kj} \Big)$$

$$T[i, i+1] = 0, \quad 1 \le i \le n - 1$$

$$O(n^3) = O(n^2) \cdot O(n)$$


THANK YOU



Office 302

Mailbox: H016

hfwei@nju.edu.cn