2-7 Discrete Probability

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TOPICS/2018

Searching an Unsorted Array (CLRS Problem $5-2\ (f)$)

procedure DETERMINISTIC-SEARCH($A[1 \cdots n], x$)
    $i \leftarrow 1$
    while $i \leq n$ **do**
        if $A[i] = x$ **then**
            return *true*
        $i \leftarrow i + 1$
    return *false*

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(f)

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&= \sum_{y=1}^{n-k+1} y \frac{\binom{n-y}{k-1}}{\binom{n}{k}}
\end{aligned}$$

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&= \sum_{y=1}^{n-k+1} y \frac{\binom{n-y}{k-1}}{\binom{n}{k}} = \frac{1}{\binom{n}{k}} \sum_{y=1}^{n-k+1} y \binom{n-y}{k-1}
\end{aligned}$$

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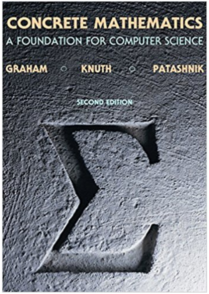
\end{aligned}$$

$$k = 1 \implies \mathbb{E}[Y] = \frac{n+1}{2}, \qquad k = n \implies \mathbb{E}[Y] = 1$$

After-class Exercise

$$\sum_{y=1}^{n-k+1} y \binom{n-y}{k-1} = \binom{n+1}{k+1}$$



$$r\binom{r-1}{k-1} = k\binom{r}{k}$$

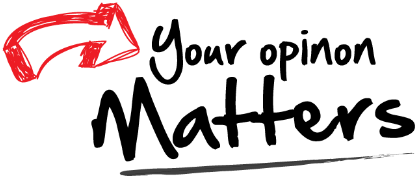
$$\sum_{0 \le k \le n} \binom{k}{m} = \binom{n+1}{m+1}$$

Chapter 5: Binomial Coefficients

$$
\begin{aligned}
\sum_{y=1}^{n-k+1} y\binom{n-y}{k-1} &= \sum_{y=0}^{n-k}(y+1)\binom{n-y-1}{k-1} \\
&= \sum_{y=0}^{n-k}\left((n+1)-(n-y)\right)\binom{n-y-1}{k-1} \\
&= \sum_{y=0}^{n-k}(n+1)\binom{n-y-1}{k-1} - \sum_{y=0}^{n-k}(n-y)\binom{n-y-1}{k-1} \\
&= (n+1)\sum_{y=0}^{n-k}\binom{n-y-1}{k-1} - k\sum_{y=0}^{n-k}\binom{n-y}{k} \\
&= (n+1)\sum_{m=k-1}^{n-1}\binom{m}{k-1} - k\sum_{m=k}^{n}\binom{m}{k} \\
&= (n+1)\binom{n}{k} - k\binom{n+1}{k+1} = \binom{n+1}{k+1}
\end{aligned}
$$

Indicator Random Variables


Thank
You!



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