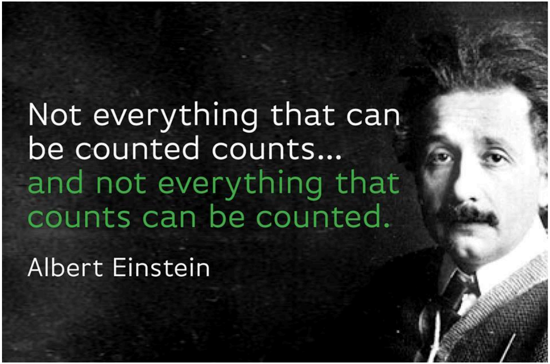
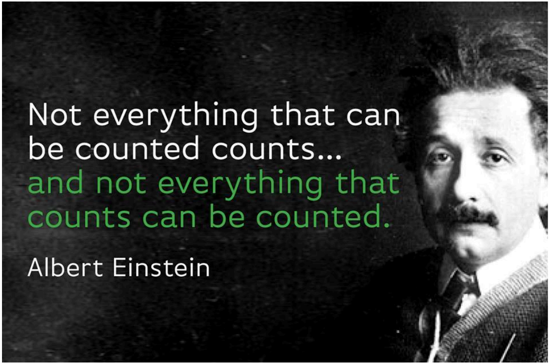
2-3 Counting

魏恒峰

hfwei@nju.edu.cn

2018 年 03 月 12 日


Not everything that can be counted counts...
and not everything that counts can be counted.
Albert Einstein



所以，学好“2-3 组合与计数”是多么重要!

Computing $\binom{n}{k}$ (CS $1.5:14$)

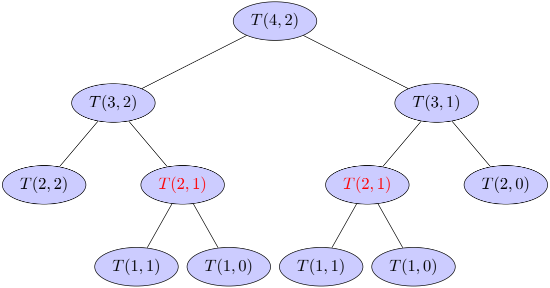
Algorithm 1 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                    ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

Computing $\binom{n}{k}$ (CS 1.5 : 14)

Algorithm 2 Computing $\binom{n}{k}$.

procedure $\textsc{Binom}(n, k)$ ▷ Required: $n \ge k \ge 0$
    if $k = 0 \lor n = k$ **then**
        return 1
    return $\textsc{Binom}(n-1, k) + \textsc{Binom}(n-1, k-1)$



Algorithm 3 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                              ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

Algorithm 4 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                          ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

(i) # of "$+$":

Algorithm 5 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                         ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

(i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

Algorithm 6 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                          ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

(i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

Algorithm 7 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                    ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

(i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of BINOM:

$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

Algorithm 8 Computing $\binom{n}{k}$.

```
procedure BINOM(n, k)                          ▷ Required: n ≥ k ≥ 0
    if k = 0 ∨ n = k then
        return 1
    return BINOM(n − 1, k) + BINOM(n − 1, k − 1)
```

(i) # of "+":

$$A(n,k) = 1 + A(n-1,k) + A(n-1,k-1)$$

(ii) # of recursive calls of $\textsc{Binom}$:

$$R(n,k) = 2 + R(n-1,k) + R(n-1,k-1)$$

$$\boxed{T(n,k) = T(n-1,k) + T(n-1,k-1) + c}$$

$$
\begin{array}{ccccccccccc}
 & & & & & \binom{0}{0} & & & & & \\
 & & & & \binom{1}{0} & & \binom{1}{1} & & & & \\
 & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & \\
 & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & \\
 & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}
$$

Algorithm 9 Computing $\binom{n}{k}$.

procedure $\textsc{Binom}(n, k)$ ▷ Required: $n \geq k \geq 0$

  for $i \leftarrow 0$ **to** n **do**

    $B[i][0] \leftarrow 1$

    $B[i][i] \leftarrow 1$

  for $i \leftarrow 2$ **to** n **do**

    for $j \leftarrow 1$ **to** k **do**

      $B[n][k] \leftarrow B[n-1][k] + B[n-1][k-1]$

  return $B[n][k]$


Thank You!