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# The Chinese Postman Problem

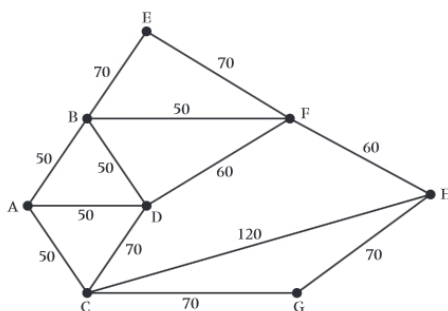
November 28, 2014 in [Real life maths](#) | Tags: [Chinese postman](#), [euler](#), [graph theory](#)



### The Chinese Postman Problem

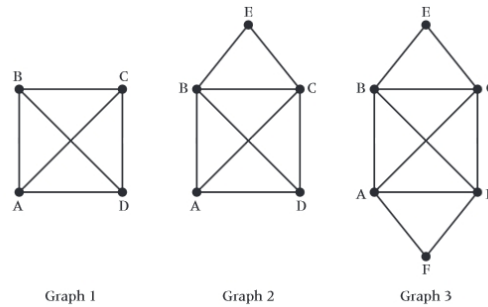
There is a fantastic [pdf resource](#) from Suffolk Maths which goes into a lot of detail on this topic – and I will base my post on their resource. Visit their site for a more in-depth treatment.

The Chinese Postman Problem was first posed by a Chinese mathematician in 1962. It involved trying to calculate how a postman could best choose his route so as to minimise his time. This is the problem that Kuan Mei-Ko tried to solve:



How could a postman visit every letter on the graph in the shortest possible time?

Solving this requires using a branch of mathematics called graph theory, created by Leonard Euler. This mathematics looks to reduce problems to network graphs like that shown above. Before we can solve this we need to understand some terminology:



Above we have 3 graphs. A graph which can be drawn without taking the pen off the paper or retracing any steps is called *traversable* (and has a Euler trail). Graph 1 is not traversable. Graph 2 is traversable as long as you start at either A or D, and Graph 3 is traversable from any point that you start. It turns out that what dictates whether a graph is traversable or not is the order of their vertices.

Looking at each letter we count the number of lines at the vertex. This is the order. For graph 1 we have 3 lines from A so A has an order of 3. All the vertices on graph 1 have an order of 3. For graph 2 we have the orders (from A, B, C, D, E in turn) 3, 4, 4, 3, 2. For graph 3 we have the orders 4, 4, 4, 4, 2, 2.

This allows us to arrive at a rule for working out if a graph is traversable.

*If all orders are even then a graph is traversable. If there are 2 odd vertices then we can find a traversable graph by starting at one of the odd vertices and finishing at the other. We need therefore to pair up any odd vertices on the graph.*

Next we need to understand how to pair the odd vertices. For example if I have 2 odd vertices, they can only be paired in one way. If I have 4 vertices (say A,B,C,D) they can be paired in 3 different ways (either AB and CD or AC and BD or AD and BC). The general term rule to calculate how many ways  $n$  odd vertices can be paired up is  $(n-1) \times (n-3) \times (n-5) \dots \times 1$ .

So now we are ready to actually solve the Chinese Postman Problem. Here is the algorithm:

To find a minimum Chinese postman route we must walk along each edge at least once and in addition we must also walk along the least pairings of odd vertices on one extra occasion.

An algorithm for finding an optimal Chinese postman route is:

**Step 1** List all odd vertices.

**Step 2** List all possible pairings of odd vertices.

**Step 3** For each pairing find the edges that connect the vertices with the minimum weight.

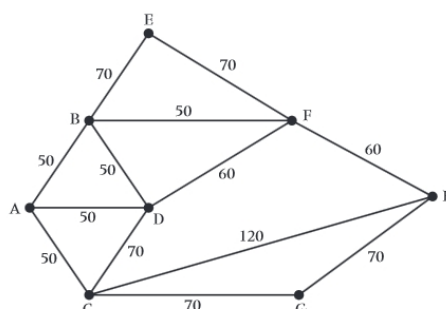
**Step 4** Find the pairings such that the sum of the weights is minimised.

**Step 5** On the original graph add the edges that have been found in Step 4.

**Step 6** The length of an optimal Chinese postman route is the sum of all the edges added to the total found in Step 4.

**Step 7** A route corresponding to this minimum weight can then be easily found.

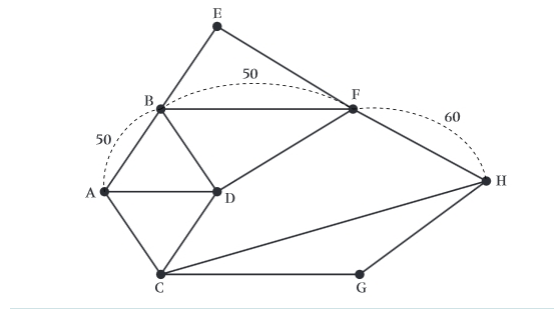
So, we can now apply this to the Chinese Postman Problem below:



Step 1: We can see that the only odd vertices are A and H.

Step 2: We can only pair these one way (AH)

Step 3 and 4: The shortest way to get from A to H is ABFH which is length 160. This is shown below:



Step 5 and 6: The total distance along all the lines in the initial diagram is 840m. We add our figure of 160m to this. Therefore the optimum (minimum) distance it is possible to complete the route is 1000m.

Step 7: We now need to find a route of distance 1000m which includes the loop ABFH (or in reverse) which starts and finishes at one of the odd vertices. One solution provided by Suffolk Maths is ADCGHCABDFBEFHFBA. There are others!

### The Bridges of Konigsburg



Graph theory was invented as a method to solve the [Bridges of Konigsburg](#) problem by Leonard Euler. This was a puzzle from the 1700s – Konigsburg was a Russian city with 7 bridges, and the question was, could anyone walk across all 7 without walking over any bridge twice. By simplifying the problem into one of connected lines, Euler was able to prove that this was in fact impossible.

If you like this post you might also like:

[Knight's Tour](#) – This puzzle dates over 1000 years and concerns the ways in which a knight can cover all squares on a chess board.

[Game Theory and Tic Tac Toe](#) – Tic Tac Toe has already been solved using Game Theory – this topic also brings in an introduction to Group Theory.

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## 5 comments

[Comments feed for this article](#)[September 2, 2015 at 10:44 pm](#)**Mr Creamer**

Very good as ever.

But Konigsburg was Prussian not

Russian for most of the 1700s (and all the time

before Euler proved the walk was impossible). It was Russian later for about 7 years and again from 1945.

This is a touchy point for my IB students in Warsaw 😊

[September 3, 2015 at 6:38 am](#)[Ibmathsresources.com](#)

Thanks for the clarification!

[October 4, 2015 at 1:13 pm](#)**Chanchal raj**

Thanks for the simple tutorial. The

chinese postman problem gets difficult

to solvr if there are more than 2 odd nodes.. It

wud really be nice of u, if u cud put an example of that case too..

[April 15, 2016 at 7:15 pm](#)**Bradley Rees**

Thanks friend, this was really helpful! I

hope that you are well and continue to

create such amazing examples, friend.



Your Friend,

Bradley 😊

[December 27, 2016 at 6:51 am](#)**Roger**

Hasn't someone made progress on this

recently? It's very similar to the

traveling salesman problem.



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