3-10 Traversability

(Part I: Eulerian Graphs)

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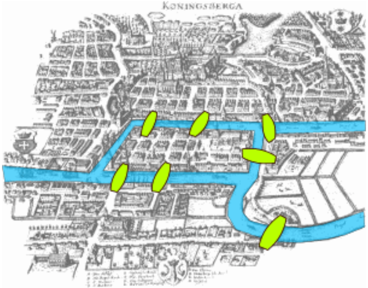
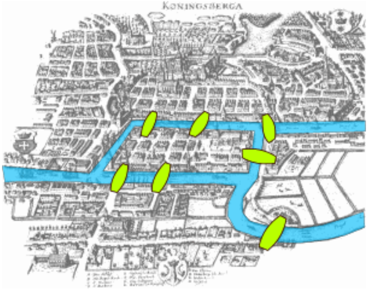


Leonhard Euler (1707 – 1783)



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Graph Theory Topology


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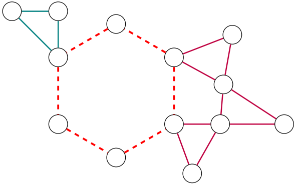

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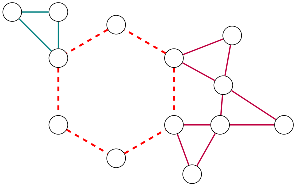

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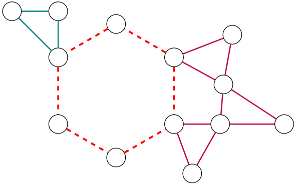

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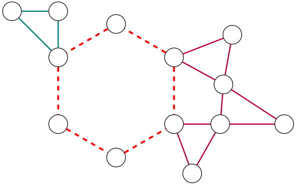

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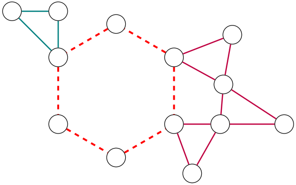

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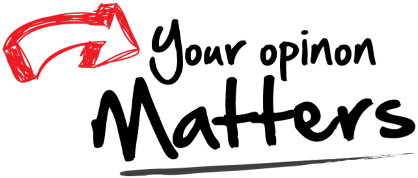
Combine each T_i with T to get an Eulerian circuit of G.

1: **procedure** HIERHOLZER(G)

procedure FLEURY(G, w, s)


THANK YOU



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