3-9 Connectivity

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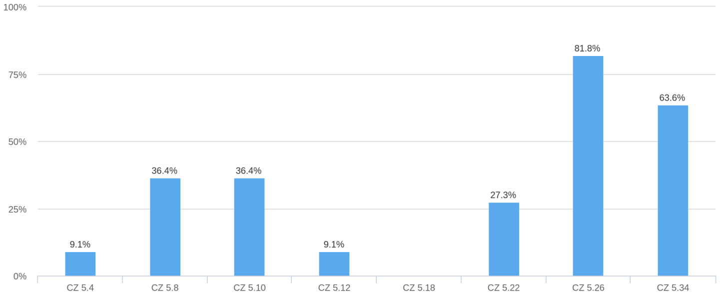
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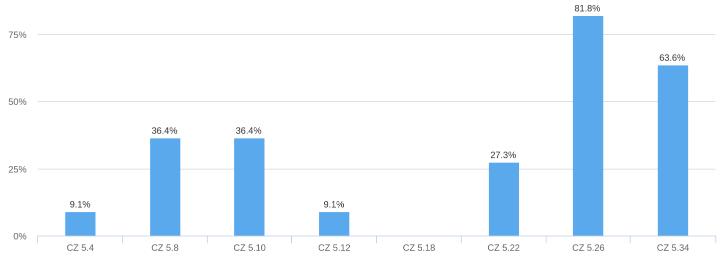
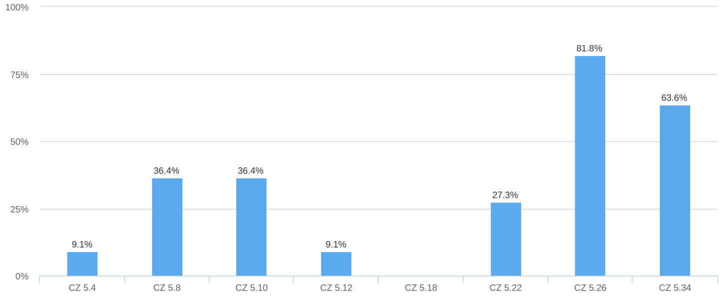


5.10 5.34 5.22 5.26



5.10 5.34 5.22 5.26

Menger's Theorem (Theorem 5.16; Theorem 5.21)

2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

Proof.

$$\text{“} \implies \text{”}$$

2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\Longleftrightarrow$$

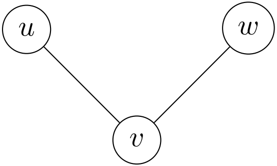
any two *adjacent* edges of G lie on a common cycle of G.

Proof.

"$\Longrightarrow$"

G is nonseparable

$\Longrightarrow$ u, w lie on a common cycle



□

2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

Proof.

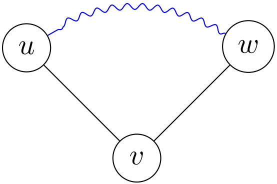
"$\implies$"

G is nonseparable

$\implies$ u, w lie on a common cycle

$\implies$ $\exists$ path $u \sim w$

$\implies$ $\exists$ cycle $u - v - w \sim u$



$\square$

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A connected graph G with $m \geq 2$ is *nonseparable*

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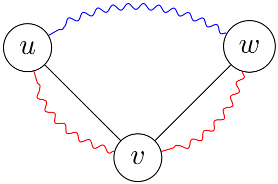

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A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

Proof.

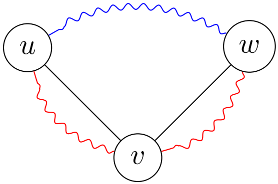
" $\implies$ "

G is nonseparable

$\implies$ u, w lie on a common cycle

$\implies$ $\exists$ path $u \sim w$ that does not contain v

$\implies$ $\exists$ cycle $u - v - w \sim u$



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2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

Proof.

"$\Longleftarrow$"

By Contradiction.

2-Connectivity (Problem 5.10)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two *adjacent* edges of G lie on a common cycle of G.

Proof.

"$\Longleftarrow$"

By Contradiction.

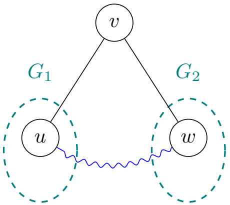
Suppose v is a cut-vertex of G

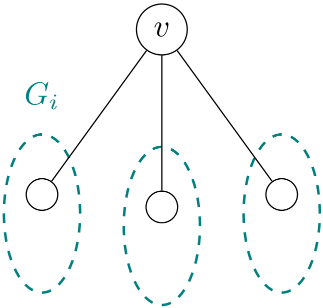
$\implies G - v$ contains ≥ 2 comps $G_1, G_2, \cdots$

$\implies \exists u \in G_1, w \in G_2 : v - u \wedge v - w$

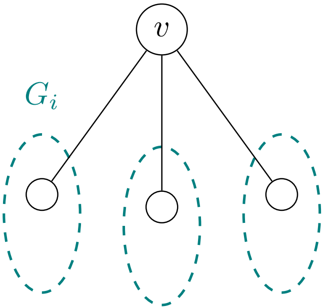
$\implies v - u, v - w$ lie on a common cycle

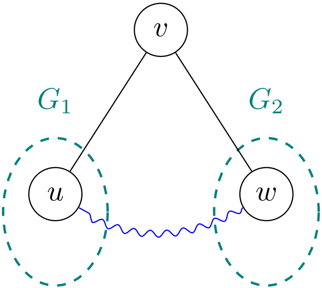
$\implies \exists$ path $u \sim w$ that does not contain v



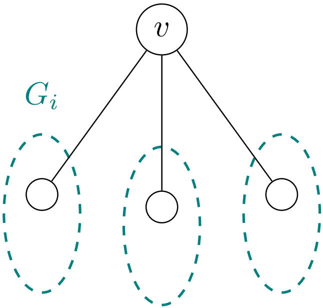


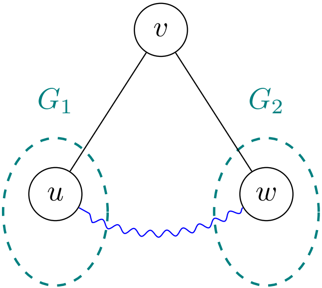
$$\forall G_i \; \exists v_i \in G_i \; v - v_i$$





$$\forall G_i \; \exists v_i \in G_i \; v - v_i$$





$\forall G_i \; \exists v_i \in G_i \; v - v_i$

$\color{red}{\forall v \in S} \; \forall G_i \; \exists v_i \in G_i \; v - v_i$

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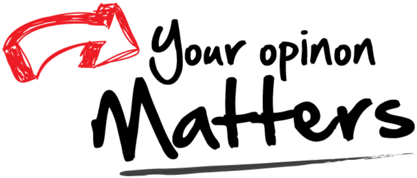
2-Connectivity (Extended Problem)

A connected graph G with $m \geq 2$ is *nonseparable*

$$\iff$$

any two edges of G lie on a common cycle of G.


THANK YOU



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