

3-1 Dynamic Programming

(Part II: “Theory”)

Hengfeng Wei

hfwei@nju.edu.cn

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Definition (Optimal Substructure)

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Relative to Subproblems

Rod Cutting



Optimal Substructure of Rod-Cutting (Problem 15.3-5)

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$$n = 4$$

length i	1	2	3	4
price p_i	1	1	1	1

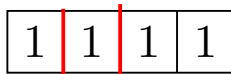
length i	1	2	3	4
limit l_i	2	1	1	1

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$$R(4) = 3$$

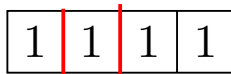
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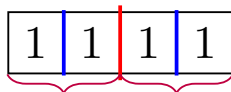
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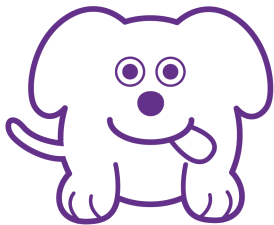


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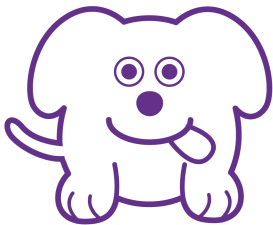
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$$R(2) = 2 \quad R(2) = 2$$

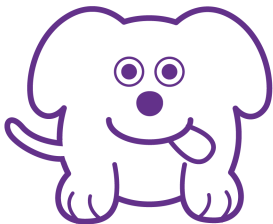


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Where is the leftmost cut?

$$R(i, L) = \max_{1 \leq j \leq i} \left(p_j + R(i - j, L[j \mapsto L_j - 1]) \right)$$





Office 302

Mailbox: H016

hfwei@nju.edu.cn