3-11 Matchings and Factors

(Part II: Perfect Matchings)

Hengfeng Wei

hfwei@nju.edu.cn

December 24, 2018

5.10 5.34 5.22 5.26

Chinese Postman Problem (CPP)

(Postman Tour Problem, Route Inspection Problem)

管梅谷(1934-)

第 10 卷 第 3 期 数 学 学 报 Vol. 10, No. 3
1960 年 12 月 ACTA MATHEMATICA SINICA Dec., 1960

奇 偶 点 图 上 作 业 法*

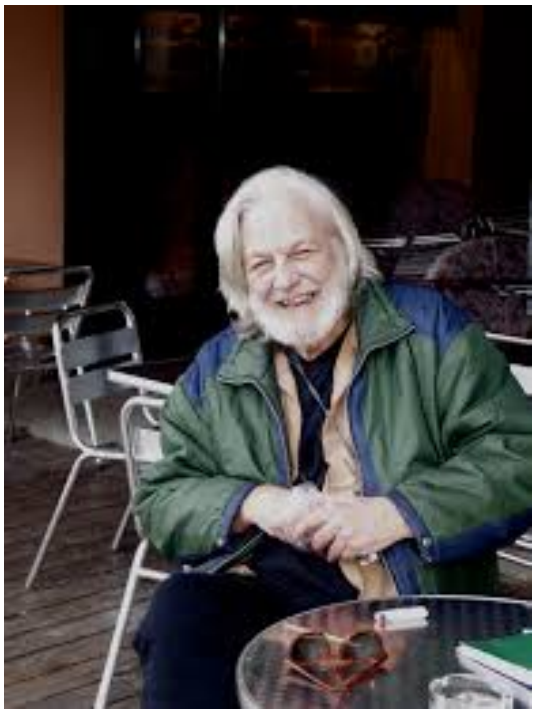
管 梅 谷
(山 东 师 范 学 院)

§1. 問 題 的 提 出

在邮局搞綫性規划时,发現了下述問題:"一个投递員每次上班,要走遍他負責送信的段[1],然后回到邮局. 問应該怎样走才能使所走的路程最短."

《奇偶点图上作业法》, 1960

Translated into English in 1962



Jack Edmonds (1934-)

MATCHING, EULER TOURS AND THE CHINESE POSTMAN

Jack EDMONDS
University of Waterloo, Waterloo, Ontario, Canada

and

Ellis L. JOHNSON
IBM Watson Research Center, Yorktown Heights, New York, U.S.A.

Received 20 May 1972
Revised manuscript received 3 April 1973

The solution of the Chinese postman problem using matching theory is given. The convex hull of integer solutions is described as a linear programming polyhedron. This polyhedron is used to show that a good algorithm gives an optimum solution. The algorithm is a specialization of the more general *b*-matching blossom algorithm. Algorithms for finding Euler tours and related problems are also discussed.

"Matching, Euler Tours and the Chinese Postman", 1973

Definition (Chinese Postman Problem)

Given an undirected weighted graph G with $w(e) > 0$,

to find the shortest tour such that each edge is traversed at least once.

Definition (Chinese Postman Problem)

Given an undirected weighted graph G with $w(e) > 0$,

to find the shortest tour such that each edge is traversed at least once.

Q : What is the relation between Postman Tour and Eulerian Tour?

P: A postman tour of G

P: A postman tour of G

P contains every edge e at least once.

P: A postman tour of G

P contains every edge e at least once.

Let $1 + x_e$ be the number of times edge e is in P.

P: A postman tour of G

P contains every edge e at least once.

Let $1 + x_e$ be the number of times edge e is in P.

Construct $G' = G + e \cdot x_e$

P: A postman tour of G

P contains every edge e at least once.

Let $1 + x_e$ be the number of times edge e is in P.

Construct $G' = G + e \cdot x_e$

P is an Eulerian tour of G'.

P: A postman tour of G

P contains every edge e at least once.

Let $1 + x_e$ be the number of times edge e is in P.

Construct $G' = G + e \cdot x_e$

P is an Eulerian tour of G'.

Definition (Chinese Postman Problem)

Given an undirected weighted graph with $w(e) > 0$,

to find the subset of edges

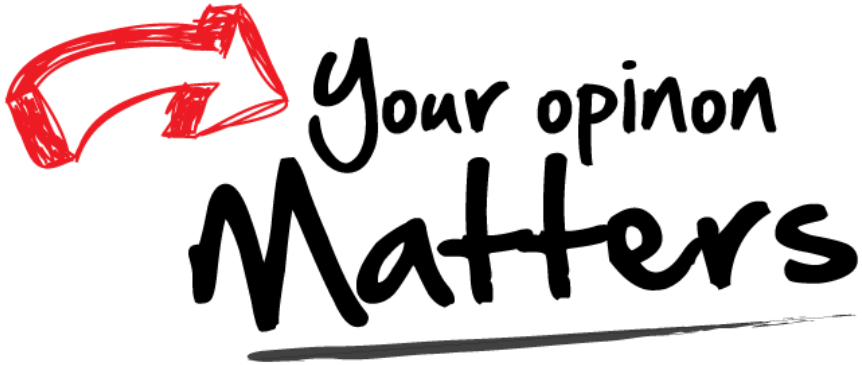
1: $V_o \leftarrow \{v : \deg(v) \text{ is odd}\}$ ▷ Collect all odd vertices

2: Construct a complete weighted graph G_p with vertices V_o:

3: $\quad \forall u, v \in V_o : w(u, v) \leftarrow$ the shortest path between u and v

4: Find a minimum-weighted perfect matching M of G_p


THANK YOU



Office 302

Mailbox: H016

hfwei@nju.edu.cn