What We Talk About When We Talk About Isomorphism Theorems

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Q : Do isomorphic groups behave exactly the same?

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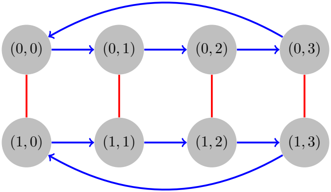

$K = \{(0,0),(0,2)\}$

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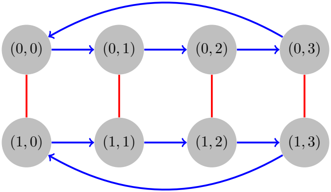

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Problem 9.3-23

$$G \times H \cong H \times K \Longrightarrow G \cong K$$

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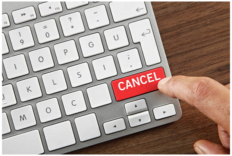

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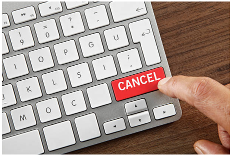

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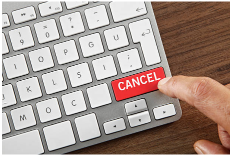

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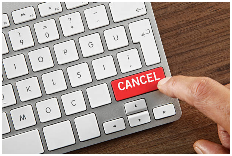

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"On Cancellation in Groups" by R. Hirshon, 1969

$$G \times H \cong H \times K, \ |K| < \infty \Longrightarrow G \cong K$$

Problem 11.4-17

$$\phi : G_1 \to G_2 \text{ is a surjective group homomorphism.}$$

$$H_1 \lhd G_1, \quad \phi(H_1) = H_2 \implies G_1/H_1 \cong G_2/H_2$$

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$$G_1 = \mathbb{Z}_2, \quad G_2 = \{e\}\,, \quad H_1 = \{0\}\,, \quad H_2 = \{e\}$$

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Find all homomorphisms from $\mathbb{Z}_{24}$ to $\mathbb{Z}_{18}$.

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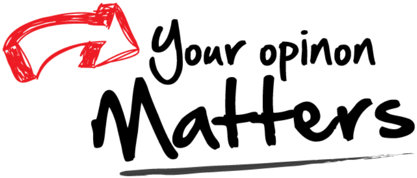
$$\phi(1) = 0, 9, 6, 12, 3, 15$$


THANK YOU



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