Graph Theory (21-484) – Spring 2002

Fleury's Algorithm

I apologize profusely for not being able to finish the Fleury's Algorithm proof. Here it is in an nice clean summary:

Algorithm (Fleury). The algorithm is:

- Begin with a vertex v.
- Having constructed $W = ve_1v_1 \cdots e_iv_i$, let $G_i = G - \{e_1, \ldots, e_i\}$ and choose e_{i+1} so that:
  1. e_{i+1} is incident to v_i.
  2. e_{i+1} is not a cut-edge of G_i unless there is no other choice.

Theorem. Fleury's algorithm produces a Euler tour in every eulerian graph.

Proof. Let G be eulerian. Let $W = v_0e_1v_1 \cdots e_nv_n$ be a trail chosen in G via Fleury's algorithm until no new edges can be chosen. Suppose W is not an Euler tour.

Let $G_i = G - \{e_1, \ldots, e_i\}$. Let $S = \{v : \deg_{G_n}(v) > 0\}$. Then, $v_n \in \overline{S}$ and $v_n = v_0$.

Because no edges are left to be chosen and the degrees of G are even.

$S \neq \emptyset$.

Because W is not an Euler tour.

Let v_m be the last vertex in W to be in S. e_{m+1} is a cut-edge of G_m.

Each vertex in $\overline{S}$ has edges incident to it in W. So, each edge in $[S, \overline{S}]$ is in W. Furthermore, since each e_j has both endvertices in $\overline{S}$ for $j > m$, then e_{i+1} is the only edge of G_m in $[S, \overline{S}]$.

$\exists e \in E\left(G_m[S]\right)$ incident to v_m that is a cut-edge of G_m.

Because $\deg_{G_n}(v_m) > 0$, there is some edge e of G_n incident to v_m. But, since e_{m+1} is a cut-edge of G_m, e must have both endvertices in S.

$G_m[S] = G_n[S]$.

G_n differs from G_m by removing e_{m+1} plus possibly some vertices with both endvertices in $\overline{S}$.

$G_m[S]$ has all vertices of even degree.

Clearly, every vertex of G_n must have even degree. Since G_n contains no edges in $[S, \overline{S}]$, $\deg_{G_n[S]}(v)$ is even for every $v \in S$. Thus, $\deg_{G_n[S]}(v) = \deg_{G_n}(v)$ for all $v \in S$.

No graph with all vertex degrees even can have a cut-edge.

This is an exercise.

Thus, $G_m[S]$ cannot have a cut edge, a contradiction to the existence of e. $\square$