

第 3 讲: 常用的证明方法

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评分: _____ 评阅: _____

2019 年 10 月 14 日

请独立完成作业, 不得抄袭。
若得到他人帮助, 请致谢。
若参考了其它资料, 请给出引用。
鼓励讨论, 但需独立书写解题过程。

- 反证法是你最好的朋友
- 数学归纳法是你最最好的朋友
- 鸽笼原理, 哦, 有点高冷, 这个朋友不好交
(看似具体, 实则抽象; 看似容易, 实则困难)

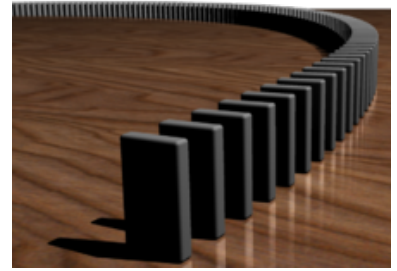


图 1: 数学归纳法的“多米诺骨牌效应”

1 作业 (必做部分)

题目 1 (UD Problem 5.12: $3k + 2$)

解答:

题目 2 (UD Problem 5.24: Squaring)

解答:

题目 3 (Primes 3 (Mod 4) Theorem)

请证明: There are infinitely many primes that are congruent to 3 modulo 4.

解答:

题目 4 (改编自 UD Problem 18.20 与 UD Problem 18.26)

请证明:

- (1) “The first principle of mathematical induction” (Theorem 18.1) 与 “The second principle of mathematical induction” (Theorem 18.9) 等价。
- (2) “The second principle of mathematical induction” 蕴含 “Well-ordering principles of the natural numbers” (in Chapter 12)。

解答:

题目 5 (Lines in the Plane)

- (1) What is the maximum number L_n of regions determined by n straight lines in the plane?

(注: 直线两端可以无限延长)

- (2) What is the maximum number Z_n of regions determined by n bent lines, each containing one “zig”, in the plane?

(注: 两端可以无限延长)

- (3) What's the maximum number ZZ_n of regions determined by n “zig-zag” lines in the plane?

(注: 两端可以无限延长)

解答:

题目 6 (ES Problem 24.4: Distance in Square)

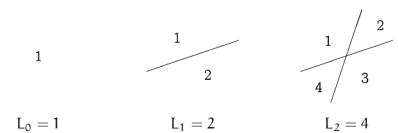
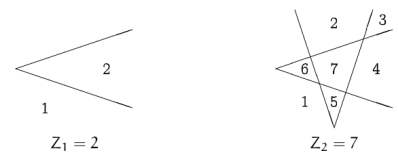
解答:

题目 7 (ES Problem 24.6: Lattice Points)

解答:

题目 8 (ES Problem 24.7: Monotone Subsequence)

解答:

图 2: Examples for L_0 , L_1 , and L_2 .图 3: Examples for Z_1 and Z_2 .图 4: Example for ZZ_2 .

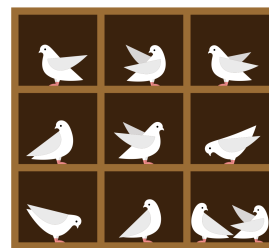
2 作业 (选做部分)

题目 9 (Numbers)

Suppose $A \subseteq \{1, 2, \dots, 2n\}$ with $|A| = n + 1$. Please prove that:

- (1) There are two numbers in A which are relatively prime (互素).
- (2) There are two numbers in A such that one divides (整除) the other.

解答:



3 Open Topics

Open Topics 1 (Coq)

请介绍如何在 Coq 中使用数学归纳法。

参考资料:

- [Induction.v](#)

解答:

Open Topics 2 (Double Counting)

“Double Counting” 是一种神奇、漂亮的组合证明技巧。请了解 Double Counting 并以 “Counting Trees” 为例介绍这种证明技巧。

参考资料:

- 电影 “Good Will Hunting” (心灵捕手)
- Chapter 30 “Cayley’s formula for the number of trees” of “Proofs from THE BOOK” (Fourth Edition)
- [Counting trees @ wiki](#)

解答:



图 5: 电影《心灵捕手》截图

4 订正

5 反馈

你可以写 ^① :

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- ...

^① 优先推荐 [ProblemOverflow](#)