

## 第 3 讲: 常用的证明方法

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评分: \_\_\_\_\_ 评阅: \_\_\_\_\_

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请独立完成作业, 不得抄袭。  
若得到他人帮助, 请致谢。  
若参考了其它资料, 请给出引用。  
鼓励讨论, 但需独立书写解题过程。

- 反证法是你最好的朋友
- 数学归纳法是你最最好的朋友
- 鸽笼原理, 哦, 有点高冷, 这个朋友不好交  
(看似具体, 实则抽象; 看似容易, 实则困难)

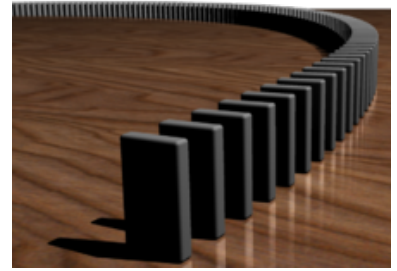


图 1: 数学归纳法的“多米诺骨牌效应”

## 1 作业 (必做部分)

题目 (UD Problem 5.12:  $3k + 2$ )

解答:

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题目 (UD Problem 5.24: Squaring)

解答:

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题目 (Primes 3 (Mod 4) Theorem)

请证明: There are infinitely many primes that are congruent to 3 modulo 4.

解答:

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**题目 (改编自 UD Problem 18.20 与 UD Problem 18.26)**

请证明:

- (1) “The first principle of mathematical induction” (Theorem 18.1) 与 “The second principle of mathematical induction” (Theorem 18.9) 等价。
- (2) “The second principle of mathematical induction” 蕴含 “Well-ordering principles of the natural numbers” (in Chapter 12)。

解答:

**题目 (Lines in the Plane)**

- (1) What is the maximum number  $L_n$  of regions determined by  $n$  straight lines in the plane?

(注: 直线两端可以无限延长)

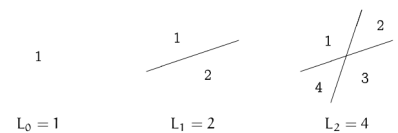
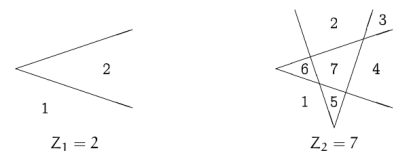
- (2) What is the maximum number  $Z_n$  of regions determined by  $n$  bent lines, each containing one “zig”, in the plane?

(注: 两端可以无限延长)

- (3) What's the maximum number  $ZZ_n$  of regions determined by  $n$  “zig-zag” lines in the plane?

(注: 两端可以无限延长)

解答:

图 2: Examples for  $L_0$ ,  $L_1$ , and  $L_2$ .图 3: Examples for  $Z_1$  and  $Z_2$ .图 4: Example for  $ZZ_2$ .**题目 (ES Problem 24.4: Distance in Square)**

解答:

**题目 (ES Problem 24.6: Lattice Points)**

解答:

**题目 (ES Problem 24.7: Monotone Subsequence)**

解答:

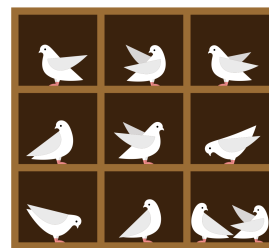
## 2 作业 (选做部分)

### 题目 (Numbers)

Suppose  $A \subseteq \{1, 2, \dots, 2n\}$  with  $|A| = n + 1$ . Please prove that:

- (1) There are two numbers in  $A$  which are relatively prime (互素).
- (2) There are two numbers in  $A$  such that one divides (整除) the other.

解答:



## 3 Open Topics

### Open Topics 1 (Coq)

请介绍如何在 Coq 中使用数学归纳法。

参考资料:

- [Induction.v](#)

解答:

### Open Topics 2 (Double Counting)

“Double Counting” 是一种神奇、漂亮的组合证明技巧。请了解 Double Counting 并以 “Counting Trees” 为例介绍这种证明技巧。

参考资料:

- 电影 “Good Will Hunting” (心灵捕手)
- Chapter 30 “Cayley’s formula for the number of trees” of “Proofs from THE BOOK” (Fourth Edition)
- [Counting trees @ wiki](#)

解答:



图 5: 电影《心灵捕手》截图

## 4 订正

## 5 反馈

你可以写 <sup>①</sup> :

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- ...

<sup>①</sup> 优先推荐 [ProblemOverflow](#)