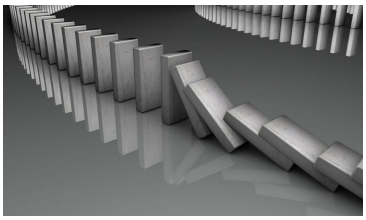
第 3 讲: 常用的证明方法

姓名: 魏恒峰　　**学号:** hfwei@nju.edu.cn

评分: _____　　**评阅:** _____

2019 年 10 月 5 日

请独立完成作业，不得抄袭。
若得到他人帮助, 请致谢。
若参考了其它资料，请给出引用。
鼓励讨论，但需独立书写解题过程。



- 数学归纳法是你最好的朋友
- 反证法也是你最好的朋友
- 鸽笼原理, 哦, 有点高冷, 这个朋友不好交

1　作业 (必做部分)

题目 (UD Problem 5.12: $3k+2$)

解答:

题目 (UD Problem 5.24: Squaring)

解答:

题目 (Primes 3 (Mod 4) Theorem)
请证明: There are infinitely many primes that are congruent to 3 modulo 4.

解答:

题目 (改编自 UD Problem 18.20 与 UD Problem 18.26)
请证明:

(1) "The first principle of mathematical induction" (Theorem 18.1) 与 "The second principle of mathematical induction" (Theorem 18.9) 等价。

(2) "The second principle of mathematical induction" 蕴含 "Well-ordering principles of the natural numbers" (in Chapter 12)。

解答:

题目 (UD Problem 18.25 (c, d): Binomial)

解答:

题目 (Lines in the Plane)

(1) What is the maximum number L_n of regions determined by n straight lines in the plane?

(注: 直线两端可以无限延长)

(2) What is the maximum number Z_n of regions determined by n bent lines, each containing one "zig", in the plane?

(注: 两端可以无限延长)

(3) What's the maximum number ZZ_n of regions determined by n "zig-zag" lines in the plane?

(注: 两端可以无限延长)

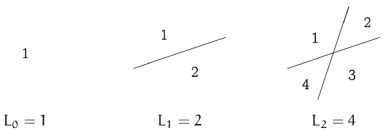


图 1: Examples for L_0, L_1, and L_2.

图 2: Examples for Z_1 and Z_2.

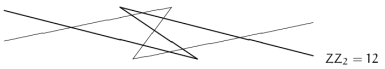


图 3: Example for ZZ_2.

解答:

题目 (ES Problem 24.4: Distance in Square)

解答:

题目 (ES Problem 24.6: Lattice Points)

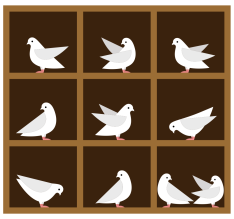
解答:

题目 (ES Problem 24.7: Monotone Subsequence)

解答:

---

2 作业 (选做部分)



题目 (Numbers)
Suppose $A \subseteq \{1, 2, \cdots, 2n\}$ with $|A| = n + 1$. Please prove that:

(1) There are two numbers in A which are relatively prime (互素).

(2) There are two numbers in A such that one divides (整除) the other.

解答:

---

3 Open Topics

Open Topics 1 (Coq)
请介绍如何在 Coq 中使用数学归纳法。
参考资料:

- Induction.v

解答:

Open Topics 2 (Double Counting)
"Double Counting" 是一种神奇、漂亮的组合证明技巧。请了解 Double Counting 并以 "Counting Trees" 为例介绍这种证明技巧。
参考资料:

- 电影 "Good Will Hunting" (心灵捕手)
- Chapter 30 "Cayley's formula for the number of trees" of "Proofs from THE BOOK" (Fourth Edition)
- Counting trees @ wiki

解答:

---

4 订正

5 反馈

你可以写 ①：

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- ···

① 优先推荐 ProblemOverflow