1-8 Set Theory: Axioms and Operations

魏恒峰

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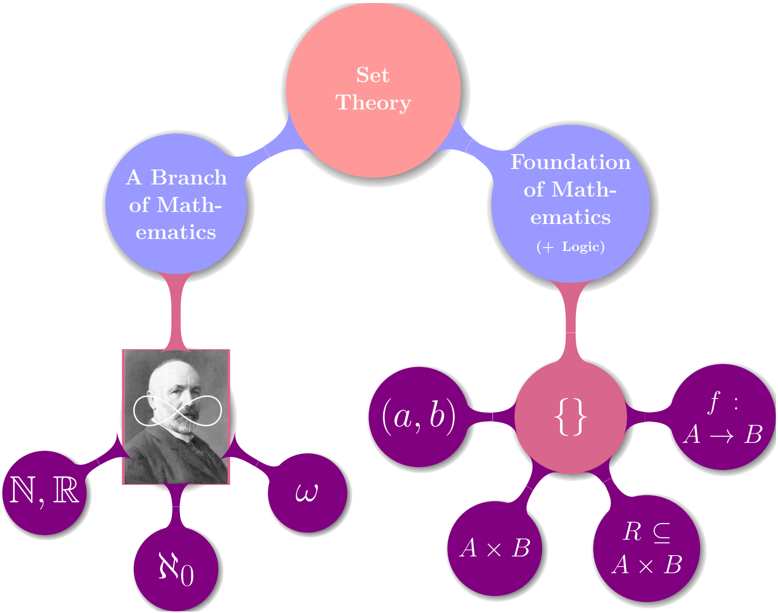
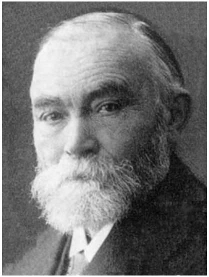
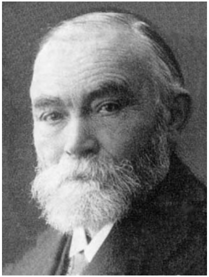

Set Theory
A Branch of Math-ematics
Foundation of Math-ematics
(+ Logic)
$\mathbb{N}, \mathbb{R}$
$\aleph_0$
ω
(a, b)
$\{\}$
$f : A \to B$
$A \times B$
$R \subseteq A \times B$



Gottlob Frege (1848–1925)



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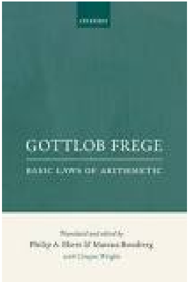
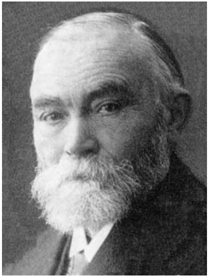


“Basic Laws of Arithmetic”



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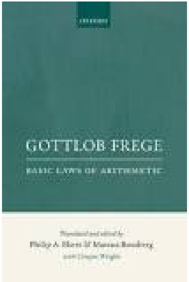
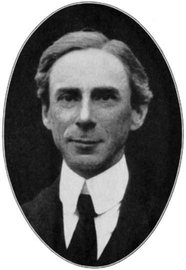
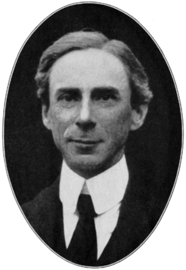


“Basic Laws of Arithmetic”

对于一个科学工作者来说，最不幸的事情莫过于：当他的工作接近完成时，却发现那大厦的基础已经动摇。 – 《附录二》



Bertrand Russell (1872–1970)



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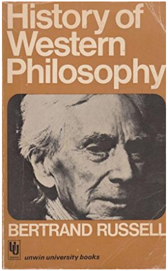

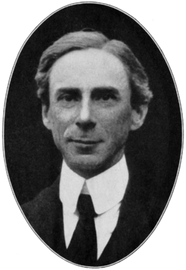

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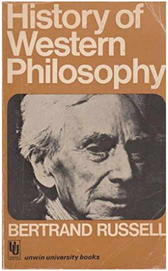

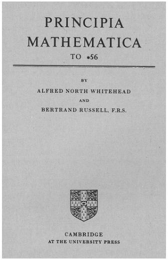

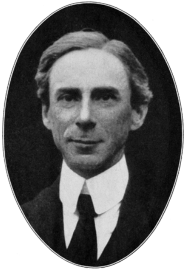

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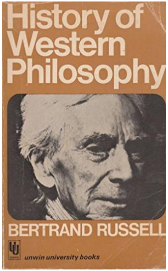

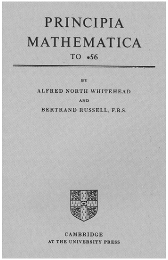
PRINCIPIA MATHEMATICA
TO *56

BY
ALFRED NORTH WHITEHEAD
AND
BERTRAND RUSSELL, F.R.S.

CAMBRIDGE
AT THE UNIVERSITY PRESS

我们将集合理解为任何将我们思想中那些确定而彼此独立的对象放在一起而形成的聚合。

— *Georg Cantor*《超穷数理论基础》

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Theorem (概括原则)

$$\forall \psi(x)\ \exists X : X = \{x \mid \psi(x)\}.$$

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Definition (Russell's Paradox)

$$\psi(x) = \text{“}x \notin x\text{”}$$

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$$R = \{x \mid x \notin x\}$$

$$Q : R \in R\ ?$$

Q : 既然朴素集合论存在悖论，你是如何做作业的?


YOU HAVE
TO FIX IT!

Theorem

$\{x \mid x \notin x\}$ **is not a set.**

Axiomatic Set Theory (ZFC)

Ernst Zermelo (1871–1953)

Abraham Fraenkel (1891–1965)

First-order Language

Parentheses: $(,)$

Variables: $x, y, z, \cdots$

Connectives: $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$

Quantifiers: $\forall$, $\exists$

Equality: $=$

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Constants:	$a, b, c, \cdots$
Functions:	$f, g, h, \cdots$
Predicates:	$R, P, Q, \cdots$

First-order Language for Sets $\mathcal{L}_{Set}$

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Quantifiers: $\forall, \exists$

Equality: $=$

Constants:

Functions:

Predicates: $\in$

Everything we consider in $\mathcal{L}_{Set}$ is a set.

Q : What is "$\in$"?

Q : What are "sets"?

We don't define them directly.

We only describe their properties in an axiomatic way.

1. To draw a straight line from any point to any point.
2. To extend a finite straight line continuously in a straight line.
3. To describe a circle with any center and radius.
4. That all right angles are equal to one another.
5. The parallel postulate.

$$E, E; P, U, R, P; I, C; F$$

Definition ($\notin$)

$$x \notin A \triangleq \neg(x \in A).$$

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Definition ($\subseteq, \subset$)

$$A \subseteq B \triangleq \forall x(x \in A \implies x \in B)$$

$$A \subset B \triangleq A \subseteq B \land A \neq B$$

Axiom (Axiom of Extensionality)

If two sets have exactly the same members, then they are equal.

$$\forall A\,\forall B\,\big(\forall x(x \in A \iff x \in B)\big) \implies A = B.$$

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$$\forall A\, \forall B\, \big(\forall x (x \in A \iff x \in B)\big) \iff A = B.$$

Axiom (Empty Set Axiom)

There is a set having no members:

$$\exists B\ \forall x(x \notin B).$$

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Proof.

By the Axiom of Extensionality. $\square$

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Proof.

By the Axiom of Extensionality. □

Definition ($\emptyset$)

$$\emptyset \triangleq \text{ the unique unique empty set.}$$

Axiom (Paring Axiom)

For any sets x and y, there is a set having as members just x and y:

$$\forall x\, \forall y\, \exists B\, \big(\forall z (z \in B \iff z = x \lor z = y)\big).$$

Axiom (Paring Axiom)

For any sets x and y, there is a set having as members just x and y:

$$\forall x\, \forall y\, \exists B\, (\forall z (z \in B \iff z = x \lor z = y)).$$

Definition ("$\{x, y\}$")

$$\{x, y\} \triangleq \text{ the unique set obtained by paring } x \text{ and } y.$$

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$$\{x, y\} = \{y, x\}.$$

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Definition ($\{x\}$)

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Axiom (Union Axiom (Simplified Version))

For any sets x and y, there is a set whose members are the elements belonging either to x or to y (or both):

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Definition ($x \cup y$)

$x \cup y \triangleq$ the unique set obtained by unioning x and y.

Definition ("$\{x, y\}$")

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Definition ($\{x, y, z\}$)

$$\{x, y, z\} \triangleq \{x, y\} \cup \{z\}.$$

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We can use pairing and union together to form finite sets.

Axiom (Union Axiom (Extended Version))

For any set A, there is a set B whose elements are the members of the members of A:

$$\forall x\big(x \in B \iff \exists y \in A(x \in y)\big).$$

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Theorem

$$\bigcup \emptyset = \emptyset.$$

Axiom (Replacement Axioms (Simplified Version; Subset Axioms; Separation Axioms))

Let $\psi(u)$ be a predicate. For any set u, there is a set B which is a subset of u such that each element x of B satisfies $\psi(x)$:

$$\forall u \, \exists B \, (\forall x (x \in B \iff x \in u \land \psi(x))).$$

Definition ($\{x \in u \mid \psi(x)\}$)

$$\{x \in u \mid \psi(x)\} \triangleq \text{ the unique set obtained by separating } u \text{ with } \psi.$$

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$$\{x \in u \mid \psi(x)\} \triangleq \text{ the unique set obtained by separating } u \text{ with } \psi.$$

Definition ($u \cap v$)

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Theorem

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Theorem

Definition ($u \setminus v$)

$$u \setminus v \triangleq \{x \in u \mid x \notin v\}.$$

Theorem (No Universal Set)

There is no universal set.

$$\nexists B(\forall x(x \in B)).$$

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Proof.

For any set A, we construct a set not in A.

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For any set A, we construct a set not in A.

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$$B \in B \iff B \in A \land B \notin B$$

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For any set A, we construct a set not in A.

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$$\boxed{B \notin A}$$

$$B \in B \iff B \in A \land B \notin B$$

$$B \in A \implies (B \in B \iff B \notin B)$$

□

$$\psi(x) = x \notin x$$

Russell

Theorem

Definition

Definition ("∩")

$$\begin{aligned} A \cap B &= \{x \in A \mid x \in B\} \\ &= \{x \mid x \in A \land x \in B\} \end{aligned}$$

Definition ("\\")

$$\begin{aligned} A \setminus B &= \{x \in A \mid x \notin B\} \\ &= \{x \mid x \in A \land x \notin B\} \end{aligned}$$

Definition (“$\cap$”)

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Definition (“$\setminus$”)

$$\begin{aligned} A \setminus B &= \{x \in A \mid x \notin B\} \\ &= \{x \mid x \in A \land x \notin B\} \end{aligned}$$

> *We can never look for objects “not in B” unless we know where to start looking. So we use A to tell us where to look for elements not in B.* – *UD (Chapter 6)*

Set Operations

$\cap \qquad \cup \qquad \setminus$

UD 7.1 (b): Distributive Property

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Theorem (Distributive Property (Theorem 7.1))

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proof.

If $x \in A \cup (B \cap C)$, then $x \in A$ or $x \in B \cap C$. Suppose first that $x \in A$. Then $x \in A \cup B$ and $x \in A \cup C$. In this first case, we see that $x \in (A \cup B) \cap (A \cup C)$. Now suppose that $x \in B \cap C$. Then $x \in B$ and $x \in C$. Since $x \in B$, we see that $x \in A \cup B$. Since we also have $x \in C$, we see that $x \in A \cup C$. Therefore, $x \in (A \cup B) \cap (A \cup C)$ in this case as well. In either case $x \in (A \cup B) \cap (A \cup C)$ and we may conclude that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

To complete the proof, we must now show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. So if $x \in (A \cup B) \cap (A \cup C)$, then $x \in A \cup B$ and $x \in A \cup C$. It is, once again, helpful to break this into two cases, since we know that either $x \in A$ or $x \notin A$. Now if $x \in A$, then $x \in A \cup (B \cap C)$. If $x \notin A$, then the fact that $x \in A \cup B$ implies that x must be in B. Similarly, the fact that $x \in A \cup C$ implies that x must be in C. Therefore, $x \in B \cap C$. Hence $x \in A \cup (B \cap C)$. In either case $x \in A \cup (B \cap C)$ and we may conclude that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

Since we proved containment in both directions we may conclude that the two sets are equal. ■

UD 7.1 (c): DeMorgan's Law

Let X denote a set, and $A, B \subseteq X$.

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$$

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$$Q : A, B \subseteq X?$$

UD 7.1(d)

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$Q : A, B \subseteq X?$

UD 7.1(d)

Let X denote a set, and $A, B \subseteq X$.

$$A \subseteq B \iff (X \setminus B) \subseteq (X \setminus A)$$

For any given $x, \cdots$

$Q: A, B \subseteq X?$ (“ $\Leftarrow: X = \emptyset$”)

Equivalence: UD 7.8

Consider the following sets:

(i) $(A \cap B) \setminus (A \cap B \cap C)$

(ii) $A \cap B \setminus (A \cap B \cap C)$

(iii) $A \cap B \cap C^c$

(iv) $(A \cap B) \setminus C$

(v) $(A \setminus C) \cap (B \setminus C)$

(a) Which of the sets above are written ambiguously, if any?

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$$\text{(ii)} : (A \cap B) \setminus (A \cap B \cap C) \ \textit{vs.}\ A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

Equivalence: UD 7.8

Consider the following sets:

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$$\text{(ii)} : (A \cap B) \setminus (A \cap B \cap C) \ \textit{vs.} \ A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

$$\text{(none)} : (A \cap B) \setminus (A \cap B \cap C) = A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

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$$\text{(none)} : (A \cap B) \setminus (A \cap B \cap C) = A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

(none): from the left to the right

Equivalence: UD 7.8

Consider the following sets:

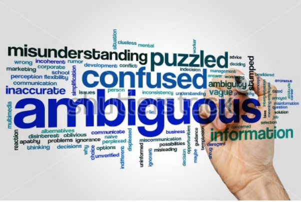
(i) $(A \cap B) \setminus (A \cap B \cap C)$

(ii) $A \cap B \setminus (A \cap B \cap C)$

(iii) $A \cap B \cap C^c$

(iv) $(A \cap B) \setminus C$

(v) $(A \setminus C) \cap (B \setminus C)$



(a) Which of the sets above are written ambiguously, if any?

$$\text{(ii)} : (A \cap B) \setminus (A \cap B \cap C) \ \textit{vs.}\ A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

$$\text{(none)} : (A \cap B) \setminus (A \cap B \cap C) = A \cap \Big(B \setminus (A \cap B \cap C)\Big)$$

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(c) Prove that $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$.

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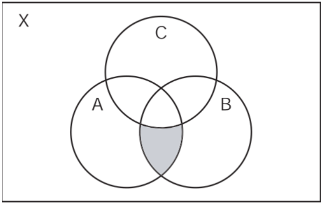

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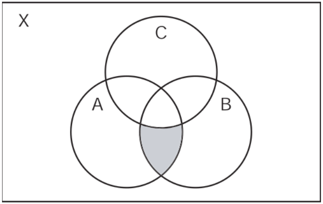

(c) Prove that $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$.

$$A \setminus C = \{x \mid x \in A \land x \notin C\}$$

Equivalence: UD 7.8

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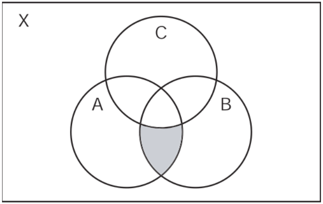

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UD 7.9

Prove that the union of two sets can be rewritten as the union of two disjoint sets.

(a) Prove that $(A \setminus B) \cap B = \emptyset$

(b) Prove that $A \cup B = (A \setminus B) \cup B$

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By contradiction.

$(A \setminus B) \cup B = \cdots$

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Thank You!