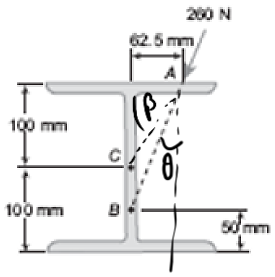
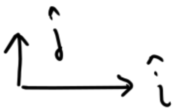
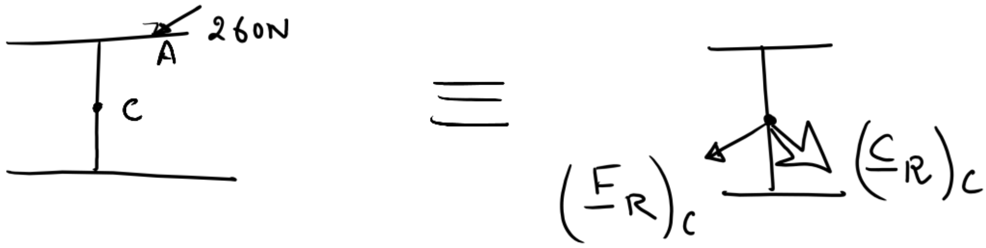
Set 4A & B



PROBLEM 3.81

A 260-N force is applied at A to the rolled-steel section shown. Replace that force with an equivalent force-couple system at the center C of the section.





1^{st} condition of equivalence:

$$(\underline{F}_R)_C = -260\sin\theta\,\hat{i} \cancel{+} - 260\cos\theta\,\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{62.5}{150}\right) = 0.395 \text{ rad}$$

$$(\underline{F}_R)_C = -100\,\hat{i} - 239.98\,\hat{j} \text{ N.}$$

2nd condition of equivalence:

Match $\underline{M}_C = \underline{M}_C'$

$$\underline{r}_{AC} \times \underline{F} = (\underline{C}_R)_C$$

$$\underline{r}_{AC} = \sqrt{62.5^2 + 100^2}\,[\cos\beta\,\hat{i} + \sin\beta\,\hat{j}]$$

$$= .0625\,\hat{i} + .1\,\hat{j} \quad (m)$$

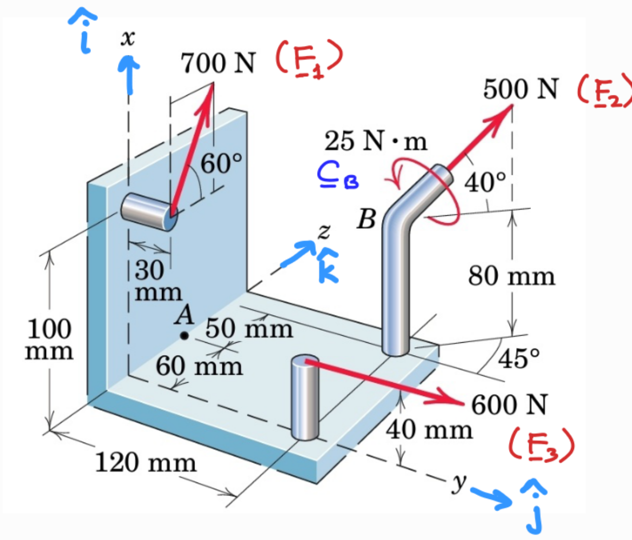
$$(\underline{C}_R)_C = (.0625\,\hat{i} + .1\,\hat{j}) \times (-100\,\hat{i} - 240\,\hat{j})$$

$$= -.0625 \times 240\,\hat{k} + 10\,\hat{k}$$

$$(\underline{C}_R)_C = -5 \text{ Nm } \hat{k}$$

Ca

2) Find the equivalent force system at point A



The given system has three forces and a couple. First, we need to write the forces and the couple in terms of its components.

$$\underline{F}_1 = 700 \sin 60^\circ \,\hat{i} + 700 \cos 60^\circ \,\hat{k} \quad (N)$$

$$= 606.22 \,\hat{i} + 350 \,\hat{k} \;(N)$$

$$\underline{F}_2 = 500 \sin 40^\circ \,\hat{i} + 500 \cos 40^\circ \cos 45^\circ \,\hat{j} + 500 \cos 40^\circ \sin 45^\circ \,\hat{k}$$

$$= 321 \,\hat{i} + 270.84 \,\hat{j} + 270.84 \,\hat{k} \quad (N)$$

$$\underline{F}_3 = 600 \,\hat{j} \quad (N)$$

$$\underline{C}_B = -25 \sin 40^\circ \,\hat{i} - 25 \cos 40^\circ \cos 45^\circ \,\hat{j} - 25 \cos 40^\circ \sin 45^\circ \,\hat{k}$$

$$= -16.07 \,\hat{i} - 13.54 \,\hat{j} - 13.54 \,\hat{k} \quad (Nm)$$

Using the same idea of equivalent force system

$$\underline{F}_R = \sum_{i=1}^{3} \underline{F}_i$$

$$\Rightarrow \quad (\underline{F}_R)_A = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= 927.22 \,\hat{i} + 870.84 \,\hat{j} + 620.84 \,\hat{k}$$

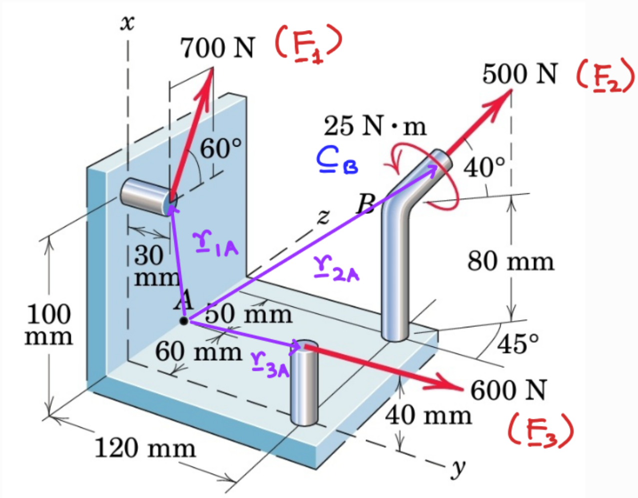
Using the equivalent of moment about pt A

$$(\underline{C}_R)_A = \sum_{i=1}^{3} (\underline{r}_{iA} \times \underline{F}_i) + \underline{C}_B$$

$$\underline{r}_{1A} = 0.1\,\hat{i} + 0.03\,\hat{j} - 0.06\,\hat{k} \quad (m)$$

$$\underline{r}_{2A} = 0.08\,\hat{i} + 0.12\,\hat{j} + 0.05\,\hat{k} \quad (m)$$

$$\underline{r}_{3A} = 0.04\,\hat{i} + 0.12\,\hat{j} - 0.06\,\hat{k} \quad (m)$$



$$\underline{r}_{1A} \times \underline{F}_1 = (0.1\,\hat{i} + 0.03\,\hat{j} - 0.06\,\hat{k}) \times (606.22\,\hat{i} + 350\,\hat{k})$$

$$= 10.5\,\hat{i} - 71.37\,\hat{j} - 18.19\,\hat{k} \quad (Nm)$$

$$\underline{r}_{2A} \times \underline{F}_2 = (0.08\,\hat{i} + 0.12\,\hat{j} + 0.05\,\hat{k}) \times (321\,\hat{i} + 270.84\,\hat{j} + 270.84\,\hat{k})$$

$$= 18.96\,\hat{i} - 5.62\,\hat{j} - 16.85\,\hat{k} \quad (Nm)$$

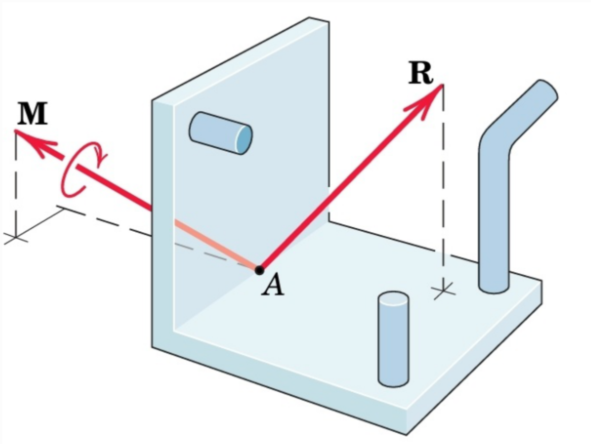
$$\underline{r}_{3A} \times \underline{F}_3 = (0.04\,\hat{i} + 0.12\,\hat{j} - 0.06\,\hat{k}) \times 600\,\hat{j}$$

$$= 36\,\hat{i} + 24\,\hat{k} \quad (Nm)$$

$$\therefore \quad (C_R)_A = (65.4\,\hat{i} - 76.99\,\hat{j} - 11.04\,\hat{k}) +$$

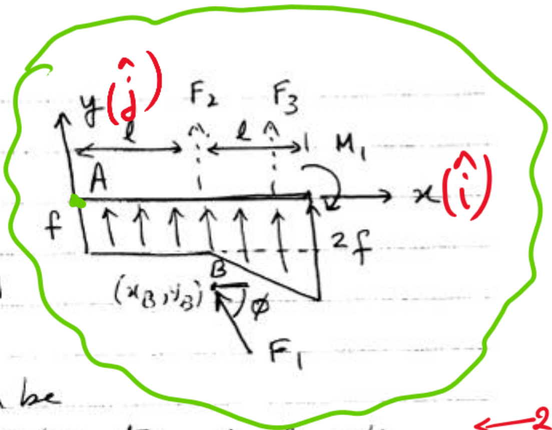
$$(-16.07\,\hat{i} - 13.54\,\hat{j} - 13.54\,\hat{k})$$

$$= 49.33\,\hat{i} - 90.53\,\hat{j} - 24.58\,\hat{k} \quad (\text{Nm})$$



↓ RB

Find the resultant force system at A.



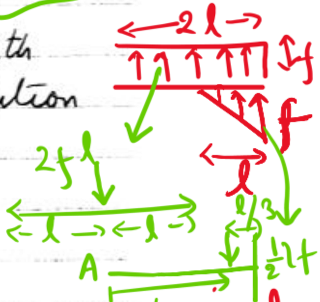
Q2.8 d (p. 125/126)

$\vec{F}_1 = (-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) \times 100$ kN

$\cos\phi = 0.8 \Rightarrow \vec{F}_1 = (-80\hat{i} + 60\hat{j})$ kN

$\vec{M}_1 = -60$ kNm $\hat{k}$.

The distributed force system can be split up into a rectangular distribution of length $2l$ and density f and a triangular distribution of length l and maximum density f.



$\vec{F}_2 = 2lf\,\hat{j} = 120$ kN $\hat{j}$; $\vec{F}_3 = \frac{1}{2} l f\,\hat{j} = 30$ kN $\hat{j}$.

$\Rightarrow \vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-80\hat{i} + 210\hat{j})$ kN

→ = moment due to all forces about A in the original force system

$(\vec{C}_R)_A = \vec{M}_1 + (x_B\hat{i} + y_B\hat{j}) \times (\vec{F}_1) + l\hat{i} \times \vec{F}_2 + (l + \frac{2}{3}l)\hat{i} \times \vec{F}_3$

$= [-60\hat{k} + (6\hat{i} - 5\hat{j}) \times (-80\hat{i} + 60\hat{j}) + 720\hat{k} + 300\hat{k}]$ kNm

$= 920\,\hat{k}$ kNm

$l + \frac{2l}{3}$

Since $\vec{C}_{RA}$ is $\perp$r to $\vec{F}_R$ we can combine the force and torque into a single force at (x_0, y_0). Can you determine x_0 and y_0?

$$(\underline{C}_R)_A = (x_0\hat{i} + y_0\hat{j}) \times \underline{F}_R.$$

$$920\,\hat{k}\ (\text{kNm}) = (x_0\hat{i} + y_0\hat{j}) \overset{m}{\times} (-80\hat{i} + 210\hat{j})\ (\text{kN})$$

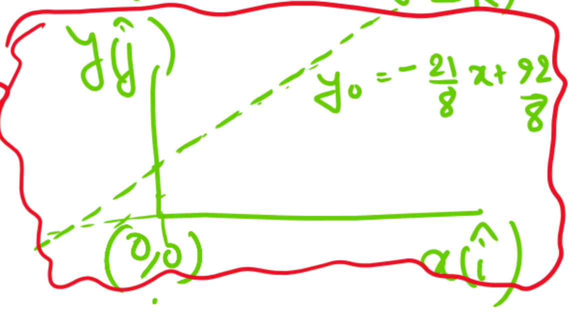
$$920\,\hat{k} = x_0(210)\hat{k} - 80y_0(-\hat{k})$$

$$920 = 210x_0 + 80y_0$$

$$y_0 = -\frac{210}{80}x_0 + \frac{920}{80}.$$

(x_0, y_0) on a straight line (the line of action of $\underline{F}_R$)

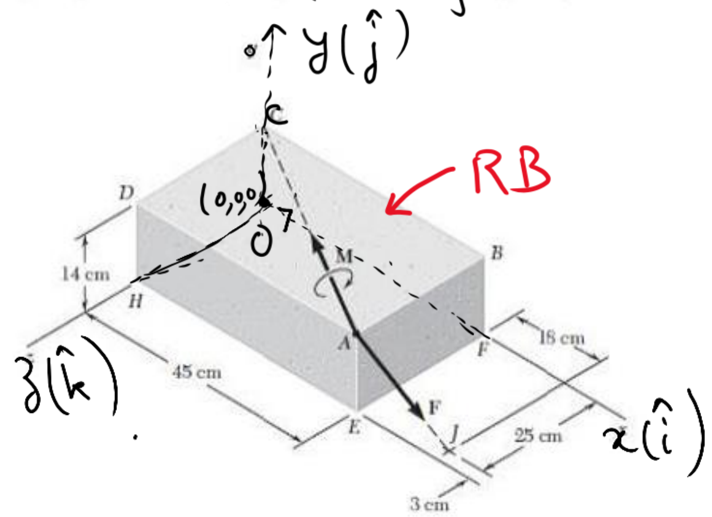
RB.

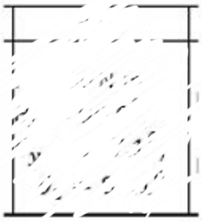


Set 4 B

PROBLEM 3.98

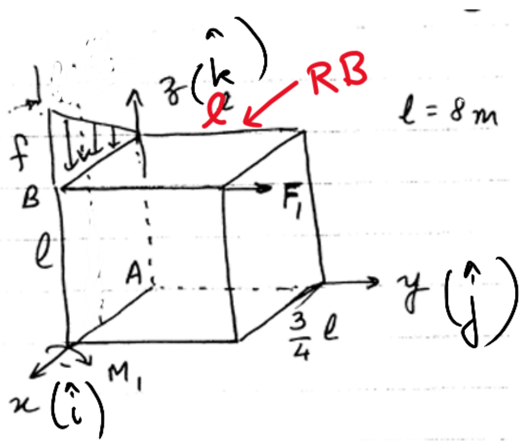
A 46-N force F and a 21.2-N·m torque M are applied to corner A of the block shown. Replace the given force-couple system with the resultant force system at corner H.





	3.98
	$=(36\text{ N})\hat{i}-(28\text{ N})\hat{j}-(6\text{ N})\hat{k} = (F_R)_H$
	$-(18.8\text{ N·m})\hat{i}+(2.7\text{ N·m})\hat{j}-(28.8\text{ N·m})\hat{k} = (C_R)_H$

Question 2



$\ell = 8\,m$

Q2.8 b (p 125/126) Find the resultant force system at A.

Given **: F1=20 kN, M1=40kN-m, f=4kN/m.**

Answer: $(20\hat{j}-12\hat{k})$ kN, $(-200\hat{i}+48\hat{j}+120\hat{k})$ kN-m.

Hint: To simplify the algebra: Before finding $(C_R)_A$, replace the distributed force system by a single point force at an appropriate location.