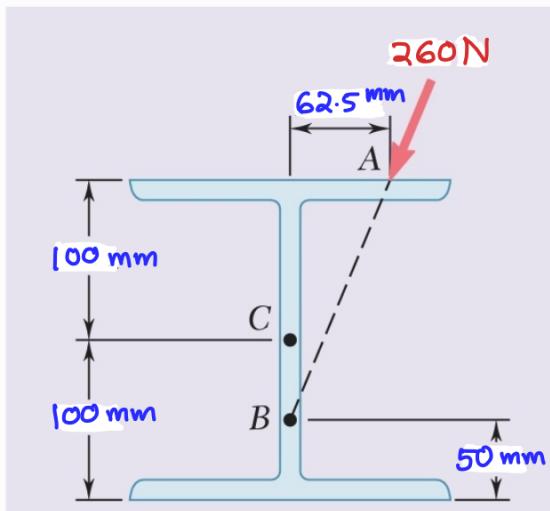


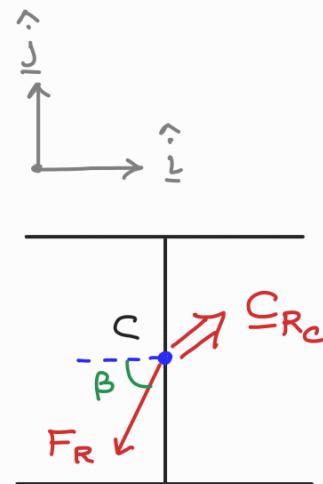
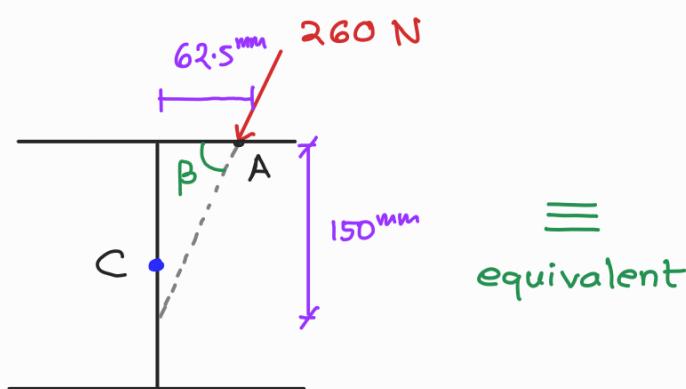
Tutorial 4 solutions

1>



A 260 N force is applied at A to the rolled-steel section.

Replace that force with an equivalent force-couple system at the center C of the section.



To get the equivalent force system, we need to satisfy two conditions

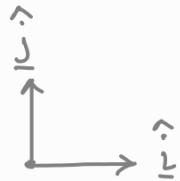
$$\textcircled{a} \quad \underline{F}_R = \underline{F}'_R$$

$$\textcircled{b} \quad \underline{M}_A = \underline{M}'_A$$

Recall

$$[\text{Force}_A] \xrightarrow[\text{to } C]{\text{transfer}} [\text{Same force at } C] + [\text{One couple}]$$

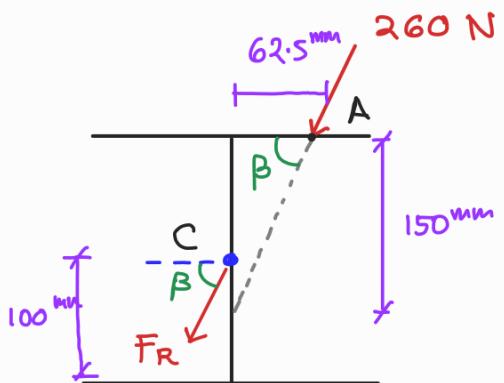
Let's get angle β : $\beta = \tan^{-1} \left(\frac{150}{62.5} \right)$
 $= 1.176 \text{ rad}$



Using ①, get force at C

$$(\underline{F}_R)_C = -260 \cos \beta \hat{i} - 260 \sin \beta \hat{j}$$

$$= -100 \hat{i} - 240 \hat{j} \text{ (N)}$$



Using ②, we get the resultant couple at C

$$(\underline{M}_R)_C = \underline{x}_{AC} \times \underline{F}_A = \left(\frac{62.5}{1000} \hat{i} + \frac{100}{1000} \hat{j} \right) \times (-100 \hat{i} - 240 \hat{j})$$

$$= -5 \hat{k} \text{ (Nm)}$$

2) Find the equivalent force system at point A

The given system has three forces and a couple. First, we need to write the forces and the couple in terms of its components.

$$\underline{F}_1 = 700 \sin 60^\circ \hat{i} + 700 \cos 60^\circ \hat{k} \text{ (N)}$$

$$= 606.22 \hat{i} + 350 \hat{k} \text{ (N)}$$

$$\underline{F}_2 = 500 \sin 40^\circ \hat{i} + 500 \cos 40^\circ \cos 45^\circ \hat{j} + 500 \cos 40^\circ \sin 45^\circ \hat{k}$$

$$= 321 \hat{i} + 270.84 \hat{j} + 270.84 \hat{k} \text{ (N)}$$

$$\underline{F}_3 = 600 \hat{j} \text{ (N)}$$

$$\underline{C}_B = -25 \sin 40^\circ \hat{i} - 25 \cos 40^\circ \cos 45^\circ \hat{j} - 25 \cos 40^\circ \sin 45^\circ \hat{k}$$

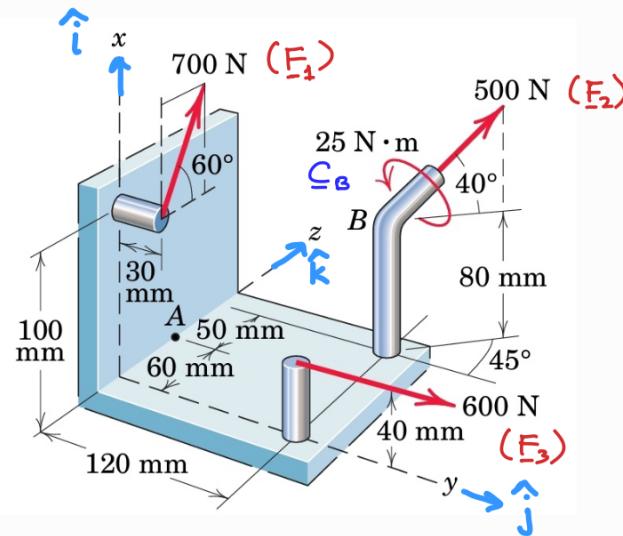
$$= -16.07 \hat{i} - 13.54 \hat{j} - 13.54 \hat{k} \text{ (Nm)}$$

Using the same idea of equivalent force system

$$\underline{F}_R = \sum_{i=1}^3 \underline{F}_i$$

$$\Rightarrow (\underline{F}_R)_A = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= 927.22 \hat{i} + 870.84 \hat{j} + 620.84 \hat{k}$$



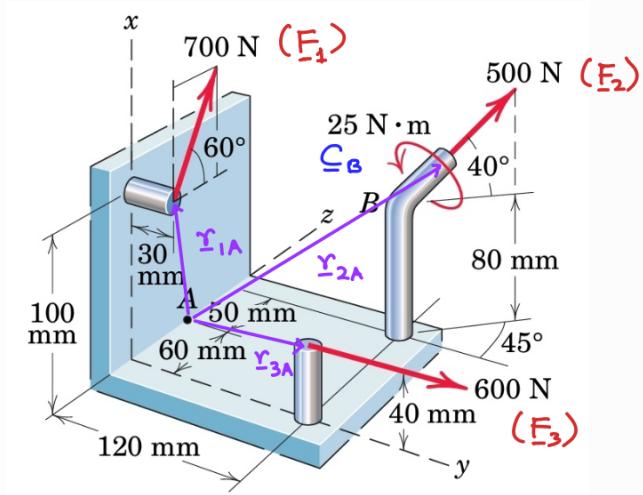
Using the equivalent of moment about pt A

$$(\underline{C}_R)_A = \sum_{i=1}^3 (\underline{r}_{iA} \times \underline{F}_i) + \underline{C}_B$$

$$\underline{r}_{1A} = 0.1\hat{i} + 0.03\hat{j} - 0.06\hat{k} \text{ (m)}$$

$$\underline{r}_{2A} = 0.08\hat{i} + 0.12\hat{j} + 0.05\hat{k} \text{ (m)}$$

$$\underline{r}_{3A} = 0.04\hat{i} + 0.12\hat{j} - 0.06\hat{k} \text{ (m)}$$

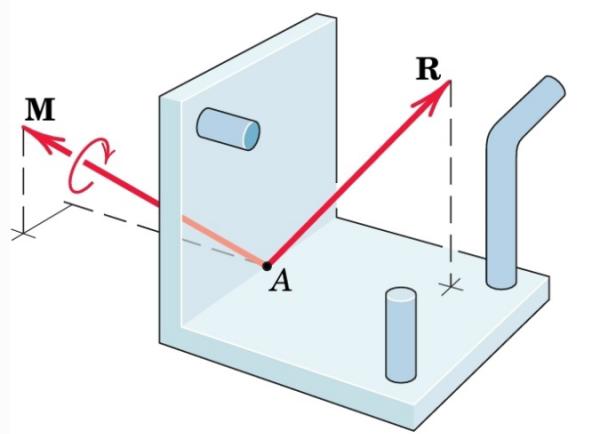


$$\begin{aligned} \underline{r}_{1A} \times \underline{F}_1 &= (0.1\hat{i} + 0.03\hat{j} - 0.06\hat{k}) \times \\ &\quad (606.22\hat{i} + 350\hat{k}) \\ &= 10.5\hat{i} - 71.37\hat{j} - 18.19\hat{k} \text{ (Nm)} \end{aligned}$$

$$\begin{aligned} \underline{r}_{2A} \times \underline{F}_2 &= (0.08\hat{i} + 0.12\hat{j} + 0.05\hat{k}) \times \\ &\quad (321\hat{i} + 270.84\hat{j} + 270.84\hat{k}) \\ &= 18.96\hat{i} - 5.62\hat{j} - 16.85\hat{k} \text{ (Nm)} \end{aligned}$$

$$\begin{aligned} \underline{r}_{3A} \times \underline{F}_3 &= (0.04\hat{i} + 0.12\hat{j} - 0.06\hat{k}) \times 600\hat{j} \\ &= 36\hat{i} + 24\hat{k} \text{ (Nm)} \end{aligned}$$

$$\begin{aligned}
 \therefore (\underline{\underline{C}}_R)_A &= (65.4 \hat{i} - 76.99 \hat{j} - 11.04 \hat{k}) + \\
 &\quad (-16.07 \hat{i} - 13.54 \hat{j} - 13.54 \hat{k}) \\
 &= 49.33 \hat{i} - 90.53 \hat{j} - 24.58 \hat{k} \text{ (Nm)}
 \end{aligned}$$



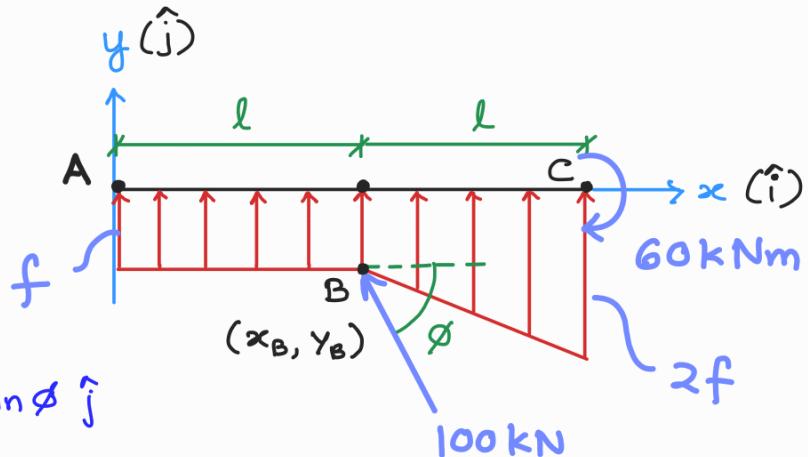
3> Find the equivalent force system at point A and the wrench location.

Given: $l = 6 \text{ m}$, $f = 10 \text{ kN}$

$$\cos \phi = 0.8 \Rightarrow \sin \phi = 0.6$$

$$x_B = 6 \text{ m}, y_B = -5 \text{ m}$$

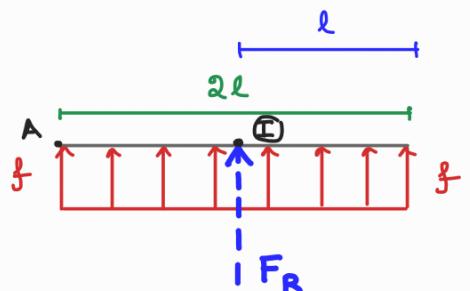
$$\begin{aligned} F_B &= -100 \cos \phi \hat{i} + 100 \sin \phi \hat{j} \\ &= -80 \hat{i} + 60 \hat{j} \text{ (kN)} \end{aligned}$$



$$C_c = -60 \hat{k} \text{ (kNm)}$$

This problem tests how to find equivalence of distributed force systems. The given parallel and coplanar distributed force can be split up into two sets:

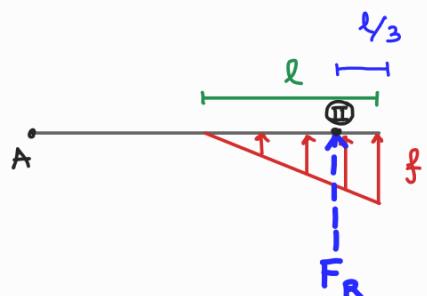
- (I) Rectangular/Uniform distribution of magnitude 'f' and length '2l'



$$\text{Resultant, } F_R^{(I)} = f(2l) \hat{j} = 120 \hat{j} \text{ (kN)}$$

$$\text{Moment arm from A, } \Sigma_{(I)A} = l \hat{i} \text{ (m)}$$

- (II) Triangular distribution of maximum magnitude 'f' and length 'l'



$$\text{Resultant, } F_R^{(II)} = \frac{1}{2} f l \hat{j} = 30 \hat{j} \text{ (kN)}$$

$$\text{Moment arm from A, } \Sigma_{(II)A} = \left[l + \left(l - \frac{l}{3} \right) \right] \hat{i} = \frac{5l}{3} \hat{i} \text{ (m)}$$

The total resultant force of the equivalent force system is:

$$\begin{aligned}
 \underline{F}_R &= \underline{F}_B + \underline{F}_R^{(I)} + \underline{F}_R^{(II)} \\
 &= -80\hat{i} + 60\hat{j} + 120\hat{j} + 30\hat{j} \\
 &= -80\hat{i} + 210\hat{j} \text{ (kN)}
 \end{aligned}$$

The total resultant couple of the equivalent force system at point A is:

$$\begin{aligned}
 (\underline{C}_R)_A &= \underline{x}_B \times \underline{F}_B + \underline{x}_{(I)A} \times \underline{F}_R^{(I)} + \underline{x}_{(II)A} \times \underline{F}_R^{(II)} + \underline{C}_c \\
 &= (x_B\hat{i} + y_B\hat{j}) \times (-80\hat{i} + 60\hat{j}) + (l\hat{i}) \times (120\hat{j}) \\
 &\quad + \left(\frac{5}{3}l\hat{i}\right) \times (30\hat{j}) - 60\hat{k} \\
 &= (6\hat{i} - 5\hat{j}) \times (-80\hat{i} + 60\hat{j}) + (6\hat{i}) \times (120\hat{j}) \\
 &\quad + \left(\frac{5}{3} \cdot 6\hat{i}\right) \times (30\hat{j}) - 60\hat{k} \\
 &= -40\hat{k} + 720\hat{k} + 300\hat{k} - 60\hat{k} \\
 &= 920\hat{k} \text{ (Nm)}
 \end{aligned}$$

Note that $(\underline{C}_R)_A$ is perpendicular to \underline{F}_R , i.e.

$$\underline{C}_{RA}'' = 0$$

Therefore, the wrench position \underline{r}_{AW} can be found as

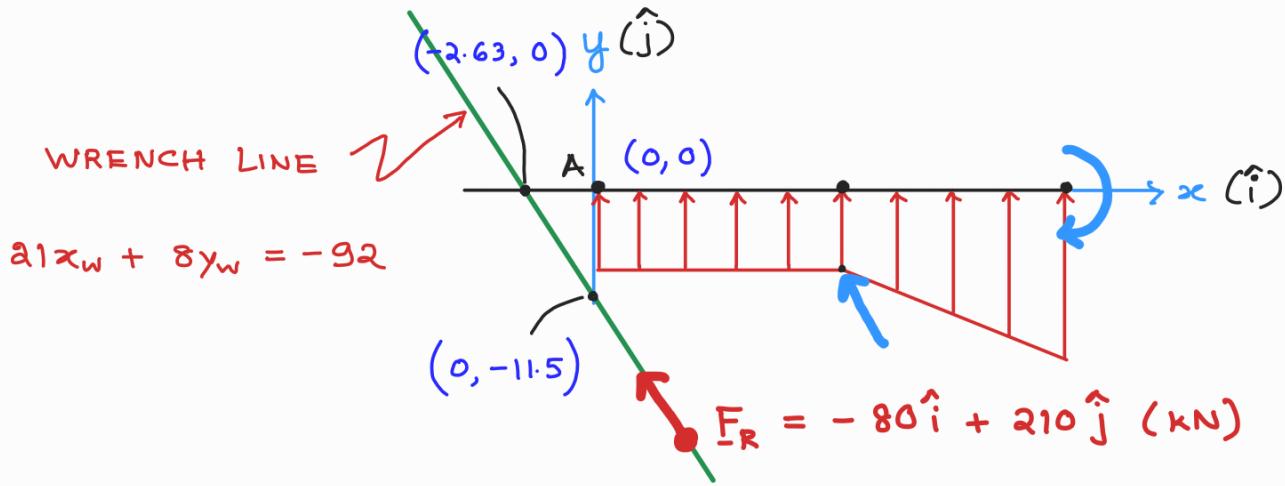
$$\underline{r}_{AW} \times \underline{F}_R + \underline{c}_{RA} \hat{k} = \underline{0}$$

$$\Rightarrow (x_w \hat{i} + y_w \hat{j}) \times (-80 \hat{i} + 210 \hat{j}) + 920 \hat{k} = \underline{0}$$

$$\Rightarrow (210x_w + 80y_w + 920) \hat{k} = \underline{0}$$

Evaluating only the k th component from both sides, we get the equation of the WRENCH LINE:

$$21x_w + 8y_w + 92 = 0$$



You can place \underline{F}_R at any point on the wrench line and you will still get the same resultant couple $\underline{c}_{RA} \hat{k}$ at A.

This is because the location of the force does not matter as long as the force appears along the same line of action!