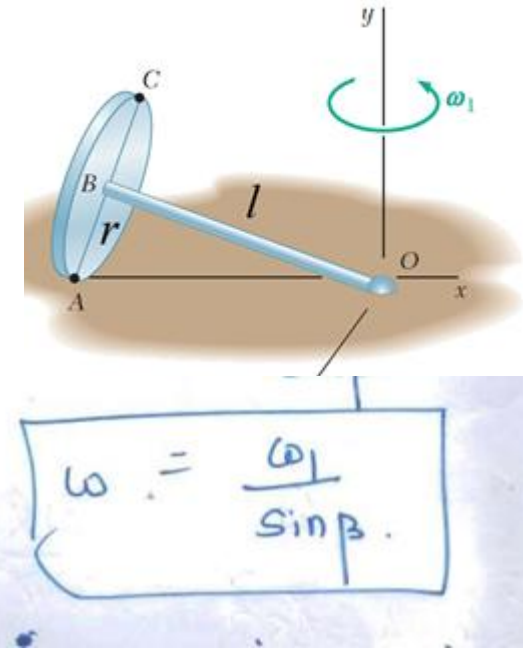
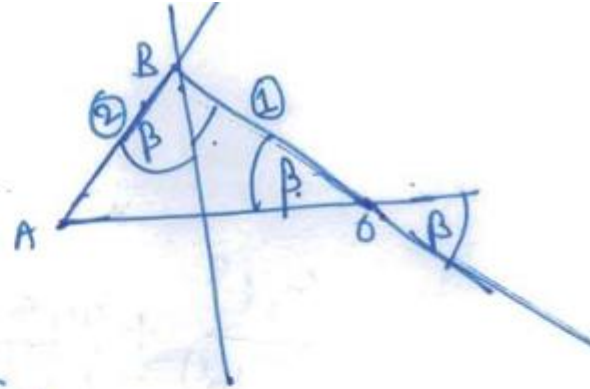


A wheel of radius $r = 3\text{m}$ is mounted on an axle OB of length $l = 4\text{m}$. The point O is fixed to ground. The wheel is free to rotate relative to the axle BO. Further, the wheel rolls without slipping on the horizontal floor, and the axle is always perpendicular to the plane of the wheel. Knowing that the axle BO has angular velocity (relative to the ground frame) as 5 rads^{-1} (constant), the magnitude of the angular acceleration of the wheel w.r.t. ground is (in rads^{-2})



$$\omega = \frac{\omega_1}{\sin \beta}$$

$$\begin{aligned} \underline{V}_{A/F} &= \underline{V}_{B/F} + \underline{\omega}_{2/F} \times \underline{r}_{AB} \\ &= \underline{V}_{B/F} + (\underline{\omega}_{2/I} + \underline{\omega}_{I/F}) \times \underline{r}_{AB} \\ &= \underline{V}_{B/F} + \underline{\omega}_{2/I} \times \underline{r}_{AB} + \underline{\omega}_{I/F} \times \underline{r}_{AB} \end{aligned}$$



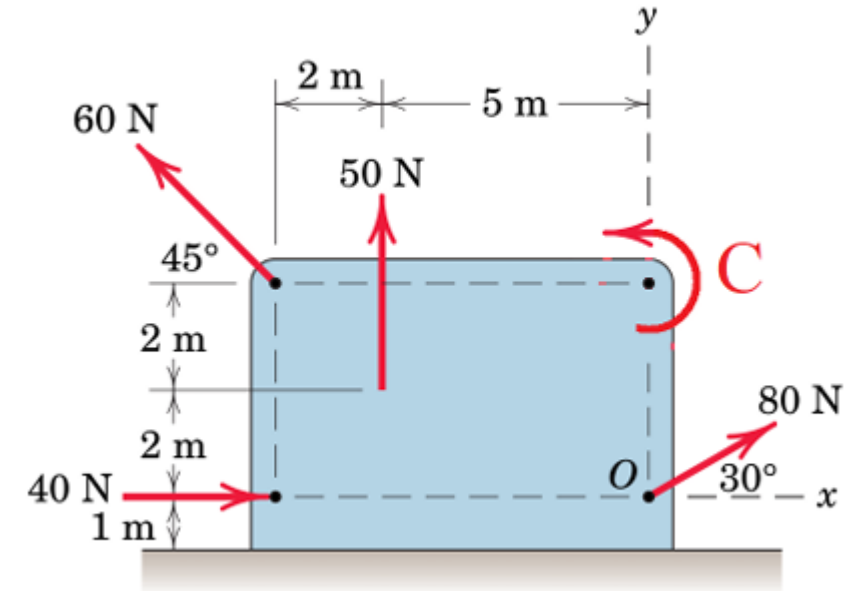
$$\underline{V}_{A/F} = \underline{V}_{O/F} + \underline{\omega}_{I/F} \times \underline{r}_{BO} + \underline{\omega}_{2/I} \times \underline{r}_{AB} + \underline{\omega}_{I/F} \times \underline{r}_{AB}$$

$$\begin{aligned} 0 &= 0 + \underline{\omega}_{I/F} \times \underline{r}_{BO} + \underline{\omega}_{2/I} \times \underline{r}_{AB} + \underline{\omega}_{I/F} \times \underline{r}_{AB} \\ \checkmark 0 &= \omega_1 \hat{j} \times (l)(-\cos \beta \hat{i} + \sin \beta \hat{j}) + \omega (\cos \beta \hat{i} - \sin \beta \hat{j}) \times (-r \sin \beta \hat{i} - r \cos \beta \hat{j}) \\ &\quad + \omega_1 \hat{j} \times (-r \sin \beta \hat{i} - r \cos \beta \hat{j}) \end{aligned}$$

$$\begin{aligned} \dot{\underline{\omega}}_{2/I} &= \dot{\underline{\omega}}_{2/I} + \dot{\underline{\omega}}_{I/I} + \underline{\omega}_{I/I} \times \underline{\omega}_{2/I} \\ &= 0 + 0 + \omega_1 \hat{j} \times \frac{\omega_1}{\sin \beta} (\cos \beta \hat{i} - \sin \beta \hat{j}) \\ &= -\omega_1^2 \cot \beta \cdot \hat{k} \quad \checkmark \quad = -\frac{\omega_1^2}{\sin \beta} \hat{k} \end{aligned}$$

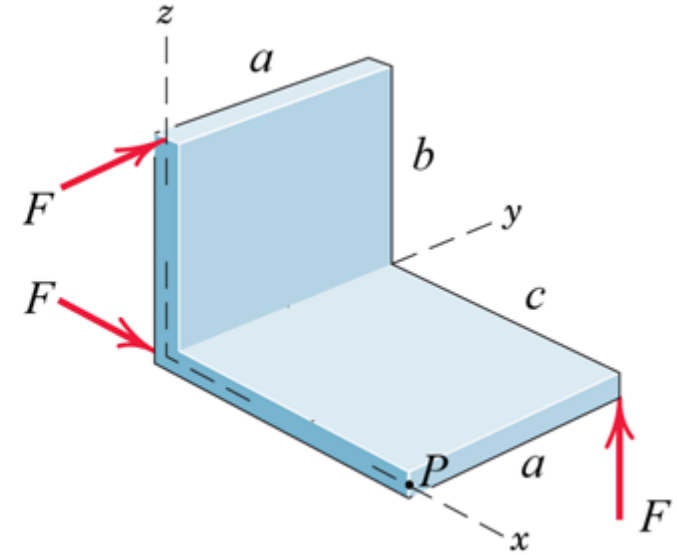
The point O is the origin of a right-handed coordinate system shown. Let (x_a, y_a) represent a point A where merely a single force can be applied to arrive at a new force system that is equivalent to the co-planar force system shown in the figure. Choose the correct relationship that exists between x_a and y_a when, $C = 140 \text{ Nm}$.

$$\begin{aligned} \sum_{i=1}^n \vec{F}_i &= \vec{F}_R \\ \vec{F}_R &= 80 \cos 30^\circ \hat{i} + 80 \sin 30^\circ \hat{j} \\ &\quad - \frac{60}{\sqrt{2}} \hat{i} + \frac{60}{\sqrt{2}} \hat{j} + 40 \hat{i} + 0 \\ &= \hat{i} \left(80 \cos 30^\circ - \frac{60}{\sqrt{2}} + 40 \right) + \hat{j} \left(80 \sin 30^\circ + \frac{60}{\sqrt{2}} \right) \\ \vec{F}_R &= 66.87 \hat{i} + 132.41 \hat{j} \quad \vec{F}_R = 66.85 \hat{i} + 132.43 \hat{j} \\ \underline{M}_O &= \underline{M}'_O \\ C \hat{k} + (-50 \times 5) \hat{k} + (-7 \hat{i} + 4 \hat{j}) \times \left(\frac{60}{\sqrt{2}} \hat{i} + \frac{60}{\sqrt{2}} \hat{j} \right) &= \\ C \hat{k} - 250 \hat{k} + (-7 \hat{i} + 4 \hat{j}) \times (-42.43 \hat{i} + 42.43 \hat{j}) &= \\ &= x_a (132.41) \hat{k} - y_a (66.87) \hat{k} \\ C \hat{k} - 250 \hat{k} + (-7 \times 42.43) \hat{k} + 4 \times 42.43 \hat{k} &= x_a (132.41) \hat{k} - y_a (66.87) \hat{k} \\ C - 250 - 297.01 + 169.72 &= x_a (132.41) - y_a (66.87) \\ C - 377.29 &= x_a (132.41) - y_a (66.87) \\ 0 &= x_a (132.41) - y_a (66.87) + 377.29 - C \\ 0 &= x_a 132.4 - 66.9 y_a + (377.29 - C) \end{aligned}$$



Find the moment due to a couple that must be put on the shown rigid body to replace the shown force system by a resultant force system at point P. The symbols \hat{i} , \hat{j} , and \hat{k} are the unit vectors along the axes shown x -, y -, and z -, respectively. Use $a = 10m$, $b = 2m$ and $c = 5m$. F is in Newtons.

- ☐ $-12F\hat{i} - 5F\hat{k}$
- ☐ $-10F\hat{i} - 2F\hat{j} - 5F\hat{k}$
- ☐ $6F\hat{i} - 15\hat{j}$
- ☐ None of these is correct.



Handwritten solution:

$$\vec{M}_P = \vec{M}'_P$$

$$(\hat{j} + c\hat{i}) \times F\hat{k} + b\hat{k} \times F\hat{j} = (\vec{C}_R)_P$$

$$aF\hat{i} - cF\hat{j} - bF\hat{i} = (\vec{C}_R)_P$$

$$(a-b)F\hat{i} - cF\hat{j} = (\vec{C}_R)_P$$

Also written: $a-b = a+b$
 $b=0$

The rod AB slides down (as shown in the figure). At the current instant, the velocity of point B w.r.t ground is non-zero and is directed along the horizontal slope (as shown in the figure). The instantaneous center of rotation (ICR) of the rod AB is at

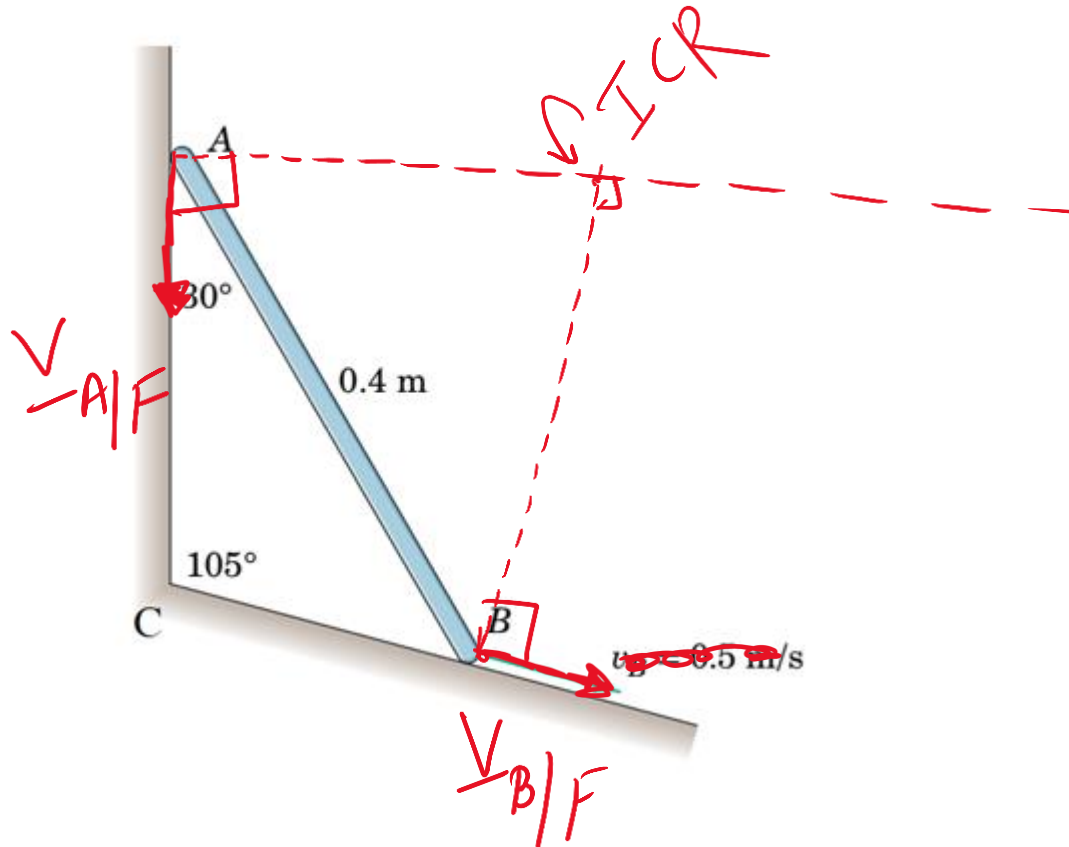


Diagram of a beam of length L fixed at point A and free at point B . A distributed load $w(x)$ acts downwards on the beam, starting at w_0 at A and increasing to $w = w_0(1 + kx^3)$ at B . The load is represented by a red curve and downward arrows.

$$\begin{aligned} \frac{F}{R} &= -j \int_0^L (\omega_0 + \omega_0 k x^3) dx = -j \int_0^L \omega_0 dx - j \omega_0 k \int_0^L x^3 dx \\ &= -j \omega_0 x \Big|_0^L - j \omega_0 k \frac{x^4}{4} \Big|_0^L \\ &= -j \omega_0 L - j \omega_0 k \frac{L^4}{4} \end{aligned}$$

Match $\underline{M}_A = \underline{M}_A$

$$-\frac{1}{k} \int_0^L (w_0 x + w_0 k x^4) dx = +X_R F_R \quad (\hat{i} x - \hat{j})$$

$$\frac{1}{k} \left(W_0 x^2 + W_0 k \frac{x^5}{5} \right) \Big|_0^L = X_R F_R (-k)$$

$$\frac{\frac{W_0 L^2}{2} + W_0 k \frac{L^5}{5}}{F_R} = \frac{W_0 L \left[\frac{L}{2} + \frac{k L^4}{5} \right]}{W_0 L + W_0 k \frac{L^4}{4}}$$

$$= \frac{W_0 L [\cdot 5L + \cdot 2kL^4]}{W_0 L [1 + \cdot 25L^3]} = \boxed{\frac{\cdot 5L + \cdot 2kL^4}{1 + \cdot 25L^3 k}}$$