

(a) "Wheel" = "disk"

Initial Energy E_{wheel} :

$$\begin{aligned} E_{\text{wheel}} &= \frac{1}{2}m(r\Omega_0)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\Omega_0^2 \\ &= \frac{1}{2}mr^2\Omega_0^2 + \frac{1}{4}mr^2\Omega_0^2 \\ &= \left(\frac{3}{4}\right)mr^2\Omega_0^2 \end{aligned}$$

Thus:

$$mgh_{\text{wheel}} = \frac{3}{4}mr^2\Omega_0^2$$

Thus:

$$h_{\text{wheel}} = \frac{3}{4} \frac{r^2\Omega_0^2}{g}$$

(b) Ring

Initial Energy E_{ring} :

$$\begin{aligned} E_{\text{ring}} &= \frac{1}{2}m(r\Omega_0)^2 + \frac{1}{2}(mr^2)\Omega_0^2 \\ &= \frac{1}{2}mr^2\Omega_0^2 + \frac{1}{2}mr^2\Omega_0^2 \\ &= mr^2\Omega_0^2 \end{aligned}$$

Thus:

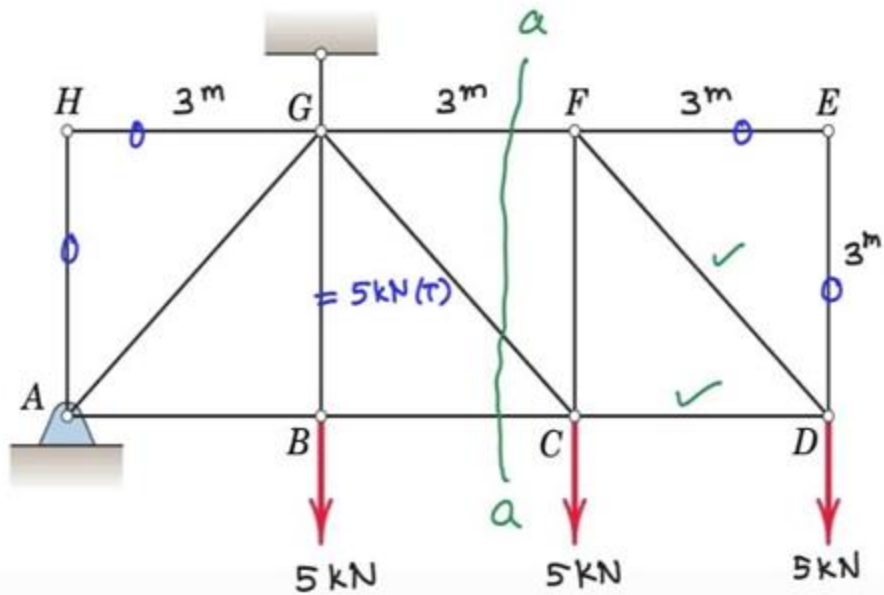
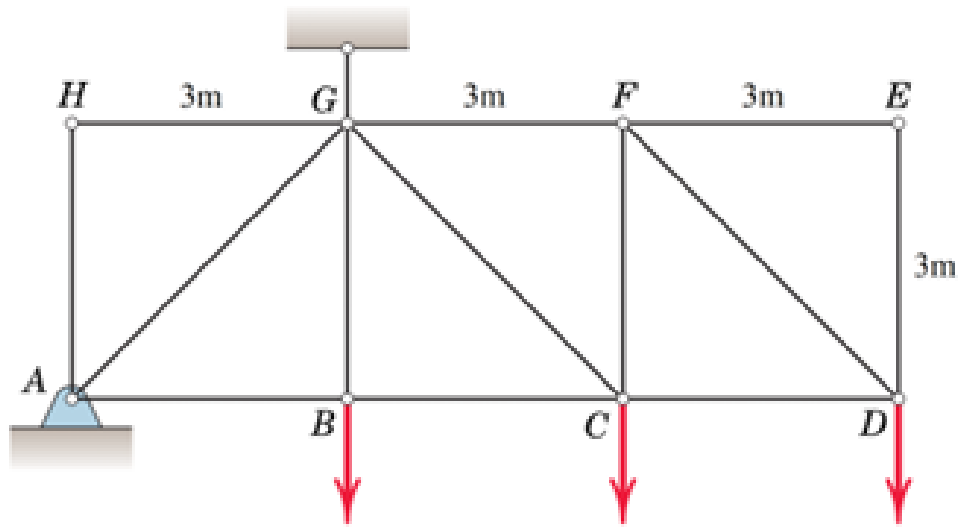
$$mgh_{\text{ring}} = mr^2\Omega_0^2$$

Thus:

$$h_{\text{ring}} = \frac{r^2\Omega_0^2}{g}$$

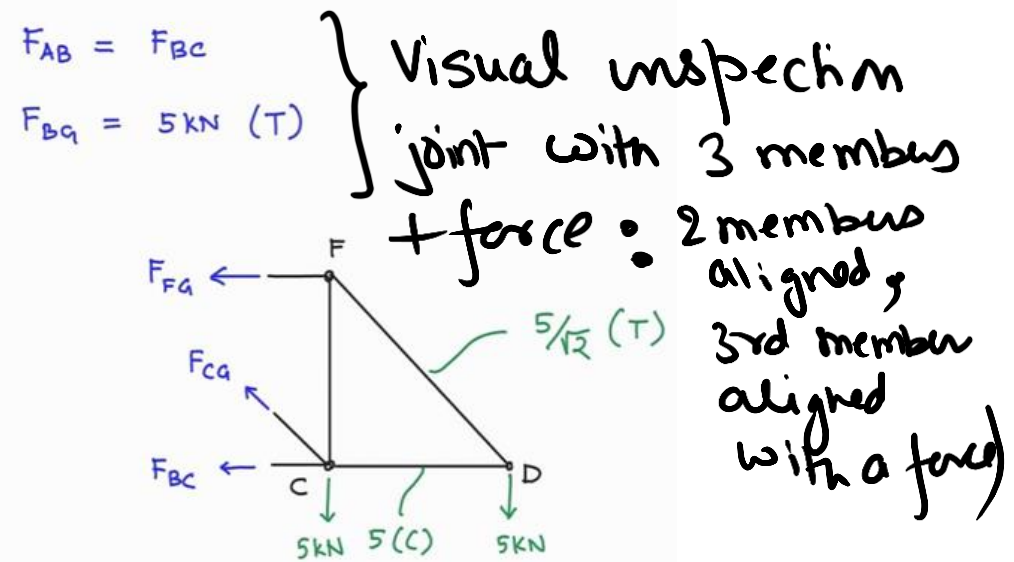
Comparing the heights achieved by the two bodies, the ring reaches a higher level by 33% times than the wheel

Answer



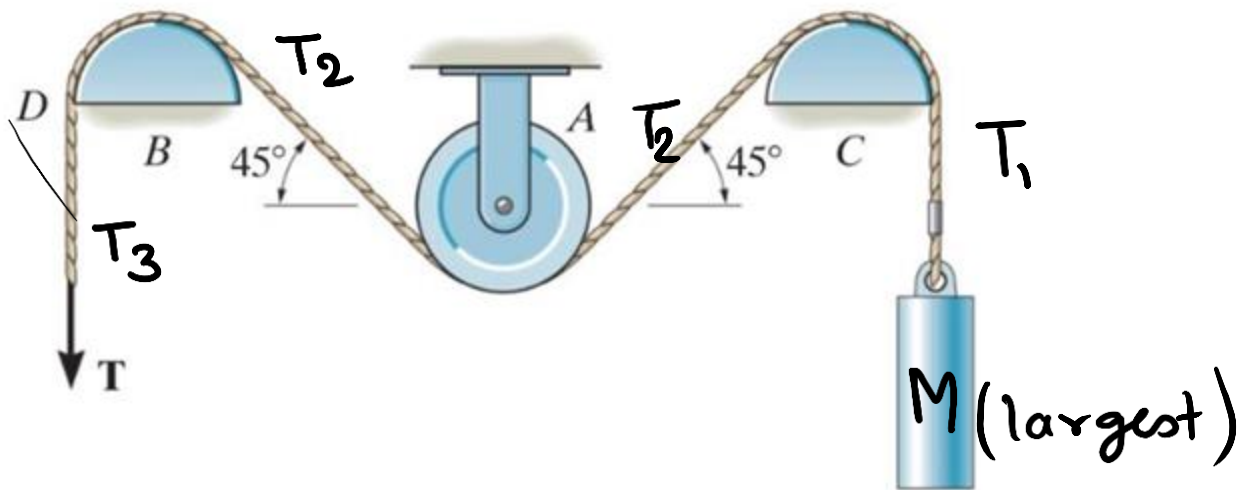
AH, HG, EF *Answer*
 ED \rightarrow zero-force members
 (visual inspection)
 *2 members, non-collinear at H.

2 members - not aligned



$$\begin{aligned}
 +\uparrow \sum F_y = 0 &\Rightarrow F_{Ca}/\sqrt{2} - 10 \text{ kN} = 0 \\
 &\Rightarrow F_{Ca} = 10\sqrt{2} \text{ kN (T)}
 \end{aligned}$$

Answer

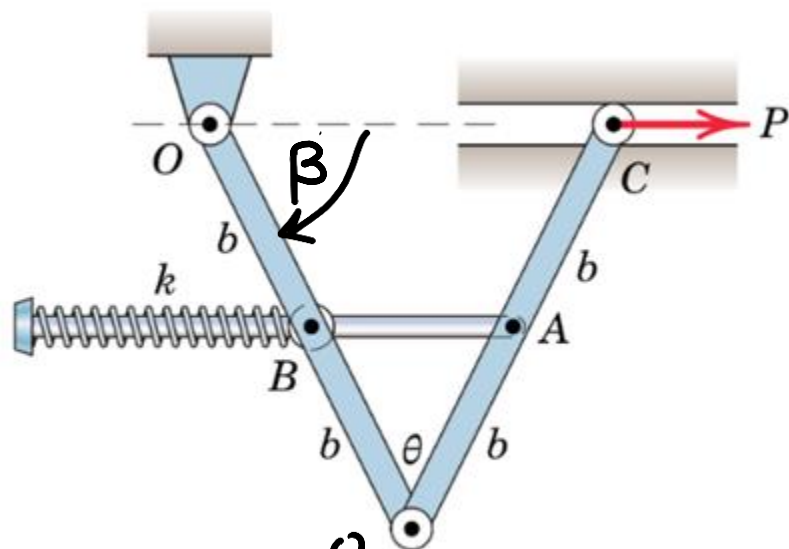


$$\frac{T_3}{T_2} = e^{\mu 3\pi/4}, \quad \frac{T_2}{T_1} = e^{\mu 3\pi/4}$$

$$T_1 = Mg, \quad T_3 = 500 \text{ N} \Rightarrow M = 15.69 \text{ kg}$$

$$T_3 = 600 \text{ N} \Rightarrow M = 18.82 \text{ kg}$$

Answer



O_x, O_y : Workless forces, mg, mg, F_s
Conservative forces.

Potential function, Spring
 $[V_e = \frac{1}{2}kx^2]$ $V_e = \frac{1}{2}k \left(2b \sin \frac{\theta}{2}\right)^2 = 2kb^2 \sin^2 \frac{\theta}{2}$

With the datum for zero gravitational potential energy taken through the support at O for convenience, the expression for V_g becomes

$$[V_g = mgh] \quad V_g = 2mg \left(-b \cos \frac{\theta}{2}\right)$$

The distance between O and C is $4b \sin \frac{\theta}{2}$, so that the virtual work done by P is

$$\delta U' = P \delta \left(4b \sin \frac{\theta}{2}\right) = 2Pb \cos \frac{\theta}{2} \delta \theta$$

The virtual-work equation now gives

$$[\delta U' = \delta V_e + \delta V_g]$$

$$\begin{aligned} 2Pb \cos \frac{\theta}{2} \delta \theta &= \delta \left(2kb^2 \sin^2 \frac{\theta}{2}\right) + \delta \left(-2mgb \cos \frac{\theta}{2}\right) \\ &= 2kb^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \delta \theta + mgb \sin \frac{\theta}{2} \delta \theta \end{aligned}$$

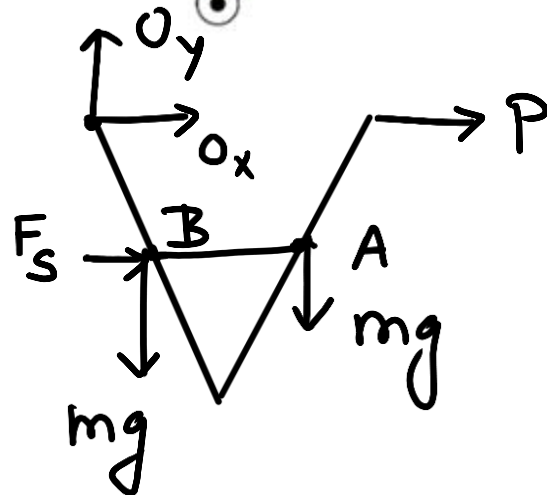
Simplifying gives finally

$$P = kb \sin \frac{\theta}{2} + \frac{1}{2}mg \tan \frac{\theta}{2}$$

Ans.

(Both sets)

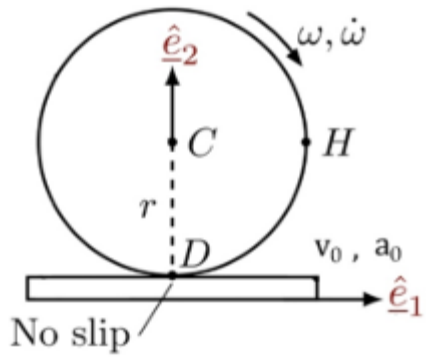
FBD



System
= {two rods
of mass
m +
massless rod.}

F_s : Spring force.

Single DOF System



$$\underline{a}_{H|I} = (\dot{\omega} r - \omega^2 r + a_0) \underline{\hat{e}}_1 - \dot{\omega} r \underline{\hat{e}}_2$$

H is a valid point if $\dot{\omega} = 0$

$$\Rightarrow \underline{a}_{H|I} = (a_0 - \omega^2 r) \underline{\hat{e}}_1 \quad // \quad \underline{r}_{CH}$$

and passing
through C

Answer

Point H is a valid point, only if $\dot{\omega} = 0$

The Euler's first axiom and the modified Euler's second axiom for an RB in static equilibrium with respect to an inertial reference frame I simplify to: $\underline{F} = 0$ and $\underline{M}_A = 0$, where \underline{F} is the net external force acting on the RB, and \underline{M}_A represents the net moment due to all forces acting on the RB about point A. Choose the correct statement.

MCQ. We know

$$\frac{d}{dt} \{ \underline{H}_{A/I} \} = \underline{M}_A - \underline{r}_{CA} \times m \underline{a}_{A/I} \quad (i) \quad A: \text{any arbit point}$$

also, $\underline{H}_{A/I} = \underline{H}_{C/I} + m \underline{r}_{CA} \times \underline{v}_{C/I}$

$$\underline{H}_{C/I} = \underline{I}^C \underline{\omega}_{m/I} \quad \text{In static equilibrium, } \underline{\omega}_{m/I} = 0$$

$$\Rightarrow \underline{H}_{C/I} = 0 \quad \text{Further } \underline{v}_{C/I} = 0 \text{ in static eqn}$$

$$\Rightarrow \underline{H}_{A/I} = m \underline{r}_{CA} \times \underline{v}_{C/I}$$

$$\therefore \frac{d}{dt} \{ \underline{H}_{A/I} \} \Big|_I = \frac{d}{dt} (\quad) \Big|_I$$

$$= m \underline{v}_{C/I} \times \underline{v}_{C/I} + m \underline{r}_{CA} \times \underline{a}_{C/I}$$

$$= m \underline{r}_{CA} \times (\underline{a}_{C/I} - \underline{a}_{A/I}) \rightarrow (ii)$$

But in static equilibrium (RB) $\underline{a}_{C/I} = 0$.

$$\Rightarrow \frac{d}{dt} \{ \underline{H}_{A/I} \} \Big|_I = -m \underline{r}_{CA} \times \underline{a}_{A/I} \rightarrow (ii)$$

Subtract (ii) from (i)

$$0 = \underline{M}_A - m \underline{r}_{CA} \times \underline{a}_{A/I} + m \underline{r}_{CA} \times \underline{a}_{A/I}$$

$$\underline{M}_A = 0 \quad A: \text{no restriction}$$

Answer

Acceleration of A w.r.t. frame I may or may not be zero.

Consider a rigid body which has its angular velocity always along a direction (\hat{e}_3) which is fixed w.r.t. to the inertial reference frame in context. The net moment due to all external forces is also known to be aligned along \hat{e}_3 . P is a point fixed to the RB as well as to the frame I. We choose a body-fixed Cartesian coordinate system (CSYS) $Px_1(\hat{e}_1)x_2(\hat{e}_2)x_3(\hat{e}_3)$ as our working CSYS, such that P lies in the $x_1(\hat{e}_1) - x_2(\hat{e}_2)$ plane at all time instants. The modified Euler's axiom for this body is known to reduce to only one non-trivial scalar equation at all time instants: $M_{P,3} = I_{33}^P \dot{\omega}$. Based on the given information, choose the correct option.

$$M_{P,1} = I_{13}^P \dot{\omega} - I_{23}^P \omega^2 \quad (P \text{ is a valid point})$$

$$M_{P,2} = I_{23}^P \dot{\omega} + I_{13}^P \omega^2$$

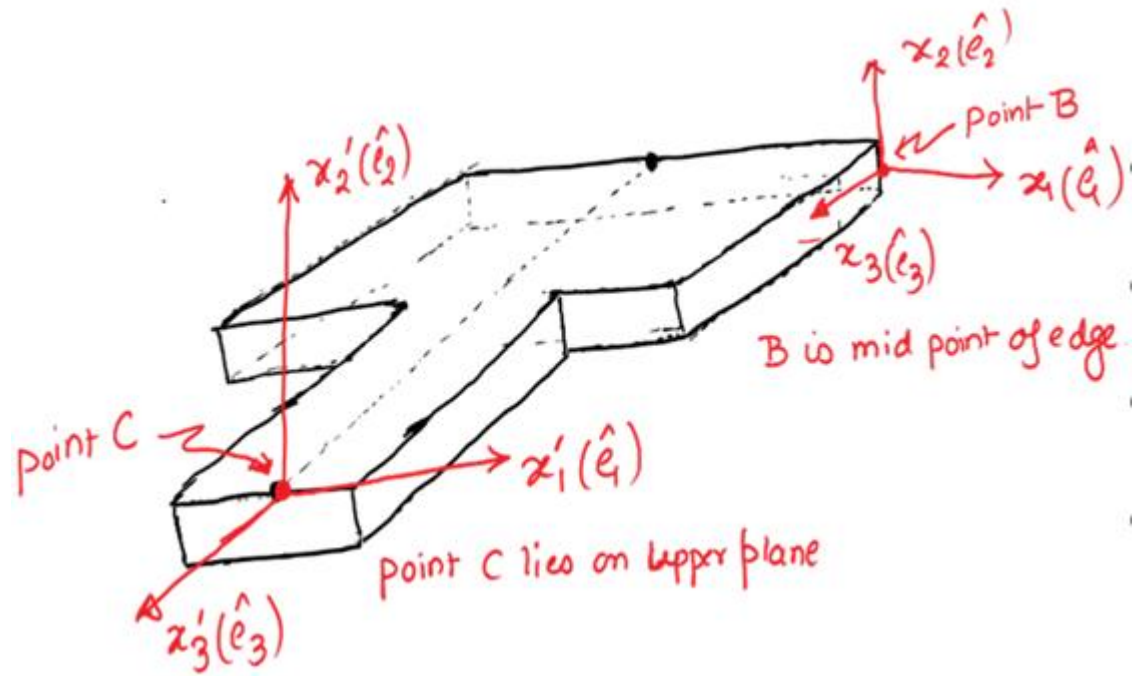
$$M_{P,3} = I_{33}^P \dot{\omega} . \quad \text{1st 2 eqns become trivial when}$$

$$I_{13}^P = 0, \quad I_{23}^P = 0,$$

$x_3(\hat{e}_3)$ principal axis of inertia at P .

The axis $x_3(\hat{e}_3)$ must be a principal axis of the inertia tensor of the body at P .

Answer

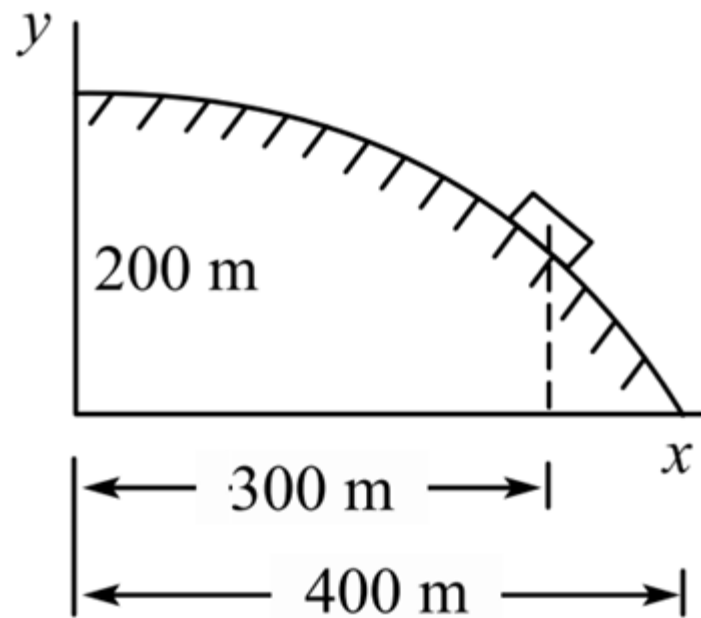


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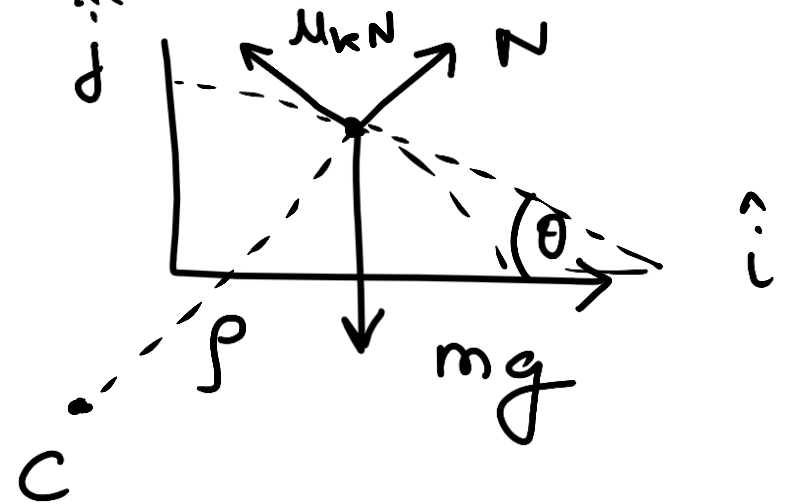
only $x_1(\hat{e}_1) - x_3(\hat{e}_3)$ plane is a symm. plane at B.
 only $x_2(\hat{e}_2)$ is a p-axis — based on visual inspection

None of these is correct.

Answer



FBD of block (point mass)



$$mg \sin \theta - \mu_k N = m \ddot{s}$$

$$mg \cos \theta - N = \frac{m \dot{s}^2}{\rho}$$

ρ : given 1597.5 m.

Eliminate $N \Rightarrow \ddot{s} = g \sin \theta - \mu_k (\cos \theta + \dot{s}^2 / \rho)$

$$y = 200 \cos(\pi x / 800) \Rightarrow dy/dx = -\tan \theta = \frac{\pi}{4} \sin \frac{3\pi}{8} \Rightarrow \sin \theta = 0.58, \cos \theta = 0.81$$

If $v = 100 \text{ m s}^{-1} \Rightarrow \ddot{s} = 5.45 \text{ m s}^{-2}$

If $v = 150 \text{ m s}^{-1} \Rightarrow \ddot{s} = 7.01 \text{ m s}^{-2}$.

A force depends only on the Cartesian coordinates x , y and z : $\underline{F} = (-2xy + z)\hat{i} + (-x^2 + xz - z)\hat{j} + (xy - y)\hat{k}$.

Check if $\nabla \times \underline{F} = 0$

For the given \underline{F} ,

$$\nabla \times \underline{F} \neq 0$$

$\Rightarrow \underline{F}$ is not conservative.

$\Rightarrow V$ does not exist

