Coordinate Systems

1) Cartesian csys:

$$\mathbf{v}_{P|F} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$
 $\mathbf{a}_{P|F} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

2) Cylindrical polar csys:

$$egin{align*} \hat{m{v}}_{P|F} &= \dot{r}\hat{m{e}}_r + r\dot{\phi}\hat{m{e}}_\phi + \dot{z}\hat{m{e}}_z \ m{a}_{P|F} &= \left(\ddot{r} - r\dot{\phi}^2\right)\hat{m{e}}_r + \left(2\dot{r}\dot{\phi} + r\ddot{\phi}\right)\hat{m{e}}_\phi + \ddot{z}\hat{m{e}}_z \end{split}$$

3) Path csys:

$$egin{aligned} oldsymbol{v}_{P|F} &= \dot{s} \hat{oldsymbol{e}}_t \ oldsymbol{a}_{P|F} &= \ddot{s} \hat{oldsymbol{e}}_t + rac{\dot{s}^2}{
ho} \hat{oldsymbol{e}}_n \ rac{1}{
ho} &= rac{ig|oldsymbol{v}_{P|F} imes oldsymbol{a}_{P|F}ig|}{ig|oldsymbol{v}_{P|F}ig|^3} \end{aligned}$$

- Velocity and Acceleration Transfer

$$egin{aligned} \dot{m{A}}_{|F} &= \dot{m{A}}_{|m} + m{\omega}_{m|F} imes m{A} \ m{v}_{P|F} &= m{v}_{P|m} + m{v}_{A|F} + m{\omega}_{m|F} imes ec{r}_{PA} \ m{a}_{P|F} &= m{a}_{P|m} + m{a}_{A|F} + \dot{m{\omega}}_{m|F} imes ec{r}_{PA} + 2m{\omega}_{m|F} imes m{v}_{P|m} + m{\omega}_{m|F} imes (m{\omega}_{m|F} imes m{r}_{PA}) \end{aligned}$$

Composition of angular vel. and acc.

$$egin{aligned} oldsymbol{\omega}_{3|1} &= oldsymbol{\omega}_{3|2} + oldsymbol{\omega}_{2|1} \ \dot{oldsymbol{\omega}}_{3|1} &= \dot{oldsymbol{\omega}}_{3|2} + \dot{oldsymbol{\omega}}_{2|1} + oldsymbol{\omega}_{2|1} imes oldsymbol{\omega}_{3|2} \end{aligned}$$

Forces and Moments

1) Equivalent Force Systems:

$$oldsymbol{F}_{R} = oldsymbol{F}_{R}^{'} \ oldsymbol{M}_{A} = oldsymbol{M}_{A}^{'}$$

2) Linear Momentum:

$$oldsymbol{p}_{|F} = \int_m oldsymbol{v}_{P|F} \ dm = m oldsymbol{v}_{C|F}$$

3) Angular Momentum:

$$egin{align} m{H}_{A|F} &= \int_m m{r}_{PA} imes m{v}_{PA|F} \; dm \ & m{H}_A &= m{I}^A m{\omega}_{m|F} \ & (m{H}_A)_i &= I^A_{ii} m{\omega}_{ij} \ \end{aligned}$$

Axioms

1) Euler's Axioms (point O fixed in 'I')

$$\begin{split} \left. \frac{d}{dt} \left\{ \boldsymbol{p}_{|I} \right\} \right|_{I} &= \dot{\boldsymbol{p}}_{|I} = \boldsymbol{F}_{R} \\ \left. \frac{d}{dt} \left\{ \boldsymbol{H}_{O|I} \right\} \right|_{I} &= \dot{\boldsymbol{H}}_{O|I} = \boldsymbol{M}_{O} \end{split}$$

2) Modified Euler's 2nd axiom

$$\left. rac{d}{dt} \left\{ oldsymbol{H}_{A|I}
ight\}
ight|_I = \dot{oldsymbol{H}}_{A|I} = oldsymbol{M}_A - oldsymbol{r}_{CA} imes moldsymbol{a}_{A|I}$$

3) Coulomb friction axiom:

Static: $|\mathbf{f}| \leq \mu_s |\mathbf{N}|$ Kinetic: $|\mathbf{f}| = \mu_k |\mathbf{N}|$

Rigid Body Dynamics -

1) Inertia Tensor (and matrix components)

$$I_{ij}^A = \int_m \left(r^2 \delta_{ij} - x_i x_j \right) dm$$

 $\mathbf{I}^A = \int_m \left\{ \left(\boldsymbol{r}_{PA} \cdot \boldsymbol{r}_{PA} \right) \mathbf{I} - \boldsymbol{r}_{PA} \otimes \boldsymbol{r}_{PA} \right\} dm$

- 2) Parallel Axes Theorem
- C is the COM, and the origin is at A

$$I_{11}^{A} = I_{11}^{C} + m \left(x_{C_{2}}^{2} + x_{C_{3}}^{2} \right)$$

$$I_{22}^{A} = I_{22}^{C} + m \left(x_{C_{1}}^{2} + x_{C_{3}}^{2} \right)$$

$$I_{33}^{A} = I_{33}^{C} + m \left(x_{C_{1}}^{2} + x_{C_{2}}^{2} \right)$$

$$I_{ij}^{A} = I_{ij}^{C} - m x_{C_{i}} x_{C_{j}} \quad i \neq j$$

- Inertia Tensor for Common RBs

1) Rectangular Cuboid $(m = \rho whl)$

$$\left[\mathbf{I}^{C}\right] = m \operatorname{diag}\left(\left[\frac{(w^{2} + h^{2})}{12}, \frac{(l^{2} + h^{2})}{12}, \frac{(l^{2} + w^{2})}{12}\right]\right)$$

2) Circular Solid Cylinder $(m = \rho \pi R^2 l)$

$$\left[\mathbf{I}^{C}\right] = m \operatorname{diag}\left(\left[\left(\frac{R^{2}}{4} + \frac{l^{2}}{12}\right), \left(\frac{R^{2}}{4} + \frac{l^{2}}{12}\right), \frac{R^{2}}{2}\right]\right)$$

'diag' denotes the diagonal components of $\left[\mathbf{I}^{C}\right]$