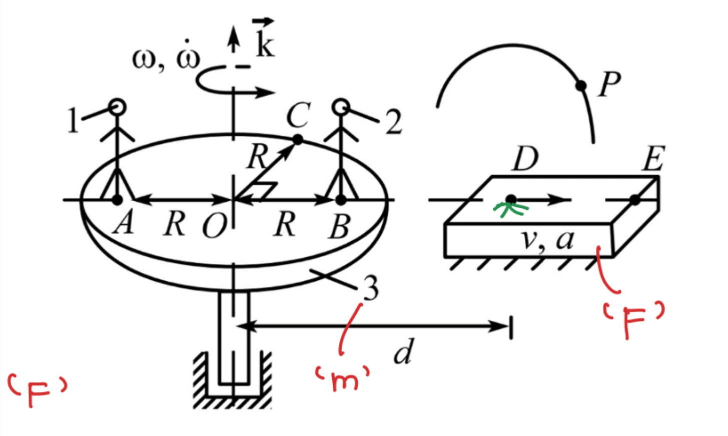
Tutorial 2 (Part A)

1) (a) Do the two persons observe the same velocity and acceleration of the projectile P? Justify your answer.
(b) Find the velocity and acceleration of ant D observed with respect to the platform.
(c) Find the velocity of point B relative to point A with respect to (i) the platform frame and (ii) the ground frame.
(d) Find the velocity and acceleration of point C as seen by the first person.

We call the rotating disc as moving frame 'm'

We call the platform DE that is fixed to ground as the fixed frame 'F'



a) To answer the first question, write the position vector of point P w.r.t. the two persons A (origin at A) and B (origin at B), and then take a time-derivative to compute the velocity. For this, we consider a cylindrical polar csys with origin once at A, calculate $\underline{r}_{PA}$ and then with origin at B and calculate $\underline{r}_{PB}$. Since both the persons are attached to the same moving ref. frame 'm', and time-derivative of a vector is ref. dependent BUT not dependent on the origin (or orientation) of the csys, therefore the two persons will observe the same velocity of the projectile P.

Mathematically, this can be shown as follows:

$$\underline{r}_{PA} = \underline{r}_{PB} + \underline{r}_{BA}$$

does not change with time

$$\underline{v}_{P|m} \text{ (observer at A)} = \frac{d}{dt}(\underline{r}_{PA})\Big|_m = \frac{d}{dt}(\underline{r}_{PB})\Big|_m + \cancel{\frac{d}{dt}(\underline{r}_{BA})\Big|_m}^{\,\underline{0}}$$

$$= \frac{d}{dt}(\underline{r}_{PB})\Big|_m$$

$$= \underline{v}_{P|m} \text{ (observer at B)}$$

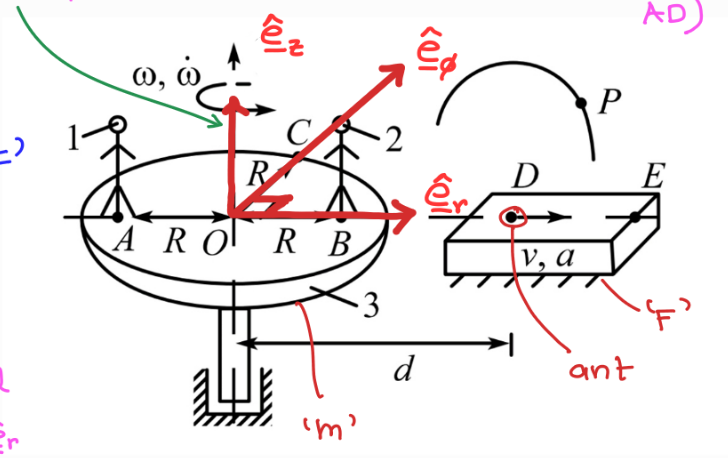
Hence, the velocities of P as seen by the two persons A & B are the SAME!

By the same argument, you can show that

$$\frac{d^2}{dt^2}(\underline{r}_{PA})\Big|_m = \frac{d^2}{dt^2}(\underline{r}_{PB})\Big|_m \Rightarrow$$

Accelerations of P are also same for the two persons A & B.

* Note: we orient the csys at the given instant in a way to help us with our calculations ($\hat{e}_r$ along AD)



b) $\left.\begin{array}{l}\underline{v}_{D|F}\\ \underline{a}_{D|F}\end{array}\right\}$ Velocity & acceleration of ant D w.r.t fixed frame 'F'

[Values are given]

$$\underline{v}_{D|F} = v\,\hat{e}_r$$

$$\underline{a}_{D|F} = a\,\hat{e}_r$$

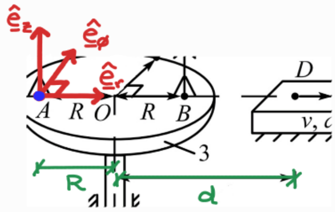
At the instant shown, $\underline{v}_{D|F}$ and $\underline{a}_{D|F}$ are along $\hat{e}_r$

$\left.\begin{array}{l}\underline{v}_{D|m}\\ \underline{a}_{D|m}\end{array}\right\}$ Velocity and acceleration of ant D w.r.t moving frame 'm'

[Needs to be computed]

We will use the velocity and acceleration transfer relationships:

We need to fix an origin; lets consider the origin at A (you could also choose 'O' as the origin) and calculate the position vector needed for velocity computation.



Velocity-transfer

$$\underline{v}_{D|F} = \underline{v}_{D|m} + \underline{v}_{A|F} + \underline{\omega}_{m|F} \times \underline{r}_{DA}$$

$$\underline{r}_{DA} = (R+d)\,\hat{e}_r$$

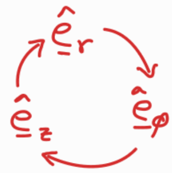
$$\underline{v}_{A|F} = \cancel{\underline{v}_{A|m}}^{\,0} + \underbrace{\underline{\omega}_{m|F}}_{\omega \hat{e}_z} \times \underbrace{\underline{r}_{AO}}_{-R\hat{e}_r} = -\omega R\,\hat{e}_\phi$$

$$\underline{\omega}_{m|F} \times \underline{r}_{DA} = \omega\hat{e}_z \times (R+d)\hat{e}_r = \omega(R+d)\,\hat{e}_\phi$$

$$\Rightarrow v\,\hat{e}_r = \underline{v}_{D|m} + \left[-\omega \cancel{R} + \omega(\cancel{R}+d)\right]\hat{e}_\phi$$

$$\Rightarrow v\,\hat{e}_r = \underline{v}_{D|m} + \omega d\,\hat{e}_\phi$$

$$\Rightarrow \boxed{\underline{v}_{D|m} = v\,\hat{e}_r - \omega d\,\hat{e}_\phi}$$



Acceleration-transfer

$$\underbrace{\underline{a}_{D|F}}_{\text{Acc of D meas wrt fixed frame 'F'}} = \underbrace{\underline{a}_{A|F}}_{\text{Acc of origin A of moving frame wrt fixed frame}} + \underbrace{\underline{a}_{D|m}}_{\text{Acc of D meas. wrt moving frame}}^{??} + \underbrace{(\dot{\underline{\omega}} \times \underline{r}_{DA})}_{\text{Tangential component}} + \underbrace{\underline{\omega} \times (\underline{\omega} \times \underline{r}_{DA})}_{\text{Centripetal/ Normal component}} + \underbrace{2(\underline{\omega} \times \underline{v}_{D|m})}_{\text{CORIOLIS Acc.}}$$

angular acc.

$\omega_{m|F}$

$$-\omega\,\hat{e}_r = \omega\hat{e}_z \times \hat{e}_\phi = \underline{\omega} \times \hat{e}_\phi$$

$$\underline{a}_{A|F} = \frac{d}{dt}(v_{A|F})\Big|_F = \frac{d}{dt}(-\omega R\,\hat{e}_\phi)\Big|_F = -\dot{\omega}R\,\hat{e}_\phi - \omega R\,\frac{d\hat{e}_\phi}{dt}\Big|_F$$

$$= -\dot{\omega}R\,\hat{e}_\phi + \omega^2 R\,\hat{e}_r$$

another way is to sub A in place of D in the acc. relation

$$\underline{a}_{A|F} = \cancel{\underline{a}_{O|F}}^{\,0} + \cancel{\underline{a}_{A|m}}^{\,0} + (\dot{\underline{\omega}} \times \underline{r}_{AO}) + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{AO}) + 2(\underline{\omega} \times \cancel{\underline{v}_{A|m}}^{\,0})$$

$$\underline{r}_{AO} = -R\,\hat{e}_r$$

$$\underline{a}_{A|F} = \left(\dot{\omega}\,\hat{\underline{e}}_z \times -R\,\hat{\underline{e}}_r\right) + \left\{\omega\,\hat{\underline{e}}_z \times \left(\omega\,\hat{\underline{e}}_z \times -R\,\hat{\underline{e}}_r\right)\right\} = -\dot{\omega} R\,\hat{\underline{e}}_\phi + \omega^2 R\,\hat{\underline{e}}_r$$

$$\therefore\ \underline{a}_{D|m} = \underline{a}_{D|F} - \underline{a}_{A|F} - \left(\dot{\underline{\omega}} \times \underline{r}_{DA}\right) - \underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{DA}\right) - 2\left(\underline{\omega} \times \underline{v}_{D|m}\right)$$

$$\dot{\underline{\omega}} \times \underline{r}_{DA} = \dot{\omega}\,\hat{\underline{e}}_z \times (R+d)\,\hat{\underline{e}}_r = \dot{\omega}(R+d)\,\hat{\underline{e}}_\phi$$

$$\underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{DA}\right) = \omega\,\hat{\underline{e}}_z \times \left\{\omega(R+d)\,\hat{\underline{e}}_\phi\right\} = -\omega^2(R+d)\,\hat{\underline{e}}_r$$

$$\underline{\omega} \times \underline{v}_{D|m} = \omega\,\hat{\underline{e}}_z \times \left\{v\,\hat{\underline{e}}_r - \omega d\,\hat{\underline{e}}_\phi\right\} = \omega v\,\hat{\underline{e}}_\phi + \omega^2 d\,\hat{\underline{e}}_r$$

$$\Rightarrow \underline{a}_{D|m} = a\,\hat{\underline{e}}_r - \left[-\cancel{\dot{\omega} R}\,\hat{\underline{e}}_\phi + \cancel{\omega^2 R}\,\hat{\underline{e}}_r\right] - \left[\dot{\omega}(\cancel{R}+d)\,\hat{\underline{e}}_\phi\right] - \left[-\omega^2(\cancel{R}+d)\,\hat{\underline{e}}_r\right]$$
$$- 2\left[\omega v\,\hat{\underline{e}}_\phi + \omega^2 d\,\hat{\underline{e}}_r\right]$$

$$\boxed{\underline{a}_{D|m} = \left(a - \omega^2 d\right)\hat{\underline{e}}_r - \left(\dot{\omega} d + 2\omega v\right)\hat{\underline{e}}_\phi}$$

c) $\underline{v}_{BA}|_F$: Velocity of point B relative to A w.r.t. frame 'F'

$\underline{v}_{BA}|_m$: Velocity of point B relative to A w.r.t. frame 'm'

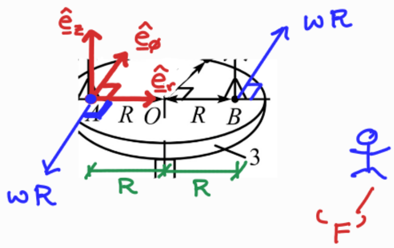
At the given instant:

$$\underline{v}_{BA}|_F = \underline{v}_B|_F - \underline{v}_A|_F$$

$$= \cancelto{0}{\underline{v}_B|_m} + \underline{\omega} \times \underline{r}_B - \cancelto{0}{v_A|_m} - \underline{\omega} \times \underline{r}_A$$

$$= \underline{\omega} \times \left(\underline{r}_B - \underline{r}_A\right)$$

$$= \omega\,\underline{e}_z \times (2R)\,\hat{\underline{e}}_r$$



$$\boxed{\underline{v}_{BA}|_F = 2\omega R\,\hat{\underline{e}}_\phi}$$

$$\boxed{\underline{v}_{BA}|_m = \cancelto{\underline{0}}{v_B|_m} - \cancelto{\underline{0}}{v_A|_m} = \underline{0}}$$ [$\because$ A and B are fixed to 'm']

d) Since the point C is fixed to the disc, therefore to an observer tied to the disc, there will be no velocity or acc. of C w.r.t to the moving disc frame 'm'

$$\therefore \quad \underline{v}_c|_m = \underline{0} \quad \text{and} \quad \underline{a}_c|_m = \underline{0}$$

2) An amusement park ride, shown in the figure below, consists of several connected rotating parts:

(i) The cockpit (where passengers sit) rotates relative to the telescopic arm AB at a given rate. → RB 3

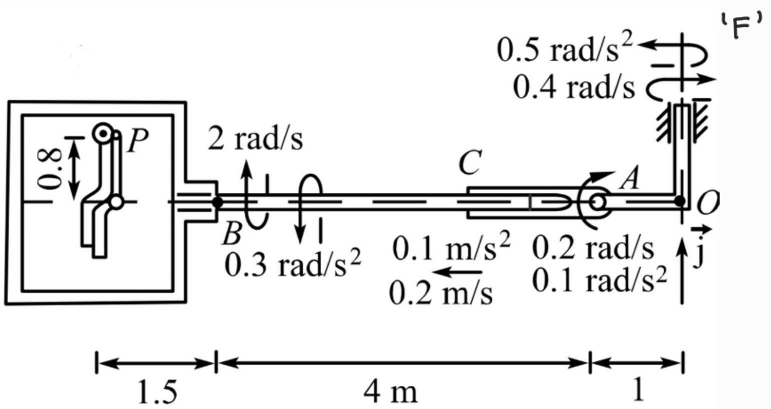
(ii) The telescopic arm AB swings relative to the arm OA at another specified rate. → RB 2

(iii) The length of the telescopic arm AB changes over time at a known rate.

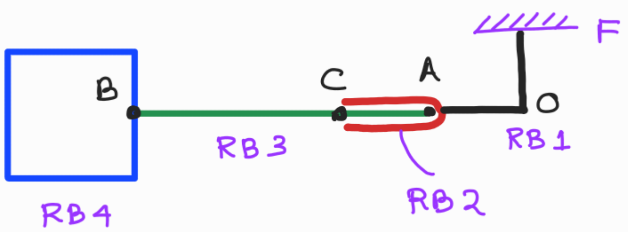
(iv) The arm OA rotates around a fixed vertical axis at a given rate. RB 1

Point P represents the center of the passenger's eye, who is seated in the cockpit. Find the acceleration of P w.r.t. the ground frame.

In these problems, it is important to first identify the reference frames needed to solve the problem!



Choosing # of frames



* RB 3 (rod ACB) is rotating and extending outwards wrt RB 2

⇓

* RB2 is rotating abt pt A and RB3 being fixed into RB 2 will also have the same rotation abt A.

From the point of view of calculating motion of pt P we would not need motion of pt C, and therefore we can take RB 2 and RB 3 together as one RB ≡ RB 2

Call RBs : Ground → frame 'F'

OA → frame '1'

AB → frame '2'

Cockpit → frame '3'

Select a single csys!

Given:

$$\underline{\omega}_{1|F} = 0.4\ \hat{e}_3 \ (\text{rad/s})$$

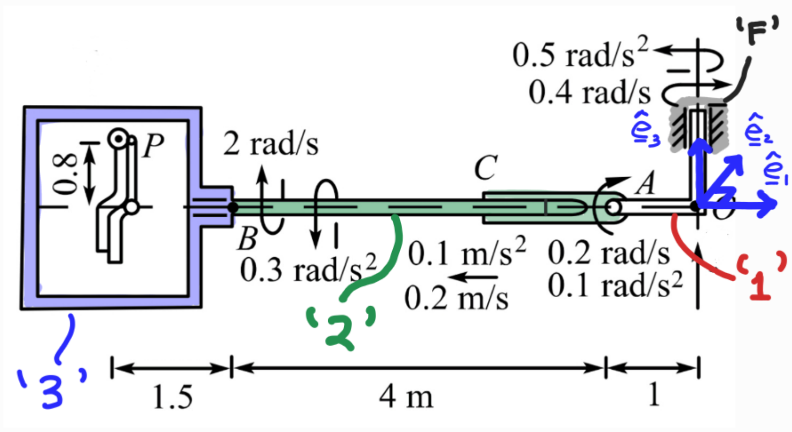
$$\dot{\underline{\omega}}_{1|F} = -0.5\ \hat{e}_3 \ (\text{rad/s}^2)$$

$$\underline{\omega}_{2|1} = 0.2\ \hat{e}_2 \ (\text{rad/s})$$

$$\dot{\underline{\omega}}_{2|1} = 0.1\ \hat{e}_2 \ (\text{rad/s}^2)$$

$$\underline{\omega}_{3|2} = -2\ \hat{e}_1 \ (\text{rad/s})$$

$$\dot{\underline{\omega}}_{3|2} = 0.3\ \hat{e}_1 \ (\text{rad/s}^2)$$



Velocity and acc of a particle (stuck at point B of rod AC) w.r.t to the RB '2':

$$\underline{v}_{B|2} = -0.2\ \hat{e}_1 \ (\text{m/s})$$

$$\underline{a}_{B|2} = -0.1\ \hat{e}_1 \ (\text{m/s}^2)$$

In the original figure:

$$\hat{i} \equiv -\hat{e}_1 \ , \quad \hat{j} \equiv \hat{e}_3 \ , \quad \hat{k} \equiv \hat{e}_2$$

Since we need the acceleration of P (part of RB '3') w.r.t 'F', we will have to find $\underline{\omega}_{3|F}$ and $\dot{\underline{\omega}}_{3|F}$ using composition of angular velocities and angular accelerations. After that, we can apply acceleration transfer relationship to get $\underline{a}_{P|F}$

Using composition of angular velocities:

$$\underline{\omega}_{2|F} = \underline{\omega}_{2|1} + \underline{\omega}_{1|F} = 0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3$$

$$\underline{\omega}_{3|F} = \underline{\omega}_{3|2} + \underline{\omega}_{2|F} = -2\,\hat{\underline{e}}_1 + 0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3$$

Using composition of angular accelerations:

$$\dot{\underline{\omega}}_{2|F} = \dot{\underline{\omega}}_{1|F} + \dot{\overline{\underline{\omega}_{2|1}}}\Big|_F$$

$$= \dot{\underline{\omega}}_{1|F} + \dot{\underline{\omega}}_{2|1} + \underline{\omega}_{1|F} \times \underline{\omega}_{2|1}$$

$$= -0.5\,\hat{\underline{e}}_3 + 0.1\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3 \times 0.2\,\hat{\underline{e}}_2$$

$$= -0.08\,\hat{\underline{e}}_1 + 0.1\,\hat{\underline{e}}_2 - 0.5\,\hat{\underline{e}}_3 \quad \left(0.08\,\hat{i} - 0.5\,\hat{j} + 0.1\,\hat{k}\right)$$

$$\dot{\underline{\omega}}_{3|F} = \dot{\underline{\omega}}_{2|F} + \dot{\overline{\underline{\omega}_{3|2}}}\Big|_F$$

$$= \dot{\underline{\omega}}_{2|F} + \dot{\underline{\omega}}_{3|2} + \underline{\omega}_{2|F} \times \underline{\omega}_{3|2}$$

$$= \left(-0.08\,\hat{\underline{e}}_1 + 0.1\,\hat{\underline{e}}_2 - 0.5\,\hat{\underline{e}}_3\right) + 0.3\,\hat{\underline{e}}_1 + \ldots$$

$$\ldots \left(0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3\right) \times \left(-2\,\hat{\underline{e}}_1\right)$$

$$= 0.22\,\hat{\underline{e}}_1 + 0.1\,\hat{\underline{e}}_2 - 0.5\,\hat{\underline{e}}_3 + 0.4\,\hat{\underline{e}}_3 - 0.8\,\hat{\underline{e}}_2$$

$$= 0.22\,\hat{\underline{e}}_1 - 0.7\,\hat{\underline{e}}_2 - 0.1\,\hat{\underline{e}}_3 \quad \left(-0.22\,\hat{i} - 0.1\,\hat{j} - 0.7\,\hat{k}\right)$$

A way to get vel. and acc. of a point is to start from the ground frame and find the vel and acc. of end points of successive RBs sequentially.

In this problem (say), we will start with the acc. of A (on RB '1'), then acc of B (on RB '2'), and then acc of P (on RB '3'), recursively applying vel/acc transfer relation and using the previous calculated acc. as $\underline{a}_{A|F}$

This A is going to be the I.P.

Acc transfer relation

$$\underline{a}_{P|F} = \underline{a}_{A|F} + \underline{a}_{P|m} + (\dot{\underline{\omega}}_{m|F} \times \underline{r}_{PA}) + \underline{\omega}_{m|F} \times (\underline{\omega}_{m|F} \times \underline{r}_{PA}) + 2(\underline{\omega}_{m|F} \times \underline{v}_{P|m})$$

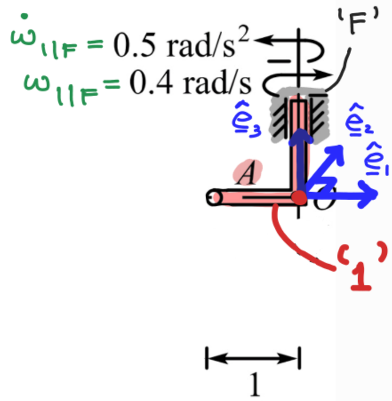
1) Acceleration of pt A on RB '1' wrt 'F'

$\Rightarrow$ moving frame, $m \equiv$ '1'

$\Rightarrow$ $\underline{\omega}_{m|F} \equiv \underline{\omega}_{1|F}$, $\dot{\underline{\omega}}_{m|F} \equiv \dot{\underline{\omega}}_{1|F}$

I.F. ≡ Intermediate frame

I.P. ≡ " pt (∈ I.F.)



Choose I.F. = '1', and

I.P. = 'O' (∈ I.F.)

$$\underline{a}_{A|F} = \cancel{\underline{a}_{O|F}}^{\,0} + \cancel{\underline{a}_{A|1}}^{\,0} + (\dot{\underline{\omega}}_{1|F} \times \underline{r}_{OA}) + \underline{\omega}_{1|F} \times (\underline{\omega}_{1|F} \times \underline{r}_{OA}) + 2(\underline{\omega}_{1|F} \times \cancel{\underline{v}_{A|1}}^{\,0})$$

$\because$ point 'O' has zero acc w.r.t F

point 'A' is fixed to RB '1', and hence $\underline{a}_{A|1} = \underline{0}$

$\underline{v}_{A|1} = \underline{0}$

$\underline{r}_{OA} = -1\,\hat{e}_1$ (m)

$\dot{\underline{\omega}}_{1|F} \times \underline{r}_{OA} = -0.5\,\hat{e}_3 \times -1\,\hat{e}_1 = 0.5\,\hat{e}_2$ (m/s²)

$\underline{\omega}_{1|F} \times (\underline{\omega}_{1|F} \times \underline{r}_{OA}) = 0.4\,\hat{e}_3 \times (0.4\,\hat{e}_3 \times -1\,\hat{e}_1)$

$= 0.4\,\hat{e}_3 \times -0.4\,\hat{e}_2$

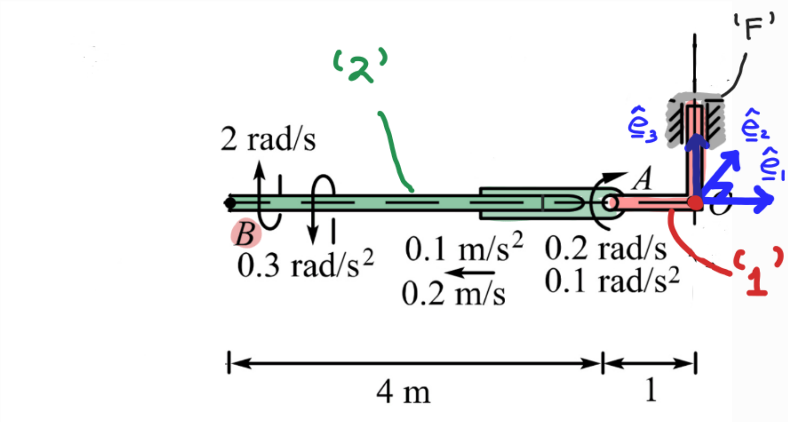
$= 0.16\,\hat{e}_1$

$\underline{a}_{A|F} = 0.16\,\hat{\underline{e}}_1 + 0.5\,\hat{\underline{e}}_2 \quad (m/s^2)$

2) Acceleration of pt 'B' on RB '2' wrt 'F'

$\Rightarrow$ moving frame, $m \equiv$ '2'

$\Rightarrow \underline{\omega} \equiv \underline{\omega}_{2|F}$, $\dot{\underline{\omega}} \equiv \underline{\omega}_{2|F}$



Choose I.F. $\equiv$ '1', and

I.P. $\equiv$ 'A' ($\in$ I.F.)

$\underline{r}_{BA}$ = Position vector from A to B = $\underline{r}_{B|O} - \underline{r}_{A|O} = -4\,\hat{\underline{e}}_1$ (m)

$$\underline{a}_{B|F} = \underline{a}_{A|F} + \underline{a}_{B|2} + (\dot{\underline{\omega}}_{2|F} \times \underline{r}_{BA}) + \underline{\omega}_{2|F} \times (\underline{\omega}_{2|F} \times \underline{r}_{BA}) + 2(\underline{\omega}_{2|F} \times \underline{v}_{B|2})$$

$$= (0.16\,\hat{\underline{e}}_1 + 0.5\,\hat{\underline{e}}_2) + (-0.1\,\hat{\underline{e}}_1)$$

$$+ \left[(-0.08\,\hat{\underline{e}}_1 + 0.1\,\hat{\underline{e}}_2 - 0.5\,\hat{\underline{e}}_3) \times (-4\,\hat{\underline{e}}_1)\right]$$

$$+ (0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3) \times \left\{(0.2\,\hat{\underline{e}}_1 + 0.4\,\hat{\underline{e}}_3) \times (-4\,\hat{\underline{e}}_1)\right\}$$

$$+ 2(0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3) \times (-0.2\,\hat{\underline{e}}_1)$$

$$= (0.16\,\hat{\underline{e}}_1 + 0.5\,\hat{\underline{e}}_2) + (-0.1\,\hat{\underline{e}}_1) + (2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3) + (0.8\,\hat{\underline{e}}_1)$$

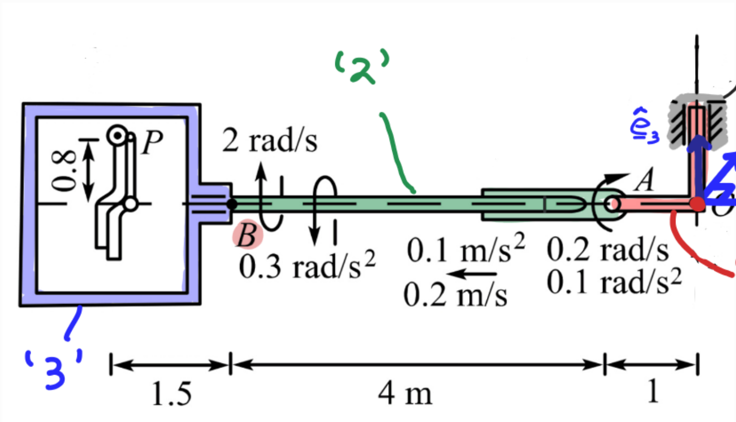
$$(-0.16\,\hat{\underline{e}}_2 + 0.08\,\hat{\underline{e}}_3)$$

$$= 0.86\,\hat{\underline{e}}_1 + 2.34\,\hat{\underline{e}}_2 + 0.48\,\hat{\underline{e}}_3 \quad (m/s^2)$$

3) Acceleration of pt 'P' wrt 'F'

$\Rightarrow$ moving frame, $m \equiv$ '3'

$\Rightarrow \quad \underline{\omega} \equiv \underline{\omega}_{3|F} \quad , \quad \dot{\underline{\omega}} \equiv \dot{\underline{\omega}}_{3|F}$



Choose I.F. $\equiv$ '3' , and

I.P. $\equiv$ 'B' ($\in$ I.F.)

$\underline{r}_{PB}$ = Position vector from B to P = $(-1.5\,\hat{\underline{e}}_1 + 0.8\,\hat{\underline{e}}_3)$

$$\underline{a}_{P|F} = \underline{a}_{B|F} + \cancelto{0}{\underline{a}_{P|3}} + (\dot{\underline{\omega}}_{3|F} \times \underline{r}_{PB}) + \underline{\omega}_{3|F} \times (\underline{\omega}_{3|F} \times \underline{r}_{PB})$$
$$+ 2\,(\underline{\omega}_{3|F} \times \cancelto{0}{\underline{v}_{P|3}})$$

$$= (0.86\,\hat{\underline{e}}_1 + 2.34\,\hat{\underline{e}}_2 + 0.48\,\hat{\underline{e}}_3)$$
$$+ (0.22\,\hat{\underline{e}}_1 - 0.7\,\hat{\underline{e}}_2 - 0.1\,\hat{\underline{e}}_3) \times (-1.5\,\hat{\underline{e}}_1 + 0.8\,\hat{\underline{e}}_3)$$
$$+ (-2\,\hat{\underline{e}}_1 + 0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3) \times \{(-2\,\hat{\underline{e}}_1 + 0.2\,\hat{\underline{e}}_2 + 0.4\,\hat{\underline{e}}_3)$$
$$\times (-1.5\,\hat{\underline{e}}_1 + 0.8\,\hat{\underline{e}}_3)\}$$

$$= (0.86\,\hat{\underline{e}}_1 + 2.34\,\hat{\underline{e}}_2 + 0.48\,\hat{\underline{e}}_3)$$
$$+ (-0.56\,\hat{\underline{e}}_1 - 0.026\,\hat{\underline{e}}_2 - 1.05\,\hat{\underline{e}}_3)$$
$$+ (-0.34\,\hat{\underline{e}}_1 + 0.664\,\hat{\underline{e}}_2 - 2.032\,\hat{\underline{e}}_3)$$

$$\underline{a}_{P|F} = -0.04\,\hat{\underline{e}}_1 + 2.978\,\hat{\underline{e}}_2 - 2.602\,\hat{\underline{e}}_3 \quad (m/s^2)$$

$$(0.04\,\hat{i} - 2.602\,\hat{j} + 2.978\,\hat{k})$$

3) Members OA and DP rotate at constant rate of 2 rad/s and 3 rad/s. The pin at P can slide in the circular slot of radius 4 m in plate AC. Find the angular acceleration of the plate AC in the given configuration when AP=1 m. OA = DP = 2 m

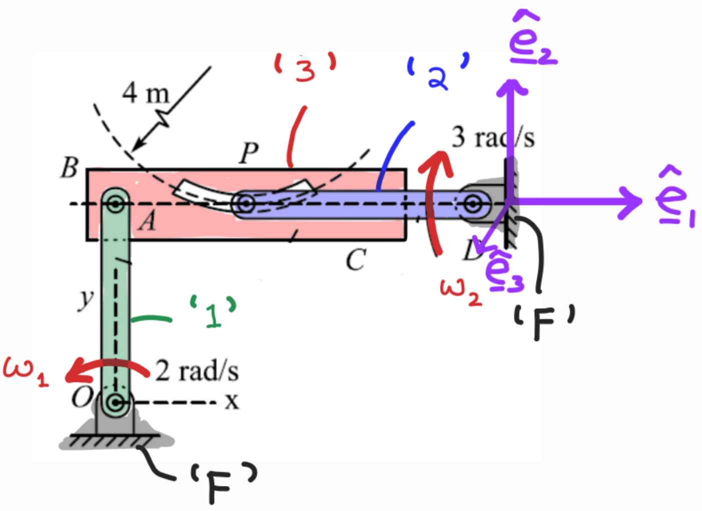
* Let's first fix the ref. frames

Hinge fixed to ground : 'F'

Rod OA : RB 1

Rod DP : RB 2

Plate AC : RB 3



* Establish a csys!

Given info: $\underline{\omega}_{1|F} = 2\,\hat{e}_3$ (rad/s)

$\underline{\omega}_{2|F} = -3\,\hat{e}_3$ (rad/s)

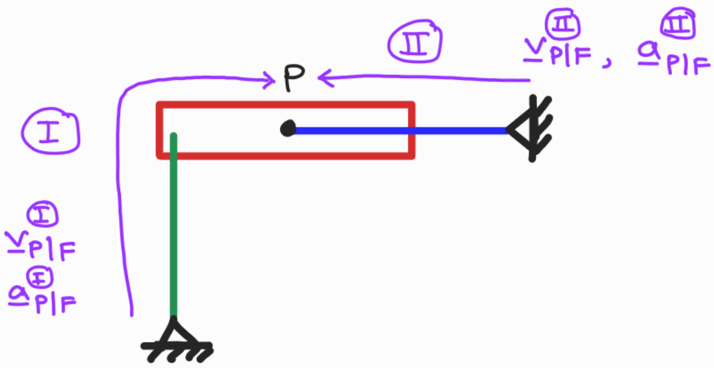
Question : $\underline{\dot{\omega}}_{3|F}$?

Note that point P is fixed to RB 2 (rod DP)

The idea of this problem is to find velocity and acceleration of a certain point (say P) from both sides and then match the two quantities.

$$\underline{v}_{P|F}^{(I)} = \underline{v}_{P|F}^{(II)}$$

$$\underline{a}_{P|F}^{(II)} = \underline{a}_{P|F}^{(I)}$$



From these two SETS you should be able to find the required unknowns.

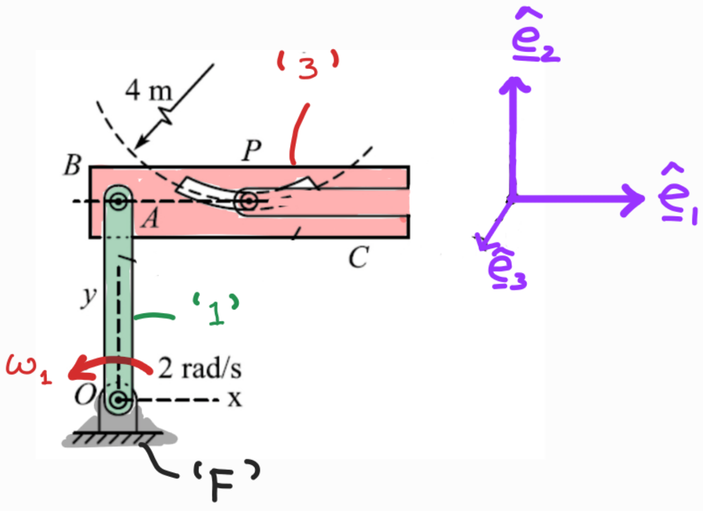
Since each vector will have three components each, so in total six unknown components can be found

Velocity transfer from side (I)

Target $\underline{v}_{P|F}^{(I)}$

$m \equiv$ '3'

I.P. $\equiv$ 'A' ($\in$ RB '3')



$$\underline{v}_{P|F} = \overset{?}{\underline{v}_{P|3}} + \overset{?}{\underline{v}_{A|F}} + \overset{?}{\underline{\omega}_{3|F}} \times \overset{\checkmark}{\underline{r}_{PA}}$$

At the given instant (see picture)

$\underline{v}_{P|3} = \underbrace{u\,\hat{e}_1}$ unknown (horizontal motion)

$\underline{v}_{A|F}$ can be obtained by treating I.F. $\equiv$ '1' and I.P. $\equiv$ 'O'

$$\underline{v}_{A|F} = \cancel{\underline{v}_{A|1}}^{0} + \cancel{\underline{v}_{O|F}}^{0} + \underline{\omega}_{1|F} \times \underline{r}_{AO}$$

$\underline{r}_{AO} = 2\,\hat{e}_2$ (m)

$$= 2\,\hat{e}_3 \times 2\,\hat{e}_2$$

$$= -4\,\hat{e}_1 \text{ (m/s)}$$

$\underline{r}_{PA} = 1\,\hat{e}_1$ (m)

$$\therefore \underline{v}_{P|F} = u\,\hat{e}_1 - 4\,\hat{e}_1 + \omega_3\hat{e}_3 \times 1\,\hat{e}_1$$

$$= (u-4)\,\hat{e}_1 + \omega_3\,\hat{e}_2$$

Since the motion is happening only in x-y plane, therefore the rotation must be abt the $\hat{e}_3$ axis

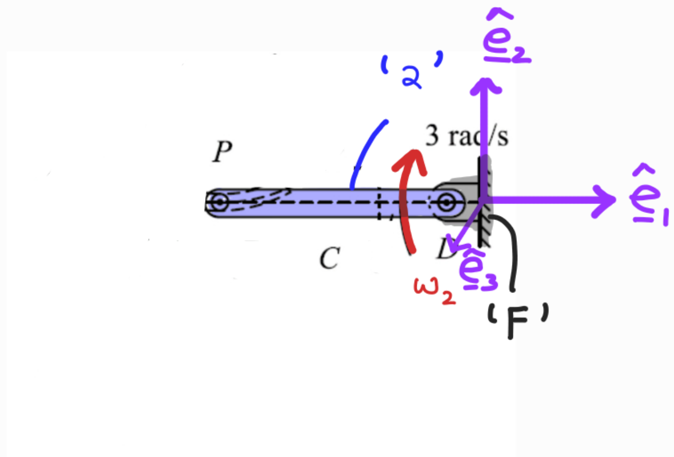
$\Rightarrow \underline{\omega}_{3|F} = \omega_3\hat{e}_3$

Velocity transfer from side (II)

Target: $\underline{v}_{P|F}^{(II)}$

$m \equiv 2$

I.P $\equiv$ D ($\in$ RB '2')



$$\underline{v}_{P|F} = \cancel{\underline{v}_{P|2}}^{0} + \cancel{\underline{v}_{D|F}}^{0} + \underline{\omega}_{2|F} \times \underline{r}_{PD}$$

$$= -3\,\hat{e}_3 \times -2\,\hat{e}_1$$

$$= 6\,\hat{e}_2 \text{ (m/s)}$$

$\underline{r}_{PD} = \underline{r}_P - \underline{r}_D = -2\,\hat{e}_1$ (m)

Match $\underline{v}_{P|F}^{(I)}$ to $\underline{v}_{P|F}^{(II)}$

$$(u-4)\,\hat{\underline{e}}_1 + \omega_3\,\hat{\underline{e}}_2 = 6\,\hat{\underline{e}}_2$$

$$\Rightarrow \quad u = 4 \text{ m/s} \quad \text{and} \quad \underbrace{\omega_3 = 6 \text{ rad/s}}$$

$$\Downarrow$$

$$\underline{\omega}_{3|F} = 6\,\hat{\underline{e}}_2 \text{ (rad/s)}$$

Repeat the same exercise for acceleration $\underline{a}_{P|F}$ and you will be able to obtain $\dot{\underline{\omega}}_{3|F}$!