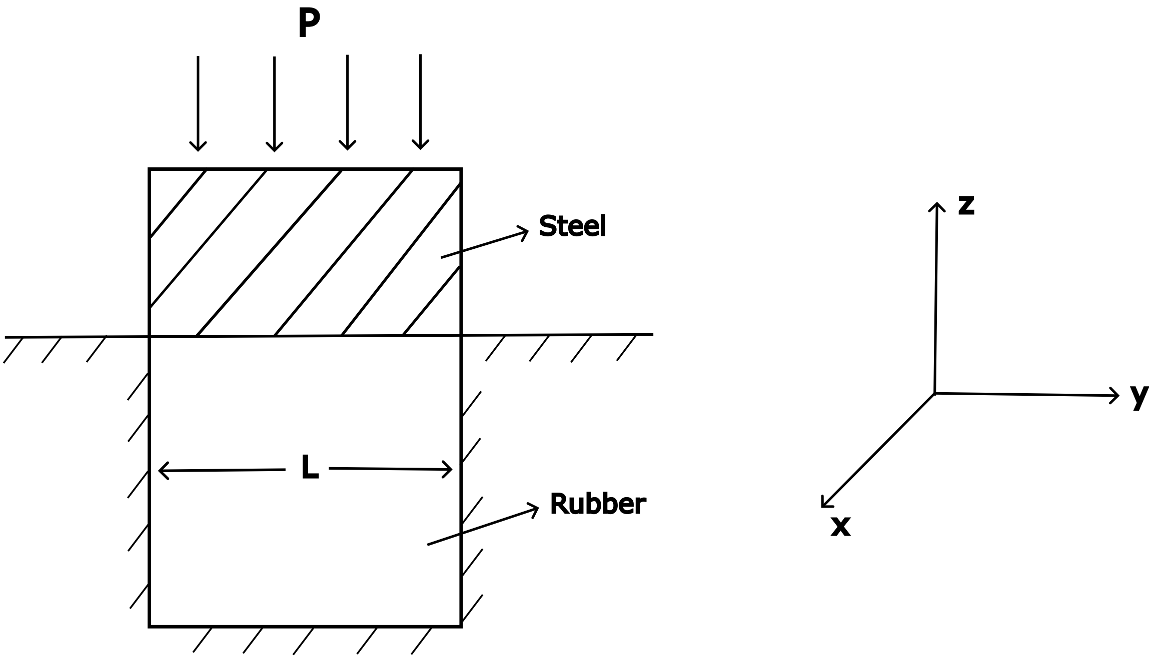
Tutorial 7

APL 104 - 2022 (Solid Mechanics)

1. Think of a rubber cube which is inserted within a cavity in steel block - the cavity has the same form and size as that of rubber cube (as shown below). The top surface of rubber cube is pressed by another steel block with a pressure of 'p' pascals. Assume the steel to be rigid and that there is no friction between steel and rubber.



   (a) Find the relation between normal stresses in x and y directions in this problem.

   (b) Find the volumetric strain.

2. A sample is subjected to a biaxial test under plane stress condition ($\sigma_z = \tau_{zx} = \tau_{zy} = 0$) using a special loading frame that maintains an in-plane loading constraint of $\sigma_x = 2\sigma_y$.

   (a) Find the slope of σ_{xx} vs ϵ_{xx}.

   (b) Find the ratio between ϵ_{xx} and ϵ_{yy} in terms of ν.

3. Think of an isotropic square plate which is clamped along one of its edges and subjected to uniform normal compressive load on the edge opposite to the clamped edge. The other two edges are traction free. Suppose that no out-of-plane displacement generates. Furthermore, the displacement components u_x and u_y are not functions of 'z' either.
   (a) Write down the strain and stress matrix for this problem.
   (b) Show that the z-component of equation is automatically satisfied.

(c) What boundary condition will be used to solve this deformation problem?

4. Think of a thin rectangular plate being compressed along its four edges but not allowed to expand or contract in its thickness direction. Assume the thickness direction is along z-axis. The following components of stress and strain matrices are given:

$$\sigma_{xx} = \sigma_{yy} = -p, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0, \qquad \epsilon_{zz} = 0.$$

   Assuming the material to be isotropic, find out the remaining components of strain matrix. Also obtain change in area divided by original area of the face of the plate (z-plane).

5. Show that in case of isotropic bodies, the stress tensor and the strain tensor will both have the same set of principal directions. Further show that the set of planes whose normals are parallel to one of the principal directions do not slide relative to each other.

6. Think of a solid beam having square cross-section of side length h and axial length L. The beam's axis lies along z axis while its cross-section's sides lie along x and y axes. Suppose the beam is stretched by applying axial force P to it such that its cross-section remains square and planar even after deformation. Also assume the deformation to be axially homogeneous. Let us think of using Cartesian coordinate system.

   (a) What coordinates (x, y, z) will the displacement functions (u_x, u_y, u_z) depend on? Give reasons for your answer.

   (b) Find out the strain matrix and the stress matrix in terms of displacement functions and the material parameters (λ, μ) in Cartesian coordinate system.

   (c) Substitute the expressions for stress components in the equilibrium equation (assume no body force/acceleration) and obtain the equations. Write down the boundary conditions too. Solve them to obtain the three displacement functions.