

Tutorial 2: Traction vector

APL 104 - 2022 (Solid Mechanics)

Q1. Show that $\underline{t}^n = \sum_i \underline{t}^i (\underline{n} \cdot \underline{e}_i) = \sum_i \hat{\underline{t}}^i (\underline{n} \cdot \hat{\underline{e}}_i)$, i.e, the formula is independent of what three planes are chosen to determine \underline{t}^n !

Q2. Suppose $[t^1] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $[t^2] = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $[t^3] = \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system.

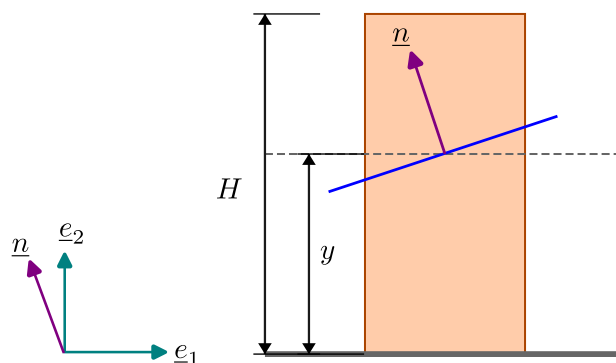
What will be the traction on a plane with normal $\underline{n} = \hat{\underline{e}}_1$ where $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$ is obtained from rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about \underline{e}_3 by 45° ? What are the normal and shear components of traction on this plane?

Q3. Show that the component of a traction vector, acting on a plane with normal \underline{n} , in the direction \underline{m} equals the component of a traction on \underline{m} -plane in the direction \underline{n} , i.e, $\underline{t}^n \cdot \underline{m} = \underline{t}^m \cdot \underline{n}$.

Q4. Consider a vertical bar having mass density ρ . Assume its length be to H and is subjected to uniform body force due to gravity. Find the traction vector on an internal section of the bar with outward normal

$$\underline{n} = -\sin \theta \underline{e}_1 + \cos \theta \underline{e}_2$$

and at a height of y from the base (see figure). Also find the normal and tangential components of the traction vector on this plane.



Q5. Think of a bar lying along \underline{e}_1 axis and loaded axially. We will learn later in the class that during the tensile loading of a bar, the traction on a section with normal along \underline{e}_1 has no shear components of traction. Also, as the bar is allowed to contract freely in the transverse direction, the traction on sections having normals perpendicular to \underline{e}_1 completely vanish. What will be the traction on the plane whose normal makes an angle θ from axial direction. What are the normal and shear components of traction on this plane?