Tutorial 6: Strain

APL 104 - 2022 (Solid Mechanics)

1. Think of the following displacement field in the body:

$$
\begin{aligned}
u_1 &= 0.05x_1 + 0.03x_2^2,\\
u_2 &= 0.07x_1x_2 + 0.08x_1^2,\\
u_3 &= 0
\end{aligned}
$$

   (a) Find the longitudinal strain of a line element along $\underline{e}_1$ direction <u>at any point</u> in the body.

   (b) Determine the shear strain between line elements along $\underline{e}_1$ and $\underline{e}_3$

   (c) Find volumetric strain for this displacement field. Does it vary from point to point?

   (d) What is the shear strain between line elements along $\underline{e}_1$ and $\underline{e}_3$ at any point (x_1, x_2)?

   (e) Determine the local rigid rotation.

2. The displacement field for a body is given by

$$\underline{u} = k(x^2 + y)\hat{\imath} + k(y + z)\hat{\jmath} + k(x^2 + 2z^2)\hat{k}$$

   Find the volumetric strain, shear strain γ_{xy} and γ_{yz}, and the axial vector of local infinitesimal rotation tensor of the body at a point $(2, 2, 3)$.

3. The displacement gradient matrix at a point in a body is given by

$$H = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   Write the condition for zero infinitesimal rotation.

4. For a circular rod subjected to a torque (shown in Figure), the displacement components obtained at any point (x, y, z) are obtained as:

$$
\begin{aligned}
u_x &= \breve{}\tau yz + ay + bz + c\\
u_y &= \tau xz \breve{}ax + ez + f\\
u_z &= \breve{}bx \breve{}ey + k
\end{aligned}
$$

   where a, b, c, e, f and k are constants, and τ is the shear stress.

(a) Select the constants a, b, c, e, f, k such that the end section $z = 0$ is fixed in the following manner:
   - Point o has no displacement.
   - The element Δz of the axis rotates neither in the plane xoz nor in the plane yoz
   - The element Δy of the axis does not rotate in the plane xoy.

(b) Determine the strain components.

(c) Verify whether these strain components satisfy the compatibility conditions.

5. For the displacement field $u_x = k(x^2 + 2z)$, $u_y = k(4x + 2y^2 + z)$, $u_z = 4kz^2$ with $k = 0.001$, determine the change in angle between two lines segments PQ and PR at $P(2, 2, 3)$ having direction cosines before deformation as

   (a) PQ: $n_{x1} = 0, \quad n_{y1} = n_{z1} = \frac{1}{\sqrt{2}}$
   PR: $n_{x2} = 1, \quad n_{y2} = n_{z2} = 0$

   (b) PQ: $n_{x1} = 0, \quad n_{y1} = n_{z1} = \frac{1}{\sqrt{2}}$
   PR: $n_{x2} = 0.6, \quad n_{y2} = 0, \quad n_{z2} = 0.8$

6. Verify whether the following strain field satisfies the equation of compatibility. Here p is a constant.

$$\epsilon_{xx} = py, \quad \epsilon_{yy} = px, \quad \epsilon_{zz} = 2p(x + y)$$
$$\gamma_{xy} = p(x + y), \quad \epsilon_{yz} = 2pz, \quad \epsilon_{zx} = 2pz$$

7. Given the following set of strain components:

$$\begin{aligned}
\epsilon_{xx} &= 5 + x^2 + y^2 + x^4 + y^4, \\
\epsilon_{yy} &= 6 + 3x^2 + 3y^2 + x^4 + y^4, \\
\gamma_{xy} &= 10 + 4xy(x^2 + y^2 + 2), \\
\epsilon_{zz} &= \gamma_{yz} = \gamma_{zx} = 0
\end{aligned}$$

   (a) Determine whether the above strain field is possible. If it is possible, determine the displacement components in terms of x and y. Assume that $u_x = u_y = 0$ and $\omega_{xy} = 0$ at the origin.

   (b) For the state of strain given in previous problem, write down the spherical and deviatoric parts and also determine the volumetric strain.