Tutorial 2: Traction vector

APL 104 - 2022 (Solid Mechanics)

Q1. Show that $\underline{t}^n = \sum_i \underline{t}^i \, (\underline{n} \cdot \underline{e}_i) = \sum_i \hat{\underline{t}}^i \, (\underline{n} \cdot \hat{\underline{e}}_i)$, i.e, the formula is independent of what three planes are chosen to determine $\underline{t}^n$!

Q2. Suppose $[\underline{t}^1] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $[\underline{t}^2] = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $[\underline{t}^3] = \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system.
What will be the traction on a plane with normal $\underline{n} = \hat{\underline{e}}_1$ where $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$ is obtained from rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about $\underline{e}_3$ by 45°?

Q3. Show that the component of a traction vector, acting on a plane with normal $\underline{n}$, in the direction $\underline{m}$ equals the component of a traction on $\underline{m}$-plane in the direction $\underline{n}$, i.e, $\underline{t}^n \cdot \underline{m} = \underline{t}^m \cdot \underline{n}$.

Q4. Consider a vertical bar having mass density ρ. Assume its length be to H and is subjected to uniform body force due to gravity. Find the traction vector on an internal section of the bar with outward normal

$$\underline{n} = -\sin\theta \; \underline{e}_1 + \cos\theta \; \underline{e}_2$$

and at a height of y from the base (see figure). Also find the normal and tangential components of the traction vector on this plane.