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-0.5 for every incorrect option

APL104: Quiz 1 (9th Sep 2022)

Q1. Prove that $\left[(\underset{\sim}{u}\cdot\underset{\sim}{a})\left(\underset{\approx}{A}^T\underset{\sim}{b}\right)\right]\cdot\underset{\sim}{n}=\underset{\sim}{a}\cdot\left[\left(\underset{\sim}{u}\otimes\underset{\approx}{A}\underset{\sim}{n}\right)\right]\underset{\sim}{b}$ (2)

Q2. If $\underset{\approx}{A}$ is a symmetric tensor and $\underset{\approx}{B}$ is an anti-symmetric tensor, then show that $\underset{\approx}{A}:\underset{\approx}{B}=0$. (2)

Q3. Given a rotation tensor $\underset{\approx}{R}$ that maps orthonormal triads of $(\underset{\sim}{e}_1-\underset{\sim}{e}_2-\underset{\sim}{e}_3)$ to $(\hat{\underset{\sim}{e}}_1-\hat{\underset{\sim}{e}}_2-\hat{\underset{\sim}{e}}_3)$, show that the matrix form of $\underset{\approx}{R}$ in the coordinate system of two orthonormal triads will be the same! (3)

Q4. Think of a bar lying along $\underset{\sim}{e}_1$ axis and loaded axially. We will learn later in the class that during the tensile loading of a bar, the traction on a section with normal along $\underset{\sim}{e}_1$ has no shear components of traction. Also, as the bar is allowed to contract freely in the transverse direction, the traction on sections having normals perpendicular to $\underset{\sim}{e}_1$ completely vanish. What will be the traction on the plane whose normal makes an angle θ from axial direction. What are the normal and shear components of traction on this plane? (2)

Q5. Which of the following statements are true: (1)

✓ (a) Scalars are zeroth order tensors
✓ (b) Tensors are independent of the coordinate system
(c) Vectors are more general than tensors
(d) One can add tensors of different orders

Q6. Traction at a point on planes which have oppositely directed normals (1)

(a) are not related
(b) are negative of each other due to Newton's 2^{nd} law
✓ (c) are negative of each other due to Newton's 3^{rd} law
(d) are related by acceleration of the body

Q7. The formula to obtain traction on an arbitrary plane at a point requires the following: (1)

✓ (a) Information of traction on three perpendicular planes at the same point
(b) Information of traction on planes at an adjacent point
(c) Magnitude of body force at that point
(d) Acceleration of the body at that point

Q8. The state of stress at a point (in MPa) is as follows: (2)

$$[\underset{\approx}{\sigma}]_{(e_1,e_2,e_3)}=\begin{bmatrix}10 & 50 & -50\\ 50 & 0 & 0\\ -50 & 0 & 0\end{bmatrix}$$

What will be the value of normal component of traction on a plane whose normal is $\underset{\sim}{e}'_1$ (it is in the direction of$(\sqrt{3}\underset{\sim}{e}_1+2\underset{\sim}{e}_2+3\underset{\sim}{e}_3)$?

third coordinate axis is also principal axis

Q9. Think of a stress tensor whose matrix form has zero off-diagonal components in the third column. Now, think of a new coordinate system which is obtained by rotating the original coordinate system relative to its third axis. What can you say about the stress matrix in the new coordinate system? (2)

- (a) the new stress matrix also has zero off-diagonal entries in third column
- (b) we cannot answer this question unless we are specified the angle by which the old coordinate system is rotated to obtain the new one.
- (c) the new stress matrix also has zero off-diagonal entries in third row
- (d) all entries of the new stress matrix will be non-zero

Q10. Suppose we have a general stress matrix in ($\underline{e}_1$-$\underline{e}_2$-$\underline{e}_3$) coordinate system. A new coordinate system is obtained by rotating the original coordinate system by 180° about $\underline{e}_2$ axis and the stress matrix in the new coordinate system is evaluated. Which of the following is true? (2)
(Hint: You can use stress transformation equation but you can also think geometrically.)

$\underline{e}_1 \to -\underline{e}_1$

$\underline{e}_2 \to \underline{e}_2$

$\underline{e}_3 \to -\underline{e}_3$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \downarrow \begin{bmatrix} \sigma_{11} & -\sigma_{12} & \sigma_{31} \\ -\sigma_{21} & \sigma_{22} & -\sigma_{32} \\ \sigma_{31} & -\sigma_{32} & \sigma_{33} \end{bmatrix}$$

- (a) The second row of the stress matrix remains unchanged after the change of the coordinate system
- (b) The second column of the stress matrix remains unchanged after the change of coordinate system
- (c) Only the diagonal elements of the stress matrix remain unchanged after the change of coordinate system
- (d) None of the above

Q11. Suppose a circular-cross-section straight beam is clamped at the one end and subjected to transverse load at other end. Its lateral surface is not subjected to any external load. What can we say about traction in its cross-section? (2)

- (a) Traction will vary in the cross-section
- (b) Traction at peripheral points of the cross-section will not have any shear component
- (c) The shear component of traction at peripheral points of the cross-section will have no radial component
- (d) None of these

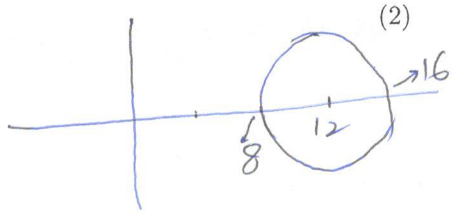
unknowns (3 independent)

$$\begin{bmatrix} - & - & \times \\ - & - & \times \\ \times & \times & \times \end{bmatrix}$$

Q12. Suppose we know the all the traction components on e_3 plane and we want to write stress matrix at a point in ($e_1 - e_2 - e_3$) coordinate system. How many independent stress components of the stress matrix are still unknown? (2)
(a) 6 (b) 2 (c) 5 (d) 3

Q13. Given below a stress matrix. What will be the difference between maximum and minimum principal stresses? (2)

$$\sigma = \begin{bmatrix} 12 & 4 & 0 \\ 4 & 12 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$



(a) 2 (b) 8 (c) 6 (d) none of these

Q14. The plane on which the shear component of traction attains its extremum value (2)

- (a) has its normal at 45° relative to those of principal planes
- (b) does not have normal component of traction
- (c) also has normal component of traction and equals the average of principal stress components
- (d) None of these

Q1

$$\mathbf{a}\cdot\left[(\mathbf{u}\otimes\mathbf{A}\mathbf{n})\,\mathbf{b}\right] = \mathbf{a}\cdot\left[\mathbf{u}\,\underbrace{(\mathbf{A}\mathbf{n}\cdot\mathbf{b})}_{\text{scalar}}\right]$$

$$= (\mathbf{a}\cdot\mathbf{u})\,(\mathbf{A}\mathbf{n}\cdot\mathbf{b})$$

$$= (\mathbf{u}\cdot\mathbf{a})\,(\mathbf{n}\cdot\mathbf{A}^T\mathbf{b})$$

$$= \underbrace{(\mathbf{u}\cdot\mathbf{a})}_{\text{scalar}}\,(\mathbf{A}^T\mathbf{b}\cdot\mathbf{n})$$

$$= \left[(\mathbf{u}\cdot\mathbf{a})(\mathbf{A}^T\mathbf{b})\right]\cdot\mathbf{n}$$

Q2

$$\mathbf{A}:\mathbf{B} = \sum_i\sum_j A_{ij}B_{ij}$$

Switching i & j since they are dummy indices

$$\sum_i\sum_j A_{ij}B_{ij} = \sum_i\sum_j A_{ji}B_{ji}$$

$$= -\sum_i\sum_j A_{ij}B_{ij} \qquad \left\{\because \begin{array}{l} A_{ij} = A_{ji} \\ B_{ij} = -B_{ji} \end{array}\right\}$$

$$\Rightarrow \quad 2\sum_i\sum_j A_{ij}B_{ij} = 0$$

$$\Rightarrow \quad \mathbf{A}:\mathbf{B} = 0$$

(3) Given $\hat{\underline{e}}_i = \underline{\underline{R}}\, \underline{e}_i \Rightarrow \underline{e}_i = \underline{\underline{R}}^{-1} \hat{\underline{e}}_i$

$$R_{ij} = (\underline{\underline{R}}\, \underline{e}_j) \cdot \underline{e}_i = \hat{\underline{e}}_j \cdot \underline{e}_i \quad -(1)$$

$\hat{\underline{\underline{R}}} \rightarrow$ Represents the rotation tensor in $\hat{\underline{e}}_1 - \hat{\underline{e}}_2 - \hat{\underline{e}}_3$

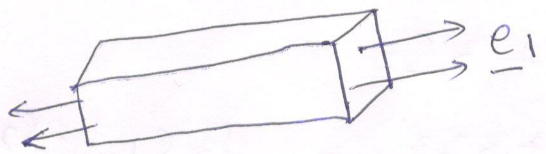
$$\hat{R}_{ij} = (\underline{\underline{R}}\, \hat{\underline{e}}_j) \cdot \hat{\underline{e}}_i = \hat{\underline{e}}_j \cdot \underline{\underline{R}}^T \hat{\underline{e}}_i = \hat{\underline{e}}_j \cdot \underline{e}_i \quad -(2)$$

$\because$ $\underline{\underline{R}}$ is an orthogonal tensor $\underline{\underline{R}}\,\underline{\underline{R}}^T = \underline{\underline{I}} \Rightarrow \underline{\underline{R}}^T = \underline{\underline{R}}^{-1}$

and $\underline{\underline{A}}\, \underline{b} \cdot \underline{c} = \underline{b} \cdot \underline{\underline{A}}^T \underline{c}$

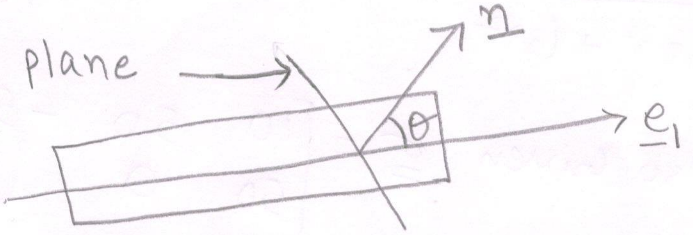
From (1) & (2) $\boxed{R_{ij} = \hat{R}_{ij}}$ Hence proved.

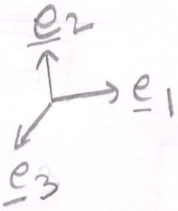
(4)



Given $\underline{\underline{\sigma}}\, \underline{e}_1 = \overset{\text{scalar}}{k}\, \underline{e}_1$

Also, $\underline{\underline{\sigma}}\, \underline{e}_2 = \underline{\underline{\sigma}}\, \underline{e}_3 = \underline{0}$





Normal of the plane $\underline{n} = \cos\theta\, \underline{e}_1 + \sin\theta\, \underline{e}_2$

Now traction is desired on $\underline{n}$

$$\underline{t}^n = \sum_i \underline{t}^i (\underline{n} \cdot \underline{e}_i) = \underline{t}^1 (\underline{n} \cdot \underline{e}_1) + \cancel{\underline{t}^2}^{\,0} (\underline{n} \cdot \underline{e}_2) + \cancel{\underline{t}^3}^{\,0} (\underline{n} \cdot \underline{e}_3)$$

$$\underline{t}^n = \underline{t}^1 (\cos\theta\, \underline{e}_1 + \sin\theta\, \underline{e}_2) \cdot \underline{e}_1$$

$$\underline{t}^n = \underline{t}^1 \cos\theta$$

$$\boxed{\underline{t}^n = k \cos\theta\, \underline{e}_1}$$

Normal component of traction $t_n = \underline{t}^n \cdot \underline{n}$

$t_n = k\cos\theta\, \underline{e}_1 \cdot (\cos\theta\, \underline{e}_1 + \sin\theta\, \underline{e}_2)$

$$\boxed{t_n = k\cos^2\theta}$$

Shear component of traction $t_s = \sqrt{|\underline{t}^n|^2 - t_n^2}$

$t_s = \sqrt{(k\cos\theta)^2 - (k\cos^2\theta)^2}$

$t_s = k\cos\theta\sqrt{1-\cos^2\theta}$

$$\boxed{t_s = k\cos\theta\sin\theta}$$

$$\boxed{t_s = \frac{1}{2}k\sin 2\theta}$$

(8) Given $\underline{e}_1' = \sqrt{3}\,\underline{e}_1 + 2\,\underline{e}_2 + 3\,\underline{e}_3$

$\underline{n} = \dfrac{\underline{e}_1'}{|\underline{e}_1'|} = \dfrac{\sqrt{3}\,\underline{e}_1 + 2\,\underline{e}_2 + 3\,\underline{e}_3}{\sqrt{(\sqrt{3})^2+(2)^2+(3)^2}} = \dfrac{\sqrt{3}\,\underline{e}_1 + 2\underline{e}_2 + 3\underline{e}_3}{4}$

$\underline{n} = \dfrac{1}{4}\begin{bmatrix}\sqrt{3}\\2\\3\end{bmatrix}$, Also Given $\underline{\underline{\sigma}} = \begin{bmatrix}10 & 50 & -50\\50 & 0 & 0\\-50 & 0 & 0\end{bmatrix}$

$$\sigma_{nn} = (\underline{\underline{\sigma}}\,\underline{n})\cdot\underline{n} = \frac{\begin{bmatrix}10 & 50 & -50\\50 & 0 & 0\\-50 & 0 & 0\end{bmatrix}\begin{bmatrix}\sqrt{3}\\2\\3\end{bmatrix}\cdot\begin{bmatrix}\sqrt{3}\\2\\3\end{bmatrix}}{4^2}$$

$$\sigma_{nn} = \frac{1}{16}\begin{bmatrix}10\sqrt{3}-50\\50\sqrt{3}\\-50\sqrt{3}\end{bmatrix}\cdot\begin{bmatrix}\sqrt{3}\\2\\3\end{bmatrix} = \frac{1}{16}\begin{bmatrix}10\sqrt{3}-50 & 50\sqrt{3} & -50\sqrt{3}\end{bmatrix}\begin{bmatrix}\sqrt{3}\\2\\3\end{bmatrix}$$

$$\sigma_{nn} = \frac{30-50\sqrt{3}+100\sqrt{3}-150\sqrt{3}}{16} = \frac{30-100\sqrt{3}}{16}$$