Tutorial 1: Mathematical Preliminaries

APL 104 - 2022 (Solid Mechanics)

Q1. Show that $\underline{a} \cdot \left(\underline{\underline{A}}\,\underline{b}\right) = \left(\underline{\underline{A}}^T \underline{a}\right) \cdot \underline{b}$

Q2. There exists a tensor $\underline{\underline{A}}$ such that $\underline{\underline{A}} \cdot \underline{e}_1 = \underline{a}$, $\underline{\underline{A}} \cdot \underline{e}_2 = \underline{b}$, $\underline{\underline{A}} \cdot \underline{e}_3 = \underline{c}$. What will be the matrix form of $\underline{\underline{A}}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system?

Q3. Show that

(a) $(\underline{a} \otimes \underline{b})\,\underline{\underline{C}} = \underline{a} \otimes \left(\underline{\underline{C}}^T \underline{b}\right)$

(b) $\underline{\underline{C}}\,(\underline{a} \otimes \underline{b}) = \left(\underline{\underline{C}}\,\underline{a}\right) \otimes \underline{b}$

Q4. Given an anti-symmetric tensor $\underline{\underline{A}}$, prove that $\left(\underline{\underline{A}}\,\underline{x}\right) \cdot \underline{x} = 0 \;\; \forall\, \underline{x}$

Q5. In class we learnt that a unique rotation tensor $\underline{\underline{R}}$ can be associated with transforming a set of orthonormal triad into anoother say $(\underline{e}_1, \underline{e}_2, \underline{e}_3) \rightarrow (\underline{\hat{e}}_1, \underline{\hat{e}}_2, \underline{\hat{e}}_3)$. In particular, we discussed a specific case where $(\underline{\hat{e}}_1, \underline{\hat{e}}_2, \underline{\hat{e}}_3)$ is obtained by rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about $\underline{e}_3$ axis by angle θ. Find the matrix form of this rotation tensor $\underline{\underline{R}}$ in $(\underline{\hat{e}}_1, \underline{\hat{e}}_2, \underline{\hat{e}}_3)$ coordinate system.