APL104: Quiz 2 (6th Nov 2022) Full Marks: 25

Instruction: There can be multiple correct options. For every incorrect choice, you will be given negative marks equal to half of the marks assigned to that particular question.

1. While deriving linear momentum balance in cylindrical coordinate system, we get extra terms such as $\frac{\sigma_{rr}}{r}$, $\frac{\tau_{\theta r}}{r}$ and $\frac{\tau_{zr}}{r}$ which are not present in the Linear momentum balance equation in Cartesian coordinate system. This happened because (1)

   (a) In cylindrical coordinate system, the areas of +r plane and -r plane are not the same.
   (b) In cylindrical coordinate system, the areas of +r plane and -r plane are same.
   (c) In cylindrical coordinate system, the areas of $+\theta$ plane and $-\theta$ planes are not the same.
   (d) None of these.

2. If the three displacement components in cylindrical coordinate system are all constant

   (a) All strain components will be zero
   (b) All except hoop strain will be zero
   (c) Both $\epsilon_{\theta\theta}$ and $\epsilon_{r\theta}$ will be non-zero
   (d) none of these

$$\begin{bmatrix} \cancel{\frac{\partial u_r}{\partial r}} & \frac{1}{r}\left(\cancel{\frac{\partial u_r}{\partial \theta}} - u_\theta\right) + \cancel{\frac{\partial u_\theta}{\partial r}} & \cancel{\frac{\partial u_r}{\partial z}} + \cancel{\frac{\partial u_z}{\partial r}} \\ & \frac{1}{r}\cancel{\frac{\partial u_\theta}{\partial \theta}} + \frac{u_r}{r} & \cancel{\frac{\partial u_\theta}{\partial z}} + \cancel{\frac{1}{r}\frac{\partial u_z}{\partial \theta}} \\ & & \cancel{\frac{\partial u_z}{\partial z}} \end{bmatrix}$$

3. The deformation of a body is such that it follows the following displacement field:
   $u_r = 0.05r^2 + 0.2\theta$, $u_\theta = 0.4\theta + .5z$, $u_z = 0.02z + 0.03z^2r + .05\theta$.
   The circumferential strain is given by (1)

   (a) $\frac{1}{r}(0.4 + 0.05r^2)$
   (b) $\frac{1}{r}(0.4 + 0.05r + 0.2\theta)$
   (c) $\frac{1}{r}(0.4 + 0.05r^2 + 0.2\theta)$
   (d) None of these

$$\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} = \frac{.4}{r} + .05r + \frac{.2\theta}{r}$$

4. While solving the extension-torsion-inflation problem in a hollow cylinder, we assumed the axial displacement u_z to depend only on 'z' because (1)

   (a) the dependence of u_z on 'r' would lead to warping in the cross-section
   (b) the dependence of u_z on would violate axisymmetry
   (c) the dependence of u_z on 'r' would lead to axial inhomogeneity
   (d) none of these

5. Think of a solid circular shaft which is pressurised by 'p' on its outer surface. No axial force or torque acts on it. What can you say about the distribution of radial stress σ_{rr}? (2)

   (a) σ_{rr} will be varying in the cylinder along its radius

$\sigma_{rr} = A + B/r^2$

At $r = 0 \Rightarrow \sigma_{rr}$ should be finite $\Rightarrow B = 0$

Apply bc $\Rightarrow A = -P$

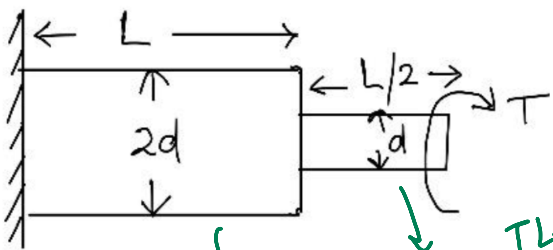
$\sigma_{rr} = -p$

(b) σ_{rr} will be constant in the cylinder and equal to -p

(c) σ_{rr} will be constant but equal to p

(d) none of these

6. A Torque 'T' is applied at the free end of a stepped rod of circular cross-sections as shown in the figure. The shear modulus of the material of the rod is G. The twisting rotation 'Ω' at the free end is (2)



$\theta_1 = \frac{TL_1}{GJ_1}$ $\quad \theta_2 = \frac{TL_2}{GJ_2}$

$\Rightarrow \Omega: \theta_1 + \theta_2 = \frac{T}{G}\frac{L \cdot 32}{\pi (2d)^4} + \frac{T}{G}\frac{L/2 \cdot 32}{\pi d^4}$

$= \frac{2TL}{\pi G d^4} + \frac{16TL}{G\pi d^4}$

$= \frac{18TL}{\pi G d^4}$

(a) $\frac{32TL}{\pi G d^4}$

(b) $\frac{18TL}{\pi G d^4}$

(c) $\frac{16TL}{\pi G d^4}$

(d) $\frac{2TL}{\pi G d^4}$

7. A beam with a rectangular cross-section of depth 'h' and width '2h' is hinged at both ends (simply supported) and carries a vertical point load 'P' at its mid-point. The length of the beam is '50h'. The ratio of the maximum bending stress (σ_{xx}) to the maximum shear stress (τ_{xy}) is (2)

P $\quad$ P/2 $\quad$ P/2

$\sigma_{xx_{max}} = \frac{M_{max} y_{max}}{I} = \frac{\frac{P}{2} \cdot \frac{50h}{2} \frac{h}{2}}{\frac{1}{12} 2h\, h^3} = \frac{75P}{2h^2}$ $\quad , \quad \tau_{max} = \frac{3}{2}\frac{V_{max}}{A} = \frac{3}{2} \cdot \frac{P/2}{2h^2} = \frac{3P}{8h^2}$

(a) 1000

(b) 100

(c) 10

(d) 1

8. Think of two square cross-sectional beams: (i) Beam A is made up of a material with Young's modulus E but its cross-sectional size is 'a'. (ii) Beam B is made up of a material with Young's modulus 4E but its cross-sectional size is 'a/2'. Which of the following statements is/are correct? (2)

(a) Large bending moment is required to bend beam B than beam A.

(b) Large bending moment is required to bend beam A than beam B.

(c) Beam A has higher bending modulus than beam B.

(d) Beam B has higher bending modulus than beam A.

$(EI)_A = E \frac{1}{12} a^4$

$(EI)_B = 4E \frac{1}{12} \left(\frac{a}{2}\right)^4$

$\Rightarrow (EI)_A = 4(EI)_B$

$\Rightarrow$ A is stiffer in bending

9. Think of an arbitrary cross-section such that $I_{yy} = 1$, $I_{yz} = 1$. Further, suppose that the bending moment acts along the z-axis and beam lies along X-axis. What would be the angle between neutral axis and the Y-axis of the cross-section (2)

   (a) we also need to know I_{zz}
   (b) 45°
   (c) 90°
   (d) 0°

$\tan\theta = \frac{I_{yz}}{I_{yy}} = 1$

$\Rightarrow \theta = 45^\circ$

10. Why do we consider open cross-sections while calculating shear stress distribution in case of non-uniform bending of unsymmetrical cross-section? (1)

   (a) For open cross-sections, shear stress at the open end gets known.
   (b) For open cross-sections, shear stress can be assumed to be constant along its thickness.
   (c) Both (a) and (b).
   (d) None of these.

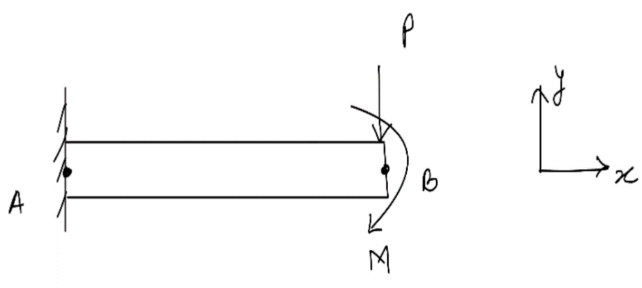
problem just says open. It does not say whether it is thick or thin!

11. Suppose a beam is clamped at one end (x=0) but at the other end (x=L), it is subjected to transverse load (P) and at the same time, it is not allowed to rotate at this end. Which of the following holds true when the beam is modelled as an Euler Bernouli beam? (2)

   (a) y(0) = 0, y'(0) = 0, y'(L)=0
   (b) y(0) = 0, y'(0) = 0, θ(L)=0
   (c) bending moment M vanishes at x=L
   (d) y(0) = 0, θ(0) = 0, θ(L)=0

$y' = \theta$ for EBT

12. Consider a uniform cantilever beam having bending modulus 'EI' and length 'L'. It is clamped at A and subjected to a concentrated force 'P' and moment 'M' applied at B as shown below. The slope at B as per Euler-Bernoulli beam theory is (3)



   (a) $-(\frac{PL^2}{2EI} + \frac{2ML}{3EI})$
   (b) $\frac{PL^2}{2EI} + \frac{ML}{EI}$
   (c) $-(\frac{PL^2}{2EI} + \frac{ML}{EI})$

$EI\frac{d^2y}{dx^2} = -M - P(L-x)$

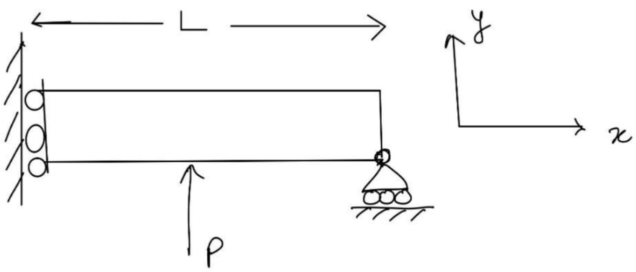
$\Rightarrow EI\frac{dy}{dx} = -Mx - PLx + \frac{Px^2}{2} + C$

$C = 0$ since $\frac{dy}{dx}(0) = 0$

$\Rightarrow \frac{dy}{dx}(L) = \frac{1}{EI}\left[-ML - PL^2 + \frac{PL^2}{2}\right]$

(d) None of these

13. Suppose a beam is kept with roller support at one end (x=0) constrained to only move in y-direction and pinned at the other end (x=L) as shown. Beam is subjected to transverse load (P) at the middle of the beam. Which of the following holds true when the beam is modeled as a Euler Bernoulli beam? (2)



for $x < L/2$, $M =$ constant

for $x > L/2$, M varies

$\frac{d}{dx}\left(EI\frac{d^2y}{dx^2}\right)=0$ at $x=0$

(a) $-\frac{d}{dx}(EI\frac{d^2y}{dx^2}) = 0$ at $x = 0$, $EI\frac{d^2y}{dx^2} = 0$ at $x = L$

(b) $\frac{dy}{dx}(0) \neq 0$, $y(L) = 0$

(c) $\frac{dy}{dx}(0) = 0$, $y(L) = 0$

(d) $-\frac{d}{dx}(EI\frac{d^2y}{dx^2}) \neq 0$ at $x = 0$, $EI\frac{d^2y}{dx^2} = 0$ at $x = L$

14. A column of length 'h' with a square cross-section of size (a×a) has a buckling load of P. If the cross-section is changed to (2a×2a) and its length changed to 3h, the buckling load of the new column will be (2)

(a) 16P/9

(b) P/4

(c) P/2

(d) 3P/2

$P_{buckling} \propto \frac{EI}{L^2}$

$P_{old} = C_1 E \frac{\frac{1}{12}a^4}{h^2}$

$P_{new} = C_1 E \frac{\frac{1}{12}(2a)^4}{(3h)^2}$

15. A straight beam is clamped at one end (s=0) while pinned at the other end (s=L). Assuming the transverse deflection is denoted by y(s) whereas the cross-sectional rotation is denoted by θ(s) for a cross-section at position 's'. Write down the boundary conditions needed to obtain deflection of this beam using Timoshenko beam theory? (1)

(a) y(0) = 0, θ(0) = 0, moment (L) = 0

(b) y(0) = 0, y$'$(0) = 0, y(L) = 0, moment (L) = 0

(c) y(0) = 0, θ(0) = 0, y(L) = 0, moment (L) = 0

(d) none of these

$\frac{P_{new}}{P_{old}} = \frac{16}{9}$