Tutorial 4: Stress equilibirum equations and Principal Stresses

APL 104 - 2022 (Solid Mechanics)

1. The cross-section of the wall of dam is shown in Fig.1.

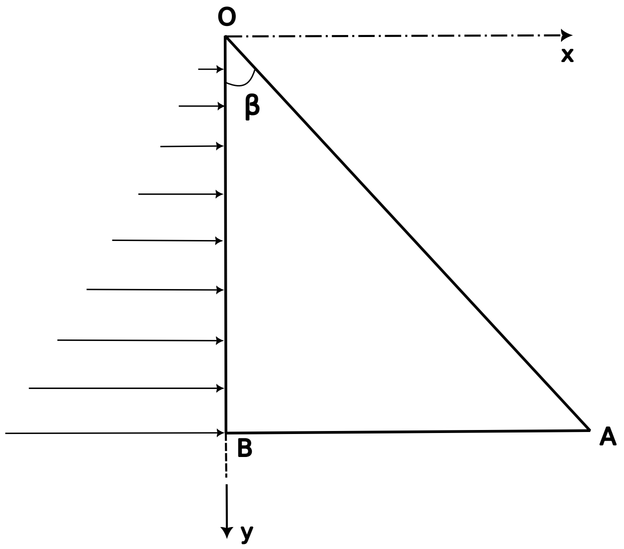

Figure 1

The pressure of water on face OB is also shown. With the axes Ox and Oy, as shown in Fig.1, the stresses at any point (x, y) are given by (γ = specific weight of water and ρ = specific weight of dam material)

$$\sigma_x = -\gamma y$$

$$\sigma_y = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = \tau_{yx} = -\frac{\gamma}{\tan^2\beta}x$$

$$\tau_{yz} = 0,\ \tau_{zx} = 0,\ \sigma_z = 0$$

Check if these stress components satisfy the differential equations of equilibrium. Also, verify if the boundary conditions are satisfied on face OB.

2. Consider the rectangular beam shown in Fig.2. According to the elementary theory of bending, the 'fibre stress' in the elastic range due to bending is given by

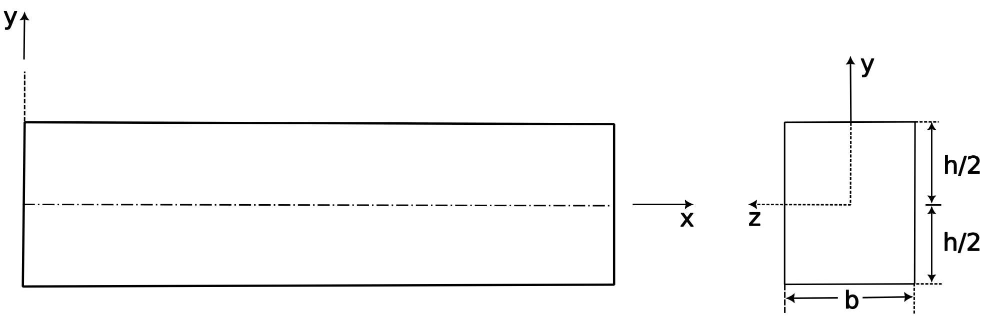

Figure 2

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3}$$

where M is the bending moment which is a function of x. Assume that $\sigma_z = \tau_{zx} = \tau_{zy} = 0$ and also that $\tau_{xy} = 0$ at the top and bottom, and further,that $\sigma_y = 0$ at the bottom. Using the differential equations of equilibrium, determine τ_{xy} and σ_y. Compare these with the values given in the elementary strength of materials.

3. A cylindrical rod (Fig.3) is subjected to a torque T. At any point P of the cross-section LN, the following stresses occur

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = \tau_{yx} = 0,\ \tau_{xz} = \tau_{zx} = -G\theta y,\ \tau_{yz} = \tau_{zy} = G\theta x$$

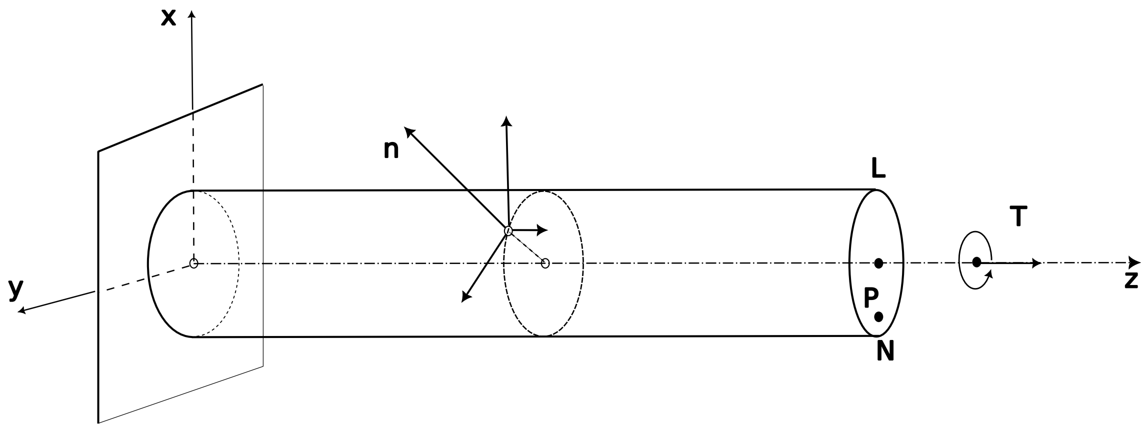

Figure 3

Check whether these satisfy the equations of equilibrium. Also show that the lateral surface is free of load, i e. show that

$$T_x^n = T_y^n = T_z^n = 0$$

4. For the state of stress given in Q3, determine the principal shear stresses, octahedral shear stress and its associated normal stress.

5. A cylindrical boiler, 180cm in diameter, is made of plates 1.8cm thick and is subjected to an internal pressure 1400 kPa. Determine the maximum shearing stress in the plate at point P and the plane on which it acts.

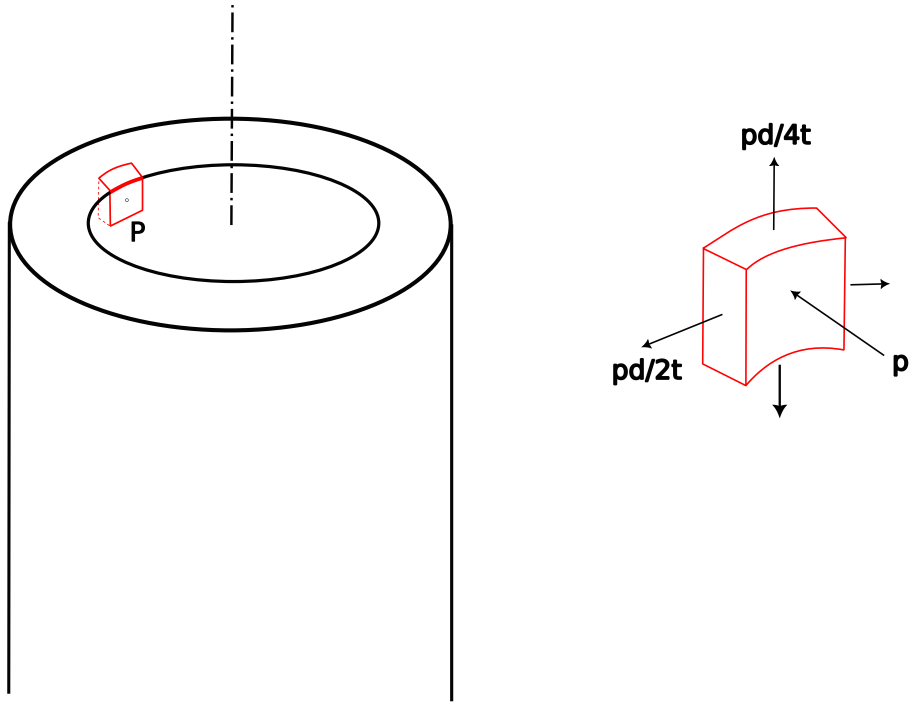

Figure 4

6. **Divergence operator**
   Divergence of a tensor is defined as follow

$$\underline{\nabla} \cdot (\circ) = \sum_i \frac{\partial}{\partial x_i} (\circ) \cdot \underline{e}_i$$

   For example

$$\begin{aligned}
\underline{\nabla} \cdot \underline{v} = \sum_i \frac{\partial}{\partial x_i} (\underline{v}) \cdot \underline{e}_i &= \sum_i \frac{\partial}{\partial x_i} \left(\sum_j v_j \underline{e}_j \right) \cdot \underline{e}_i \\
&= \sum_i \sum_j \frac{\partial \underline{v}}{\partial x_j} \delta_{ij} \\
&= \sum_i \frac{\partial v_i}{\partial x_i}
\end{aligned}$$

   Show that $\underline{\nabla} \cdot \underline{\underline{\sigma}} = \sum_i \sum_j \frac{\partial \sigma_{ji}}{\partial x_i} \underline{e}_j$