

Tutorial 6: Strain

APL 104 - 2022 (Solid Mechanics)

1. Think of the following displacement field in the body:

$$\begin{aligned}u_1 &= 0.05x_1 + 0.03x_2^2, \\u_2 &= 0.07x_1x_2 + 0.08x_1^2, \\u_3 &= 0\end{aligned}$$

- (a) Find the longitudinal strain of a line element along \underline{e}_1 direction at any point in the body.
 - (b) Determine the shear strain between line elements along \underline{e}_1 and \underline{e}_3
 - (c) Find volumetric strain for this displacement field. Does it vary from point to point?
 - (d) What is the shear strain between line elements along \underline{e}_1 and \underline{e}_3 at any point (x_1, x_2) ?
 - (e) Determine the local rigid rotation.
2. The displacement field for a body is given by

$$\underline{u} = k(x^2 + y)\hat{i} + k(y + z)\hat{j} + k(x^2 + 2z^2)\hat{k}$$

Find the volumetric strain, shear strain γ_{xy} and γ_{yz} , and the axial vector of local infinitesimal rotation tensor of the body at a point $(2, 2, 3)$.

3. The displacement gradient matrix at a point in a body is given by

$$H = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Write the condition for zero infinitesimal rotation.

4. For the displacement field $u_x = k(x^2 + 2z)$, $u_y = k(4x + 2y^2 + z)$, $u_z = 4kz^2$ with $k = 0.001$, determine the change in angle between two lines segments PQ and PR at $P(2, 2, 3)$ having direction cosines before deformation as

- (a) PQ: $n_{x1} = 0$, $n_{y1} = n_{z1} = \frac{1}{\sqrt{2}}$
PR: $n_{x2} = 1$, $n_{y2} = n_{z2} = 0$

- (b) PQ: $n_{x1} = 0$, $n_{y1} = n_{z1} = \frac{1}{\sqrt{2}}$
 PR: $n_{x2} = 0.6$, $n_{y2} = 0$, $n_{z2} = 0.8$

5. Verify whether the following strain field satisfies the equation of compatibility. Here p is a constant.

$$\begin{aligned}\epsilon_{xx} &= py, & \epsilon_{yy} &= px, & \epsilon_{zz} &= 2p(x+y) \\ \gamma_{xy} &= p(x+y), & \epsilon_{yz} &= 2pz, & \epsilon_{zx} &= 2pz\end{aligned}$$

6. Given the following set of strain components:

$$\begin{aligned}\epsilon_{xx} &= 5 + x^2 + y^2 + x^4 + y^4, \\ \epsilon_{yy} &= 6 + 3x^2 + 3y^2 + x^4 + y^4, \\ \gamma_{xy} &= 10 + 4xy(x^2 + y^2 + 2), \\ \epsilon_{zz} &= \gamma_{yz} = \gamma_{zx} = 0.\end{aligned}$$

- Determine whether the above strain field is possible. If it is possible, determine the displacement components in terms of x and y . Assume that $u_x = u_y = 0$ and $\omega_{xy} = 0$ at the origin.
- For the state of strain given in previous problem, write down the spherical and deviatoric parts and also determine the volumetric strain.