Quiz 1 Solution

Total marks: 15

Total time: 45 mins

Instructions:

- **Write your name and roll number on answer script**
- For multiple choice questions (MCQs), multiple options may be correct. For every incorrect option, you would receive $-$**0.5 mark**!
- Vectors are denoted with a single underline $\underline{a}$, and matrices by double underline, $\underline{\underline{A}}$. Scalars appear without any underline. Please follow this rule through your answer book.

1. [**1 mark**] The analysis of deformable bodies involves these fundamental steps:

   1/3 ✓ (a) Study of forces and equilibrium requirements
   1/3 ✓ (b) Study of deformation and geometric compatibility
   1/3 ✓ (c) Study of force-deformation relation
   (d) None of the above

2. [**1 mark**] Traction at a point on planes having oppositely directed normals:

   (a) are unrelated
   (b) are negative of each other due to Newton's 2nd law
   1 ✓ (c) are negative of each other due to Newton's 3rd law
   (d) are related by the deformation of the body

3. [**1 mark**] Suppose, we are given traction on two planes at a point. What can you say about traction on a plane whose normal is perpendicular to the normals of both the given planes?

   (a) It will be perpendicular to tractions of both the given planes
   (b) It will be parallel to the cross product of tractions on both the given planes
   (c) It will be equal to the sum of both the tractions
   1 ✓ (d) cannot be determined

4. [**1 mark**] The formula to obtain traction on an arbitrary plane at a point requires the following:

   1 ✓ (a) Information of traction on three perpendicular planes at the same point
   (b) Information of traction on planes at an adjacent point

(c) Magnitude of body force at that point

(d) Deformation of the body at that point

5. **[1 mark]** Which of the following statements is/are correct about stress tensor?

1/3 (a) At a point, the stress tensor is unique

1/3 (b) It can change from point to point within the body

(c) It varies for different inclined planes that pass through the point

1/3 (d) Its representation (in the form of its components) may change with a different coordinate system

6. **[1 mark]** What does the following stress matrix represent?

$$\left[\underline{\underline{\sigma}}\right]_{\underline{e}_1,\underline{e}_2,\underline{e}_3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

(a) No traction on $\underline{e}_1$ plane

(b) Traction on the $\underline{e}_3$ plane is along $\underline{e}_2$

(c) Normal stress components are all zero

(d) All of the above

7. **[1 mark]** We obtain stress tensor to be symmetric as a consequence of

(a) Balance of linear momentum (force equilibrium)

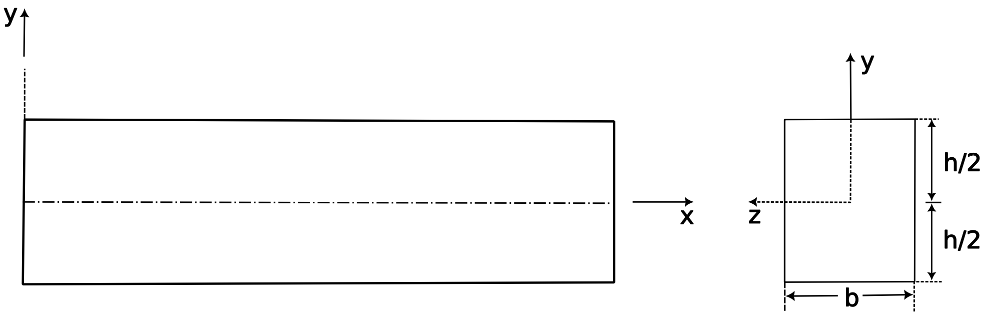
(b) Balance of angular momentum (moment equilibrium)

(c) Balance of energy

(d) None of these

8. **[2 marks]** A body having an arbitrary shape is acted upon by atmospheric pressure (p) on its boundary surface. Think of a point on the surface and a coordinate system such that its origin passes through that point and the z-axis lies along the outward surface normal whereas x and y axes lie on the tangent plane to the surface. Write down the stress matrix at this point.

9. **[3.5 marks]** Consider the rectangular beam shown below. According to the elementary theory of bending, the 'fiber stress' within the elastic range due to bending is given by

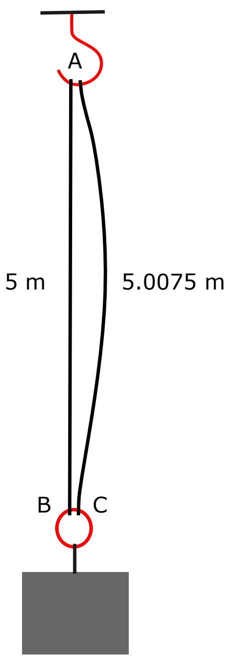


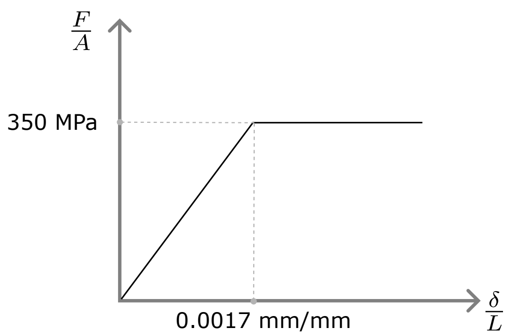
$$\sigma_{xx} = -\frac{My}{I} = -\frac{12My}{bh^3}$$

where M is the bending moment in the beam's cross-section and is a function of x. Assume that $\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$ everywhere. Furthermore $\tau_{xy} = 0$ on the top and bottom face and $\sigma_{yy} = 0$ on the bottom face. Using the differential equations of equilibrium, show that

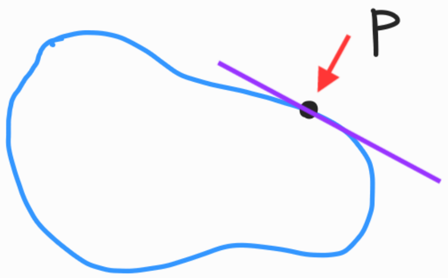
$$\begin{aligned}\tau_{xy} &= \frac{3}{2bh}\frac{\partial M}{\partial x}\left(\frac{4y^2}{h^2} - 1\right)\\ \sigma_{yy} &= -\frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{4y^3}{3h^2} - y - \frac{h}{3}\right)\end{aligned}$$

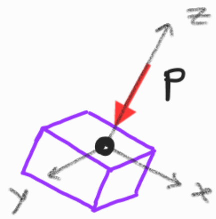
10. **[2.5 marks]** Two steel wires are used to lift a weight of 15kN, as shown below. Wire AC has an unstretched length of 5.0075m, while wire AB has an unstretched length of 5m. Each wire has a cross-sectional area of 30mm^2. Both wires are made out of steel which has an *elastic-perfectly-plastic* behavior, as shown by the normalized force-deformation relation. Determine the force in each wire and its elongation.
Assume AC to remain elastic and AB to be plastically strained.





8. **[2 marks]** A body having an arbitrary shape is acted upon by atmospheric pressure (p) on its boundary surface. Think of a point on the surface and a coordinate system such that its origin passes through that point and the z-axis lies along the outward surface normal whereas x and y axes lie on the tangent plane to the surface. Write down the stress matrix at this point.





can't determine these terms in 'X'

$$[\underline{\underline{\sigma}}]_{(x\,y\,z)} = \begin{bmatrix} X & X & 0 \\ X & X & 0 \\ 0 & 0 & -p \end{bmatrix}$$

Only the last row and column can be determined

→ 1 mark for writting the last row & column correctly

→ 1 mark for understanding that $\sigma_{11}, \sigma_{22}, \tau_{12}$ cannot be determined

9) Assume self-weight to be negligible.

Using equations of stress equilibrium along X-direction

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \cancel{\tau_{zx}}^{\,0}}{\partial z} = 0 \quad — \quad 0.5$$

Since $\tau_{yx} = \tau_{xy} = 0$ everywhere and M is a function of x

$$\frac{\partial}{\partial x}\left(-\frac{12 M(x) y}{bh^3}\right) + \frac{\partial \tau_{yx}}{\partial y} + 0 = 0$$

$$\Rightarrow \quad -\frac{12y}{bh^3}\frac{\partial M}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0$$

$$\Rightarrow \quad \tau_{yx} = \frac{\overset{6}{\cancel{12}}}{bh^3}\frac{\partial M}{\partial x}\left(\frac{y^2}{\cancel{2}}\right) + c_1(x,z) \quad — \quad 0.5$$

Here $c_1(x,z)$ is the integration constant which will be a function of x and z. To get the value of this integration constant, we use the condition that $\tau_{yx} = \tau_{xy} = 0$ on the top and bottom face of the C/s i.e. at $\left(x, y = \pm\frac{h}{2}, z\right)$ — 0.25

$$\therefore \quad c_1(x,z) = \underbrace{-\frac{6}{bh^3}\frac{\partial M}{\partial x}\left(\frac{h^2}{4}\right)}$$

this term is only a function of x and not z, there $c_1(x,z) = c_1(x)$

$$\Rightarrow \quad \boxed{c_1(x) = -\frac{3}{2bh}\frac{\partial M}{\partial x}} \quad 0.5$$

$$\therefore \quad \tau_{xy} = \frac{3}{2bh}\frac{\partial M}{\partial x}\left(\frac{4y^2}{h^2} - 1\right)$$

To obtain σ_{yy}, we use the stress equilibrium eqn in the y-direction:

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \cancel{\tau_{zy}}^{\,0}}{\partial z} = 0 \quad \text{— } 0.5$$

$$\Rightarrow \frac{\partial \sigma_{yy}}{\partial y} = -\frac{\partial \tau_{xy}}{\partial x} = -\frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{4y^2}{h^2} - 1\right)$$

$$\Rightarrow \sigma_{yy} = -\frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{4y^3}{3h^2} - y\right) + c_2(x,z)$$

0.5

where $c_2(x,z)$ is an integration constant determined using the condition that $\sigma_{yy} = 0$ on the bottom face i.e. at $\left(x, y = -\frac{h}{2}, z\right)$. 0.25

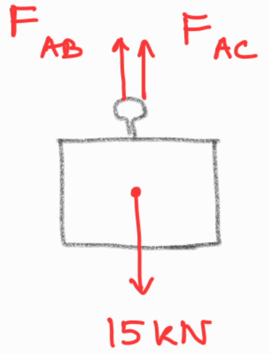
$$c_2(x,z) = \frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{4\left(-\frac{h}{2}\right)^3}{3h^2} - \left(-\frac{h}{2}\right)\right)$$

$$= \frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{h}{3}\right) = \frac{1}{2b}\frac{\partial^2 M}{\partial x^2} \quad \text{— } 0.5$$

Substituting back the expression of σ_{yy}, we get

$$\sigma_{yy} = -\frac{3}{2bh}\frac{\partial^2 M}{\partial x^2}\left(\frac{4y^3}{3h^2} - y - \frac{h}{3}\right)$$

10) Draw FBD of the suspended weight



Force equilibrium

$$+\uparrow \sum F_y = 0$$

$$\Rightarrow F_{AB} + F_{AC} = 15 \text{ kN} \quad \text{— (1)}$$

0.5

Force-deformation relation

Since AB is assumed to be plastically strained, it must support maximum load. Therefore, we must get the maximum stress that is possible in the steel wire AB. Therefore,

$$F_{AB} = \left(350 \times 10^{6} \text{ N/m}^2\right)\left(30 \times 10^{-6} \text{ m}^2\right) \quad \text{— 0.5}$$

$$= 10.5 \times 10^{3} \text{ N} = 10.5 \text{ kN}$$

Using (1), we get $F_{AC} = 4.5 \text{ kN}$ — 0.25

Since the wire AC remains elastic, the corresponding elongation in AC is

$$\delta_{AC} = \frac{F_{AC} L_{AC}}{E_{AC} A_{AC}} \quad \text{— 0.25}$$

$$E_{AC} = \frac{350 \times 10^{6} \text{ N/m}^2}{0.0017}$$

0.25

$$= \frac{\left(4.5 \times 10^{3} \text{ N}\right)\left(5.0075 \text{ m}\right)}{\left(\dfrac{350 \times 10^{6} \text{ N/m}^2}{0.0017}\right)\left(30 \times 10^{-6} \text{ m}^2\right)}$$

$$\delta_{AC} = 0.003648 \text{ m}$$

Deformation compatibility

$$\delta_{AB} - \delta_{AC} = L_{AC} - L_{AB}$$

$$\Rightarrow \quad \delta_{AB} - \delta_{AC} = 0.0075 \text{ m} \quad — 0.5$$

$$\Rightarrow \quad \delta_{AB} = 0.0075 \text{ m} + 0.003648 \text{ m}$$

$$= 0.01115 \text{ m} \quad — 0.25$$

for correct answer