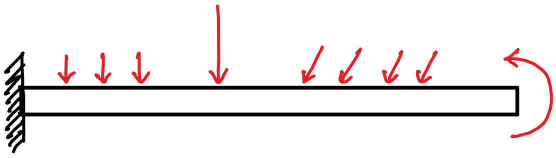
Bending of beams

When a slender member is subjected to transverse loading we say it is a beam.

loading perpendicular to beam axis



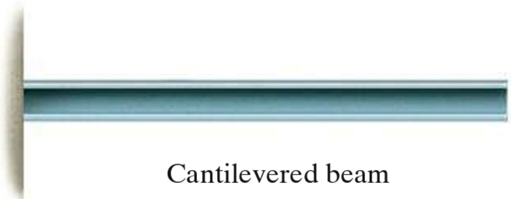
c/s

In general, beams are long, straight bars having a constant c/s area. Often they are classified as to how they are supported.

Simply supported beam ⟷ pinned at one end, roller at other end

Simply supported beam

Cantilever beam ⟷ fixed at one end, free at other end



Cantilevered beam

Overhanging beam ⟷ one end free/fixed, extended over a roller support

Overhanging beam

Beams are important structural elements. They are used to support the floor of a building, deck of a bridge, wing of an aircraft, the boom of a crane, bones in our bodies, etc.

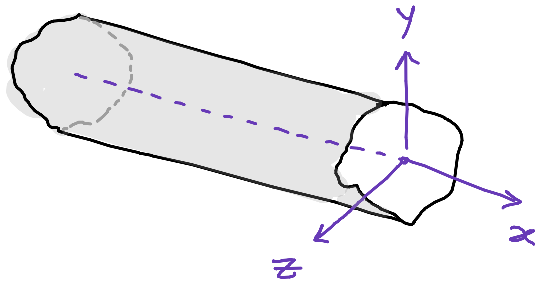
Shear force and Bending moment diagrams

Because of the applied transverse loads, beams develop an internal resistive shear force and bending moment. These internal shear force and bending moment can vary from point to point along the length (or the axis) of the beam.

The internal shear force and bending moment functions can be plotted and represented by graphs called shear force and bending moment diagrams, SFD & BMD.

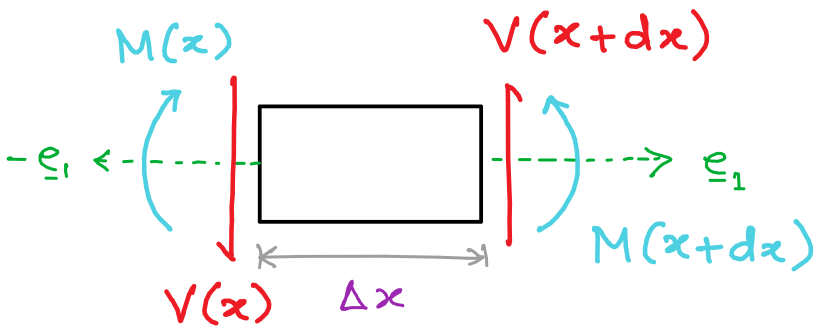
The maximum values of shear force, V and bending moment, M can be obtained from these graphs.

Procedure for drawing SFD & BMD



Beam length is along x-axis

- Determine support reactions
- For beam problems, x-axis (or $\underline{e}_1$-dir) is considered along the beam length
- Usually, we specify an origin of coordinate system at the left end
- Take sections of the beam at different values of x and draw free-body-diagrams.
- Assert a sign convention for SF and BM



Positive sign convention for SF & BM

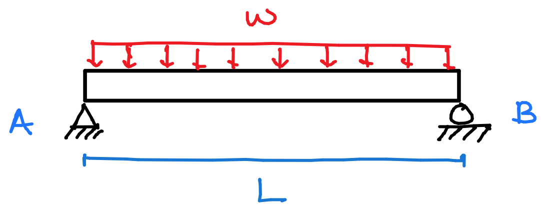
- When a force or moment acts on a positive face in a positive coordinate direction, the corresponding force/moment is positive
- When a force or moment acts on a negative face in a negative coordinate direction, the corresponding force/moment is also positive

- The shear forces are obtained by summing forces perpendicular to the beam axis

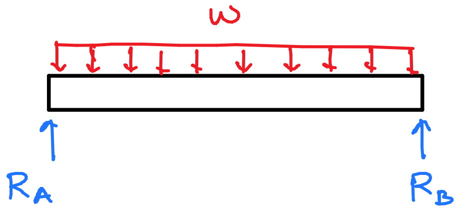
- The bending moment is obtained by summing moments abt the sectioned end of the segment

- Plot the SFD and BMD. If numerical values of V and M are positive, the values are plotted above the x-axis, whereas negative values are plotted below the axis.

Ex:



FBD

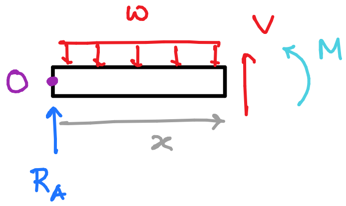


- Support reactions

$$R_A = R_B = \frac{wL}{2}$$

SFD and BMD

Consider origin at the left end and take a segment of beam



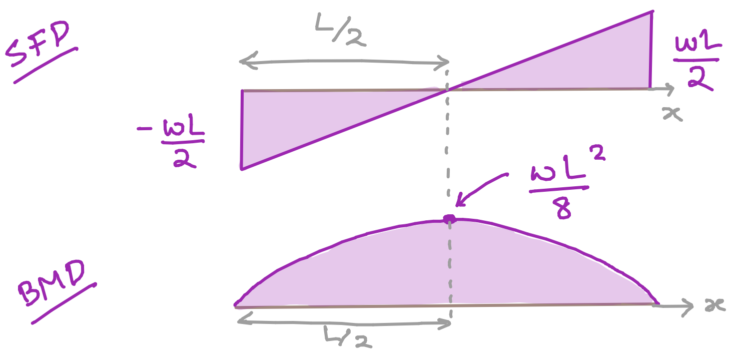
$$+\uparrow \Sigma F_y = 0 \Rightarrow \overset{\frac{wL}{2}}{R_A} + V = wx$$

$$\Rightarrow V = -w\left(\frac{L}{2} - x\right)$$

$$\circlearrowleft + \Sigma M_O = 0 \Rightarrow Vx - wx\left(\frac{x}{2}\right) + M = 0$$

$$\Rightarrow M = w\frac{L}{2}x - wx^2 + \frac{wx^2}{2}$$

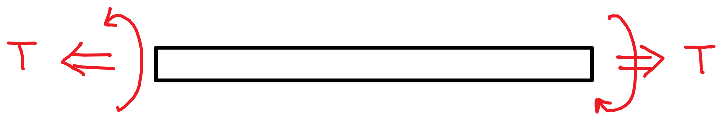
$$= \frac{w}{2}\left(Lx - x^2\right)$$



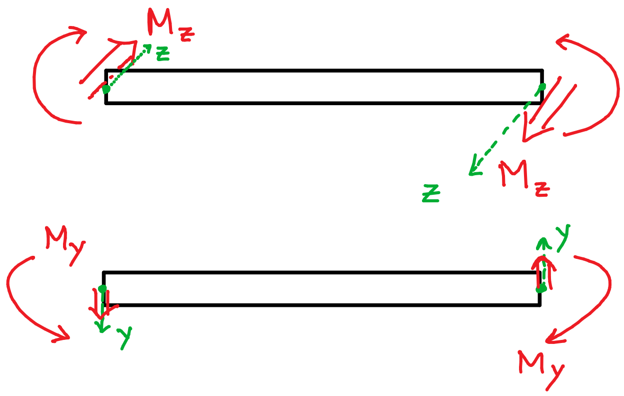
We have now learned how to draw shear force and bending moment diagrams for beams subjected to transverse loadings.

But while drawing them, we assumed the beams to not deform.

But the beams curve $\Rightarrow$ That is called BENDING

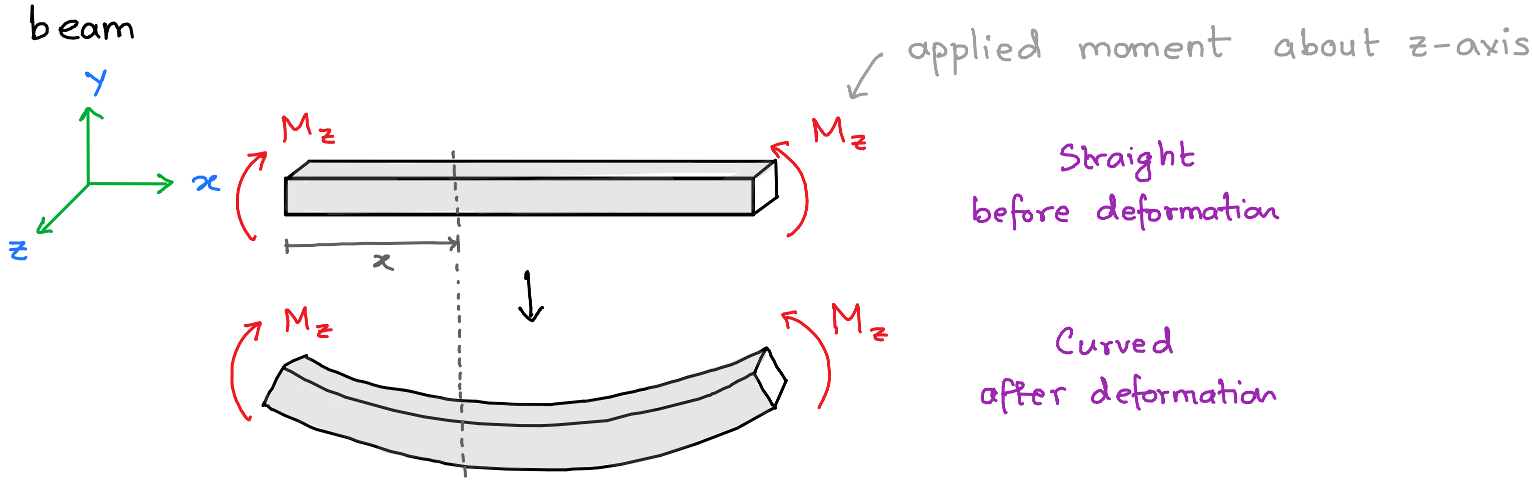


Torque moment vector acts along the length of the beam $\xrightarrow{\text{causing}}$ twist

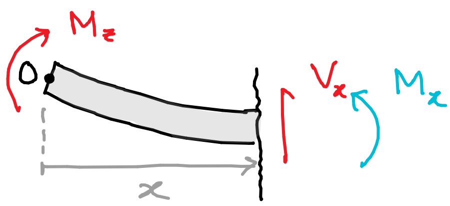


Bending moment acts $\perp$ to the axis of the beam $\xrightarrow{\text{causing}}$ bending

Now we will take another step and learn how beams deform or get curved when they are subjected to such loads. The simplest case to begin with is that what we call as PURE BENDING of beams, where a constant moment acts on the beam



We find that for this type of loading (i.e. constant moment applied to the beam), there is no shear force arising in the beam, and the beam is only subjected to a constant bending moment along its length. We can verify these by doing force and moment balance at an arbitrary section at a distance x



$V_x \rightarrow V$ at a dist x

$M_x \rightarrow M$ at a dist x

$+\uparrow \sum F_y = 0$

$\Rightarrow \quad V_x = 0$

No internal shear force

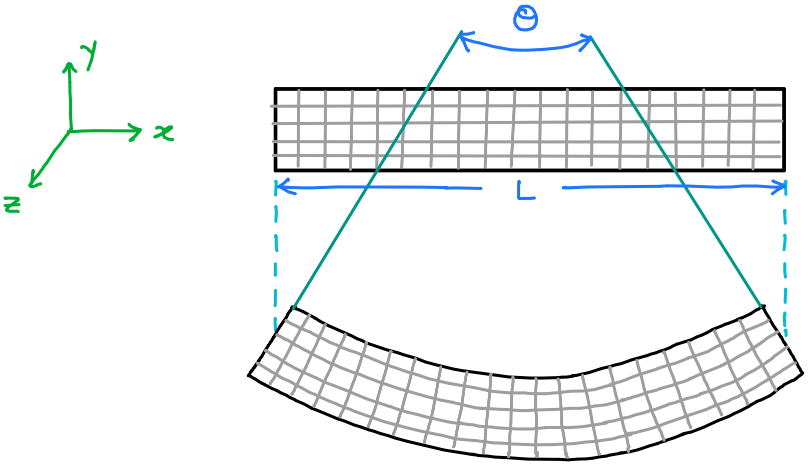
$\circlearrowleft + \sum M_o = 0$

$\Rightarrow M_x + \cancelto{0}{V_x}\, x - M_z = 0$

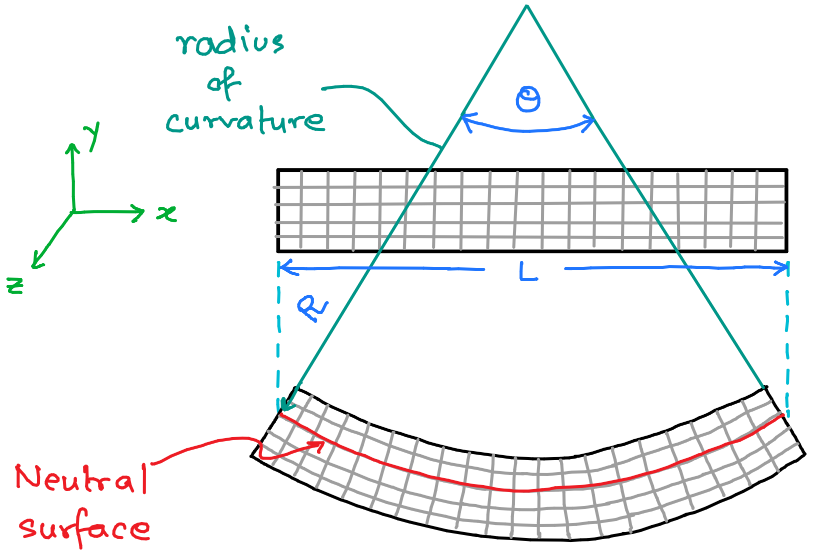
$\Rightarrow M_x = M_z$

Bending moment is constant everywhere

For such a case where there is no shear force and a constant bending moment throughout the beam with constant cross-section, the beam bends in such a way that it represents an arc of a perfect circle. As such, all lines (or fibers) of the beam along its length deform to become arcs of a circle



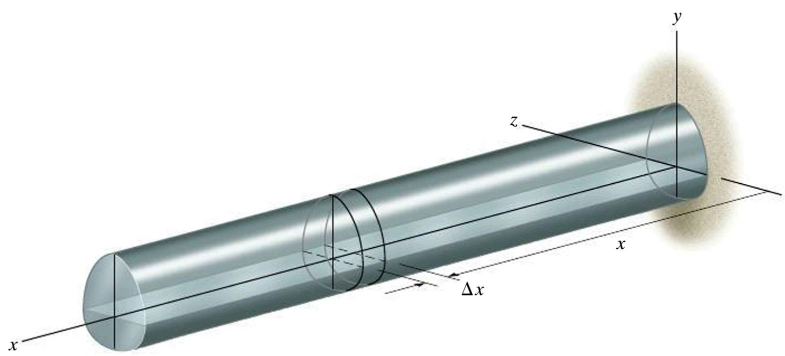
- The longitudinal lines become curved into arcs while the vertical transverse lines remain straight, yet undergo a rotation



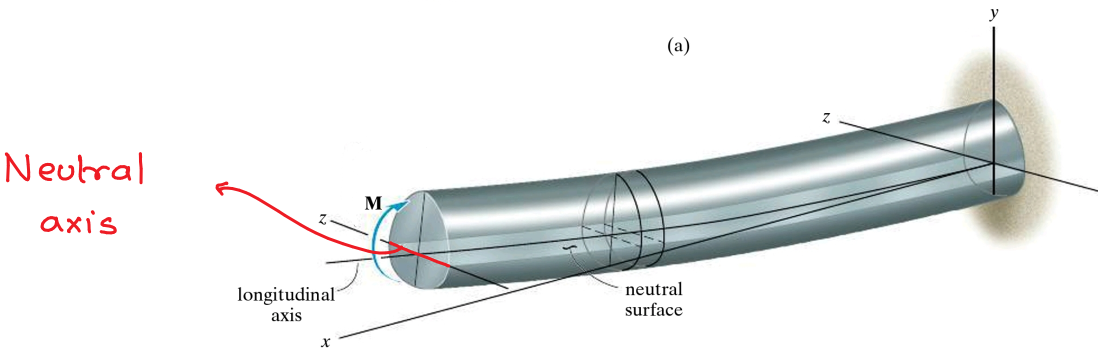
Assumption: Planar C/s remains plane after deformation

- The longitudinal lines become curved into arcs while the vertical transverse lines remain straight, yet undergo a rotation
- The bending moment causes the fibers within the top portion to shorten/compress and those in the bottom portion to stretch

- Therefore, between the two regions of fibers getting compressed and elongated, there must be a surface whose fibers do not undergo any elongation or compression. This surface is called the NEUTRAL PLANE as all longitudinal lines on this plane do not undergo any change in length. In particular, the z-axis, lying in the plane of the cross-section and about which the C/s rotates is called the neutral axis

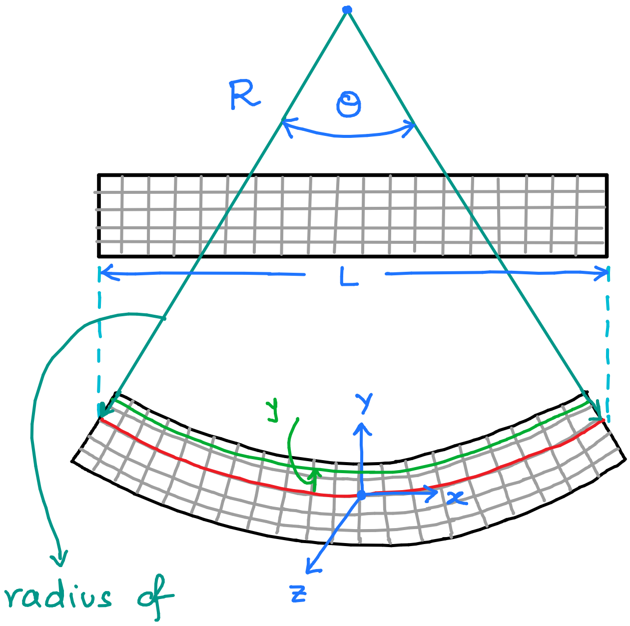


(a)



Longitudinal / bending strain

Let us now find the amount of elongation/compression of the longitudinal line elements which are parallel to the beam's axis. The neutral line is shown in red. Note that we donot know yet the location of this neutral line exactly. We will consider the x-axis to lie in the neutral surface.



Consider a longitudinal fiber (shown in green line) at a distance y from the neutral line.

If the radius of the neutral line is R, the radius of the circle corresponding to the green line will be $(R-y)$. If the angle subtended by these arcs at the center is θ, then the length of the neutral line will be

$$l_n = R\theta$$

This length of neutral line must be equal to the undeformed length of the beam, L.

$$l_n = R\theta = L$$

Similarly, the length of green line would be

$$l_g = (R-y)\,\theta = (R-y)\,\frac{R\theta}{R} = (R-y)\,\frac{L}{R} = \left(1-\frac{y}{R}\right)L$$

As the undeformed length of all longitudinal lines is L, the longitudinal strain of the green line will be

$$\underbrace{\epsilon_b}_{\text{bending strain}} = \epsilon_{xx} = \frac{\Delta L}{L} = \frac{l_g - L}{L} = -\frac{y}{R}$$

We have thus obtained longitudinal strain along x-axis (hence ϵ_{xx}) for a general longitudinal line at a distance y from the neutral line.