Quiz 2

Total marks: 15 , **Total time**: 45 mins, **Multiple options MAY BE CORRECT.**
For every incorrect option, you would lose **50% of the total marks of the question**!

1. [**1 mark**] Which of the following assumptions are required to have a *linear-elastic* stress-strain relation?

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   (a) material should be isotropic (unrelated)
   ✓(b) all strain components should be small
   (c) material should be homogeneous (unrelated)
   (d) stress-strain relation is considered only up to the elastic limit (proportionality limit)

2. [**1 mark**] For a linear elastic *anisotropic* material, the number of independent material constants are

   1

   (a) 2 ✓(b) 21 (c) 36 (d) 81

3. [**1 mark**] A material is heated up while it is rigidly restrained such that it undergoes no strain. Then which of the following could be true:

   $\frac{1}{3}$ each

   ✓(a) the material property of the bar may change (Material properties can change with temperature change)
   (b) thermal strain will be zero
   ✓(c) thermal strain could be non-zero
   ✓(d) the elastic part of the strain could be non-zero

   $\cancel{\epsilon^{total}} \to 0 = \epsilon^{elastic} + \epsilon^{thermal}$
   $\Rightarrow \epsilon^{elastic} = -\epsilon^{thermal}$

4. [**1 mark**] A body is made of an incompressible material that is non-homogeneous but isotropic in nature. Which of the following would be true: means $\nu = 0.5$ everywhere

   $\frac{1}{2}$ each

   ✓(a) the volumetric strain is zero at every point in the body (because it is incompressible everywhere)
   (b) only the total volume of the body does not change but local volume can always increase or decrease
   (c) the Poisson's ratio is different at every point within the body
   ✓(d) the Poisson's ratio is the same at every point within the body $\nu = 0.5$

5. [**3 marks**] A body is subjected to a biaxial test under plane stress condition ($\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$) with an in-plane loading such that $\sigma_{xx} = 2\sigma_{yy}$.

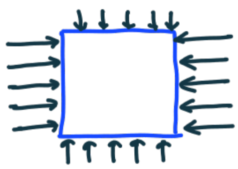
   (a) [**1.5 marks**] The ratio of σ_{xx} to ϵ_{xx} will be

   1.5

   i. $(2-\nu)/2E$
   ii. $(1-\nu)/2E$
   ✓iii. $2E/(2-\nu)$ ← $\sigma_{xx}/\epsilon_{xx}$
   iv. $2E/(1-\nu)$

   $\epsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\left(\cancel{\sigma_{yy}}^{\frac{1}{2}\sigma_{xx}} + \cancel{\sigma_{zz}}^{0}\right)\right)$

   $\epsilon_{xx} = \frac{2-\nu}{2E}\sigma_{xx}$

   (b) [**1.5 marks**] The ratio between ϵ_{xx} and ϵ_{yy} in terms of ν will be

   1.5

   i. $(1-2\nu)/(2-\nu)$
   ii. $(2-\nu)/(2+\nu)$
   iii. $(1+2\nu)/(2+\nu)$
   ✓iv. $(2-\nu)/(1-2\nu)$

   $\epsilon_{yy} = \frac{1}{E}\left(\cancel{\sigma_{yy}}^{1/2\,\sigma_{xx}} - \nu\left(\sigma_{xx} + \cancel{\sigma_{zz}}^{0}\right)\right)$

   $= \frac{1}{E}\left(\frac{1}{2}\sigma_{xx} - \nu\sigma_{xx}\right) = \frac{1-2\nu}{2E}\sigma_{xx}$

   $\therefore \frac{\epsilon_{xx}}{\epsilon_{yy}} = \frac{(2-\nu)}{(1-2\nu)}$

6. [**1 mark**] Which of the following statements is/are true regarding the partial derivative of basis vectors?

   (a) $\partial \underline{e}_r/\partial r = 0$ and $\partial \underline{e}_z/\partial z = 0$

   (b) $\partial \underline{e}_r/\partial r = 0$ and $\partial \underline{e}_\theta/\partial \theta = 0$ ×

   (c) $\partial \underline{e}_z/\partial z = 0$ and $\partial \underline{e}_\theta/\partial \theta = 0$ ×

   (d) none of these

$\left.\begin{matrix} \underline{e}_r \\ \underline{e}_\theta \end{matrix}\right] \rightarrow$ varies only with θ

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7. [**1 mark**] If the three displacement components in a cylindrical coordinate system are all constant

   (a) all strain components will be zero

   (b) all except hoop strain ($\epsilon_{\theta\theta}$) will be zero

   (c) both $\epsilon_{\theta\theta}$ and $\epsilon_{r\theta}$ will be non-zero

   (d) none of these

$$\begin{bmatrix} \cancel{\frac{\partial u_r}{\partial r}}^{0} & \frac{1}{r}\left(\cancel{\frac{\partial u_r}{\partial \theta}}^{0} - u_\theta\right) + \cancel{\frac{\partial u_\theta}{\partial r}}^{0} & \cancel{\frac{\partial u_r}{\partial z}}^{0} + \cancel{\frac{\partial u_z}{\partial r}}^{0} \\ & \frac{1}{r}\cancel{\frac{\partial u_\theta}{\partial \theta}}^{0} + \frac{u_r}{r} & \cancel{\frac{\partial u_\theta}{\partial u_z}}^{0} + \frac{1}{r}\cancel{\frac{\partial u_z}{\partial \theta}}^{0} \\ & & \cancel{\frac{\partial u_z}{\partial z}}^{0} \end{bmatrix}$$

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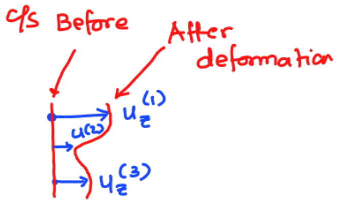
8. [**1 mark**] While solving the extension-torsion-inflation problem in a hollow cylinder, we assumed the axial displacement u_z to depend only on 'z' because

   (a) the dependence of u_z on r would lead to warping (bulging) in the cross-section

   (b) the dependence of u_z on r on would violate axisymmetry

   (c) the dependence of u_z on r would lead to axial inhomogeneity

   (d) none of these



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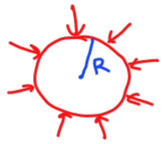
9. [**1 mark**] Think of a solid circular shaft that is pressurized by compressive p on its outer surface. No axial force or torque acts on it. What can you say about the distribution of radial stress σ_{rr}?

   (a) σ_{rr} will be varying in the cylinder along its radius

   (b) σ_{rr} will be constant in the cylinder and equal to $-p$

   (c) σ_{rr} will be constant but equal to p

   (d) None of these



At $r=0$, σ_{rr} must be finite, so $B=0$

At $r=R$, $\sigma_{rr} = -P$

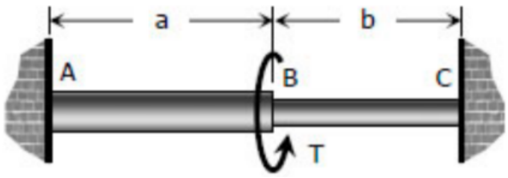
$\therefore \frac{A}{2} = -P$

$\sigma_{rr} = \frac{A}{2} + \frac{B}{r^2}$

$\therefore \sigma_{rr} = -P$ every where

1

10. [**4 marks**] The compound shaft shown in the figure below is attached to rigid supports. For the bronze segment AB, the diameter is 75mm, $\tau \leq 60$MPa, and $G = 35$GPa. For the steel segment BC, the diameter is 50mm, $\tau \leq 80$MPa, and $G = 83$GPa. Determine the ratio of lengths $\frac{b}{a}$ so that each material will be stressed to its permissible limit. What torque T is required?



Soln :

In torsion, $\tau = \dfrac{Tr}{J} \Rightarrow T = \dfrac{\tau J}{r}$

For bronze:

$$\tau_B^{max} = 60\ MPa = 60\ N/mm^2$$

$$G_B = 35\ GPa = 35 \times 10^3\ MPa = 35 \times 10^3\ N/mm^2$$

$$J_B = \frac{\pi d_b^4}{32} = \frac{\pi (75)^4}{32}$$

Maximum allowable torque for bronze, $T_b^{max} = \dfrac{\tau_b^{max} J_b}{r_b}$

$$T_b^{max} = \frac{(60)\ \frac{\pi (75)^4}{32}}{\frac{75}{2}} = 4970097\ N\ mm$$

$$= 4.970\ kNm \quad 0.5$$

For steel :

$$\tau_s^{max} = 80\ MPa = 80\ N/mm^2$$

$$G_s = 83\ GPa = 83 \times 10^3\ N/mm^2$$

$$J_s = \frac{\pi d_s^4}{32} = \frac{\pi (50^4)}{32}$$

Maximum allowable torque for steel, $T_s^{max} = \dfrac{\tau_s^{max} J_s}{r_s}$

$$T_s^{max} = \frac{(80)\ \frac{\pi (50)^4}{32}}{\frac{50}{2}} = 1963.495\ N\ mm$$

$$= 1.963\ kN\ m \quad 0.5$$

We have from geometric compatibility that the angle of twist θ from bronze & steel must be same.

$$\theta_b = \theta_s$$

$$\theta_b = \theta_s \quad \text{——} \quad 1$$

$$\Rightarrow \frac{T_b^{max} L_b}{G_b J_b} = \frac{T_s^{max} L_s}{G_s J_s}$$

$$\Rightarrow \frac{4.970 \quad (a)}{\frac{1}{\cancel{32}} \cancel{\pi} (75^4) \ (35000)} = \frac{1.963 \quad (b)}{\frac{1}{\cancel{32}} \cancel{\pi} (50^4) \ (83000)}$$

$$\Rightarrow \frac{b}{a} = 0.843 \quad \text{or} \quad \frac{a}{b} = 1.186 \quad 1$$

Torque required, $\boxed{T = T_b^{max} + T_s^{max}}$ 1

$$= 4.970 + 1.963$$

$$= 6.933 \text{ kNm}$$