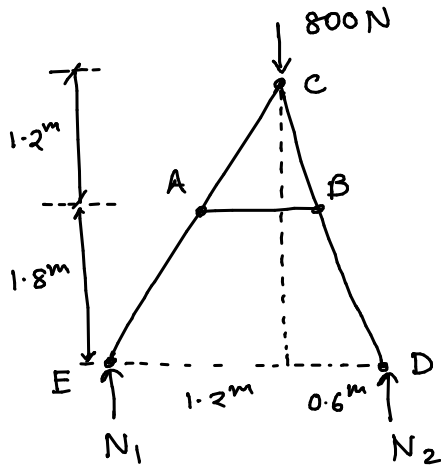# Tutorial 1 solutions

1) No Surface Friction

→ Draw FBD of the entire ladder frame

Use eqm conditions

$$\circlearrowleft + \sum M_D = 0$$

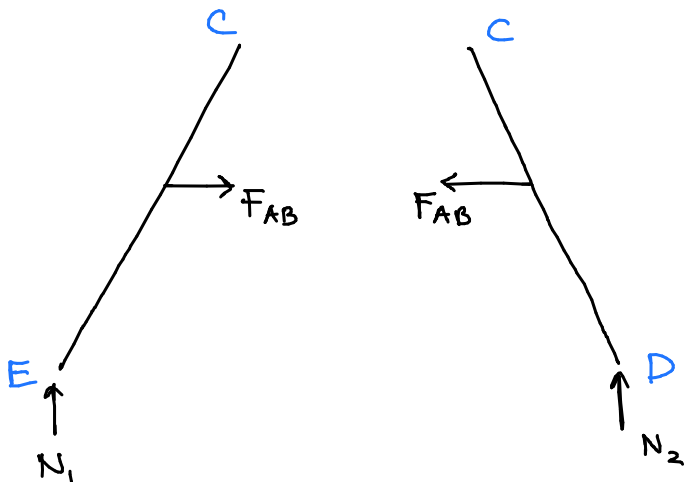$$\Rightarrow \quad 800\,(0.6) - N_1\,(1.8) = 0$$

$$\Rightarrow \quad N_1 = 266.67\ N$$

$$+\uparrow\ \sum F_y = 0$$

$$\Rightarrow \quad N_1 + N_2 - 800 = 0$$

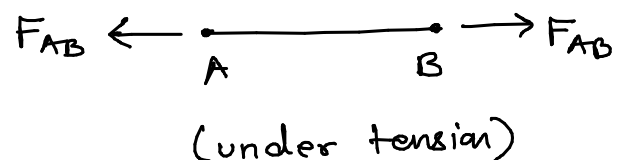$$\Rightarrow \quad N_2 = 533.3\ N$$

→ Draw FBD of isolated members
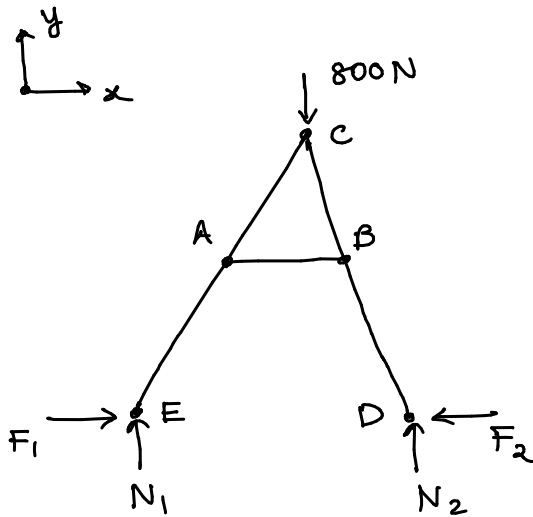

Apply eqm condition for CE

$$\circlearrowleft + \sum M_C = 0$$

$$\Rightarrow \quad F_{AB}\,(1.2) - N_1\,(1.2) = 0$$

$$\Rightarrow \quad F_{AB} = 266.67\ N$$

(under tension)

With surface friction

From eq^m of the entire system in the x-direction,

$$\xrightarrow{+} \Sigma F_x = 0$$
$$\Rightarrow F_1 = F_2$$

$$\curvearrowright + \Sigma M_E = 0$$
$$\Rightarrow N_1 = 266.67 \text{ N}$$

$$+\uparrow \Sigma F_y = 0$$
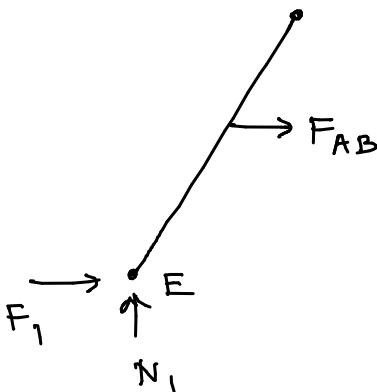$$\Rightarrow N_2 = 533.33 \text{ N}$$

The friction forces developed could be

$$|F_1| \leq 0.2 \, (266.67) = 53.33 \text{ N}$$

$$|F_2| \leq 0.2 \, (533.33) = 106.67 \text{ N}$$

Since $F_1 = F_2$, it must be that $F_1 = F_2 = 53.33$ N

To obtain F_{AB}, isolate member CE

$$\curvearrowright + \Sigma M_c = 0$$
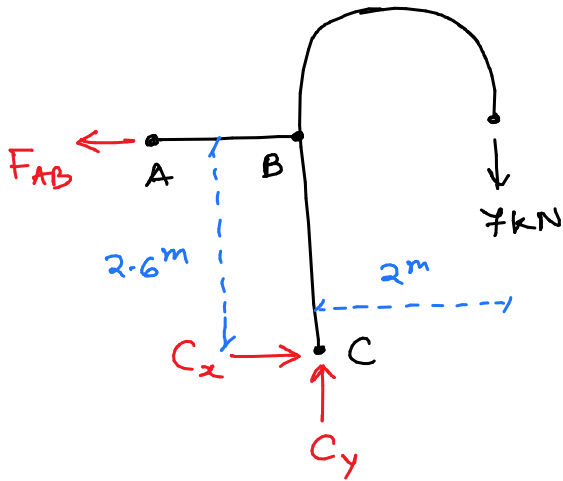$$\Rightarrow F_{AB} \, (1.2) + F_1 \, (3) - N_1 (1.2) = 0$$
$$\Rightarrow F_{AB} = 133.3 \text{ N}$$

However, if F_1 is directed opposite, then

$$F_{AB} = F_1 \, (3) + N_1 \, (1.2)$$
$$= 400 \text{ N}$$

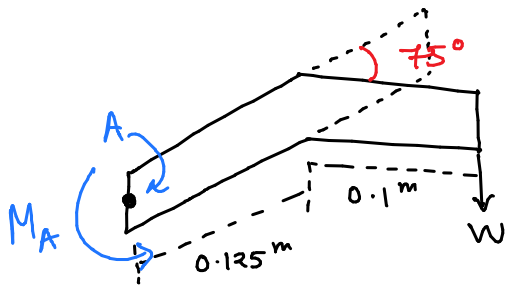The designer should account for the maximum force F_{AB}

2)

FBD of the entire system

Apply eqm condition

$$(+ \sum M_c = 0$$

$$\Rightarrow F_{AB}(2.6) - 7(2) = 0$$
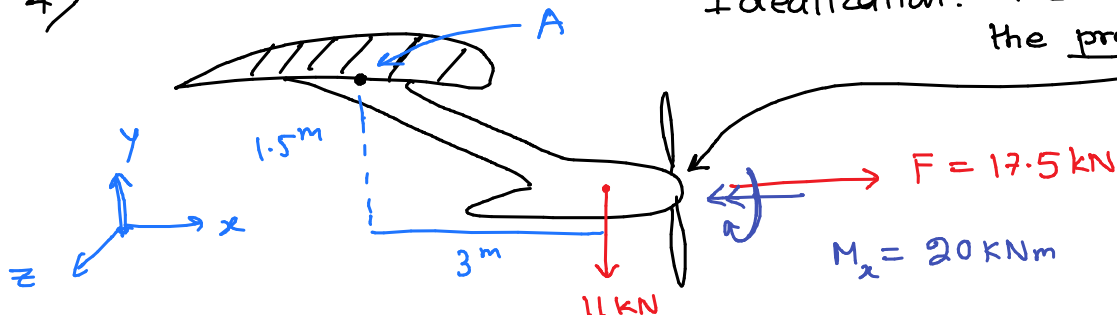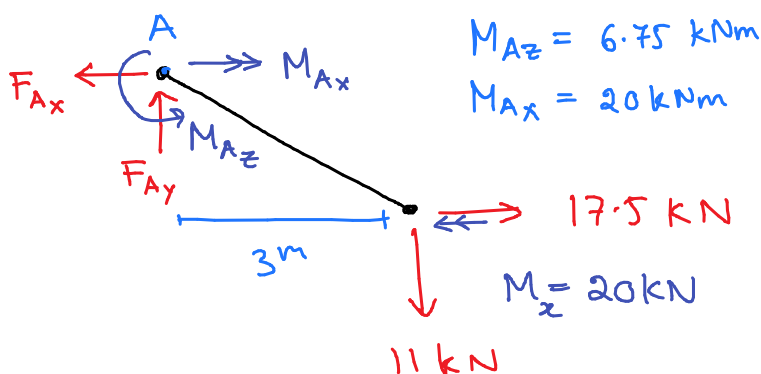
$$\Rightarrow F_{AB} = 5.385 \text{ kN}$$

3)

Maximum moment at weld

$$M_A = W(0.1\cos 75° + 0.125)$$

$$\Rightarrow 100 = W(0.1\cos 75° + 0.125)$$

$$\Rightarrow W = 632.8 \text{ N}$$

4)

Idealization: The thrust acts through the propeller hub

$F = 17.5 \text{ kN}$

$M_x = 20 \text{ kNm}$

FBD of strut only

$M_{Az} = 6.75 \text{ kNm}$

$M_{Ax} = 20 \text{ kNm}$
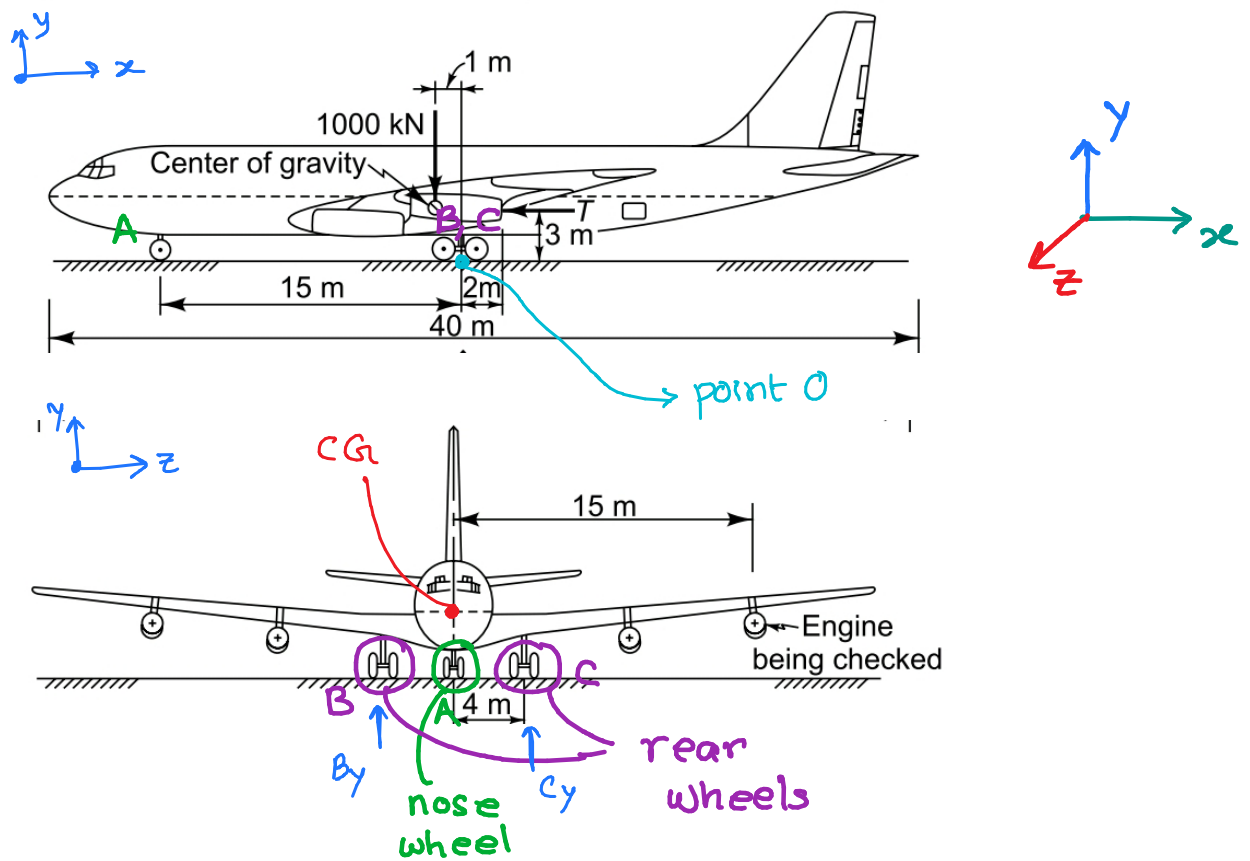
17.5 kN

$M_x = 20 \text{ kN}$

11 kN

$\begin{cases} F_A = -17.5 \text{ kN } \hat{i} + 11 \text{ kN } \hat{j} \\ M_A = 20 \text{ kNm } \hat{i} + 6.75 \hat{k} \text{ kNm} \end{cases}$

Force + moment exerted on wing

$F_A = 17.5 \hat{i} - 11 \hat{j} \text{ kN}$

$M_A = -(20 \hat{i} + 6.75 \hat{k}) \text{ kNm}$

5>

1 m

1000 kN

Center of gravity

A

B C

T

3 m

15 m

2m

40 m

→ point O

CG

15 m

Engine being checked

B

A 4 m

C

By

Cy

nose wheel

rear wheels

Idealize the two wheels on one side as one wheel

Zoomed figure of wheels

no horizontal reaction due to no brakes and castor

Ay

1000 kN

3m (y-dir)

Bx

Bz

By

8m

Cx

Cz

Cy

15ᵐ (z-dir)

11m

40kN

3ᵐ (y-dir)
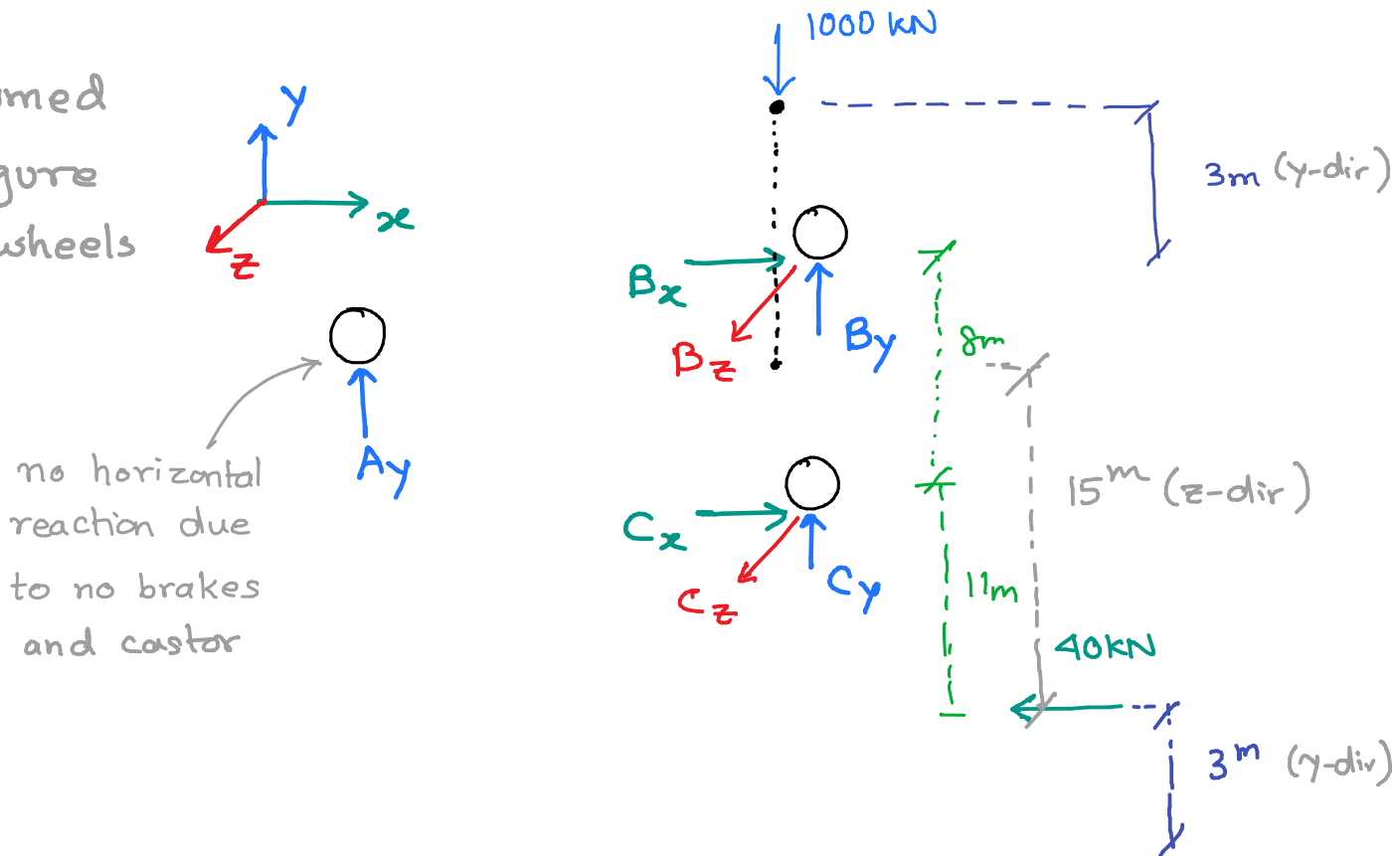
Applying eqm equations

$\xrightarrow{+} \Sigma F_x = 0 \implies C_x + B_x - 40 \text{ kN} = 0$

$\implies C_x + B_x = 40 \text{ kN} \quad - \text{①} \checkmark$

$+\uparrow \Sigma F_y = 0 \implies A_y + C_y + B_y - 1000 \text{ kN} = 0 \quad - \text{②}$

$\overset{+}{\nearrow} \Sigma F_z = 0 \implies C_z + B_z = 0$

$\implies C_z = -B_z \quad - \text{③}$

190

$\xrightarrow{+} \Sigma M_{x|0} = 0 \implies -B_y (4^m) + C_y (4^m) = 0$

$\implies B_y = C_y \quad - \text{④}$

$+\uparrow \Sigma M_{y|0} = 0 \implies C_x (4^m) - B_x (4^m) - (40 \text{ kN})(15^m) = 0$

$\implies C_x - B_x = 150 \text{ kN} \quad - \text{⑤} \checkmark$

$\overset{+}{\swarrow} \Sigma M_{z|0} = 0 \implies (40 \text{kN})(3^m) - A_y (15^m) + (1000 \text{ kN})(1^m) = 0$

$\implies A_y = 1200/15 = 80 \text{ kN} \quad - \text{⑥}$

7 Unknowns: $B_x, C_x, A_y, B_y, C_y, B_z, C_z$

Using ① & ⑤ $\implies C_x = 95 \text{kN}, \quad B_x = -55 \text{kN}$

Using ②, ④ & ⑤ $\implies C_y = 460 \text{ kN}, \quad B_y = 460 \text{ kN}$

$A_y = 80 \text{ kN}$
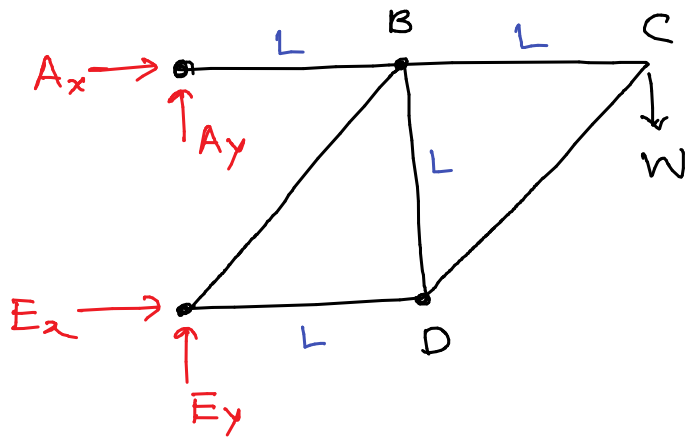
(b) To prevent slipping

$f_s \geqslant \max \left\{ \dfrac{B_x}{B_y}, \dfrac{C_x}{C_y} \right\} = \max \left\{ \dfrac{55}{460}, \dfrac{95}{460} \right\} = \max \{ 0.12, 0.2 \}$

$= 0.2$

6)
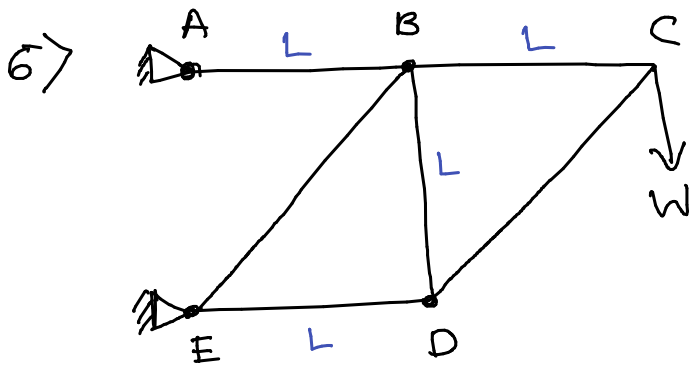
A truss consists of members which resist axial forces such as tension or compression

FBD of entire truss

$$\circlearrowright + \sum M_E = 0$$
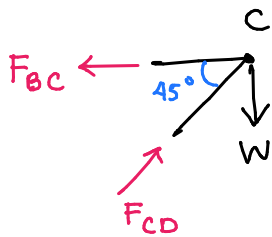$$\Rightarrow -A_x(L) - W(2L) = 0$$
$$\Rightarrow A_x = -2W$$

$$+\uparrow \sum F_y = 0$$
$$\Rightarrow E_y + A_y - W = 0$$

$$\xrightarrow{+} \sum F_x = 0$$
$$\Rightarrow A_x + E_x = 0$$

FBD of point C

$$+\uparrow \sum F_y = 0$$
$$\Rightarrow F_{CD} \cos 45° - W = 0$$
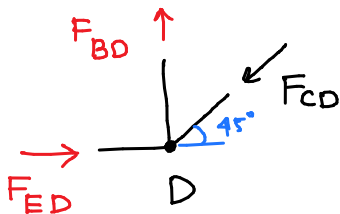$$\Rightarrow F_{CD} = \sqrt{2}\, W$$

$$\xrightarrow{+} \sum F_x = 0$$
$$\Rightarrow -F_{BC} + F_{CD} \sin 45° = 0$$
$$\Rightarrow F_{BC} = W$$

FBD of point D

$$+\uparrow \sum F_y = 0$$
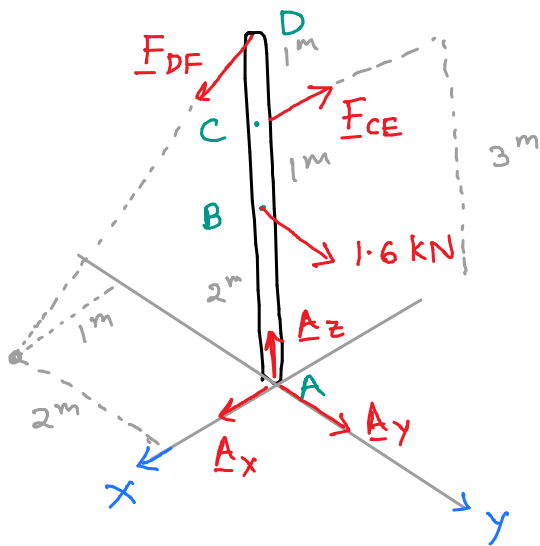$$\Rightarrow -F_{CD} \sin 45° + F_{BD} = 0$$
$$\Rightarrow F_{BD} = W$$

$$\xrightarrow{+} \sum F_x = 0$$
$$\Rightarrow F_{ED} - F_{CD} \cos 45° = 0$$
$$\Rightarrow F_{ED} = W$$

Proceed similarly !

7)

In this 3D case, it is better to use vector notation to denote the forces

- Unit vector along F_{DF}

$$= \frac{(\underline{i} - 2\underline{j} - 4\underline{k})}{\sqrt{1 + 2^2 + 4^2}}$$

$$\underline{F}_{DF} = F_{DF} (\underline{i} - 2\underline{j} - 4\underline{k})/\sqrt{21}$$

- Unit vector along F_{CE}

$$= -\underline{i}$$

$$\Rightarrow \quad \underline{F}_{CE} = F_{CE} (-\underline{i})$$

- Reaction force

$$\underline{A} = A_x \underline{i} + A_y \underline{j} + A_z \underline{k}$$

Using equilibrium of forces on the rod AD

$$\sum \underline{F} = \underline{0}$$

$$\Rightarrow \quad \underline{F}_{DF} + \underline{F}_{CE} + \underline{A} = 0$$

Componentwise forces:

x - dir: $\quad \dfrac{F_{DF}}{\sqrt{21}} - F_{CE} + A_x = 0 \quad$ —①

y - dir: $\quad -\dfrac{2}{\sqrt{21}} F_{DF} + A_y - 1.6 \, kN = 0 \quad$ —②

z - dir: $\quad -\dfrac{4}{\sqrt{21}} F_{DF} + A_z = 0 \quad$ —③

Now, using moment equilibrium equations

Taking moment about pt A,

$$\Sigma \underline{M}_A = \underline{0}$$
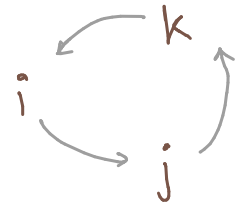
$$\underline{M}_{DF} = \underline{r}_{D|A} \times \underline{F}_{DF} = (4\underline{k}) \times \frac{F_{DF}}{\sqrt{21}}(\underline{i} - 2\underline{j} - 4\underline{k})$$

$$= \frac{1}{\sqrt{21}}(4\underline{j} + 8\underline{i}) F_{DF}$$

$$\underline{M}_{CE} = \underline{r}_{C|A} \times \underline{F}_{CE} = (3\underline{k}) \times F_{CE}(-\underline{i})$$

$$= -3F_{CE} \, \underline{j}$$

$$\underline{M}_{applied} = \underline{r}_{B|A} \times \underline{F}_{applied} = (2\underline{k}) \times (1.6 \text{ kN } \underline{j})$$

$$= -3.2 \, \underline{i} \quad kNm$$

Total sum of moments should be zero:

$$\underline{M}_{DF} + \underline{M}_{CE} + \underline{M}_{applied} = 0$$

Componentwise moments:

x-dir: $\quad \frac{8}{\sqrt{21}} F_{DF} - 3.2 \text{ kNm} = 0 \quad — \quad ④$

Y-dir: $\quad \frac{4}{\sqrt{21}} F_{DF} - 3 F_{CE} = 0 \quad — \quad ⑤$
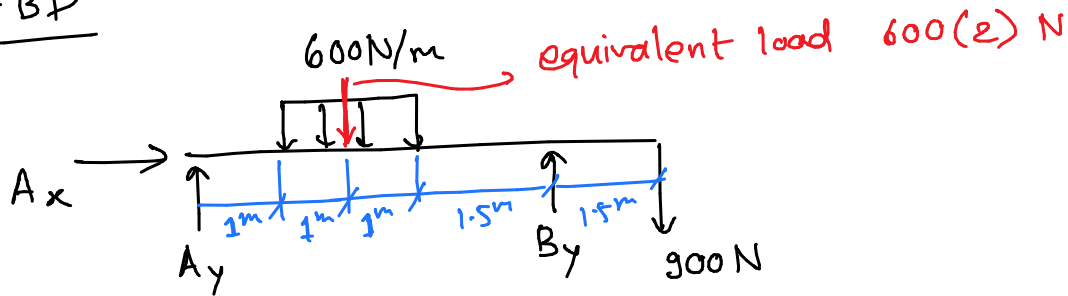
Solve ①, ②, ③, ④ & ⑤, get the values of

$F_{DF}, F_{CE}, A_x, A_y, A_z$

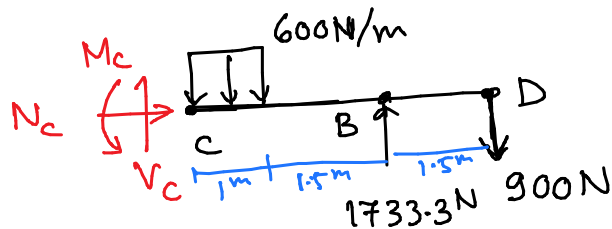8) Smooth journal bearings $\rightarrow$ only prevents translation vertically

FBP

600N/m → equivalent load 600(2) N

$A_x \rightarrow$

A_y

1m | 1m | 1m | 1.5m | 1.5m

B_y

900 N

Support reactions

$\curvearrowright + \Sigma M_A = 0$

$\Rightarrow \quad B_y (4.5) - \left(600(2)\right)(2) - 900(6) = 0$

$\Rightarrow \quad B_y = 1733.3 \text{ N}$

Cutting a section and isolating only the RHS of C

M_C

N_C

600N/m

C | B | D

V_C

1m | 1.5m | 1.5m

1733.3N | 900N

$\xrightarrow{+} \Sigma F_x = 0 \quad \Rightarrow \quad N_C = 0$

$+\uparrow \Sigma F_y = 0 \quad \Rightarrow \quad V_C - 600(1) + 1733.3 - 900 = 0$

$\Rightarrow \quad V_C = -233 \text{ N}$

$\curvearrowright + \Sigma M_C = 0 \quad \Rightarrow \quad M_C - \left(600(1)\right)(0.5) + 1733.3(2.5)$

$\qquad \qquad \qquad \qquad \qquad \qquad - 900(4) = 0$

$\Rightarrow \quad M_C = 433 \text{ N}$

9) Idealize the forearm and biceps as follows:

D

will take mostly axial load

75°

B

C

0.23m

2 (9.81) N

Draw FBD
_____

F_{BD}

75°

C_x

0.07m   0.23m

C_y

2 (9.81) N
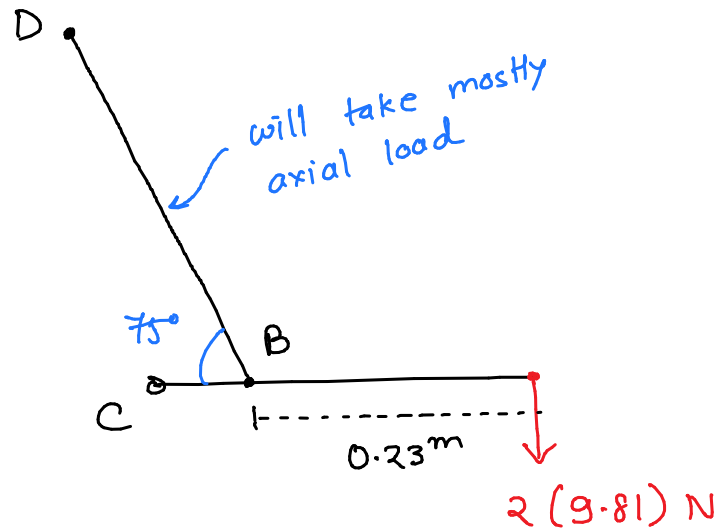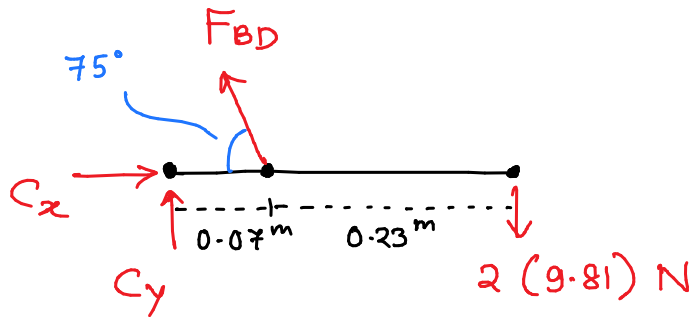
Apply equilibrium equation,

$\circlearrowleft + \Sigma M_c = 0 \Rightarrow F_{BD} \sin 75° (0.07) - 2(9.81)(0.3) = 0$

$\Rightarrow F_{BD} = 87.05 \, N$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow C_x - F_{BD} \cos 75° = 0$

$\Rightarrow C_x = 22.53 \, N$

$+\uparrow \Sigma F_y = 0 \Rightarrow C_y - 2(9.81) + 87.05 \cos 75° = 0$

$\Rightarrow C_y = -64.47 \, N$

The internal resistive forces in the bone are:

64.47

V_E

22.53 → N_E

M_E
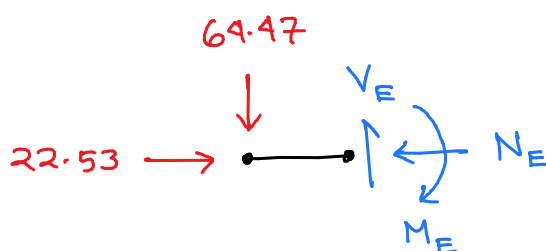
$N_E = 22.53 \, N$

$V_E =$

10) FBD of entire structure

$dF = \omega_0 (r\,d\theta)$

$r\sin\theta$

$r(1-\cos\theta)$

A_x, A_y, B_y

$\circlearrowleft + \Sigma M_A = 0$

$\Rightarrow \quad B_y(2r) - \int_0^\pi \left[(\omega_0\, r\,d\theta)\sin\theta\right] r\,(1-\cos\theta)$

$- \int_0^\pi \left[(\omega_0\, r\,d\theta)\cos\theta\right] r\sin\theta$

$\Rightarrow \quad B_y(2r) - \omega_0 r^2 \int_0^\pi \sin\theta\,d\theta = 0$

$\Rightarrow \quad B_y = \dfrac{\omega_0 r^2}{2r}\left[-\cos\theta\right]_0^\pi = \omega_0 r$
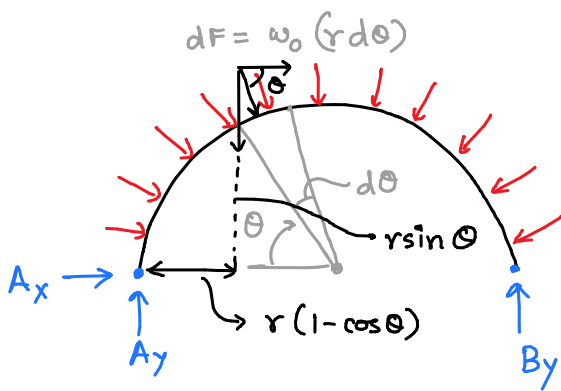
$\left(\begin{array}{l}\text{You can also find the resultant directly by looking at}\\ \text{the symmetry and seeing that forces cancel out in } x\text{-dir}\end{array}\right)$

$\xrightarrow{+}\Sigma F_x = 0$

$\Rightarrow \quad A_x = 0$

$+\uparrow \Sigma F_y = 0$

$\Rightarrow \quad A_y + B_y - \omega_0(2r) = 0$

$\Rightarrow \quad A_y = \omega_0 r$

FBD of isolated part
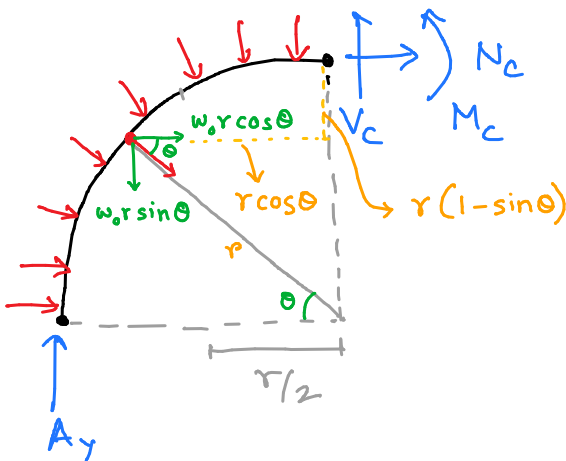
$\omega_0 r\cos\theta$

$\omega_0 r\sin\theta$

N_c, V_c, M_c

$r\cos\theta$

$r(1-\sin\theta)$

r

θ

$r/2$

A_y

$\xrightarrow{+}\Sigma F_x = 0 \quad\Rightarrow\quad N_c + \int_0^{\pi/2}\omega_0 r\cos\theta\,d\theta = 0$

$\Rightarrow \quad N_c = -\omega_0 r\left[\sin\theta\right]_0^{\pi/2}$

$= -\omega_0 r$

$+\uparrow \Sigma F_y = 0$

$\Rightarrow \quad \omega_0 r + V_c - \omega_0 r\int_0^{\pi/2}\sin\theta\,d\theta = 0$

$\Rightarrow \quad V_c = -\omega_0 r + \omega_0 r\left[-\cos\theta\right]_0^{\pi/2}$

$= -\omega_0 r - 0 + \omega_0 r = 0$

$\circlearrowleft + \Sigma M_c = 0$

$\Rightarrow -\cancel{A_y}^{\omega_0 r}(r) + M_c$

$+ \int_0^{\pi/2}(\omega_0 r\cos\theta)\, r(1-\sin\theta)$

$+ \int_0^{\pi/2}(\omega_0 r\sin\theta)\, r\cos\theta\,d\theta$

$\Rightarrow -\cancel{\omega_0 r^2} + M_c + \boxed{\int_0^{\pi/2}\omega_0 r^2\cos\theta\,d\theta} = 0$

$\overset{\omega_0 r^2}{\Uparrow}\quad \Rightarrow M_c = 0$

11) Draw FBD

$$\downarrow^+ \, \Sigma F_x = 0$$

$$\Rightarrow \quad V_{Cx} + 1000 - 750 = 0$$

$$\Rightarrow \quad V_{Cx} = -250 \, N$$

$$\searrow^+ \, \Sigma F_y = 0$$

$$\Rightarrow \quad N_{Cy} = 0$$

$$\uparrow^+ \, \Sigma F_z = 0$$

$$\Rightarrow \quad V_{Cz} + 240 = 0$$

$$\Rightarrow \quad V_{Cz} = -240 \, N$$

$$\overset{+}{\curvearrowright} \, \Sigma M_x \big|_C = 0$$

$$\Rightarrow \quad M_{Cx} + 240 \, (0.45) = 0$$

$$\Rightarrow \quad M_{Cx} = -108 \, Nm$$

$$\Sigma M_y \big|_C = 0$$

$$\Rightarrow \quad T_{Cy} = 0$$

$$\Sigma M_z \big|_C = 0$$

$$\Rightarrow \quad M_{Cz} - 1000 \, (0.2) + 750 \, (0.45) = 0$$

$$\Rightarrow \quad M_{Cz} = -138 \, Nm$$