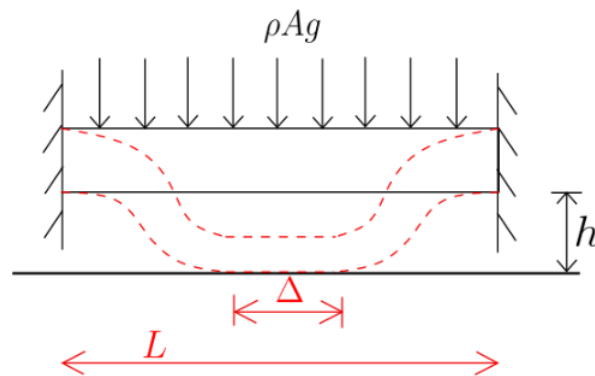


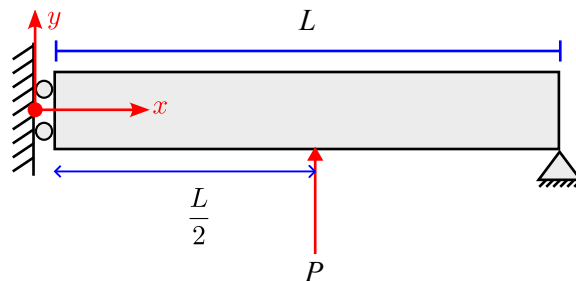
# Tutorial 11: Euler-Bernoulli beams and Energy Methods

APL 104B - 2024 (Solid Mechanics)

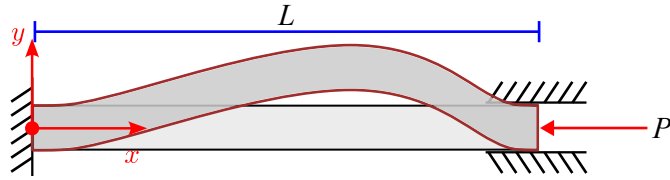
1. Consider a beam clamped at both ends. The beam sags down due to its own weight as shown in Figure 10, the distributed weight being  $\rho Ag$ . However, the ground position ( $h$  below the beam) is such that some part of the beam rests on the ground upon deformation while the remaining part just hangs. Find the length of the beam  $\Delta$  which will rest on the ground.



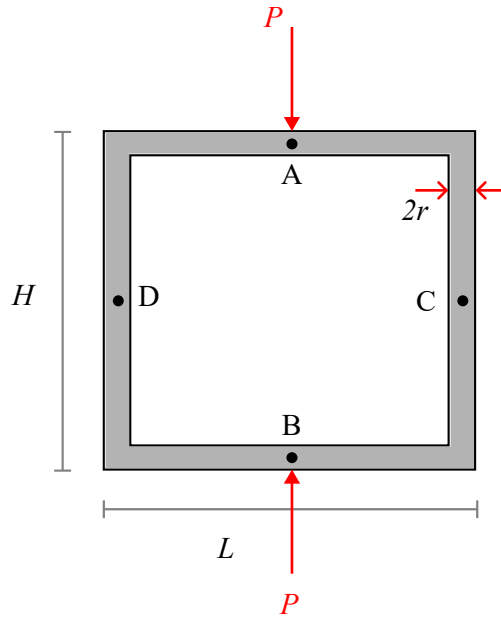
2. Suppose a beam is kept with roller support at one end ( $x = 0$ ) constrained to only move in  $y$ -direction and pinned at the other end ( $x = L$ ) as shown. The beam is subjected to transverse load ( $P$ ) at the middle of the beam. Find the deflection of the beam using the Euler-Bernoulli beam theory.



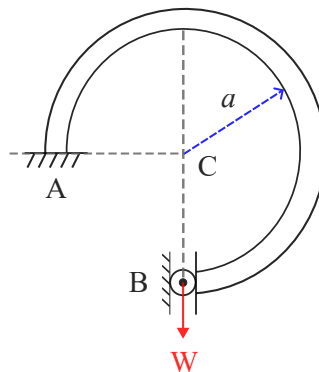
3. Think of a beam that is clamped against both transverse deflections as well as rotation at both ends. Deduce the equation which gives us the critical buckling load of the beam. (You don't have to solve it)



4. Think of a rectangular beam as shown below. Assume its cross-section to be solid circular. Suppose the ring is subjected to equal and opposite forces at 'A' and 'B'. Neglect energy in the beam's cross-section due to shear force and axial force.



- By how much will point A and point B get closer to each other?
  - By how much will points C and D get farther apart?
  - What is the internal moment in the cross-section at C?
5. Using the energy method, determine (i) the vertical deflection of point B under the action of load W and (ii) the horizontal reaction force at B. The end B is free to rotate but can move only in a vertical direction. Consider all forms of energy, i.e. bending, twisting, stretching as well as shearing energy.



6. For the structure shown, what is the vertical deflection at end A? Also, determine the ratio of  $L$  to  $r$  if the horizontal and vertical deflections of the loaded end A are equal. P is the only force acting

