Q1>

Considering the case of uniaxial loading, we can see that

$$\epsilon_{11} = \frac{1}{E}\left(\sigma_{11} - \nu(\cancelto{0}{\sigma_{22}} + \cancelto{0}{\sigma_{33}})\right)$$

$$\Rightarrow \quad \sigma_{11} = E\,\epsilon_{11}$$

$$\epsilon_{22} = \frac{1}{E}\left(\cancelto{0}{\sigma_{22}} - \nu(\sigma_{11} + \cancelto{0}{\sigma_{33}})\right) = -\frac{\nu\,\cancelto{E\epsilon_{11}}{\sigma_{11}}}{E} = -\nu\,\epsilon_{11}$$

$$\epsilon_{33} = -\nu\,\epsilon_{11}$$

Now using Lame's constant, we can write

$$\sigma_{11} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\,\epsilon_{11} \qquad \tau_{12} = \mu\,\gamma_{12}$$

$$\Rightarrow E\epsilon_{11} = \lambda(1-2\nu)\,\epsilon_{11} + 2\mu\,\epsilon_{11} \qquad \tau_{12} = G\,\gamma_{12}$$

0.5

$$\Rightarrow E = \lambda(1-2\nu) + 2\mu$$

$$\Rightarrow E = (\lambda + 2\mu) - 2\lambda\nu \quad \text{—— (1)} \qquad \therefore \quad \mu = G \quad \text{— 1}$$

$$\sigma_{22} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\,\epsilon_{22}$$

$$\Rightarrow 0 = \lambda(1-2\nu)\,\epsilon_{11} - 2\mu\nu\,\epsilon_{11}$$

1

$$\Rightarrow 0 = \lambda - 2(\lambda+\mu)\,\nu$$

$$\Rightarrow \nu = \frac{\lambda}{2(\lambda+\mu)} \quad \text{—— (2)}$$

Use (2) in (1) :

$$E = (\lambda + 2\mu) - \frac{2}{2}\,\frac{\lambda^2}{(\lambda+\mu)}$$

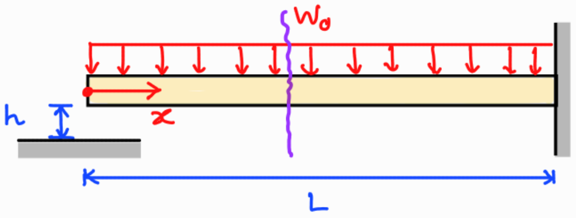
$$\Rightarrow E = \frac{(\lambda+2\mu)(\lambda+\mu) - \lambda^2}{(\lambda+\mu)}$$

$$= \frac{\lambda^2 + 3\lambda\mu + 2\mu^2 - \lambda^2}{(\lambda+\mu)}$$

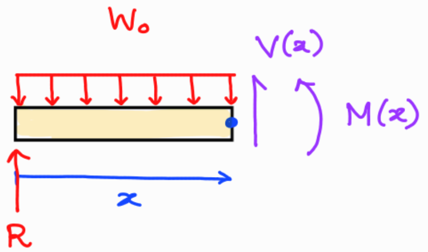
$$= \frac{\mu(3\lambda + 2\mu)}{(\lambda+\mu)}$$

0.5

Q2) a)



When the gap h is closed a reaction force R exists at the left end



$\circlearrowleft + \sum M_{right} = 0$

$$\Rightarrow \quad M(x) = Rx - \frac{W_0 x^2}{2}$$ 1

Use Euler- Bernoulli beam equation

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$\Rightarrow \quad EI \frac{d^2 v}{dx^2} = Rx - \frac{W_0 x^2}{2}$$

$$\Rightarrow \quad EI \frac{dv}{dx} = \frac{Rx^2}{2} - \frac{W_0 x^3}{6} + c_1$$

$$\Rightarrow \quad EI\, v(x) = \frac{Rx^3}{6} - \frac{W_0 x^4}{24} + c_1 x + c_2$$ 0.5

Boundary conditions

1.5

$v(0) = -h$

$v(L) = 0$

$\frac{dv}{dx}(L) = 0$

yields a solution

$$c_1 = \frac{W_0 L^3}{6} - \frac{RL^2}{2}$$

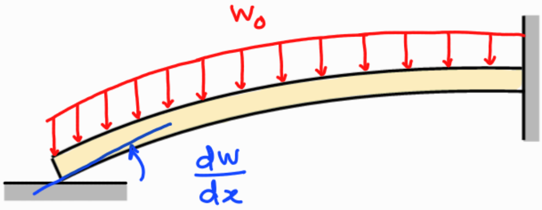
$$c_2 = -EIh$$

$$R = \frac{3}{8} W_0 L - \frac{3EIh}{L^3}$$ 1

The slope at the beam end

$$\frac{dv}{dx}(0) = \frac{c_1}{EI} = \frac{1}{EI}\left(\frac{W_0 L^3}{6} - \frac{3}{16} W_0 L^3 + \frac{3}{2} \frac{EIh}{L} \right) = \frac{3}{2}\frac{h}{L} - \frac{W_0 L^3}{48 EI}$$ 0.5

b)



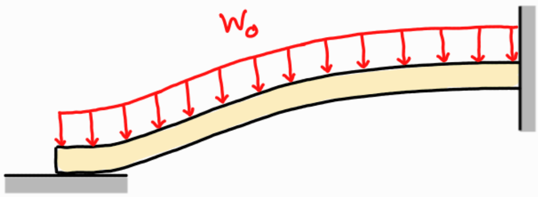
The first gap closes when $R \approx 0$ (1)

$$R = \frac{3}{8} w_0 L - \frac{3EIh}{L^3} = 0$$

$$\Rightarrow \quad w_0 = \frac{8EIh}{L^4} \quad (0.5)$$

Slope $\quad \frac{dv(0)}{dx} = \frac{c_1}{EI} = \frac{w_0 L^3}{6EI} \quad [\because R = 0]$ (0.5)

c) If we increase w_0, $\frac{dv(0)}{dx}$ decreases until it becomes 0



This occurs when,

$$\frac{dv(0)}{dx} = \frac{c_1}{EI} = 0 \quad \Rightarrow \quad c_1 = 0$$

$$\Rightarrow \quad \frac{3}{2}\frac{h}{L} - \frac{w_0 L^3}{48EI} = 0$$

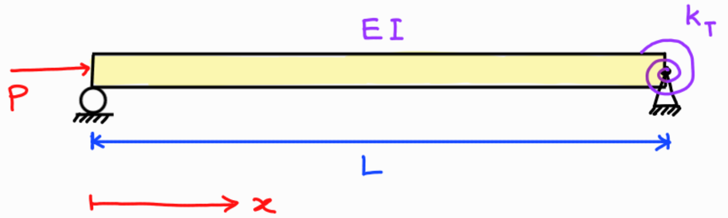
$$\Rightarrow \quad w_0 = \frac{72EIh}{L^4} \quad (1)$$

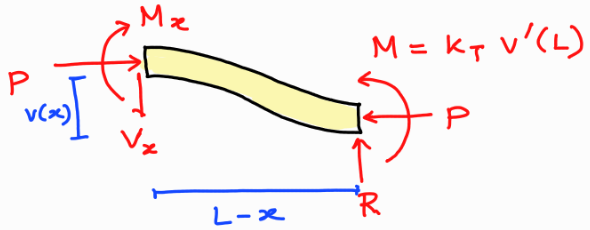
Corresponding reaction $R = \frac{3}{8} w_0 L - \frac{3EIh}{L^3}$

$$= \frac{3}{8}\left(\frac{72EIh}{L^4}\right) L - \frac{3EIh}{L^3}$$

$$= \frac{24EIh}{L^3} \quad (0.5)$$

Q3)





$$\circlearrowleft + \sum M_{left} = 0$$

$$\Rightarrow \quad M(x) = M + R(L-x) - Pv$$
$$= k_T\, v'(L) + R(L-x) - Pv \qquad (1)$$

Using Euler-Bernoulli equation, we write

$$EI \frac{d^2v}{dx^2} = M + R(L-x) - Pv$$

$$\Rightarrow \quad EI \frac{d^2v}{dx^2} + Pv = M + R(L-x) = k_T\, v'(L) + R(L-x)$$

General soln: $v(x) = C_1 \cos \lambda x + C_2 \sin \lambda x + \dfrac{M + R(L-x)}{P}$ (1)

$$v'(x) = -c_1 \lambda \sin \lambda x + c_2 \lambda \cos \lambda x - \frac{R}{P}$$

$$v''(x) = -c_1 \lambda^2 \cos \lambda x - c_2 \lambda^2 \sin \lambda x$$

Boundary conditions:

(2)

$$v(0) = 0 \Rightarrow c_1 + \frac{M + RL}{P} = 0 \quad \text{— (1)}$$

$$v(L) = 0 \Rightarrow c_1 \cos \lambda L + c_2 \sin \lambda L + \frac{M}{P} = 0 \quad \text{— (2)}$$

$$EI\, v''(0) = 0 \Rightarrow -c_1 \lambda^2 = 0 \Rightarrow c_1 = 0 \quad \text{— (3)}$$

$$EI\, v''(L) = M \Rightarrow EI\left(-c_1 \lambda^2 \cos \lambda L - c_2 \lambda^2 \sin \lambda L\right) \quad \text{— (4)}$$
$$= k_T\left(-c_1 \lambda \sin \lambda L + c_2 \lambda \cos \lambda L - \frac{R}{P}\right)$$

Using 3 in 1, 2, & 4

1 $\rightarrow$ $\dfrac{M + RL}{P} = 0 \Rightarrow R = -\dfrac{M}{L}$ — 5

2 $\rightarrow$ $c_2 \sin \lambda L + \dfrac{M}{P} = 0$ — 6

4 $\rightarrow$ $c_2(-EI\,\lambda^2 \sin \lambda L) = c_2\, k_T \lambda \cos \lambda L - k_T \dfrac{R}{P}$ — 7

1

Replace R in 7 with M from 5

7 $\rightarrow$ $c_2\left[-EI\,\lambda^2 \sin \lambda L - \lambda \cos \lambda L\right] = k_T \dfrac{M}{PL}$ — 8

Now use 6 to sub the value of $\dfrac{M}{P}$

8 $\rightarrow$ $\cancel{c_2}\left[-EI\,\lambda^2 \sin \lambda L - \lambda \cos \lambda L\right] = -\dfrac{\cancel{c_2}}{L} k_T \sin \lambda L$

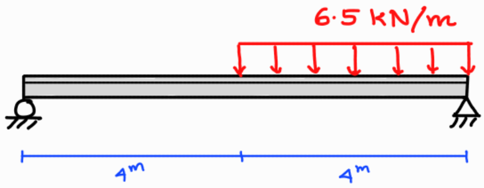
$\Rightarrow -\left(EI\,\lambda^2 - \dfrac{k_T}{L}\right) \sin \lambda L = \lambda \cos \lambda L$

$$\Rightarrow \boxed{\tan \lambda L = -\dfrac{\lambda}{EI\,\lambda^2 - \dfrac{k_T}{L}}}$$

Buckling Characteristic Equation

1

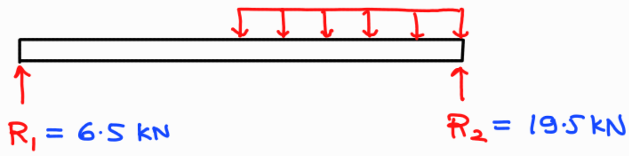
Q4) Support reactions



$\circlearrowleft + \sum M_{left} = 0$

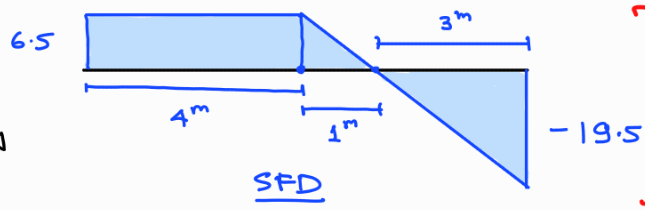
$\Rightarrow \quad R_2\,(8) - (6.5)(4)\left(\frac{4}{2}+4\right) = 0$

$\Rightarrow \quad R_2 = 19.5 \text{ kN}$



$+\uparrow \sum F_y = 0 \quad \Rightarrow \quad R_1 + R_2 = (6.5)(4)$

$\Rightarrow \quad R_1 = 26 - 19.5 = 6.5 \text{ kN}$



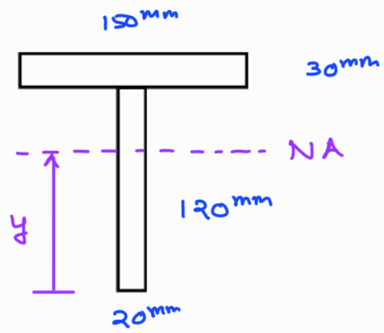
1.5

Maximum shear force will occur at the right support

Section properties

The NA lies at the centroid. The centroid of the C/s has to be determined.

Considering the reference at the bottom-most fiber, the centroid $\bar{y}$ is



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$$\bar{y} = \frac{(150)(30)\left(\frac{30}{2}+120\right) + (120)(20)\left(\frac{120}{2}\right)}{(150)(30) + (120)(20)}$$

$$= 108.91 \text{ mm}$$

The moment of inertia about the neutral axis :

1

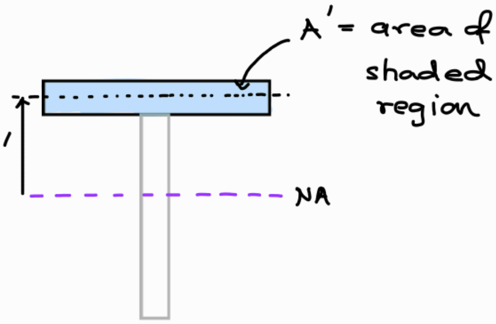
$$I_{zz} = \left[\frac{1}{12}(0.15)(0.03)^3 + (0.15)(0.03)\left(\frac{0.03}{2} + 0.12 - 0.10891\right)^2 + \frac{1}{12}(0.02)(0.12)^3 + (0.12)(0.02)\left(0.10891 - \frac{0.12}{2}\right)^2\right]$$

$$= 3.4 \times 10^{-6} \text{ m}^4 + 8.621 \times 10^{-6} \text{ m}^4$$

$$= 1.2021 \times 10^{-5} \text{ m}^4$$

It is the glue's resistance to longitudinal shear stress at the connection that holds the boards from slipping at the right hand support.

Shear stress at the level of the connection



1.5

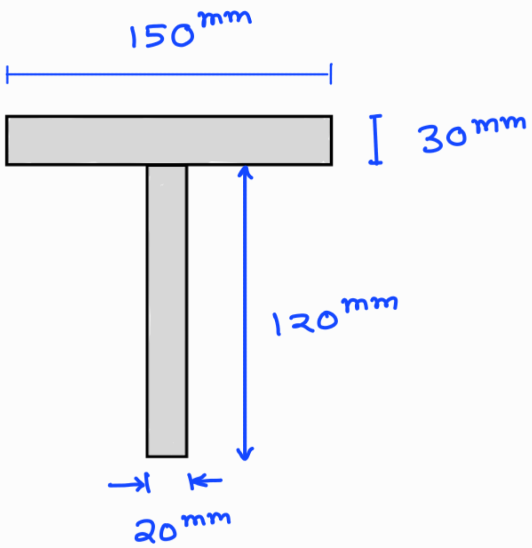
$$Q = \bar{y}'A'$$

$$= \left(0.12 + \frac{0.03}{2} - 0.10891\right)(0.15)(0.03)$$

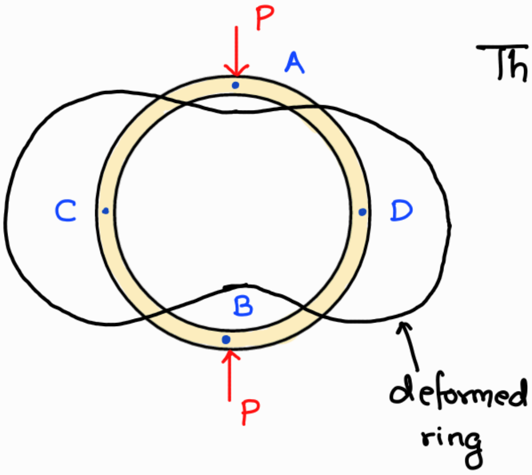
$$= 1.17405 \times 10^{-4}\ m^3$$

1

$$\tau_{max} = \frac{V_{max}\ Q}{I_{zz}\ b_{min}} = \frac{(19.5 \times 10^3\ N)(1.17405 \times \cancel{10^{-4}}\ m^3)}{(1.2021 \times \underset{10^{-1}}{\cancel{10^{-5}}}\ m^4)(0.02\ m)} = 9.52\ MPa$$

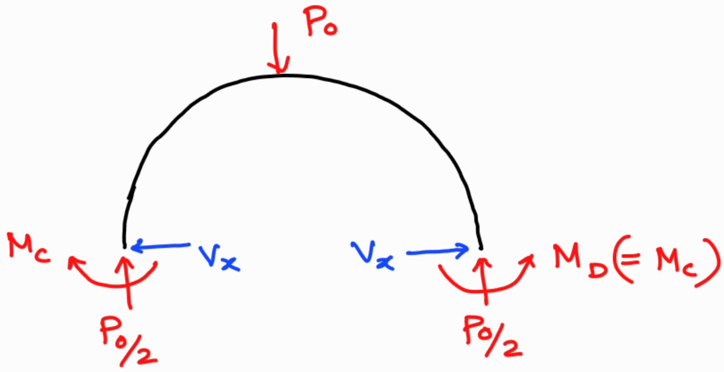


Q5)



The circular beam bends symmetrically

Cut a half section



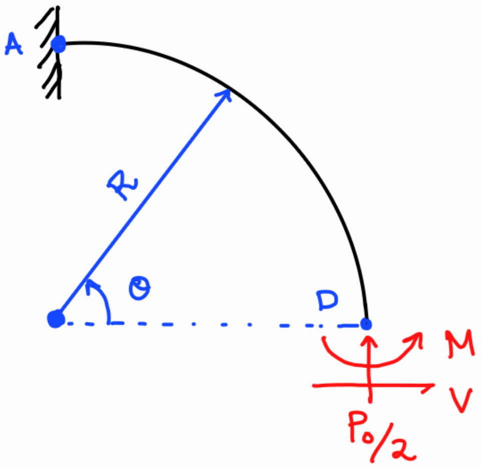
Due to symmetry,

1) $V_x = 0$

2) pt C will not rotate

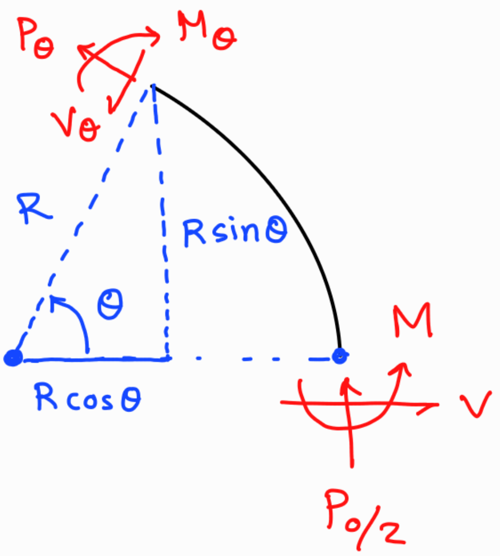
1

Consider now a quarter of the circle. The point A is fixed.



Therefore, point D can now move vertically and horizontally but will not rotate.

$$\theta_D = \frac{\partial U_i}{\partial M} = 0$$ 1



$$\circlearrowleft + \sum M_{\text{about top}} = 0$$

$$\Rightarrow -M_\theta + M + V R\sin\theta + \frac{P_o}{2} R(1-\cos\theta) = 0$$

$$\Rightarrow M_\theta = M + VR\sin\theta + \frac{P_o R}{2}(1-\cos\theta) = 0$$

1

Stored internal energy due to bending

$$U_i = \int_0^{\pi/2} \frac{M_\theta^2}{2EI} R\,d\theta$$

$$= \int_0^{\pi/2} \frac{\left[M + VR\sin\theta + \frac{P_0 R}{2}(1-\cos\theta)\right]^2}{2EI} R\,d\theta$$

To obtain the internal moment at D, we use $\frac{\partial U_i}{\partial M} = 0$

$$\cancelto{0}{\frac{\partial U_i}{\partial M}} = \int_0^{\pi/2} \frac{\cancel{2}\left[M + \cancelto{0}{V}R\sin\theta + \frac{P_0 R}{2}(1-\cos\theta)\right]}{\cancel{2}EI} R\,d\theta$$ 0.5

$$\Rightarrow \int_0^{\pi/2} \left[M + \frac{P_0 R}{2}(1-\cos\theta)\right] d\theta = 0$$

$$\Rightarrow M\left(\frac{\pi}{2}\right) = -\frac{P_0 R}{2}\left(\frac{\pi}{2} - 1\right)$$

$$\Rightarrow \boxed{M = -\frac{P_0 R}{2}\left(1 - \frac{2}{\pi}\right)}$$ 0.5

The outward displacement of point D :

$$\delta_{D_x} = \frac{\partial U_i}{\partial V} = \int_0^{\pi/2} \frac{\left[M + \cancelto{0}{V}R\sin\theta + \frac{P_0 R}{2}(1-\cos\theta)\right]}{EI} R^2 \sin\theta\,d\theta$$

$$= \frac{MR^2}{EI}\int_0^{\pi/2} \sin d\theta + \frac{P_0 R^3}{2EI}\int_0^{\pi/2} (1-\cos\theta)\sin\theta\,d\theta$$

$$= \frac{MR^2}{EI} + \frac{P_0 R^3}{4EI} = \frac{R^2}{EI}\left[M + \frac{PR}{4}\right]$$ 2

$$\delta_{AB} = 2\,\delta_{D_y} = 2\,\frac{\partial U_i}{\partial P_o}$$

$$= \int_0^{\pi/2} \frac{\left[M + \cancel{V}^{\,0} R\sin\theta + \frac{P_o R}{2}(1-\cos\theta)\right]}{EI}\;\frac{R^2}{2}(1-\cos\theta)\,d\theta$$

$$= \frac{MR^2}{2EI}\int_0^{\pi/2}(1-\cos\theta)\,d\theta + \frac{P_o R^3}{4EI}\int_0^{\pi/2}(1-\cos\theta)^2\,d\theta$$

$$= \frac{MR^2}{2EI}\left(\frac{\pi}{2}-1\right) + \frac{P_o R^3}{4EI}\left(\frac{3\pi}{4}-2\right)$$

2