Tutorial 4: Principal Stresses and Principal Planes

APL 108 - 2025 (Mechanics of Solid)

1. Show that when all eigenvalues of a stress tensor are distinct, the eigenvectors will be perpendicular to each other

2. Show that when two eigenvalues of a stress tensor are repeated, the corresponding eigenvectors are non-unique and any vector lying in the plane perpendicular to the unique eigenvector direction (corresponding to the non-repeated eigenvalue) will be a valid eigenvector.

3. For the given state of stress at a point, determine the principal stresses and their directions

$$\left[\underline{\underline{\sigma}}\right] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

4. A cylindrical rod shown in the figure below is subjected to a torque T. At any point P of the cross-section LN, the following stress components exist:

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = \tau_{yx} = 0,\ \tau_{xz} = \tau_{zx} = -G\theta y,\ \tau_{yz} = \tau_{zy} = G\theta x$$

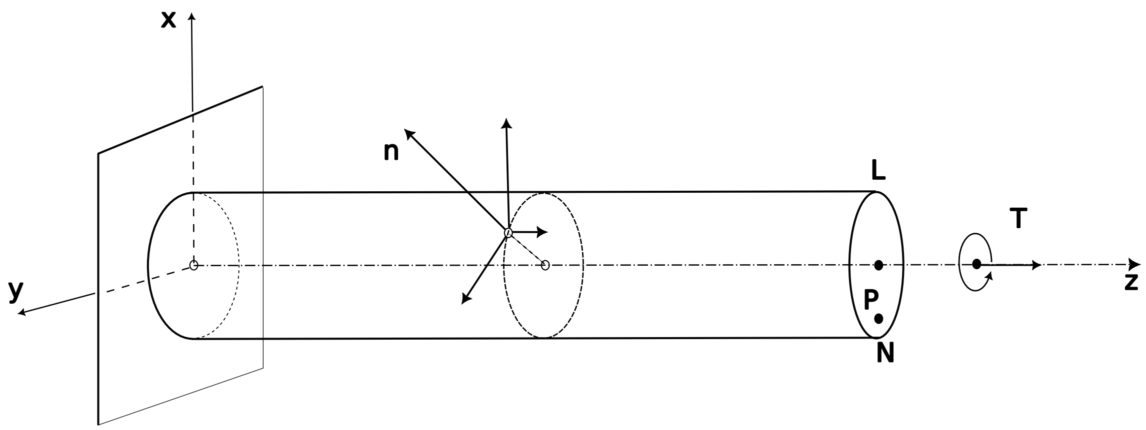

Figure 3

Check whether these satisfy the equations of equilibrium. Also, show that the above distribution implies that the lateral surface should be free of external load.

For the state of stress given, determine the principal stress components and the associated plane normals on which they are realized.

5. Consider a thin-walled cylinder of internal radius r and thickness t. If the cylinder is subjected to an internal pressure p and an axial force F, show that the r, θ, z directions are the principal stress directions. Also show that if the wall is so thin that $t/r << 1$, then the stresses in the pipe wall are given approximately by

$$\sigma_{rr} = 0$$
$$\sigma_{\theta\theta} = \frac{pr}{2t}$$
$$\sigma_{zz} = \frac{F}{2\pi rt}$$