Lecture - 18, APL 100 15/05/2023

Note Title 15-05-2023

Recap:-

$$T|_F = \tfrac{1}{2} m \vec{v}_{c|F}^{\,2} + \tfrac{1}{2} I^c_{ij}\, \omega_i \omega_j$$

$\vec{\omega} = \omega_1 \hat{e}_1 + \omega_2 \hat{e}_2 + \omega_3 \hat{e}_3$

$[I^c_{ij}]$ = inertia matrix w.r.to the axes located at the centre of mass c.

$\vec{v}_{c|F}$ = velocity of the centre of mass

m = mass of the rigid body

Centre of percussion

$$d = K^{c\,2}_{33}/h \, ,$$

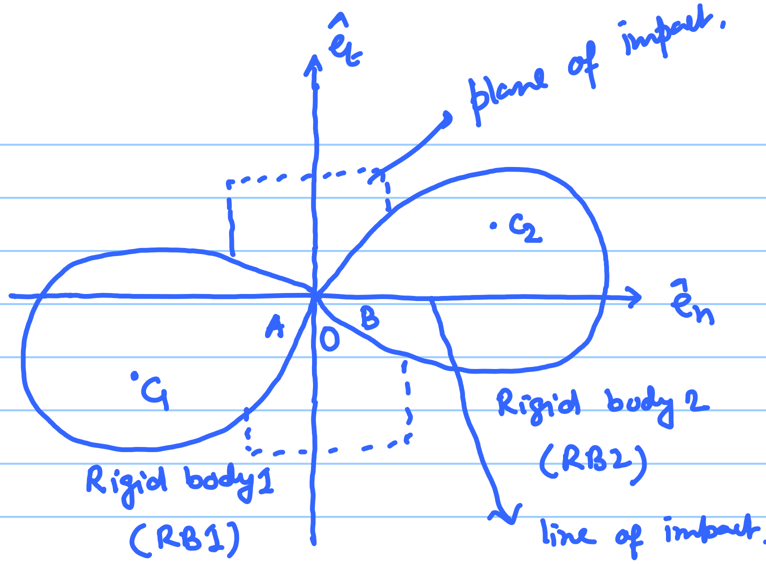
K^c_{33} = radius of gyration w.r.to $\hat{e}_3$ axis located at centre of mass

d = distance from centre of mass to the location of centre of percussion

h = distance from support to the centre of mass.

Gyroscopic couple

Impact of rigid bodies:- (Smooth impact)



Suppose, O is a fixed point space. Let the impact occurs at O. Suppose, A is the impact point for RB1 and B is the impact point for RB2. $(A \equiv B \equiv O)$.

Suppose, C_1 is the centre of mass for RB1 and C_2 is the centre of mass for RB2.

The common tangent plane at O is called plane of impact.
$\hat{e}_t$ is the unit tangent vector at O and $\hat{e}_n$ is the unit normal vector.

The line along the direction of $\hat{e}_n$ is called line of impact.

If the centre of masses of RB1 and RB2 lie on the line of impact, then impact is called central impact.

$$\hat{e}_b = \hat{e}_t \times \hat{e}_n$$

For RB1 (before impact)

$\vec{v}_{c_1} = v_{1t}\hat{e}_t + v_{1n}\hat{e}_n + v_{1b}\hat{e}_b$ = velocity of Centre of mass for RB1

$\vec{\omega}_1 = \omega_{1t}\hat{e}_t + \omega_{1n}\hat{e}_n + \omega_{1b}\hat{e}_b$ = angular velocity of RB1

For RB2 (before impact)

$\vec{v}_{c_2} = v_{2t}\hat{e}_t + v_{2n}\hat{e}_n + v_{2b}\hat{e}_b$ = Velocity of Centre of mass for RB2

$\vec{\omega}_2 = \omega_{2t}\hat{e}_t + \omega_{2n}\hat{e}_n + \omega_{2b}\hat{e}_b$ = angular velocity of RB2,

For RB1 (after impact)

$\vec{v}_{c_1}' = v_{1t}'\hat{e}_t + v_{1n}'\hat{e}_n + v_{1b}'\hat{e}_b$ = velocity of Centre of mass for RB1

$\vec{\omega}_1' = \omega_{1t}'\hat{e}_t + \omega_{1n}'\hat{e}_n + \omega_{1b}'\hat{e}_b$ = angular velocity of RB1

$\vec{v}_{c_2}' = v_{2t}'\hat{e}_t + v_{2n}'\hat{e}_n + v_{2b}'\hat{e}_b$ = velocity of centre of mass for RB2

$\vec{\omega}_2' = \omega_{2t}'\hat{e}_t + \omega_{2n}'\hat{e}_n + \omega_{2b}'\hat{e}_b$ = angular velocity of RB2.

For RB1

As there is no force in $\hat{e}_t$ direction, we can write $\boxed{v_{1t} = v_{1t}'} \rightarrow (1)$ $\{m_1 = \text{mass of the RB1}\}$

Similarly, there is no force in $\hat{e}_b$ direction we can write $\boxed{v_{1b} = v_{1b}'} \rightarrow (2)$

Similarly, for RB2, we can get $v_{2t} = v_{2t}' \rightarrow (3)$

$v_{2b} = v_{2b}' \rightarrow (4)$

Now, we consider RB1 and RB2 as a combined body, then there is no external force.

$$\therefore \quad m_1 \vec{v}_{c_1} + m_2 \vec{v}_{c_2} = m_1 \vec{v}_{c_1}' + m_2 \vec{v}_{c_2}' \qquad \{m_2 \text{ is the mass of the RB2}\}$$

$$\Rightarrow \quad m_1 v_{1n} + m_2 v_{2n} = m_1 v_{1n}' + m_2 v_{2n}' \rightarrow (5)$$

Now we will use the angular impulse about a point O.

For body 1 $\quad \vec{H}_O = \vec{H}_O' \quad (O \equiv A)$

$$\Rightarrow \vec{H}_{c_1} + \vec{r}_{c_1 O} \times m_1 \vec{v}_{c_1 O} = \vec{H}_{c_1}' + \vec{r}_{c_1 O} \times m_1 \vec{v}_{c_1 O}' \Rightarrow \vec{H}_{c_1} + \vec{OC_1} \times m_1 \vec{v}_{c_1} = \vec{H}_{c_1}' + \vec{OC_1} \times m_1 \vec{v}_{c_1}'$$

$$\Rightarrow \vec{H}_{G_1} + \vec{OG_1} \times m_1 v_{1n} \hat{e}_n = \vec{H}'_{G_1} + \vec{OG_1} \times m_1 v'_{1n} \hat{e}_n$$

Similarly, for body 2, we can write

$$\vec{H}_{C_2} + \vec{OC_2} \times m_2 v_{2n} \hat{e}_n = \vec{H}'_{C_2} + \vec{OC_2} \times m_2 v'_{2n} \hat{e}_n$$

$$e = \frac{v_s}{v_a} = \frac{\text{Velocity of separation of B from A after impact}}{\text{velocity of approach of A towards B before impact}} = \frac{\text{Impulse during restoration}}{\text{Impulse during deformation}}$$

↓

This measures the loss of kinetic energy.

$$e = \frac{v'_{Bn} - v'_{An}}{v_{An} - v_{Bn}}$$

$$v'_{Bn} = \vec{v}'_B \cdot \hat{e}_n = [\vec{v}'_{C_2} + \vec{\omega}'_2 \times \vec{C_2B}] \cdot \hat{e}_n$$

$$v'_{An} = \vec{v}'_A \cdot \hat{e}_n = [\vec{v}'_{G_1} + \vec{\omega}'_1 \times \vec{G_1A}] \cdot \hat{e}_n$$

$$v_{Bn} = \vec{v}_B \cdot \hat{e}_n = [\vec{v}_{C_2} + \vec{\omega}_2 \times \vec{C_2B}] \cdot \hat{e}_n$$

$$v_{An} = \vec{v}_A \cdot \hat{e}_n = [\vec{v}_{G_1} + \vec{\omega}_1 \times \vec{G_1A}] \cdot \hat{e}_n$$

Suppose, RB2 is a massive body, then $\vec{v}_{c_2} = \vec{v}'_{c_2}$ and $\vec{\omega}_2 = \vec{\omega}'_2$

You need only four equations to find out four knowns for RB1.

$$v_{1t} = v'_{1t} \qquad v_{1b} = v'_{1b}$$

$$\vec{H}_{G_1} + \vec{OG_1} \times m_1 v_{1n} \hat{e}_n = \vec{H}'_{G_1} + \vec{OG_1} \times m_1 v'_{1n} \hat{e}_n$$

$$e = \frac{v'_{Bn} - v'_{An}}{\cancel{A}v_{An} - v_{Bn}}$$

If RB2 is a massive body and it is at rest, $\vec{v}_{c_2} = \vec{v}'_{c_2} = 0$ and $\vec{\omega}_2 = \vec{\omega}'_2 = 0$

$$e = -\frac{v'_{An}}{v_{An}} \Rightarrow \boxed{v'_{An} = -e\, v_{An}}$$

For a central impact $\vec{H}_{C_1} = \vec{H}'_{C_1} \Rightarrow \vec{\omega}_1 = \vec{\omega}'_1$ (for RB1)

similarly, for RB2, we have $\vec{H}_{C_2} = \vec{H}'_{C_2} \Rightarrow \vec{\omega}_2 = \vec{\omega}'_2$

$$v'_{Bn} = \vec{v}'_B \cdot \hat{e}_n = \left[\vec{v}'_{C_2} + \vec{\omega}'_2 \times \vec{C_2B}\right] \cdot \hat{e}_n = \vec{v}'_{C_2} \cdot \hat{e}_n + \cancelto{0}{(\vec{\omega}'_2 \times \vec{C_2B})} \cdot \hat{e}_n = v'_{2n}$$

$$\therefore e = \frac{v'_{Bn} - v'_{An}}{v_{An} - v_{Bn}} = \frac{v'_{2n} - v'_{1n}}{v_{1n} - v_{2n}}$$