Goal

Developing a numerical solver for 2D incompressible Navier-Stokes equations. The advantage is that it has richer dynamics (like vortices, turbulences, etc), allows for clear benchmark problems (we can focus on vortices, which gives a goal in terms of accuracy and stiffness level, since low viscosity leads a dominance of convective terms over diffusive damping), and copes with elliptic pressure coupling (Poisson equation), rather than coupled density-pressure-energy for the 1D compressible case. It has a higher computational cost, but that's exactly what we want: use methods that mitigate this, while maintaining stability (and as much accuracy as possible to get "good" vortices for example).

Tasks

[1] **Temporal discretization**
Challenge. Balancing stability (stiff diffusion terms) and accuracy (nonlinear convection terms)
Numerical approach. Semi-implicit schemes, adaptive time-steps, etc.

[2] **Spatial discretization**
Challenge. Preserving divergence-free velocity fields ($\Delta \mathbf{u} = 0$) while resolving sharp vorticity gradients.
Numerical approach. Staggered grids (e.g. Marker-and-Cell), finite volume/element methods, etc.

[3] **Poisson solver**
Challenge. Pressure-velocity coupling, that is, solving the elliptic pressure equation efficiently, especially in vortical regions with strong pressure gradients.
Numerical approach. Multigrid/FFT-based solvers, projection methods (e.g. Chorin/IPCS), etc.

[4] **Validation**
Challenge. Validating our numerical model on specific benchmark problems, that would be, vortex-dominated flows like the Taylor-Green vortex and lid-driven cavity. Analyze (actual) convergence rates and scalability.