# Solving Navier Stoke Equation 2D : Incompressibility

APC 523 , Spring 2025

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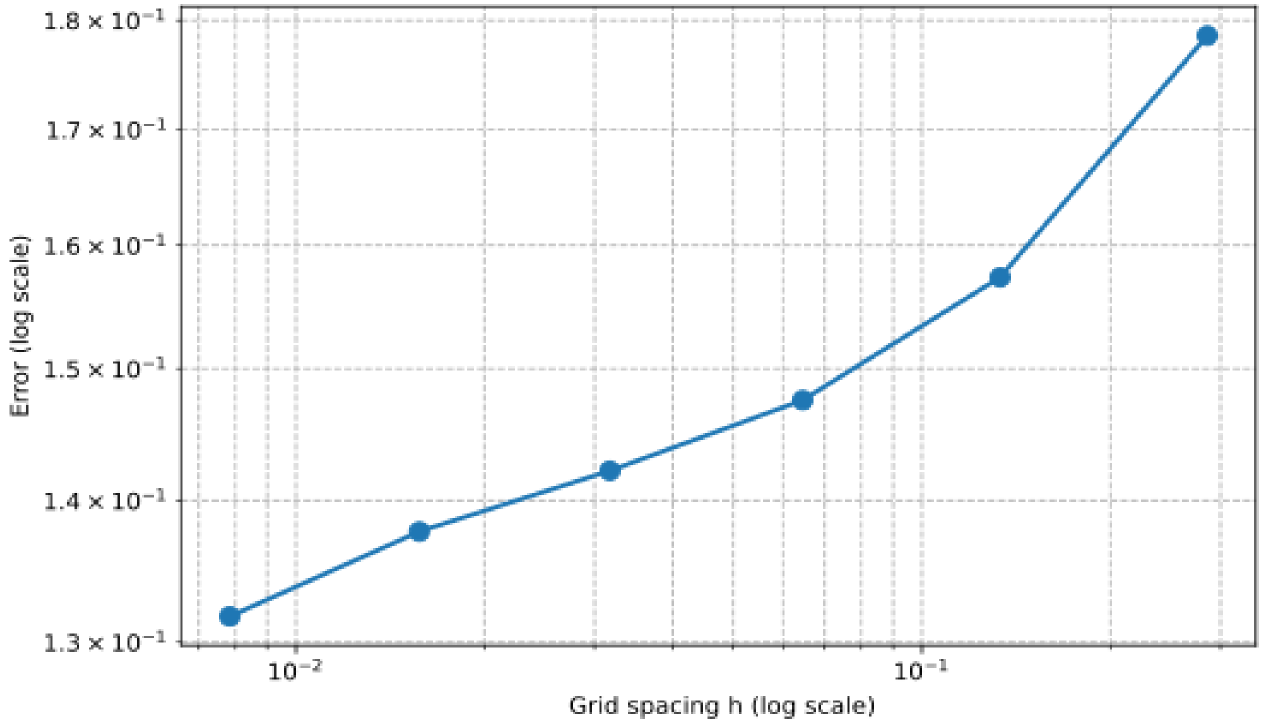
This project focuses on the two-dimensional, incompressible Navier–Stokes equations, which model the behavior of viscous Newtonian fluids with constant density and no external forces.

$$\begin{cases} \dfrac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\dfrac{1}{\rho}\nabla p + \nu \nabla^2 \vec{u}, & \text{(momentum equation)} \\ \nabla \cdot \vec{u} = 0, & \text{(continuity equation)} \end{cases}$$

# Methodology for Solving the Two-Dimensional Incompressible Navier–Stokes Equations

- **We discretize the spatial terms** (RHS) of the momentum equations, including advection, diffusion, and pressure gradient.
- **We update the velocity fields in time** using a predictor step that computes tentative velocities without enforcing incompressibility.
- **We solve a pressure Poisson equation** to enforce the incompressibility constraint.
- **We correct the velocity field** using the computed pressure to ensure a divergence-free flow.

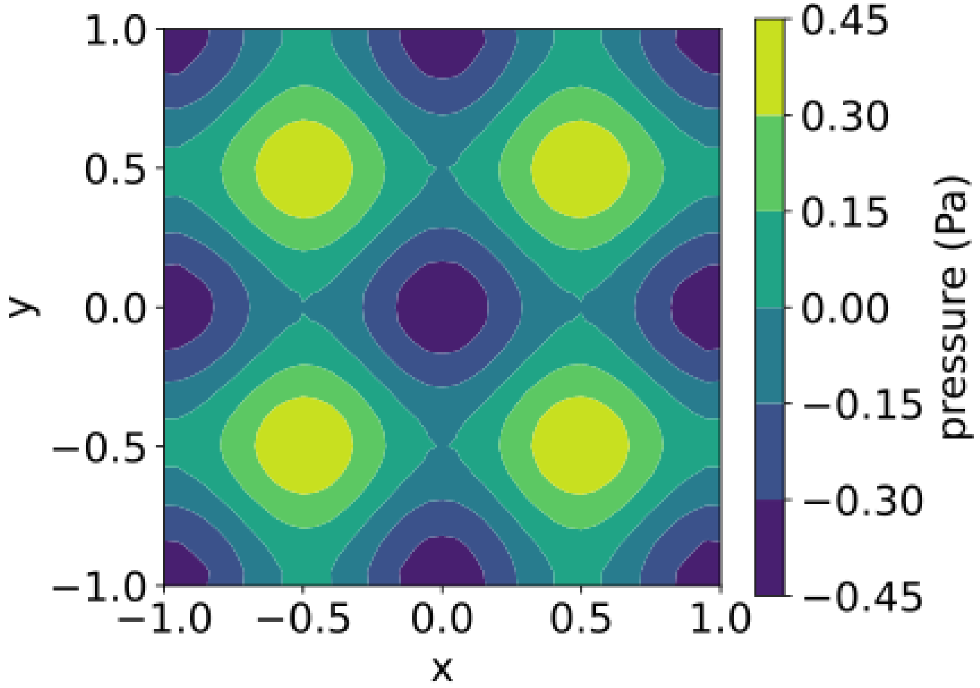
# Performance Analysis



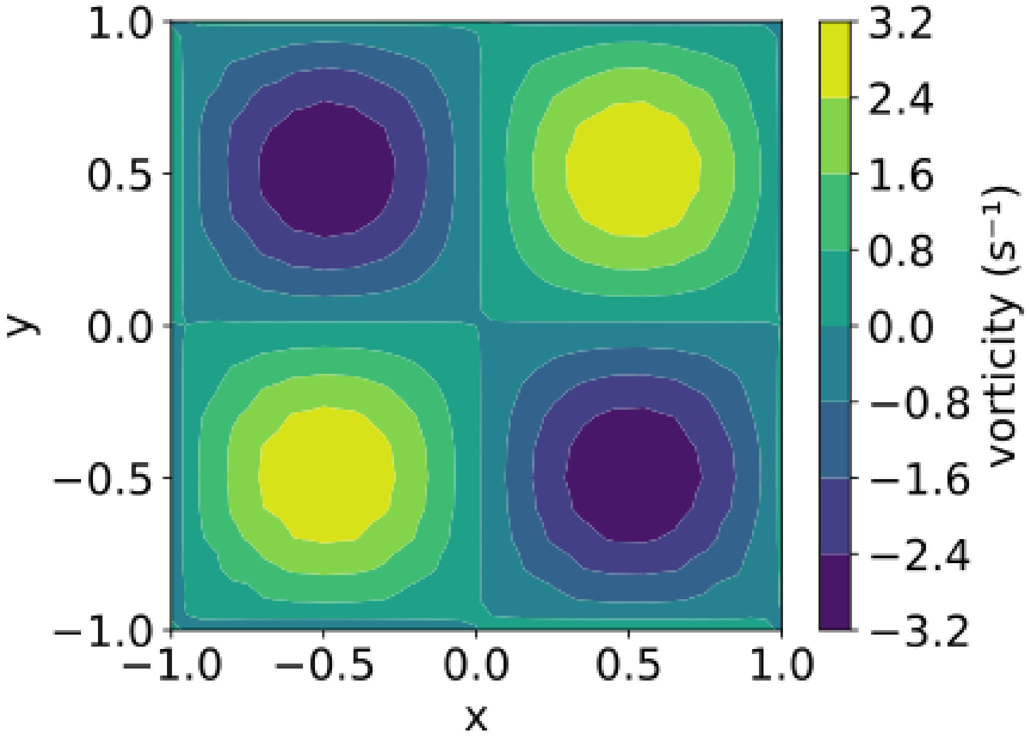

**Temporal Accuracy Evaluation**

- Time step size $\Delta t$ varied
- Grid size fixed
- Semi-implicit predictor–corrector scheme used
- Finite volume discretization for spatial terms
- Simulated up to T=0.3s
- Error recorded at final time

# TGV Plots

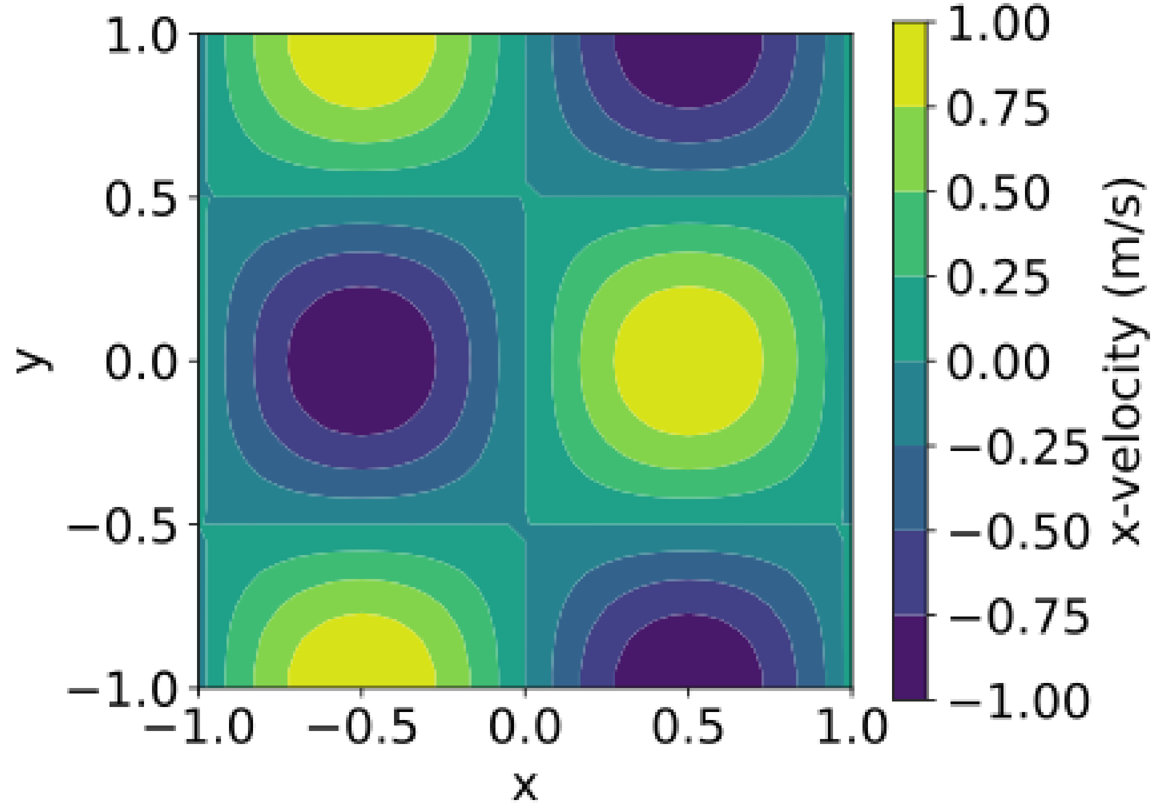


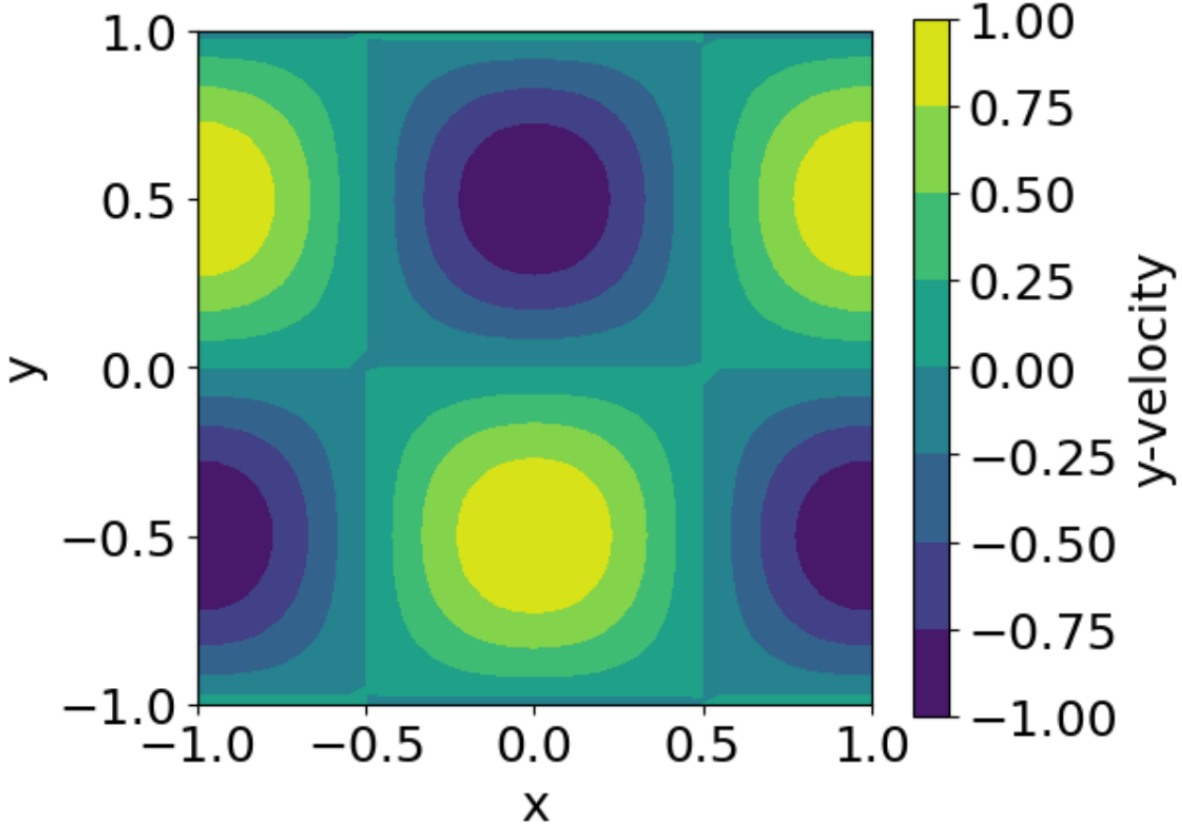
1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
x
0.45
0.30
0.15
0.00
−0.15
−0.30
−0.45
pressure (Pa)

1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
x
3.2
2.4
1.6
0.8
0.0
−0.8
−1.6
−2.4
−3.2
vorticity (s⁻¹)

# TGV Plots


1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
x-velocity (m/s)


1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y-velocity