Numerical Algorithms

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Abstract

This document presents numerical algorithms and methods for solving 2d incompressible Navier-stokes, with detailed sections on right- and left-hand side formulations, Poisson solver, and validation techniques.

1 Model

We are dealing with the 2D incompressible Navier-Stokes equations, wich gives us:

- For the x-momentum equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

- For the y-momentum equation:

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

- Additionally, the continuity equation for incompressible flow ensures mass conservation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2 Spatial Discretization

2.1 Finite Difference

2.1.1 Central Scheme

2.1.2 Upwind Scheme

2.2 Finite Volume

3 Prediction Step

3.1 Explicit Schemes

3.2 Semi-implicit Schemes

3.2.1 Explicit Advection and Implicit Diffusion

3.2.2 BDF with Extrapolation for Advection

4 Projection Step

5 Validation