Matrix World

Matrix Factorization

Matrix $(m \times n)$

$$A = CR$$

$$A = U\Sigma V^{\mathrm{T}}$$

 $row \ rank = column \ rank$

SVD: orthonormal basis U, V

 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Square Matrix $(n \times n)$

$$det(A) \neq 0$$
, all $\lambda \neq 0$ at least one $\lambda = 0$, $det(A) = 0$

$$A = QR$$
 ---- Triangularize -- $A = LU$

Gram-Schmidt

▶U has at least one zero row

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Diagonalizable

$$A = X\Lambda X^{-1}$$
 - Diagonalize $A = XJX^{-1}$

I = Iordan form

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

Symmetric

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

 $A^{\mathrm{T}}A = AA^{\mathrm{T}}$ diagonalizable by orthogonal matrix

 $S = S^{T}$, all λ are real

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

 $J = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Positive $A = Q\Lambda Q^{\mathrm{T}}$

Semidefinite

$$S = Q\Lambda Q^{\mathrm{T}}$$

Semidefiniall
$$\lambda \geq 0$$
, all A^TA

Orthogonal

 $Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$Q^{-1} = Q^{\mathrm{T}}$$

$$all |\lambda| = 1$$

Permutation

permutation of
$$I$$
 all λ are roots of 1

Projection $P^2 = P = P^T, \lambda = 1 \text{ or } 0$

$$P = P = P^{\mathrm{T}}, \lambda = 1 \text{ or } 0$$

Diagonal
$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} / \Lambda = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Positive

all $\lambda > 0$

0

Definite

 $A^{-1} = V \Sigma^{-1} U^{\mathrm{T}}$

pseudoinverse for all A

$$A^+ = V \Sigma^+ U^{\mathrm{T}}$$

Drawn by Kenji Hiranabe with the help of Prof. Gilbert Strang (v1.5, Mar.2th, 2023)