UVA DATA SCIENCE

Foundation of Computer Science for Data Science

# Binary Search Tree

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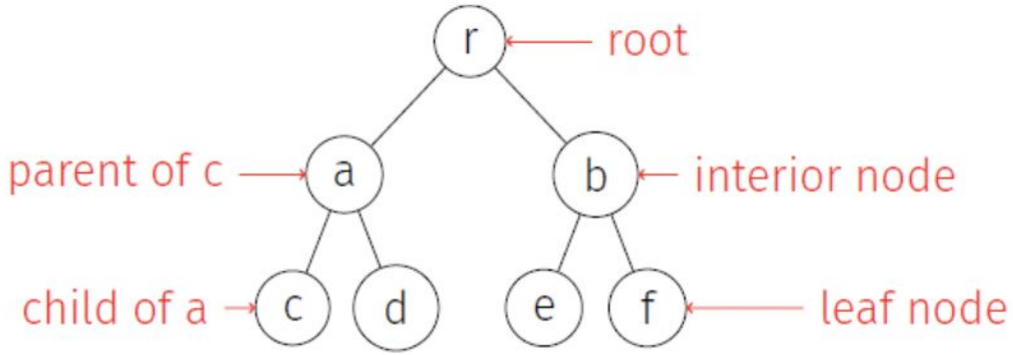
UVA DATA SCIENCE

# Learning Objectives

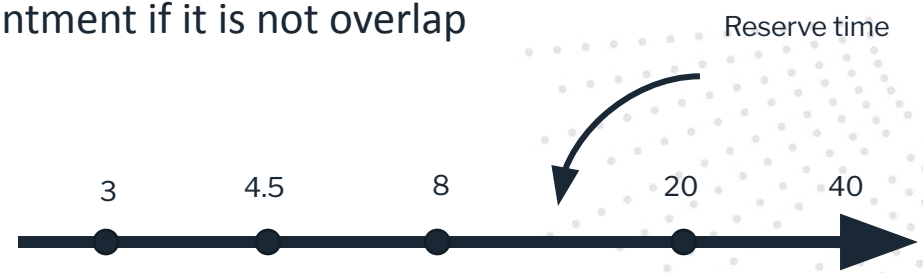
- Understand the concept of BSTs, their properties, and how they are organized
- Searching, inserting and deleting in BSTs
- Analyzing BST time complexity

# Binary Tree



# Binary Search Trees: Motivation

- Runway reservation system
  - You have to make reservation for plane landing
  - Each landing in 3 mins
  - Reserve request for future landing
  - Add time t to the set of appointment if it is not overlap



# Binary Search Trees: Motivation

- What is our possible solution and what wrong with them?
  - Unsorted array
  - Sorted array
  - Sorted linked list

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  - Unsorted array
  - Sorted array
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| Data Structure | Operation | Average Case | Worst Case |
|---|---|---|---|
| **Sorted Array** | Search | $O(\log n)$ | $O(\log n)$ |
| | Insert | $O(n)$ | $O(n)$ |
| **Unsorted Array** | Search | $O(n)$ | $O(n)$ |
| | Insert | $O(1)$ | $O(1)$ |
| **Sorted Linked List** | Search | $O(n)$ | $O(n)$ |
| | Insert | $O(n)$ | $O(n)$ |

choose this for frequent searches

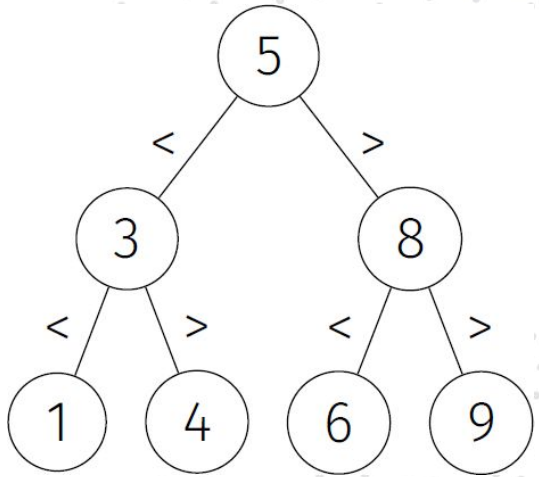
choose this for frequent insertions

# Binary Search Trees: Motivation

- Binary search tree is the one would allow us to do the above things in O(logn)

# Binary Search Tree

- Binary tree with **comparable** key values
- **Binary search tree property:**
  - Every node in the **left** subtree has key whose value is **less** than the value of the root's key value, and
  - Every node in the **right** subtree has key whose value is **greater** than the value of the root's key value.




BST visualization : https://visualgo.net/en/bst

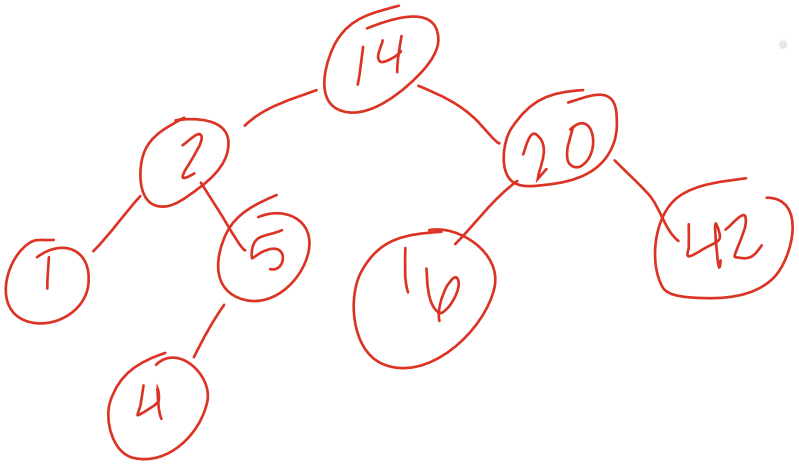
# Binary Search Tree

- How could we traverse a BST so that the nodes are visited in **sorted** order?
  - *In-order* traversal: left tree, node, right tree

# Binary Search Tree Practice

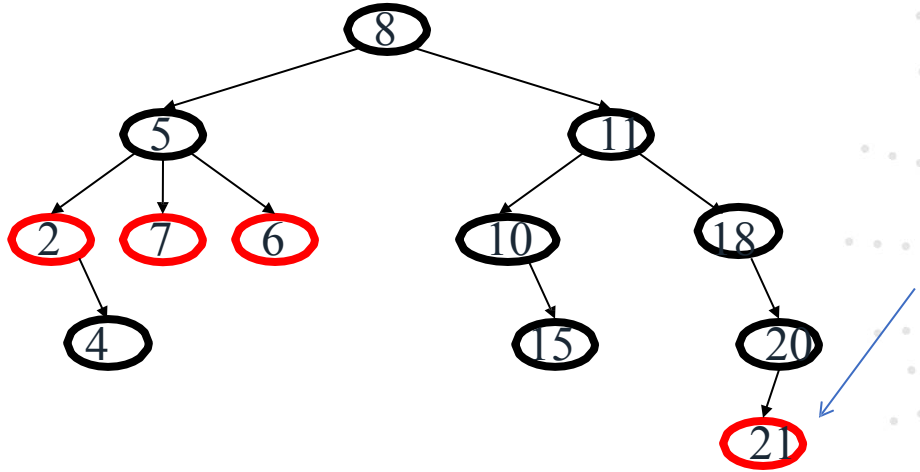
Start with an empty tree. Insert the following values, in the given order:

14, 2, 5, 20, 42, 1, 4, 16



# Binary Search Tree

Three child nodes

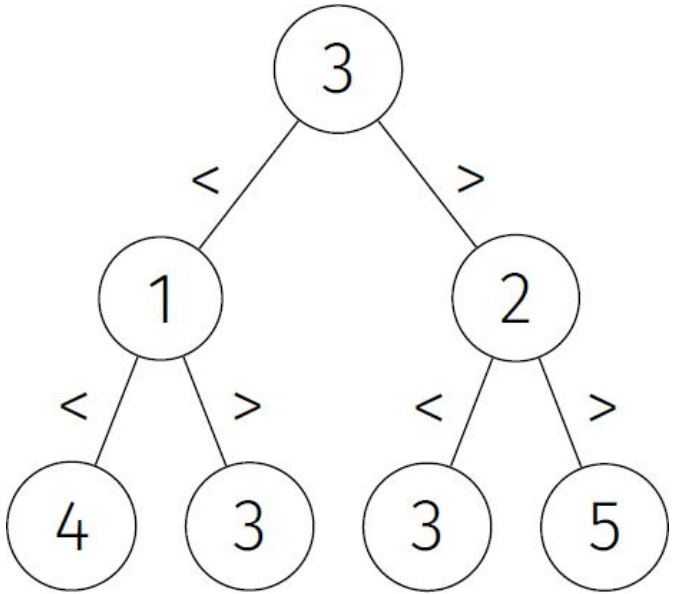



Left node not less than root

**NOT** A BINARY SEARCH TREE

# Quiz

- Is this a binary search tree?



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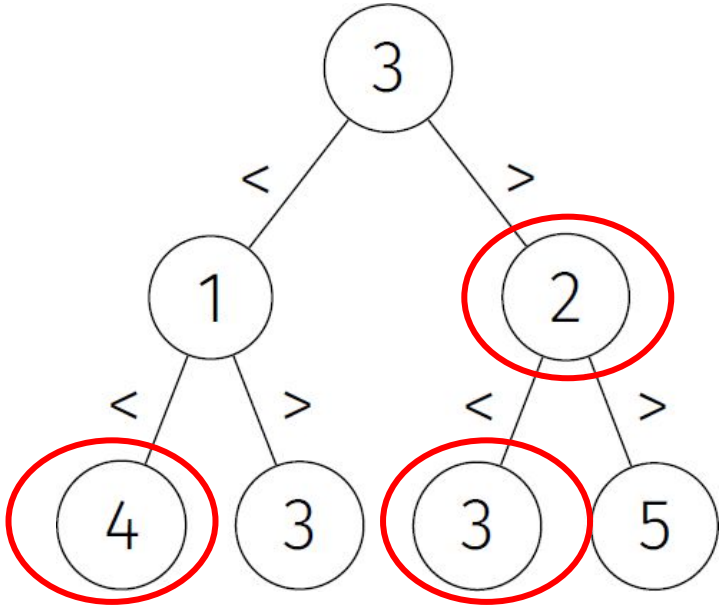


***No!*** Binary search tree property not preserved

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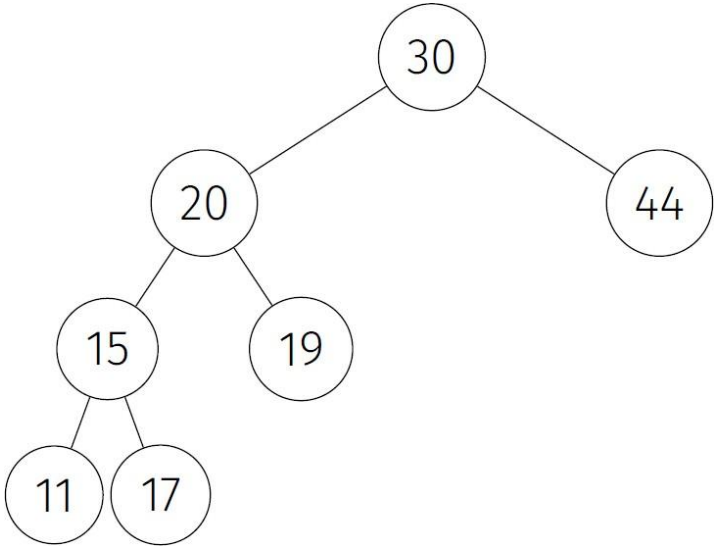

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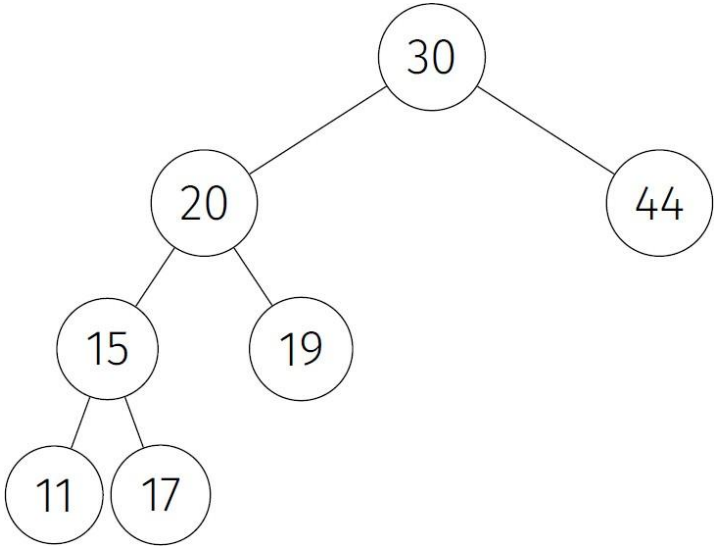


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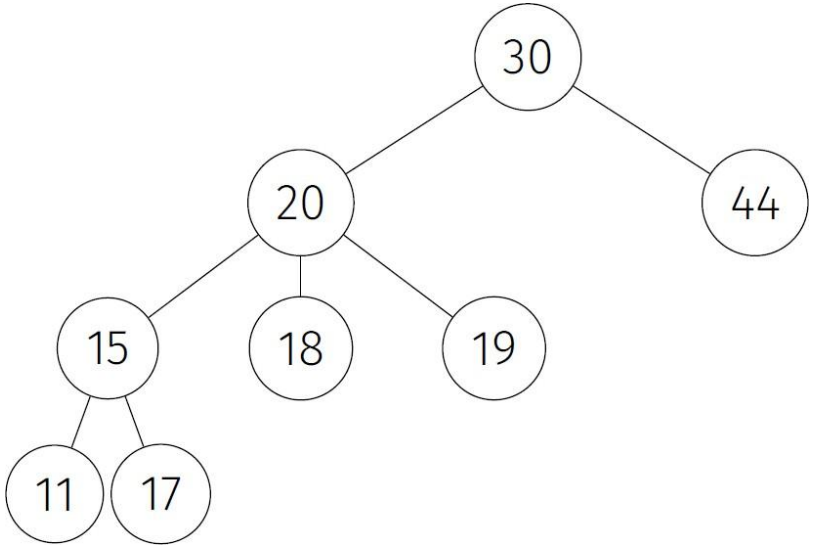

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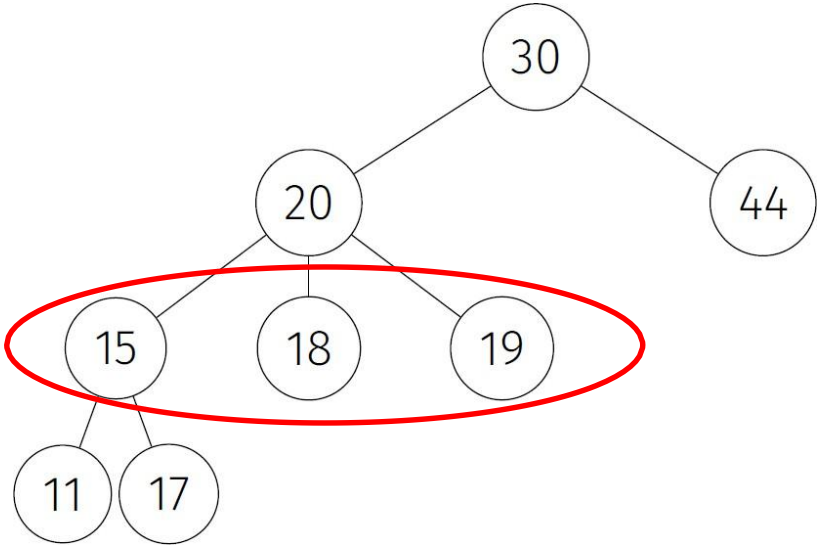


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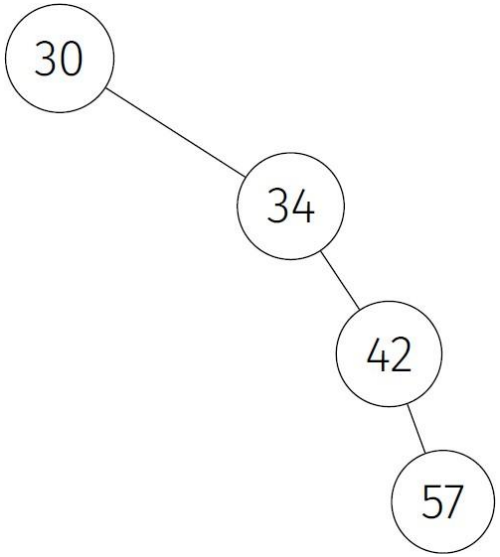
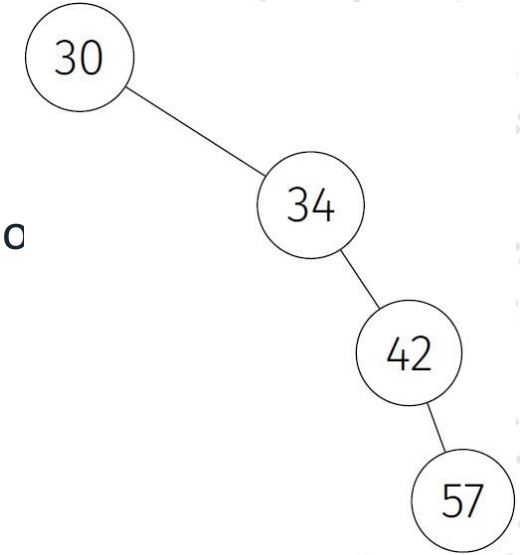
# Quiz

- Is this a binary search tree?



# Quiz

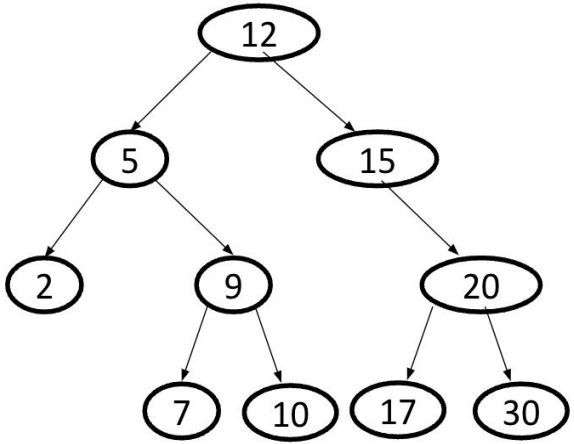
- Is this a binary search tree? ***Yes!***
- However, this tree is unbalanced!
  - This is an ordered list
- A **balanced** binary tree
- Guarantees height of child subtrees differ by no



# How do we search an element in BST?

- **Find** an element in the tree
  - Compare with root, if less traverse left, else traverse right; repeat
  - Stops when found or at a leaf

# How do we search an element in BST?




```
Data find(Data value, Node root)

    if(root == null)
      return null

    if(key < root.value)
      return find(value, root.left)

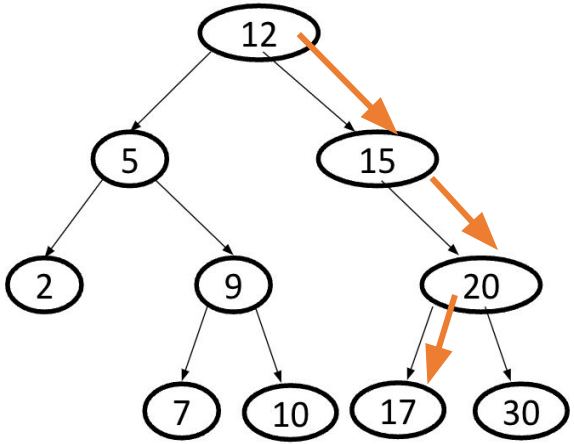
    if(key > root.value)
      return find(value, root.right)

    return root.value;
```

Sounds like **binary search**!

# How do we search an element in BST?

Find (17)

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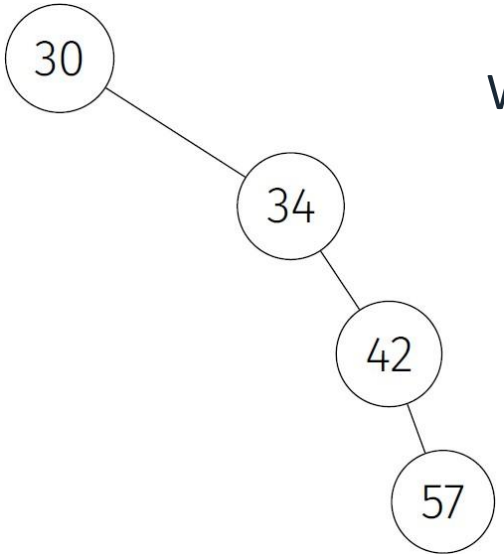
```

What is the running time?

# How do we search an element in BST?

Find (57)




What is the running time?

# How do we search an element in BST?

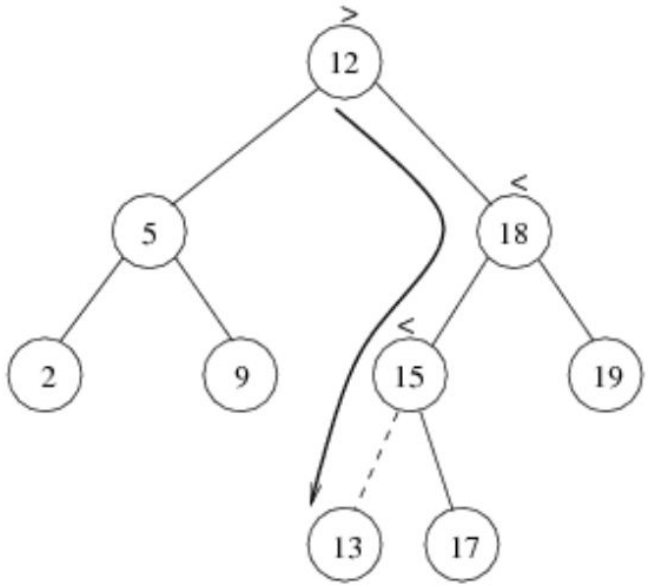
- what is *minimum* node in BST?
- what is the *maximum* node in BST?

# Time Complexity of Searching BST

| Operation | Best Case | Average Case | Worst Case |
|---|---|---|---|
| Searching | O(1) | O(log n) | O(n) |

# How do we insert an element in BST?

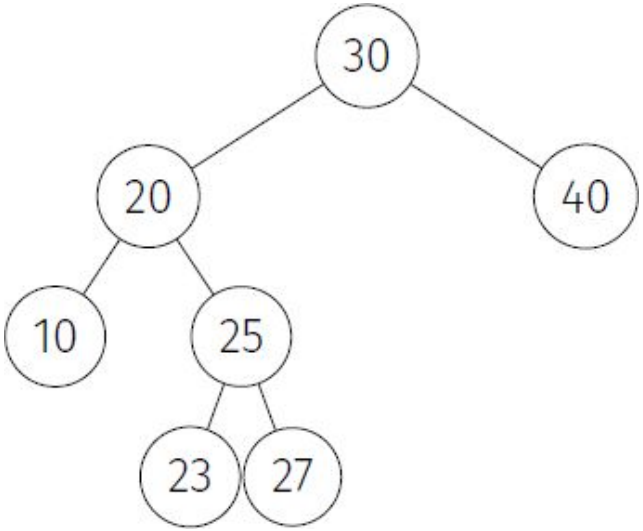
- Proceed down the tree as you would with a search
- Insert the new element at the last spot on the path traversed (add it at the leaf node)




Insert (13)

# How do we insert an element in BST?

Insert 33 into the following binary search tree



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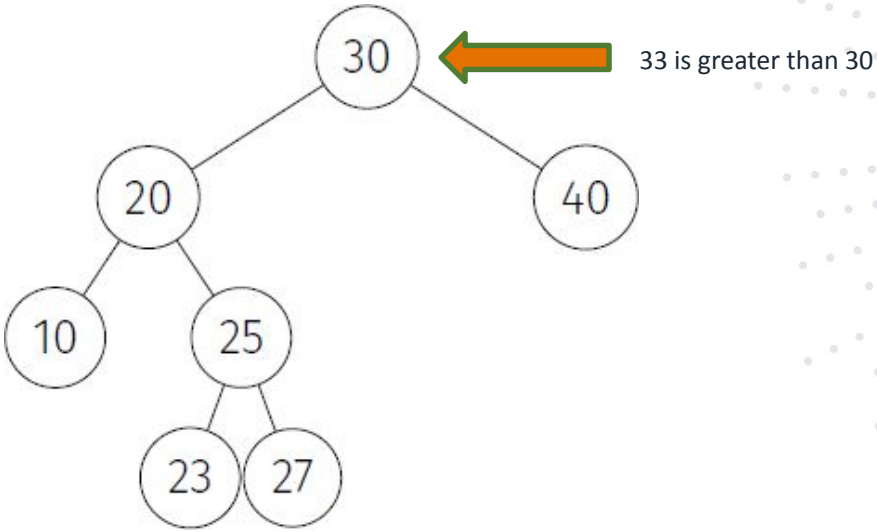

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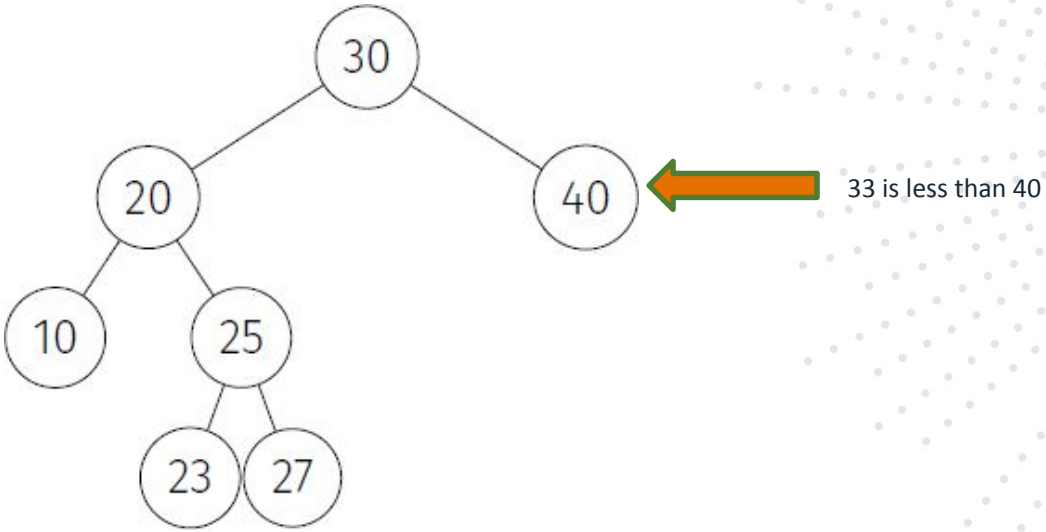

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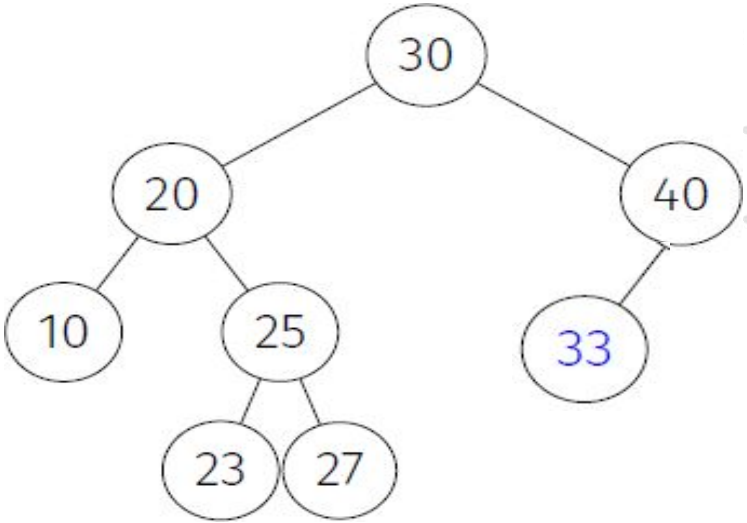


# Time Complexity of Insertion BST

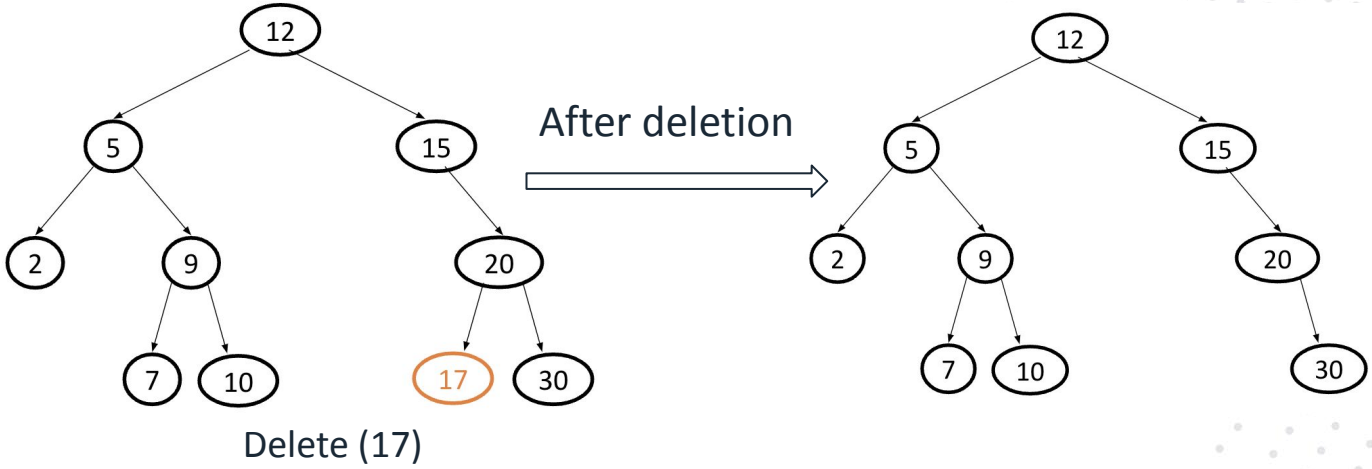
| Operation | Best Case | Average Case | Worst Case |
|---|---|---|---|
| Searching | O(1) | O(log n) | O(n) |

# How do we delete an element from BST?

- When we delete a node, we need to consider how we take care of the children of the deleted node
  - This has to be done such that the property of the search tree is maintained
  - More complicated – we need to select a node as replacement!
- Deletion under Different Cases
  - Case 1: the node is a leaf
  - Case 2: the node has one child
  - Case 3: the node has 2 children

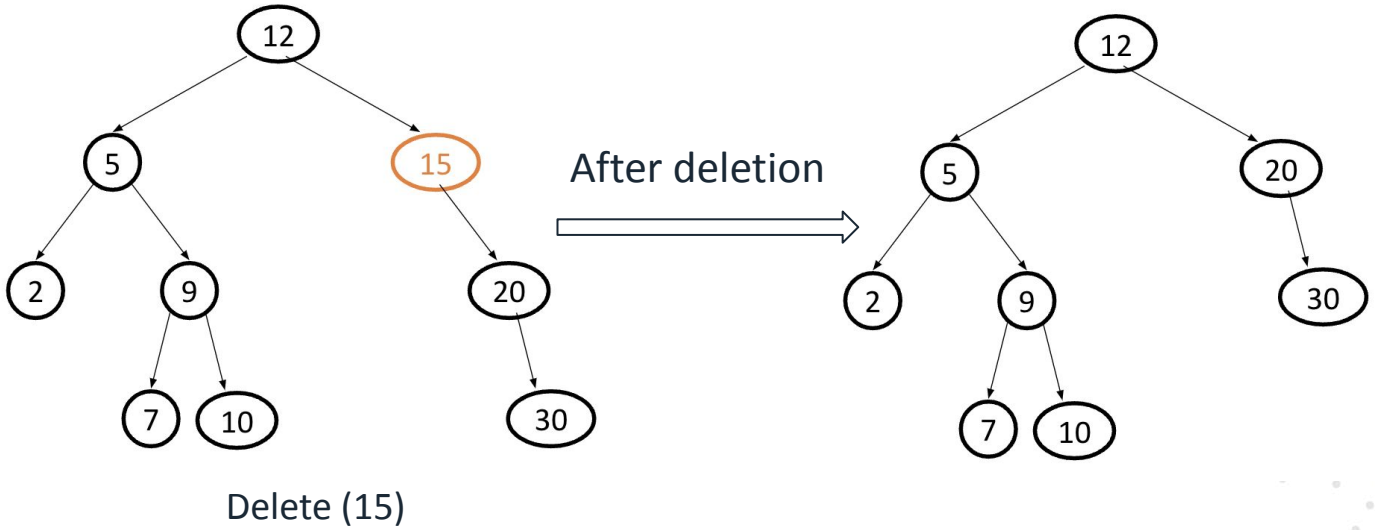
# How do we delete an element from BST?

- Case 1: the node is a leaf
  - Delete it immediately



# How do we delete an element from BST?

- Case 2: the node has one child
  - Adjust a pointer from the parent to bypass that node



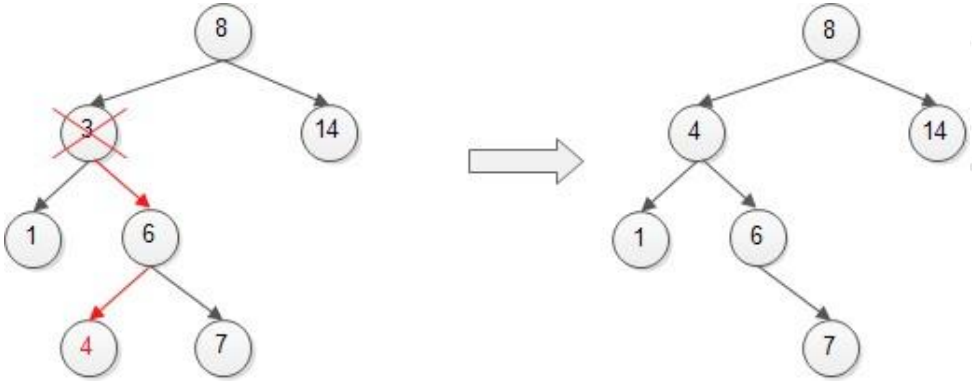

Delete (15)

# How do we delete an element from BST?

- Case 3: the node has 2 children
  - Replace the deleted node with its successor
- Where to find the successor? there are 2 options:
  - The next “largest” element - in left subtree
  - The next “smallest” element - in right subtree

# How do we delete an element from BST?

- Find successor (of 3) in it's **right** subtree (i.e. node **4**) – finding the **minimum** (*leftmost node*) of right subtree



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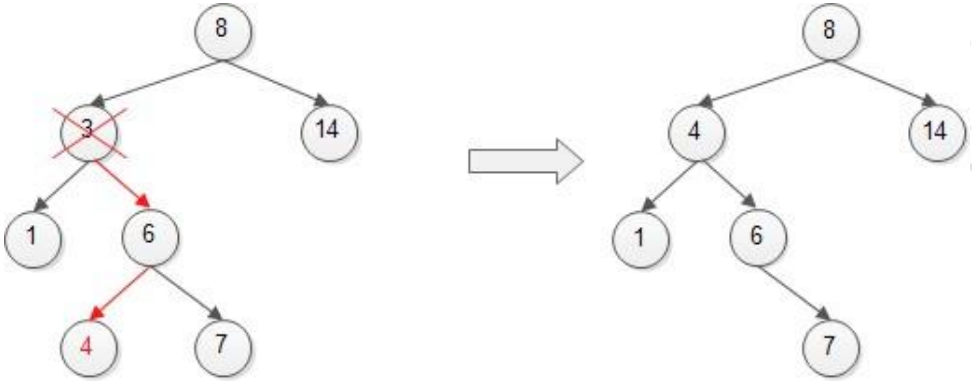
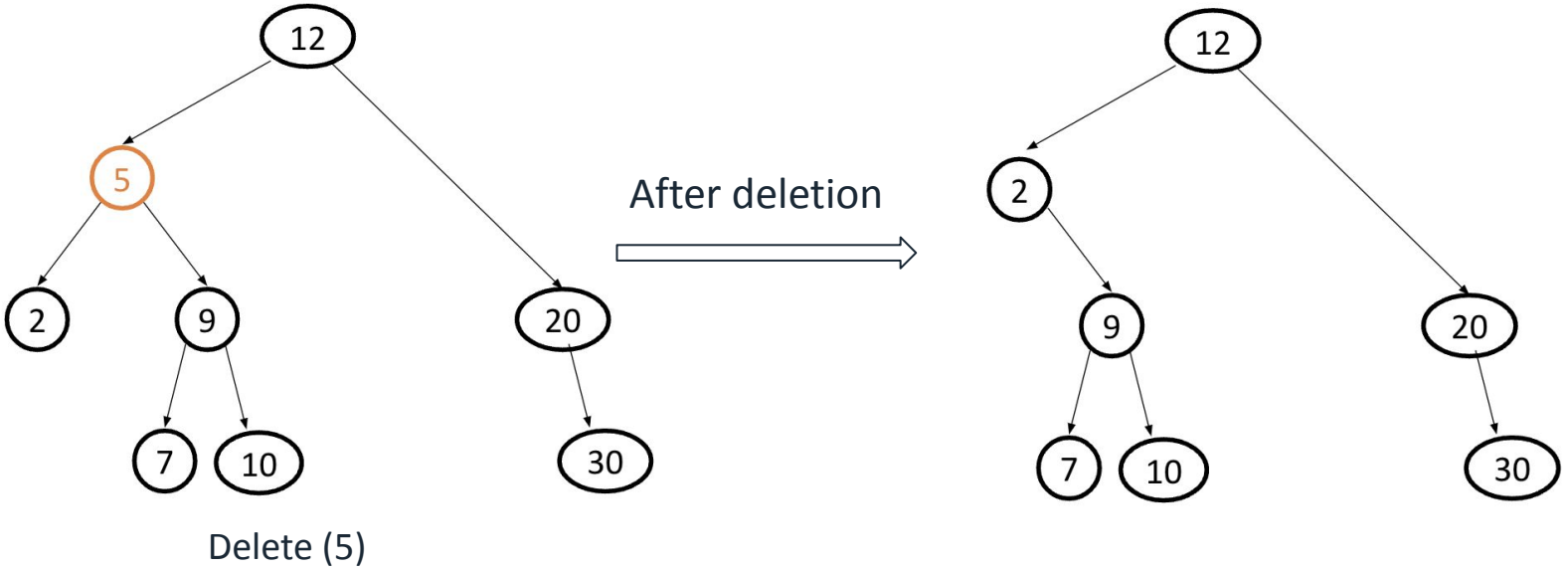


# How do we delete an element from BST?

- Largest number of **left** subtree, which is the next smallest number



# Time Complexity of Deletion BST

| Operation | Best Case | Average Case | Worst Case |
|---|---|---|---|
| Searching | O(1) | O(log n) | O(n) |

# Quiz

**What is the time complexity of building BST in average and worst cases?**