

CS 5012: Foundations of Computer Science

Asymptotic Complexity Exercise

Given the following code snippets, provide the worst case time complexity in the form of Big-O notation. Justify your response and state any assumptions made. Treat these functions as constant runtime: print(), append()

```

► def measure(inputList):
    int n = len(inputList)    O(1) --> Assignments are constant
    int sum = 0;              O(1)
    for i in range(0, n):     O(n) --> b/c the for loop runs n times
        for j in range(0, 5):    n * O(1) = O(n) --> b/c the loop runs 5 times exactly
            sum += j * inputList[i]    n * O(1) * O(1) = O(n)
            for k in range(0, n):    n * O(n) = O(n^2) --> b/c it's a nested for loop that runs n times
                sum -= inputList[k]    n * n * O(1) = O(n^2)

```

The asymptotic complexity of this algorithm is: $O(\underline{\quad n^2 \quad})$

$$T(n) = O(1) + O(1) + O(n) + O(n) + O(n) + O(n^2) + O(n^2)$$

```

► def addElement(ele):
    myList = []                O(1) --> Instantiating an empty list
    myList.append(666)         O(1) --> constant run time
    print myList               O(1) --> constant run time

```

The asymptotic complexity of this algorithm is: $O(\underline{\quad 1 \quad})$

$$T(n) = O(1) + O(1) + O(1)$$

Assume that num is a fixed!!!!

```
► num = 10 O(1)

def addOnesToTestList(num):
    testList = [] O(1) --> Instantiating an empty list
    for i in range(0,num): O(num) = O(10) = O(1)
        testList.append(1) O(1) --> constant run time
        print(testList) O(1) --> constant run time

    return testList O(1) --> constant run time
```

The asymptotic complexity of this algorithm is: $O(1)$

$T(n) = O(1) + O(1) + O(1) + O(1) + O(1) + O(1)$

Assume that num is NOT fixed!!!

```
► testList = [1, 43, 31, 21, 6, 96, 48, 13, 25, 5] O(1) --> instantiating a list

def someMethod(testList):
    for i in range(len(testList)): O(n) --> b/c the for loop runs n times
        for j in range(i+1, len(testList)): n * O(n) = O(n^2)
            if testList[j] < testList[i]: n * n * O(1) = O(n^2)
                testList[j], testList[i] = testList[i], testList[j] n * n * O(1) * O(1)
                print(testList) n * n * O(1)
```

The asymptotic complexity of this algorithm is: $O(n^2)$

$T(n) = O(1) + O(n) + O(n^2) + O(n^2) + O(n^2) + O(n^2)$

```
► def searchTarget(target_word):
    # Assume range variables are unrelated to size of aList

    for (i in range1): O(1)
        for (j in range2): O(1) * O(1) = O(1)
            for (k in range3): O(1) * O(1) * O(1) = O(1)
                if (aList[k] == target_word): O(1) * O(1) * O(1) * O(1) = O(1)
                    return 1 O(1)

    return -1 O(1)
return -1 O(1)
```

The asymptotic complexity of this algorithm is: $O(1)$

$T(n) = O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O(1)$

```

▶ def someSearch(sortedList, target):
    left = 0                                O(1)
    right = len(sortedList) - 1             O(1) --> Assignments are constant

    while (left <= right):                   O(log n) --> b/c each iteration divides by 2
        mid = (left + right) / 2             log n * O(1) = O(log n)
        if (sortedList[mid] == target):      log n * O(1) = O(log n)
            return mid                       log n * O(1) = O(log n)
        elif (sortedList[mid] < target):      log n * O(1) = O(log n)
            left = mid + 1                   log n * O(1) = O(log n)
        else:
            right = mid - 1                 log n * O(1) = O(log n)

    return -1                                O(1)

```

The asymptotic complexity of this algorithm is: $O(\log n)$

$T(n) = O(1) + O(1) + O(\log n) + O(\log n) + O(\log n) + O(\log n) + O(\log n) + O(\log n) + O(\log n) + O(1)$

```

▶ #Assume data is a list of size n
    total = 0                                O(1) --> Assignments are constant
    for j in range(n):                       O(n) --> b/c the loop runs n times
        total += data[j]                   n * O(1) = O(n)
    big = data[0]                            O(1)
    for k in range(1, n):                   O(n)
        big = max(big,                     n * O(1) = O(n)
            data[k])

```

The asymptotic complexity of this algorithm is: $O(n)$

$T(n) = O(1) + O(n) + O(n) + O(1) + O(n) + O(n)$

```

▶ powers = 0                                O(1) --> Assignments are constant
    k = 1                                    O(1)
    while k < n:                             O(log n)
        k = 2*k                             log n * O(1)
        powers += 1                         log n * O(1)

```

The asymptotic complexity of this algorithm is: $O(\log n)$

$T(n) = O(1) + O(1) + O(\log n) + O(\log n) + O(\log n)$

```

▶ k = 1                                    O(1)
    while k < n:                             O(log n)
        for j in range(k):                  log n * O(n) = O(n) --> b/c linear time has a higher complexity than log time
            steps += 1                      n * log n * O(1) = O(n)
        k = 2*k                             log n * O(1) = O(log n)

```

The asymptotic complexity of this algorithm is: $O(\underline{\hspace{1cm}n\hspace{1cm}})$

$$T(n) = O(1) + O(\log n) + O(n) + O(n) + O(\log n)$$

```
► for k in range(1,n):      O(n)
    j = 1                  n * O(1)
    while j < k:           n * O(log n) = O(n) --> b/c linear time has a higher complexity than log time
        total += 1        n * log n * O(1) = O(n)
        j = 2 * j         n * log n * O(1) = O(n)
```

The asymptotic complexity of this algorithm is: $O(\underline{\hspace{1cm}n\hspace{1cm}})$ <-- Should be $O(n \log n)$

$$T(n) = O(n) + O(n) + O(n) + O(n) + O(n)$$