

Homework 12 – Due: 12/10/2019 9:00 am

Problem 1. (40 points) Please read lecture 17 page 9-16 and follow the code example to complete this problem. Consider the Runge function:

$$f(x) = \frac{1}{1 + 25x^2}$$

- (1) Interpolate this function using the polynomial $y=a_0+a_1x+a_2x^2$ at the points $[-1, 0, 1]$, that is solving for the unknowns a_0, a_1 , and a_2 using three points $(-1, f(-1))$, $(0, f(0))$ and $(1, f(1))$.
- (2) Interpolate this function using the polynomial $y=b_0+b_1x+\dots+b_4x^4$ at the points $[-1, -.5, 0, .5, 1]$
- (3) Interpolate this function using the polynomial $y=c_0+c_1x+\dots+c_8x^8$ at the points $[-1, -.75, -.5, -.25, 0, .25, .5, .75, 1]$.
- (4) Plot the Runge function and three interpolating functions on the same plot using 101 points evenly spaced between -1 and 1. Comment on what you see.

Report your figures and comments in the write-up (Please do not submit the .fig file, instead, please insert your figure in the write-up).

Please submit your .m file as “yourLastName_hw12_prob1.m” with all the MATLAB commands you used.

Problem 2 (30 point) Please read lecture 17 page 19-25 and follow the code example to complete this problem. We have 500 pairs of samples numbered $x=[x_1; x_2; \dots; x_{500}]$ and $y=[y_1; y_2; \dots; y_{500}]$ in “hw12prob2.mat” file under “Files” tab “lab12_data”. Write a MATLAB script to find the least squares solution for $a=[a_0, a_1, \dots, a_{n-1}]$ so that the function $y_n(x) = a_0+a_1x+\dots+a_{n-1}x^{n-1}$ best fits the data for the cases $n = 2, 3, 4, 5$. Plot y vs. x as well as $y_n(x)$ vs. x on the same plot. Based on this plot, explain which polynomial function $y_n(x)$ you would choose to describe the noisy data.

Note: you may import .mat format data into your workspace as:

```
load hw12prob2.mat
```

After the loading, you will see two vectors x and y in your workspace.

Report your figures and comments in the write-up (Please do not submit the .fig file, instead, please insert your figure in the write-up).

Please submit your .m file as “yourLastName_hw12_prob2.m” with all the MATLAB commands you used.

Problem 3. [30 points] Consider a mass-spring-damper system as we described in lecture 18 slide 15. The initial displacement of the mass is $r(0) = 0.5$ m and its initial velocity is $v(0) = 1.0$ m/s. A force of 1 N is applied for the first 4 seconds.

The system has the following parameters:

```
k = 4; % spring coefficient N/m
b = 1; % damper coefficient N.s/m
m = 1; % mass in kg
```

Plot $r(t)$ vs. t and $v(t)$ vs. t on the same plot for a period of 30 seconds with 5001 samples.

Report your plot in the write-up.

Please submit your .m file as “yourLastName_hw12_prob3.m” with all the MATLAB commands you used.

Bonus Problem (+10 points) Download the file ‘butterfly.gif’ under “Files” tab “Lab12_data”. Design an image warping and shift operator so that the image rotates 90 degree clock-wise (Hint: Note that the rotation is around (0,0), you may try a few output pixels and see where they are mapped to your source image).

You may read an image file and display it as follows:

```
% Read an image
img=imread('butterfly.gif');
% .gif files are stored as uint8 (unsigned 8bit integer)
% Convert to [0,1]
img = double(img)/255;
% Show it
imshow(img);
```

Report your rotated figures (generated using your code) in the write-up (Please do not submit the .fig file, instead, please insert your figure in the write-up).
Please submit your .m file as “yourLastName_hw12_prob4.m” with all the MATLAB commands you used.

Submission Instructions:

There should be 3 files in your submission:

1. A write up (any type- .txt, .docx, .pdf are all fine) that contains your answers to all questions in problem 1-3.
2. The .m file for problem 2.
3. The .m file for problem 3.
4. If you solved the bonus problem, please submit the code and the figure as well.

Please make sure your last name is included in the filename.