

**CS&S/SOC/STAT 563 — Statistical Demography — Spring 2022 - Homework
no. 1**

Due Monday April 11 at 2:15pm on the course Canvas website.

Note: Please write your homework as a report, including figures and tables with captions. Please do not write your homework as annotated computer output. However, please do include an appendix containing the computer code that you used (if you used any — maybe no need for this homework.)

1. Suppose the instantaneous mortality rate for a population is a constant that does not depend on age, equal to 0.02 per year.
 - (a) Find analytically and plot the probability density function of age at death for this population.
 - (b) Find the probability that a member of this population is still alive at age 70.
 - (c) Find the probability that a member of this population dies before age 6.
 - (d) Find the life expectancy at birth for a member of this population.
 - (e) Find the life expectancy at age 50 for a member of this population.
 - (f) Find the median age at death for this population.
2. Suppose the instantaneous mortality rate at age x for a cohort is

$$\mu(x) = (0.0168x^2 - 0.668x + 8)/1000.$$

Denote by X the age at death of a person randomly chosen from this cohort (a random variable).

- (a) Plot the instantaneous mortality rate for this cohort against age.
- (b) Find analytically and plot the cumulative hazard function of this distribution, $\Lambda(x)$.
- (c) Find analytically and plot the survival function, $S(x)$.
- (d) Find and plot the probability density function of X , $f(x)$.
- (e) Find the life expectancy at birth of a member of this cohort.
- (f) Find the life expectancy at age 10 of a member of this cohort.
- (g) Find ${}_{45}q_{15}$ for this cohort.

3. From the UN's 2019 *World Population Prospects* extract and show:
 - (a) estimates of the male population of Peru in each of the age groups 70-74, 75-79, 80-84, 85-89, 90-94, 95+ in 2015 and 2020
 - (b) estimates of the number of deaths in each of these age groups during the period 2015–2020.
4. Use these estimates to:
 - (a) estimate ${}_5M_x$ for ages $x = 70, 75, 80, 85, 90, 95$ for males in Peru for the period 2015-2020
 - (b) estimate ${}_5q_x$ for ages $x = 70, 75, 80, 85, 90, 95$ for males in Peru for the period 2015-2020, using each of the two approximations discussed in class.
 - (c) estimate ${}_5m_x$ for the same ages.
 - (d) Compare the estimates from the two approximations. Which of the two approximations do you think is better? Why?
 - (e) Compare the estimates of ${}_5M_x$ and ${}_5m_x$ with one other, and comment on any differences.
5. For each of the two approximations:
 - (a) Derive a life table (ℓ_x only) for a cohort of 10,000 males aged 70.
 - (b) Estimate the life expectancy at age 70 from each of the two approximations, and compare the two estimates. Comment on any differences, and why they arise.